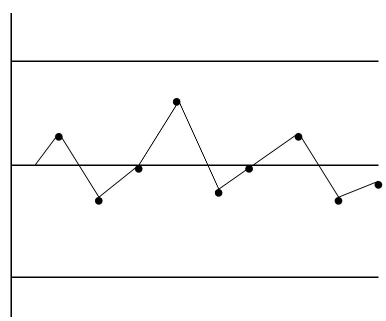
# Statistical Control Charts

## DYNAMIC ANALYSIS OF A PROCESS



Application of Process Control Charts

# Analysis of Process Variation

## Process Variation

In any process, there are two possible kinds of variability:

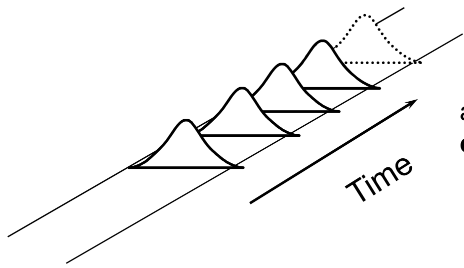
| | |
|---|---|
| Natural variability (inherent) | Unnatural variability (systematic) |
| Random causes | Assignable causes |
| Uncontrollable | Controllable |
| Acceptable | Not acceptable |

A process is in statistical control when its variation pattern, evolving over time, is subject to the forces of only chance causes. It is then performing in a consistent pattern over time, fluctuating about a fixed mean level and a constant pattern of variation.

| Random causes | Assignable causes |
|---|---|
| *Description* | |
| Consists of many individual causes. | Consists of just one or just a few individual causes. |
| Any one random cause results in a minute amount of variation. | Any one assignable cause can result in a large amount of variation. |
| Examples are human variation in setting control dials; slight vibration in machines, slight variation in raw material. | Examples are operator blunder, a faulty setup; a batch of defective raw material. |

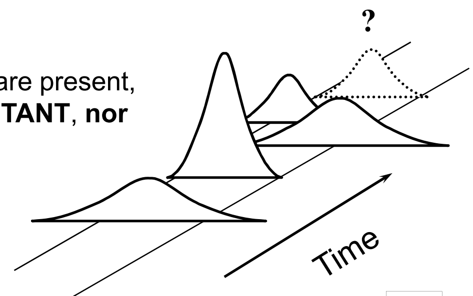
| *Interpretation* | |
|---|---|
| Random variation cannot economically be eliminated from a process. | Assignable variation can be detected; action to eliminate the causes is usually economically justified. |
| An observation within the control limits means that the process should not be adjusted. | An observation beyond the control limits means the process should be investigated and corrected. |
| With only random variation, the process is sufficiently stable to use sampling procedures to predict the quality of production or make process optimization studies. | With assignable variation present, the process is not sufficiently stable to use sampling procedures for prediction. |

# The Role of SPC




If only **common causes** of variability are present, then the process output is **constant and predictable** over time.

If only **special causes** of variability are present, the process output **is neither CONSTANT**, **nor PREDICTABLE**.




## Statistical Control of a Process

The presence of statistical control is important because

a) it signals that the process is operating in a routine fashion, as it was intended to, i.e. it is doing the best it can under the present system.

b) The process behavior is predictable and therefore its performance can be rationally assessed to determine the extent to which it is meeting the expectations of the customer.

# Control Charts in Practice

The major objective of SPC is to quickly detect the occurrence of assignable causes, so that investigation and corrective action may be taken. This can be achieved through the use of control charts.

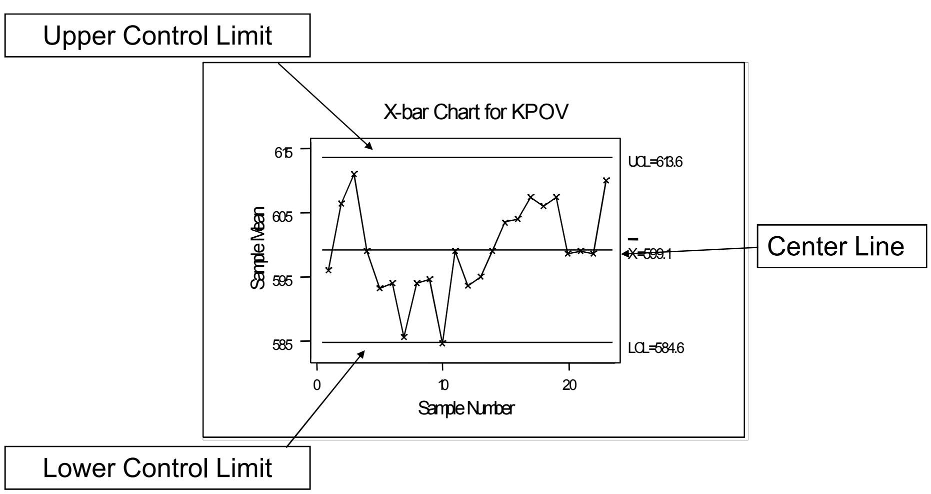
## Role of Control Charts

- Control charts only detect processes out-of-control, not why the process is out of control
- What happens after an out-of-control situation occurs is the core of a successful SPC program

# Where to Use Charts

- Place charts only where necessary (as determined via FMEA, DOE)
- Identify processes that are critical and cannot be fool-proofed
- If a chart has been implemented, do not hesitate to remove it if it is not value-added
- In early investigations place charts on output variables
- After investigations place charts on critical input variables
- The goal:  Monitor and control inputs and, over time, eliminate the need for SPC charts on outputs

# Components of a Control Chart

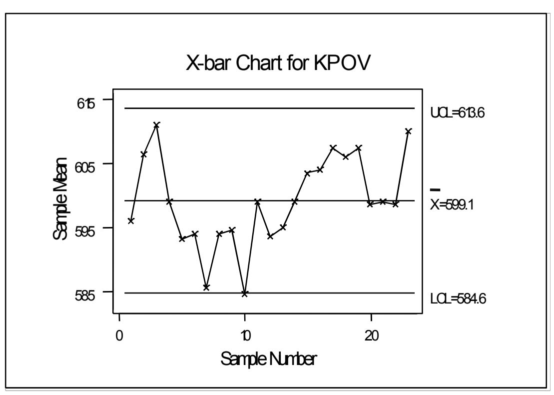


# Exercise

Indicate on the control chart:

1. What we try to control
2. What we expect
3. What we actually get
4. What we allow - maximum
5. What we allow – minimum

# A Typical Control Chart



The C**entral Limit Theorem** (CLT)

states that given a sufficiently large sample size from a population with a finite level of variance, the mean of all samples from the same population will be approximately equal to the mean of the population.

As the sample size increases, the sampling distribution of the mean, X-bar, can be approximated by a normal distribution with mean $\mu$ and standard deviation $\sigma/\sqrt{n}$ where:

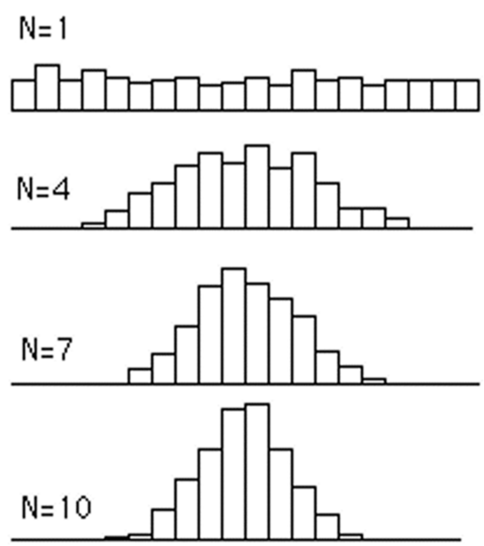
$\mu$ is the population mean
$\sigma$ is the population standard deviation
$n$ is the sample size

In other words, if we repeatedly take independent random samples of size $n$ from any population, then when $n$ is large, the distribution of the sample means will approach a normal distribution.

If the original population is a normal distribution, then the distribution of the sample means will be a normal distribution regardless of the value of $n$.

On the right are shown the resulting frequency distributions each based on 500 means. For n = 4, 4 scores were sampled from a uniform distribution 500 times and the mean computed each time. The same method was followed with means of 7 scores for n = 7 and 10 scores for n = 10.

*When n increases:*
*1. the distributions becomes "more and more normal".*
*2. the spread of the distributions decreases.*



# Statistical Model

General Model for Center Line, Upper and Lower Control Limits for a process control chart.

$$\text{UCL} = \mu + k\sigma$$

$$\text{Center Line} = \mu$$

$$\text{LCL} = \mu - k\sigma$$

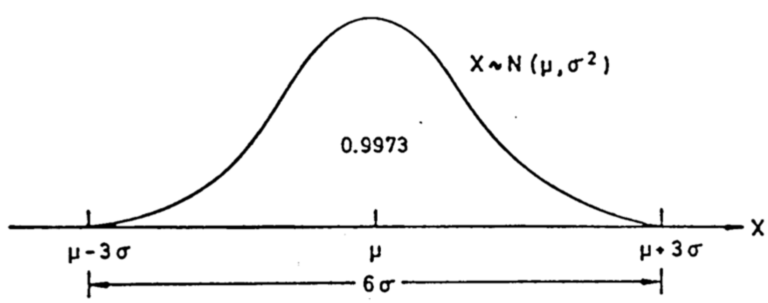
Where

$\mu$ = the population mean

$\sigma$ = the population standard deviation

k = some distance of the control limits from the center line (usually 3)

## The Normal Distribution



# Sampling for a Control Chart

- Sample Size
  - Variables chart:  5 if possible
  - Attributes Chart:  30 or more depending on percentages

- Sample Frequency

- Rational Subgroups
  - Try to capture process when things are consistent.
  - Depends on critical source of variations

# Formation of Rational Subgroups

- Select subgroup so that variability due only to chance causes is captured within the subgroups (thus members of a subgroup are to be obtained at almost the same time)

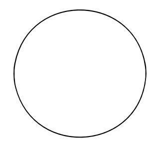
- For the control chart, the control limits need to be based on the $\sigma$ of the problem

## Selecting Rational Subgroup

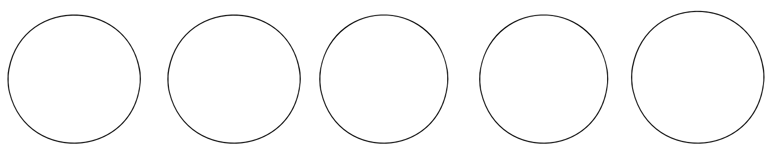
3 ways to select subgroups of size 5 for wafer production:

•Five repeated measurements at one position on wafer.

•Five different measurements on one wafer



•Five measurements on five wafer



## Identifying Improvement Goals

- By sub-grouping the data, we can set improvement goals and approximate time
- Short-term: center the process on the target value
- Medium-term: remove/reduce *between* sub-group variation
- Long-term: remove/reduce *within* sub-group variation

# Application of Control Charts

# Categories of Data

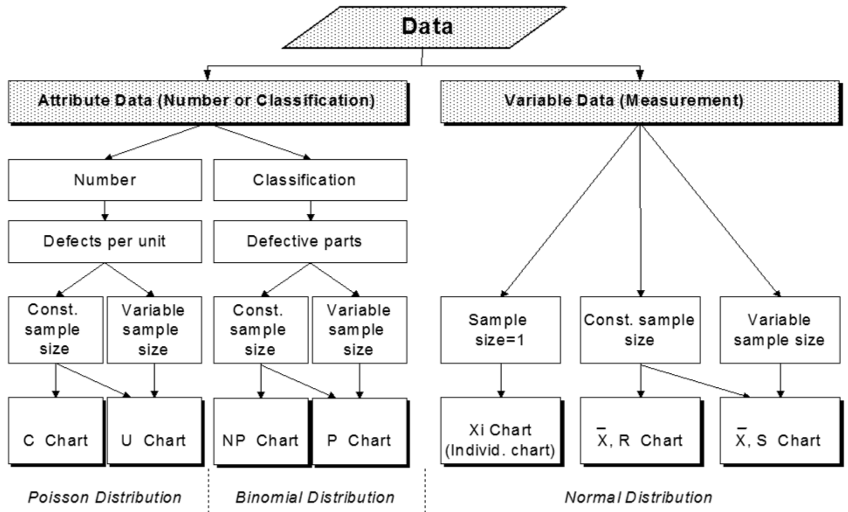
**Example: length**

- Variable
- Continuous
- Quantitative
- Measurable

**Example: pass/fail**

- Attribute
- Discrete
- Qualitative
- Countable

# Selection of SPC Charts




# Control Charts for Variable Data

- X-bar
- Range (R)
- Individuals (X)
- Moving Range (MR)

## X-bar and Range Chart

- How to set up “from scratch”?

## X-bar and Range Chart

Upper control line for means

UCL = $\overline{\overline{X}} + A_2 \overline{R}$

Lower control line for means

LCL = $\overline{\overline{X}} - A_2 \overline{R}$

Upper control line for ranges

UCL(R) = $D_4 \overline{R}$

Lower control line for ranges

LCL(R) = $D_3 \overline{R}$

## Constants for Given *n*

| $n$ | $A_2$ | $D_3$ | $D_4$ | $d_2$ |
|---|---|---|---|---|
| 2 | 1.880 | 0.000 | 3.267 | 1.128 |
| 3 | 1.023 | 0.000 | 2.574 | 1.693 |
| 4 | 0.729 | 0.000 | 2.282 | 2.059 |
| 5 | 0.577 | 0.000 | 2.115 | 2.326 |
| 6 | 0.483 | 0.000 | 2.004 | 2.534 |
| 7 | 0.419 | 0.076 | 1.924 | 2.704 |
| 8 | 0.373 | 0.136 | 1.864 | 2.847 |
| 9 | 0.337 | 0.184 | 1.816 | 2.970 |

## Oxide Thickness Example

| Lot | Wafer 1 | Wafer 2 | Wafer 3 | Wafer 4 | Average | Range |
|---|---|---|---|---|---|---|
| 1 | 946 | 966 | 977 | 943 | 958.0 | 34 |
| 2 | 961 | 977 | 963 | 956 | 964.3 | 21 |
| 3 | 915 | 948 | 905 | 906 | 918.5 | 43 |
| 4 | 968 | 970 | 989 | 969 | 974.0 | 21 |
| 5 | 950 | 962 | 960 | 945 | 954.3 | 17 |
| 6 | 984 | 967 | 956 | 991 | 974.5 | 35 |
| 7 | 993 | 984 | 1034 | 969 | 995.0 | 65 |
| 8 | 922 | 935 | 927 | 932 | 929.0 | 13 |
| 9 | 962 | 948 | 933 | 933 | 944.0 | 29 |
| 10 | 955 | 984 | 946 | 986 | 967.8 | 40 |
| 11 | 965 | 983 | 982 | 949 | 969.8 | 34 |
| 12 | 948 | 963 | 943 | 985 | 959.8 | 42 |
| 13 | 980 | 956 | 1009 | 970 | 978.8 | 53 |
| 14 | 964 | 985 | 964 | 963 | 969.0 | 22 |
| 15 | 950 | 938 | 956 | 934 | 944.5 | 22 |
| 16 | 952 | 963 | 971 | 975 | 965.3 | 23 |
| 17 | 949 | 984 | 967 | 1006 | 976.5 | 57 |
| 18 | 939 | 933 | 922 | 947 | 935.3 | 25 |
| 19 | 1010 | 987 | 972 | 966 | 983.8 | 44 |
| 20 | 969 | 958 | 977 | 952 | 964.0 | 25 |
| 21 | 995 | 999 | 998 | 980 | 993.0 | 19 |
| | | | | | 962.8 | 32.6 |

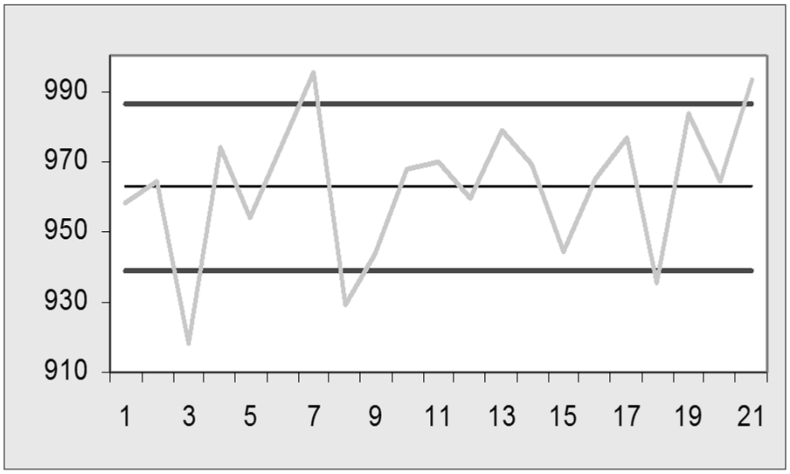
*Exercise:*

- Calculate the Center Line and Control Limits for the X-bar and R charts
- Use a blank control chart to plot the observed data

## Mean and Range Charts

- Upper control line for means
- UCL = $\overline{\overline{X}} + A_2 \overline{R}$ = 962.8+0.729*32.6
- Lower control line for means
- LCL = $\overline{\overline{X}} - A_2 \overline{R}$
- Upper control line for ranges
- UCL(R) = $D_4 \overline{R}$ = 2.282*32.6
- Lower control line for ranges
- LCL(R) = $D_3 \overline{R}$ =0

Oxide Thickness Example: X-bar

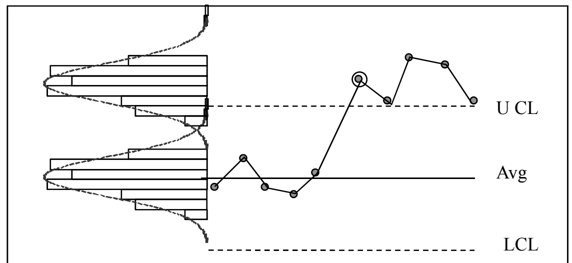



Implications

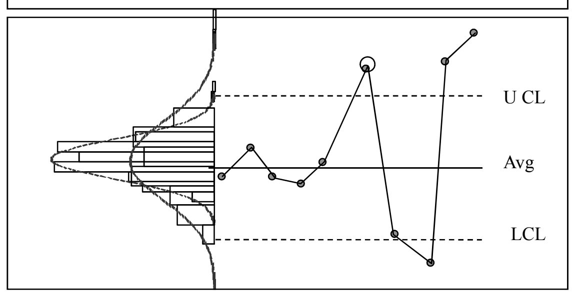
- Either the process mean, or the process sigma is changing over time.

# Interpreting Charts

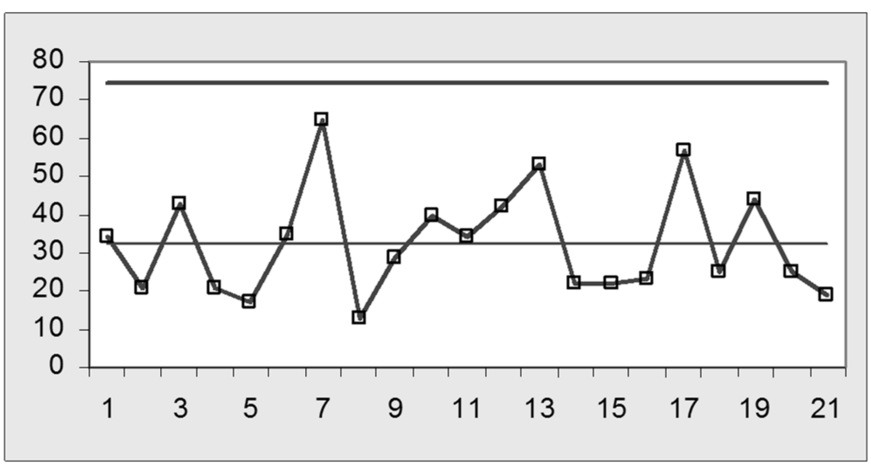
Change in level




Change in variability




## Oxide Thickness Example: Range



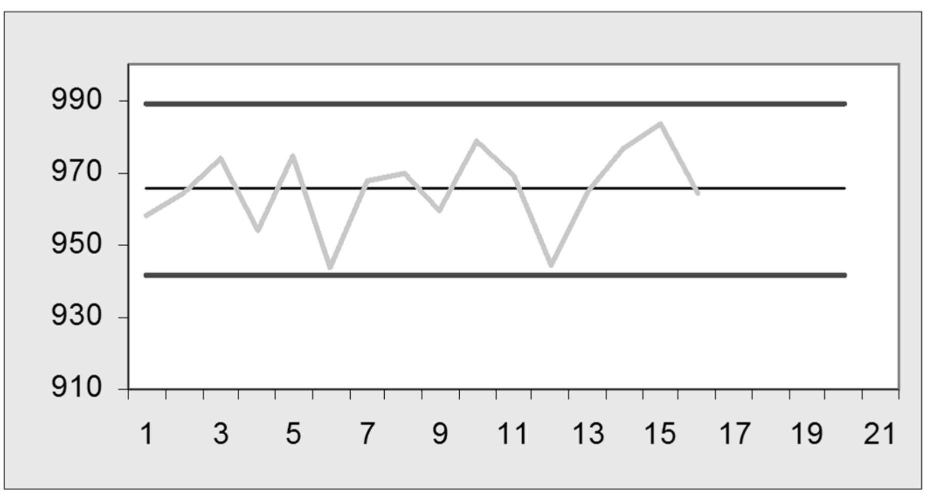
## Setting and Changing Limits

- Stage 1 :
  - Investigate outliers
  - Remove all outliers from the data.
  - Recalculate control limits and plot the remaining data points
  - Repeat until all points are in control.

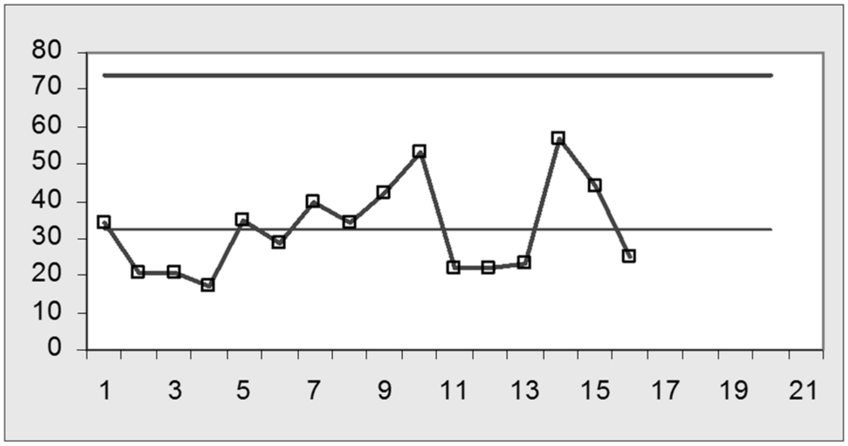
- Stage 2 :
  - Do not recalculate control limits unless process has significantly improved.
  - Control chart limits are not constantly updated with new data.
  - The goal is to improve the process over time to narrow  the control limits.

## Oxide Thickness Example: X-bar

After removing the out of control points:

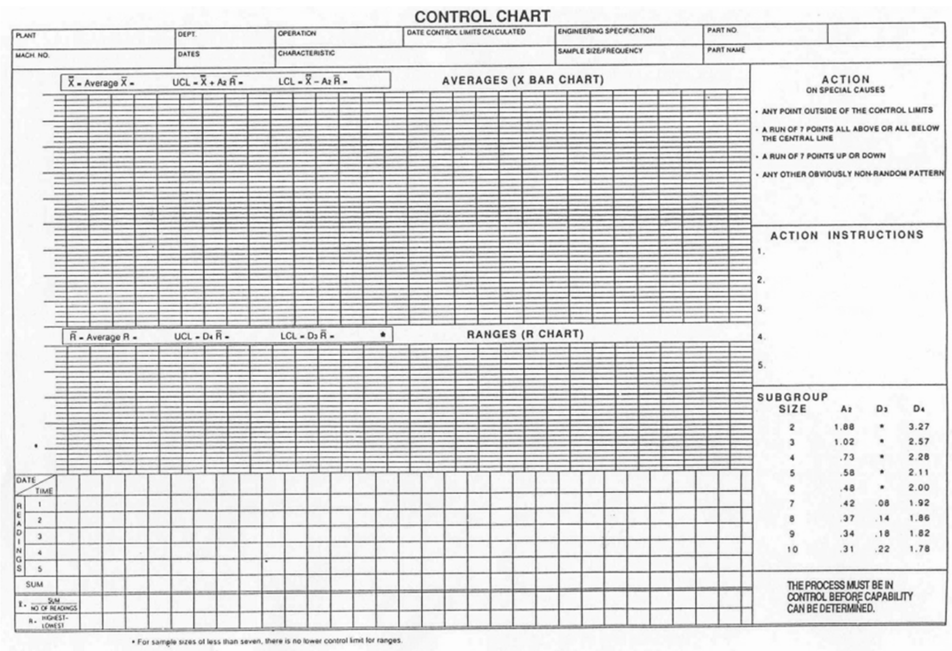


Oxide Thickness Example: Range




# Nature of the X-bar and R Charts

- Most sensitive (powerful) chart for tracking process excursions in the mean and the variation
- Assumes normal distribution of individuals
- Subgroups means tend to produce normal distribution because of the central limit theorem
- Three sigma limits used based on the number of subgroups.
- Minimum number of subgroups of 30 required to establish control limits



CONTROL CHART

PLANT | DEPT. | OPERATION | DATE CONTROL LIMITS CALCULATED | ENGINEERING SPECIFICATION | PART NO.

MACH. NO. | DATES | CHARACTERISTIC | SAMPLE SIZE/FREQUENCY | PART NAME

$\bar{\bar{X}}$ = Average $\bar{X}$ = UCL = $\bar{\bar{X}} + A_2\bar{R}$ = LCL = $\bar{\bar{X}} - A_2\bar{R}$ =

AVERAGES (X BAR CHART)

$\bar{R}$ = Average R = UCL = $D_4\bar{R}$ = LCL = $D_3\bar{R}$ = *

RANGES (R CHART)

DATE / TIME

READINGS 1 2 3 4 5

SUM

$\bar{X}$ = SUM / NO OF READINGS

R = HIGHEST-LOWEST

ACTION
ON SPECIAL CAUSES

- ANY POINT OUTSIDE OF THE CONTROL LIMITS
- A RUN OF 7 POINTS ALL ABOVE OR ALL BELOW THE CENTRAL LINE
- A RUN OF 7 POINTS UP OR DOWN
- ANY OTHER OBVIOUSLY NON-RANDOM PATTERN

ACTION INSTRUCTIONS

1.
2.
3.
4.
5.

| SUBGROUP SIZE | $A_2$ | $D_3$ | $D_4$ |
|---|---|---|---|
| 2 | 1.88 | * | 3.27 |
| 3 | 1.02 | * | 2.57 |
| 4 | .73 | * | 2.28 |
| 5 | .58 | * | 2.11 |
| 6 | .48 | * | 2.00 |
| 7 | .42 | .08 | 1.92 |
| 8 | .37 | .14 | 1.86 |
| 9 | .34 | .18 | 1.82 |
| 10 | .31 | .22 | 1.78 |

THE PROCESS MUST BE IN CONTROL BEFORE CAPABILITY CAN BE DETERMINED.

* For sample sizes of less than seven, there is no lower control limit for ranges.

## Computation of $C_p$ from Specification Limits

Two-sided specification:

$$C_p = \frac{S_U - S_L}{6\sigma}$$

One-sided specification:

$$C_p = \frac{S_U - \mu}{3\sigma} \quad \text{or} \quad C_p = \frac{\mu - S_L}{3\sigma}$$

$S_U$ : upper specification limit
$S_L$ : lower specification limit
$\mu$ : process mean
$\sigma$ : process standard deviation

## Computation of $C_p$ from Control Limits

Two-sided specification:

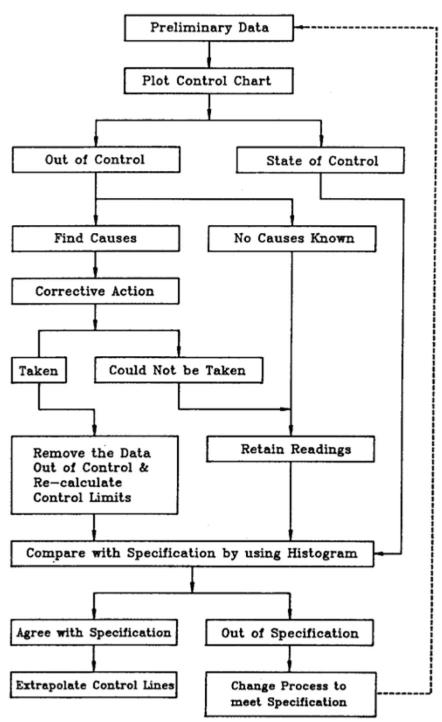
$$C_p = \frac{S_U - S_L}{(UCL - LCL)\sqrt{n}}$$

One-sided specification:

$$C_p = \frac{2(S_U - \overline{\overline{X}})}{(UCL - LCL)\sqrt{n}} \quad \text{or} \quad C_p = \frac{2(\overline{\overline{X}} - S_L)}{(UCL - LCL)\sqrt{n}}$$

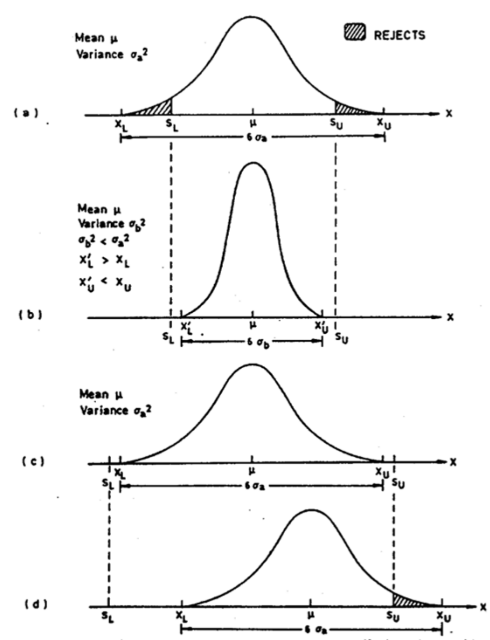
$S_U$ : upper specification limit
$S_L$ : lower specification limit
$n$ : constant subgroup size in $\overline{\overline{X}} - R$ chart
$\overline{\overline{X}}$ : process average at central line

# Decisions in Control Charting



# Assessment of Processes and Products

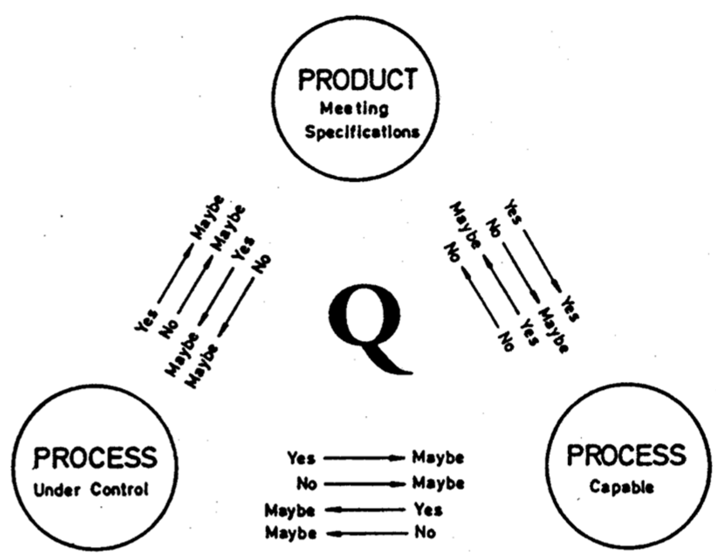
| MEANS OF ASSESSMENT | THEORETICAL JUSTIFICATIONS | PRACTICAL PURPOSE | DESIRED RESULT |
|---|---|---|---|
| Control charts | Statistical | Maintain process mean | Process under control |
| Capability studies | Statistical | Investigate potential to meet specifications | Process capable |
| Inspection specifications | Technical | Conform to quality requirements | Product meeting requirements |




Relationship between process capability and product specifications: (a) process is not capable; (b) and (c) process is capable and specifications are met; (d) process is capable but specifications are not met.

# STATISTICAL QUALITY CONTROL: QUIZ

| Is process under CONTROL? | Is process CAPABLE? | Are product specifications CONFORMED? | ACTION |
|---|---|---|---|
| No | No | No | |
| No | No | Yes | |
| No | Yes | No | |
| No | Yes | Yes | |
| Yes | No | No | |
| Yes | No | Yes | |
| Yes | Yes | No | |
| Yes | Yes | Yes | |




Implications of results of process and product assessments

## What if sub-grouping is not possible?

*Examples:*

## Control Chart for Individual Measurements

- Extension of the X-bar chart
- Individual observations plotted
- Used when subgrouping is impractical
- Not as sensitive as the X-bar chart

## Control Chart for Individual Measurements

- Control Limits:

$$UCL_X = \overline{X} + 3\overline{R} / d_2$$
$$LCL_X = \overline{X} - 3\overline{R} / d_2$$
$$UCL_R = D_4 \overline{R}$$
$$LCL_R = 0$$

- Use with moving range MR=$|X_i - X_{i-1}|$

## Constants for Given *n*

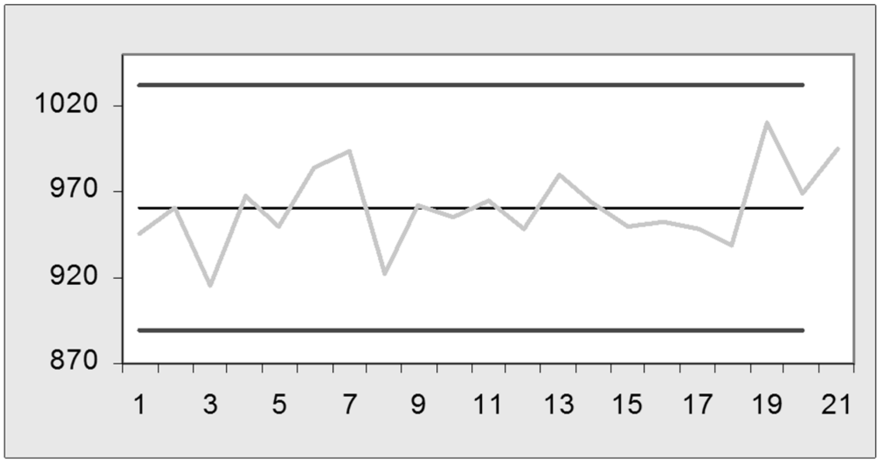
| $n$ | $A_2$ | $D_3$ | $D_4$ | $d_2$ |
|---|---|---|---|---|
| 2 | 1.880 | 0.000 | 3.267 | 1.128 |
| 3 | 1.023 | 0.000 | 2.574 | 1.693 |
| 4 | 0.729 | 0.000 | 2.282 | 2.059 |
| 5 | 0.577 | 0.000 | 2.115 | 2.326 |
| 6 | 0.483 | 0.000 | 2.004 | 2.534 |
| 7 | 0.419 | 0.076 | 1.924 | 2.704 |
| 8 | 0.373 | 0.136 | 1.864 | 2.847 |
| 9 | 0.337 | 0.184 | 1.816 | 2.970 |

## Oxide Thickness Example

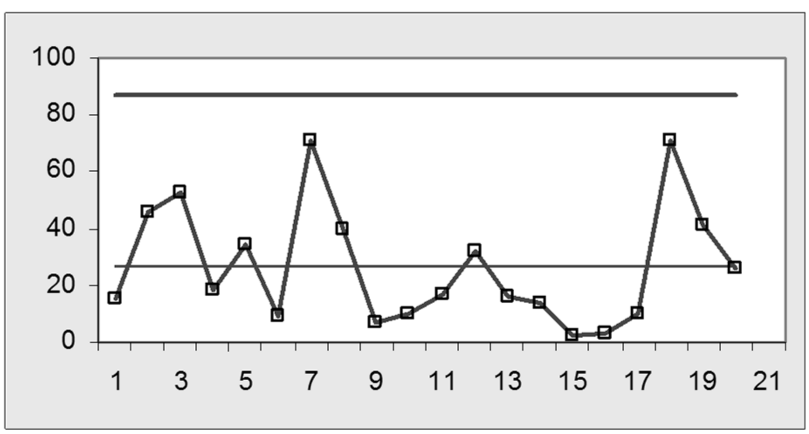
| Lot | Wafer 1 | MR |
|---|---|---|
| | | |
| 1 | 946 | |
| 2 | 961 | 15 |
| 3 | 915 | 46 |
| 4 | 968 | 53 |
| 5 | 950 | 18 |
| 6 | 984 | 34 |
| 7 | 993 | 9 |
| 8 | 922 | 71 |
| 9 | 962 | 40 |
| 10 | 955 | 7 |
| 11 | 965 | 10 |
| 12 | 948 | 17 |
| 13 | 980 | 32 |
| 14 | 964 | 16 |
| 15 | 950 | 14 |
| 16 | 952 | 2 |
| 17 | 949 | 3 |
| 18 | 939 | 10 |
| 19 | 1010 | 71 |
| 20 | 969 | 41 |
| 21 | 995 | 26 |

Exercise: Calculate the Center Line and Control Limits
for the appropriate control charts

## Oxide Thickness Example: X




## Oxide Thickness Example: Range



EXERCISE: *X* and *MR* charts

| Date | KUPH Data | Moving Range, *Rs* |
|---|---|---|
| 4/1 | 1.96 | - |
| 5/1 | 2.10 | 0.14 |
| 6/1 | 2.12 | 0.02 |
| 7/1 | 2.09 | 0.03 |
| 10/1 | 1.92 | 0.17 |
| 11/1 | 1.98 | 0.06 |
| 12/1 | 2.28 | 0.30 |
| 13/1 | 2.25 | 0.03 |
| 14/1 | 2.17 | 0.08 |
| 17/1 | 2.01 | 0.16 |

| Date | KUPH Data | Moving Range, *Rs* |
|---|---|---|
| 18/1 | 2.02 | 0.01 |
| 19/1 | 2.14 | 0.12 |
| 20/1 | 2.90 | 0.05 |
| 21/1 | 2.24 | 0.05 |
| 24/1 | 2.22 | 0.02 |
| 25/1 | 2.33 | 0.11 |
| 26/1 | 2.08 | 0.25 |
| 27/1 | 2.22 | 0.14 |
| 28/1 | 2.20 | 0.02 |
| TOTAL | $\Sigma X = 41.23$ | $\Sigma Rs = 1.76$ |
| MEAN | $\bar{X} = 2.170$ | $\bar{Rs} = 0.098$ |

# Exercise: *X* and *MR* charts

Consider a process that yielded the following 120 measured values, in order of measurement.

Plot X - MR charts for the single measurements

Plot X Bar – Range charts by taking 4 successive measurements as a subgroup.

Comment on the results.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 13 | 4 | 21 | 12 | 10 | 23 | 12 | 2 | 17 | 11 |
| 9 | 11 | 6 | 29 | 10 | 11 | 7 | 10 | 3 | 21 |
| 15 | 2 | 9 | 12 | 14 | 7 | 19 | 25 | 11 | 7 |
| 21 | 8 | 12 | 8 | 9 | 14 | 10 | 9 | 37 | |
| 13 | 26 | 6 | 20 | 16 | 8 | 9 | 15 | 12 | 14 |
| 4 | 12 | 25 | 16 | 13 | 14 | 8 | 11 | 21 | 14 |
| 13 | 19 | 35 | 9 | 10 | 11 | 30 | 16 | 13 | 6 |
| 3 | 18 | 23 | 26 | 18 | 20 | 11 | 7 | 12 | 16 |
| 11 | 11 | 4 | 19 | 10 | 9 | 22 | 15 | 15 | 8 |
| 21 | 12 | 19 | 11 | 7 | 10 | 14 | 10 | 18 | 11 |
| 8 | 9 | 11 | 12 | 15 | 10 | 29 | 13 | 8 | 15 |
| 11 | 10 | 19 | 8 | 11 | 19 | 11 | 23 | 16 | 18 |

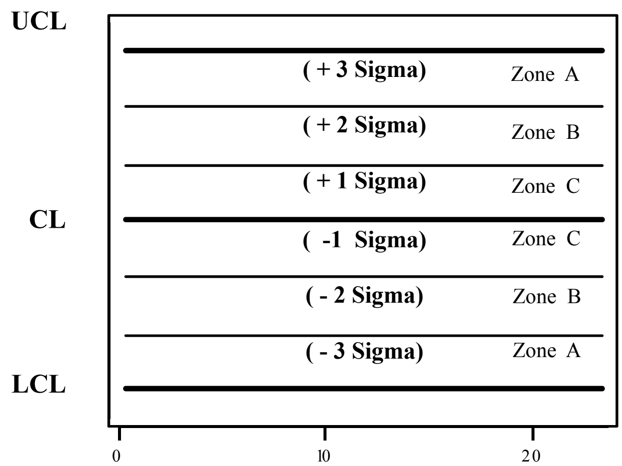
Center Line and Control Limits for the charts?

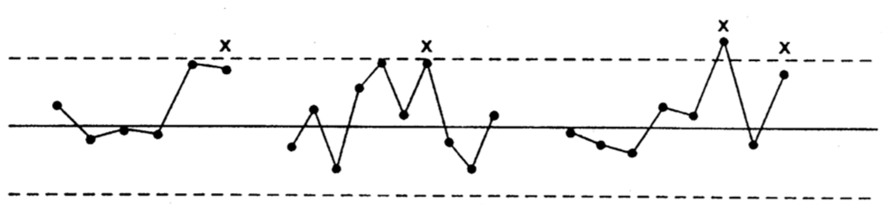
# Patterns on Control Charts

## Detection of Patterns on Control Charts

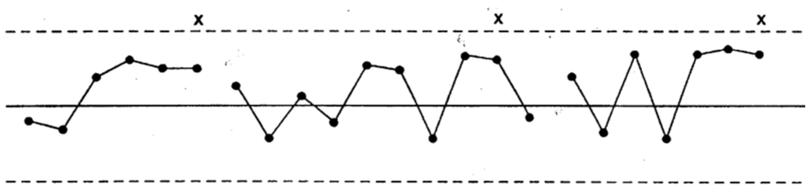
- Anything outside the upper or lower control limits are obviously out-of-control
- Western Electric Rules:
  - One point outside the 3-sigma limit
  - Two of three outside the 2-sigma limit
  - Four of five outside the one-sigma limit
  - Eight consecutive on one side of the center line

# Western Electric Rules: X-bar charts

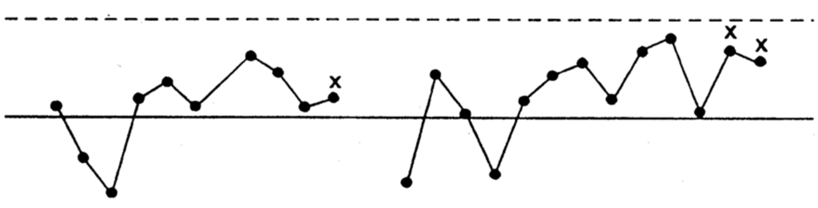



**Two out of three succcessive points outside of 2 sigma**




Four out of five successive
points outside of 1 sigma




Two reasons for marking the last point

**Eight successive points on one side of the centerline**

UNNATURAL PATTERNS

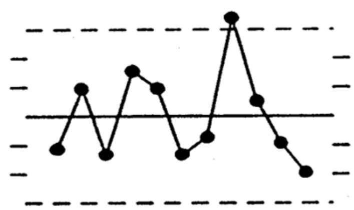


Fig. 1

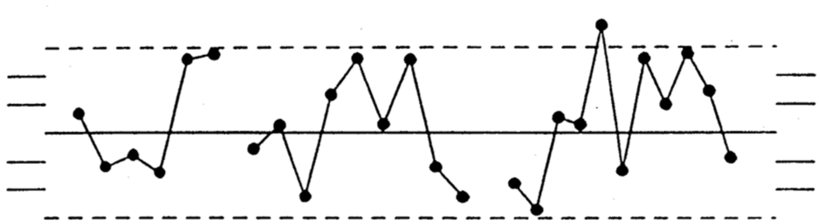


Fig. 2

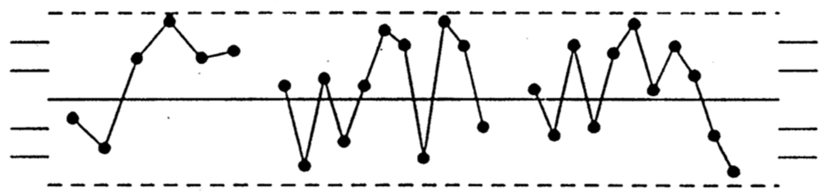


Fig. 3

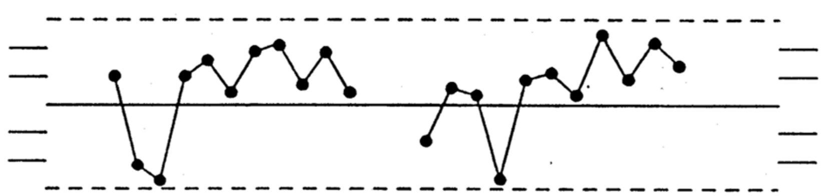
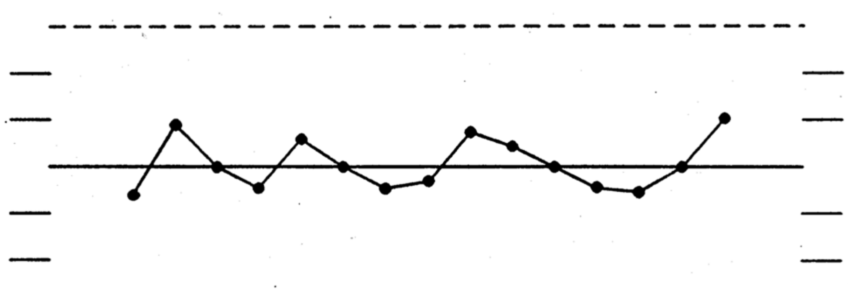


Fig. 4

# Other Possible Patterns

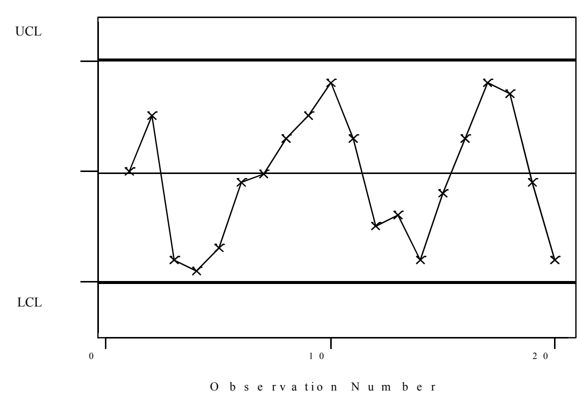


## Analysis of Patterns on Control Charts

- Cycling
- Gradual change
- Mixture
- Stratification
- Sudden shift
- Trend
- Systematic variation

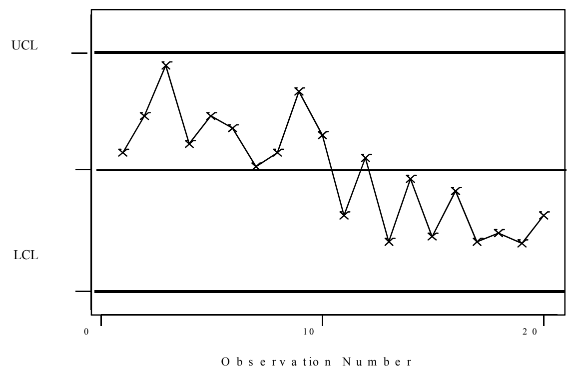
## Analysis of Patterns : Cycling

These could be caused by systematic changes like: temperature, operator fatigue, voltage or pressure fluctuations, etc.



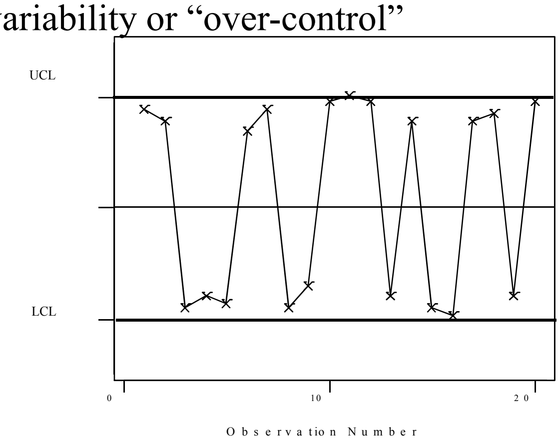
## Analysis of Patterns : Gradual change

These are usually due to the introduction of new operators, machines, procedures, etc. Can also happen because of process improvements.



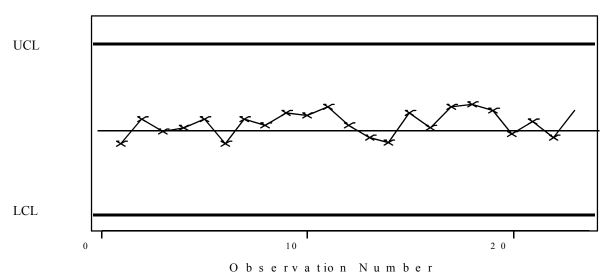

## Analysis of Patterns : Mixture

These are usually due to underestimating the process variability or “over-control”



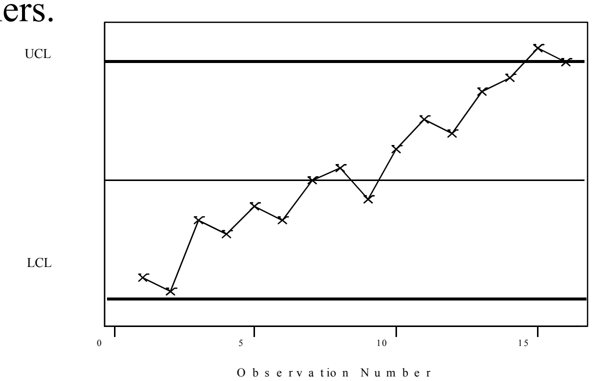
## Analysis of Patterns : Stratification

Points tend to cluster around the mean. Usually caused by: (1) incorrect control limits (overestimating process variability), (2) continuous improvements paying-off. Limits should be recalculated when this situation is found.



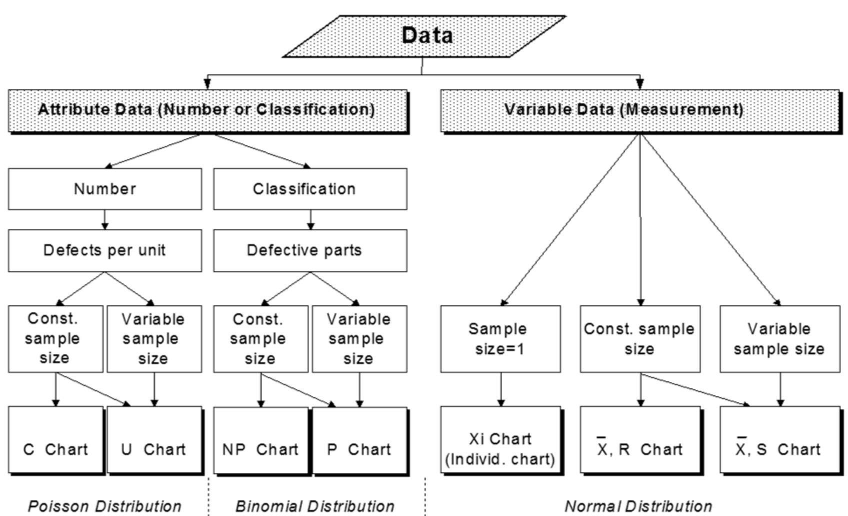

## Analysis of Patterns :Trend

These are usually caused by component wear among others.



# Charts For Attributes

## Selection of SPC Charts



# Control Charts for Attribute data

- p chart

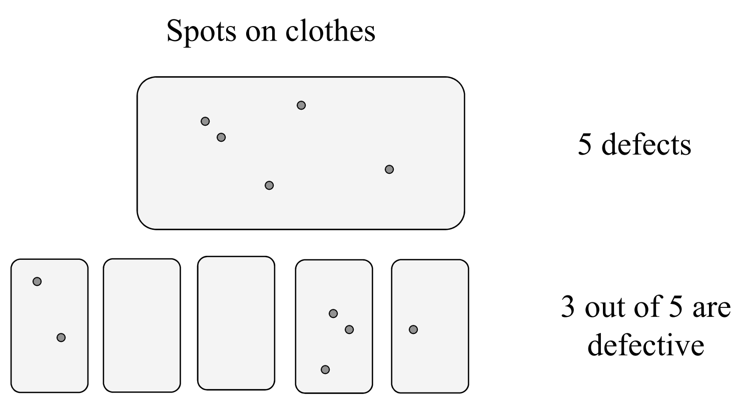
  This chart is used to track the proportion of defective product in a process, variable sample size
- np chart

  Same as previous, constant sample size
- c chart

  This chart is used to track the number of nonconformity's in a sample, constant sample size. (One unit can have more than one nonconformity)
- u chart

  Same as previous, sample size is variable

# Defects vs. Defective Units




Rule of thumb: If the number of potential occurrences is finite then you are dealing with defectives, otherwise it is defects.

Charts for Defectives

Charts for Defectives

- p chart

This chart is used to track the proportion of defective product in a process with constant or variable sample size

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\,(1 - \bar{p})}{\bar{n}}}$$

$$\text{Center line} = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\,(1 - \bar{p})}{\bar{n}}}$$

where: $\bar{p}$ = fraction percent defective
$n$ = sample size (could be nonconstant, in which case $\bar{n}$ is the average sample size)

## Example

| Subgroup, | $n_i$ | $d_i$ | $p_i$ |
|---|---|---|---|
| 1 | 126 | 15 | 0.119048 |
| 2 | 134 | 12 | 0.089552 |
| 3 | 115 | 11 | 0.095652 |
| 4 | 121 | 9 | 0.07438 |
| 5 | 142 | 18 | 0.126761 |
| 6 | 133 | 21 | 0.157895 |
| 7 | 115 | 15 | 0.130435 |
| 8 | 131 | 7 | 0.053435 |
| 9 | 126 | 20 | 0.15873 |
| 10 | 119 | 11 | 0.092437 |

## Example

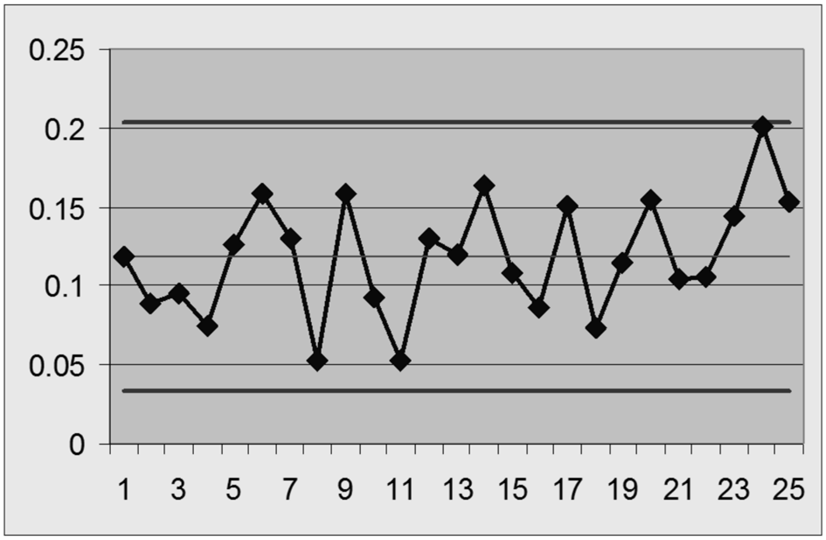
$$Center\ Line = \overline{p} = \frac{\sum D_i}{\sum n_i} = \frac{139}{1262} =$$

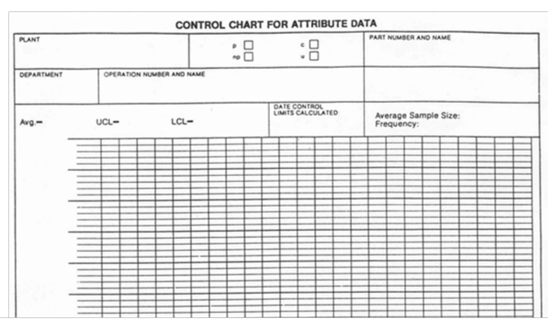
Average sample size = 126.2

(Note : applicable when fluctuation of sample size is less than 10%)

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}} =$$

# Example



**CONTROL CHART FOR ATTRIBUTE DATA**

PLANT

p ☐ np ☐ c ☐ u ☐

PART NUMBER AND NAME

DEPARTMENT

OPERATION NUMBER AND NAME

Avg.— UCL— LCL—

DATE CONTROL LIMITS CALCULATED

Average Sample Size:
Frequency:

# Charts for Defectives

- p chart with constant n

  Purpose: to track the proportion of defective product in a process with constant sample size

  $$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

  $$\text{Center line} = \bar{p}$$

  $$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

  where: $\bar{p}$ = fraction percent defective
  
  $n$ = sample size (constant)

# Exercise: Derive the "np" chart

Note: *d* = *np*

Objective: plot the number of defects *d* from a sample of size *n*

# Model for Defective Units

The Binomial distribution is the natural choice for counting *defective units*.
For n independent samples each having the same chance, p, of being defective, we have:

$$P(X = x) = \binom{n}{x} p^{x} (1 - p)^{n-x}$$

$$E(X) = np$$

$$Var(X) = np(1 - p)$$

## Model for Defects

The Poisson distribution is the simplest and yet the best model for *defects (non-conformities)*.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0,1,2,3,\Lambda$$

$$E(X) = \lambda$$

$$Var(X) = \lambda$$

## Charts for Defects

- c chart

  monitor the number of **defects** (or non-conformities) per sample with constant sample size.

$$UCL = \overline{c} + 3\sqrt{\overline{c}}$$

$$Center\ \ Line = \overline{c}$$

$$LCL = \overline{c} - 3\sqrt{\overline{c}}$$

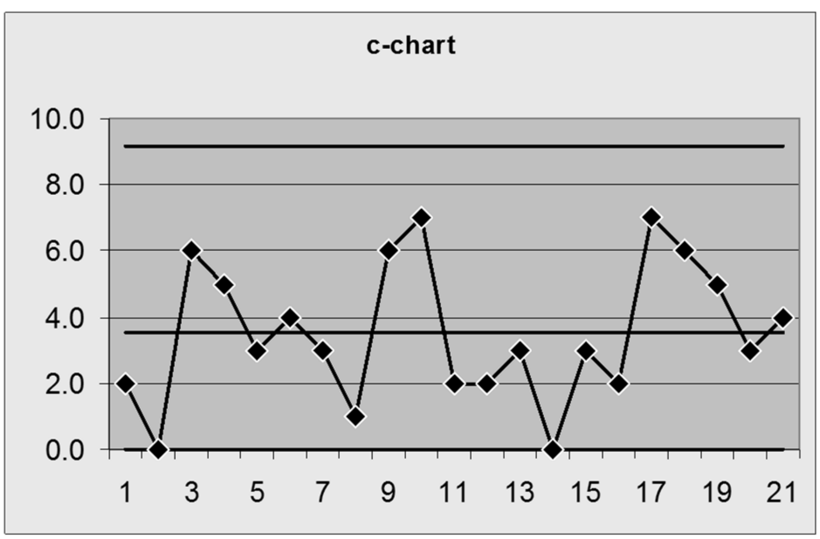
where $\overline{c}$ = average number of defects

# Example

The data captured are number of defects in a week for particular ATM machine.

| Week | # of defect in a week | LCL | UCL | CL |
|---|---|---|---|---|
| 1 | 2 | 0 | 9.2 | 3.5 |
| 2 | 0 | 0 | 9.2 | 3.5 |
| 3 | 6 | 0 | 9.2 | 3.5 |
| 4 | 5 | 0 | 9.2 | 3.5 |
| 5 | 3 | 0 | 9.2 | 3.5 |
| 6 | 4 | 0 | 9.2 | 3.5 |
| | | N | | |
| 19 | 5 | 0 | 9.2 | 3.5 |
| 20 | 3 | 0 | 9.2 | 3.5 |
| 21 | 4 | 0 | 9.2 | 3.5 |
| | | | | |
| Average = | **3.5** | | | |

# Example



## Charts for Defects per Unit

- u chart

  monitor the average number of defects per unit ( $u$ ).

$$UCL = \overline{u} + 3\sqrt{\overline{u}/n_i}$$

$$Center\ \ Line = \overline{u}$$

$$LCL = \overline{u} - 3\sqrt{\overline{u}/n_i}$$

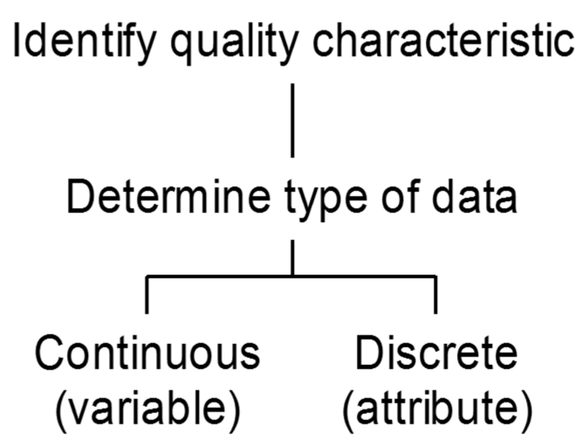
where $\overline{u}$ = average number of defects per unit
$n_i$ = sample size

## Words of Caution

- Technical requirements cannot be used to set control limits.
- Not applicable for high yield process (i.e. extremely low p).
- Use variable charts whenever possible.
- As these charts are used for monitoring undesirable outputs, they are “passive” in nature.

# Summary of Control Charts

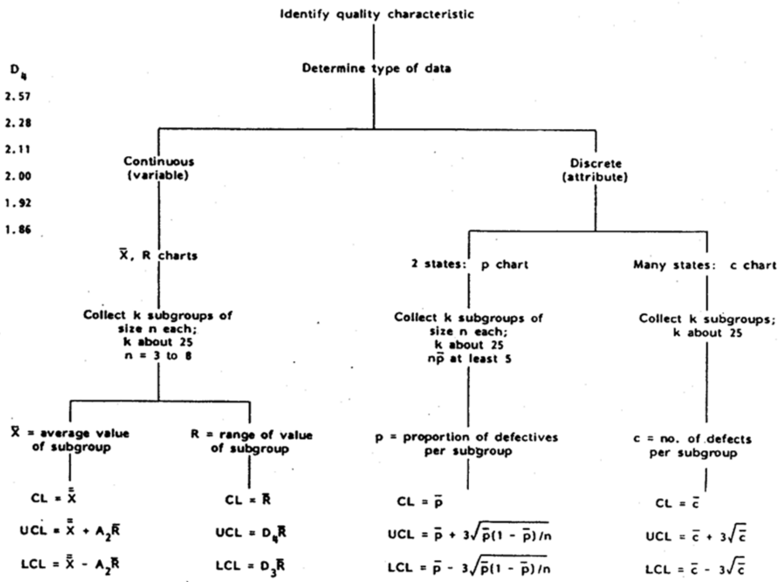
OVERVIEW



## SUMMARY

CONSTRUCTION OF CONTROL CHARTS

| n | $A_2$ | $D_3$ | $D_4$ |
|---|---|---|---|
| 3 | 1.02 | 0 | 2.57 |
| 4 | 0.73 | 0 | 2.28 |
| 5 | 0.58 | 0 | 2.11 |
| 6 | 0.48 | 0 | 2.00 |
| 7 | 0.42 | 0.08 | 1.92 |
| 8 | 0.37 | 0.14 | 1.86 |



# SQC Test II

1. In statistical quality control, a "statistic" is a value computed from measurements obtained from a sample.
2. Random variability usually refers to variability due to a wide range of unidentifiable causes none of which contributes any substantial amount to the total variability.
3. Process capability study is necessary if some units of a product have been found not to conform to specifications.
4. Shewhart control charts are designed based on three standard deviations from the mean.
5. If a process is under statistical control, only a negligible proportion, if any at all, of the product will not conform to specifications.
6. Process capability study is not necessary if a process is under statistical control.
7. 100 percent product inspection is necessary if a process is not capable.
8. A capable process is one which has a 0.9973 chance of being under statistical control.
9. A shift in process is almost always followed by a point falling out of a control limit.
10. If a process is capable and under statistical control, the product will meet specifications.
11. If a product meets specifications, it must have come from a process under statistical control.
12. All control charts have a definite upper control limit.
13. Poisson distribution is the basis of the *p*-chart.
14. A *p*-chart can be used to ensure that the proportion defective is below a stated target.
15. A *p*-chart can be used to check whether it is possible to keep the proportion defective below a stated target.
16. The *p*-chart is based on approximation of the binomial distribution by the normal distribution.
17. Poisson distribution is the basis of the *c*-chart.
18. Under certain conditions, there is no difference in results from a control chart for proportion defective and a control chart for number of defectives per sample.
19. It is more important to have a capable process than to have a process under statistical control.
20. Control charts can be used to detect sources of random variations in a process.

21. Control charts should be set up with reference to product specification limits.
22. Shewhart control charts are designed with the objective of establishing an acceptable quality level.
23. Shewhart control charts are designed with the objective of deciding when to look for causes of variation.
24. If a process is out of control, the theoretical probability that four consecutive points on an X-bar chart will fall on the same side of the mean is $(1/2)^4$.
25. A control chart for plotting the actual number of defects found during an inspection is known as the *c*-chart.
26. A process that is in statistical control will not necessarily produce products that meet specifications.
27. When a process is out of statistical control, it can be remedied by relaxing product specifications.
28. When a process is out of statistical control, it can be remedied by reducing the variance of the characteristic being measured.
29. If a process is out of statistical control, it will be immediately indicated by a point outside a control limit.
30. When no sample gives a point outside the control limits, the process is considered under statistical control.
31. When no sample gives a point outside the control limits, there is still a possibility that the process has already been out of statistical control.
32. When an unnatural pattern is detected in a control chart an assignable cause of variation should be looked for.
33. A control chart should be updated whenever there is a point outside the control limits.
34. A control chart should be updated if consecutive points show a trend.
35. A control chart should be updated whenever an assignable cause of variation is found.
36. A control chart can be used for controlling one quality characteristic at a time.
37. A control chart can be used to calculate process capability.
38. Compare to random errors, systematic errors are usually few and identifiable.
39. Control charts are useful in identifying assignable causes of variation.
40. A control chart should be used whenever a production lot is rejected by the customer.