

1. Define the energy

$$E(t) = \int_0^1 u(x, t)^2 dx$$

For the constant solution $u \equiv 1$, we have

$$E(t) = \int_0^1 1^2 dx = 1 \quad \text{for all } t \geq 0 \quad (*)$$

Since $f \equiv 0$, the energy estimate (2) becomes

$$E(t) \leq e^{-\pi t} E(0)$$

$$\Rightarrow 1 \leq e^{-\pi t} \cdot 1$$

However, for any $t > 0$, we have $e^{-\pi t} < 1$, which contradicts the inequality above. Therefore, the energy estimate (2) does not hold in this case.