Ch8 #3

Prove that $w_{n+1}'(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$, where $w_{n+1}(x) = \prod_{i=0}^{n} (x - x_i)$

$w_{n+1}(x) = \prod_{i=0}^{n} (x - x_i) = (x-x_0)(x-x_1)\ldots(x-x_{n-1})(x-x_n)$

$\Rightarrow w_{n+1}'(x) = (x-x_1)(x-x_2)(x-x_3)\ldots(x-x_{n-1})(x-x_n) +$

$(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_{n-1})(x-x_n) +$

$(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_{n-1})(x-x_n) +$

$\vdots$

$(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-2})(x-x_n) +$

$(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-2})(x-x_{n-1}) = \sum_{i=0}^{n} \left(\prod_{\substack{j=0 \\ j\neq i}}^{n} (x - x_j) \right)$

When $x = x_i$, $w_{n+1}'(x_i) = \sum_{k=0}^{n} \left(\prod_{\substack{j=0 \\ j\neq k}}^{n} (x_i - x_j) \right) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$ (其它項皆包含 $(x_i - x_i)$)