Symmetry properties of spin currents and spin polarizations in multiterminal mesoscopic spin-orbit-coupled systems

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We study theoretically some symmetry properties of spin currents and spin polarizations in a multiterminal mesoscopic spin-orbit-coupled system. Based on the scattering wave function approach, we show rigorously that no finite equilibrium terminal spin currents and/or finite equilibrium spin polarizations can survive in the equilibrium state. By use of a typical two-terminal structure as the example, we show explicitly that the nonequilibrium terminal spin currents in a multiterminal mesoscopic spin-orbit-coupled system may be nonconservative in general, and this nonconservation is intrinsic but not caused by the use of an improper definition of spin current in the calculations. We also show that the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of *finite* length of a ballistic two-dimensional electronic system with intrinsic spin-orbit coupling may be nonantisymmetric in general, suggesting that some cautions need to be taken when attributing the occurrence of nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in such a system to a spin Hall effect.

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I. INTRODUCTION

The efficient generation of finite spin polarizations and/or spin-polarized currents in nonmagnetic semiconductors by all-electrical means is one of the principal challenges in spin-based semiconductor electronics.[1,2] For this purpose, several interesting ideas were proposed recently based on the spin-orbit (SO) interaction character of electrons in some semiconducting materials.[3–6] Among them, the so-called *intrinsic spin Hall effect* has attracted much attention both theoretically[5–24] and experimentally[25,26] in the last several years, while at the same time it also raised a lot of debates and controversies.[8–10] A central problem related to these debates and controversies is that what is the correct definition of spin current in a system with intrinsic SO coupling and what is the actual relation between the spin current and the induced spin accumulation in such a system.[27] In most recent studies, the conventional (i.e., *standard*) definition of spin current was applied. However, since spin is not a conserved quantity in a system with intrinsic SO coupling, the physical meanings of spin current calculated based on the conventional definition are somewhat ambiguous and the actual relations between the spin current and the induced spin accumulation are not much clear. Indeed, as was pointed out in several recent papers,[28–31] some serious problems may occur when the conventional definition of spin current is used to describe spin-dependent transports in a system with intrinsic SO coupling. To avoid such problems, several alternative definitions of spin current were proposed in some recent papers based on different theoretical considerations, which are significantly different from the conventional one and also significantly different from each other.[28–31] Another possible way to circumvent this problem is to study a *mesoscopic* SO-coupled system attached to external leads. If no SO couplings are present in the leads or the SO couplings in the leads can be neglected, then the conventional definition of spin current can be well applied without ambiguities in the leads. Several recent works have adopted this strategy[14–18] and some interesting results were obtained. Of course, the study of spin transports in mesoscopic SO-coupled systems not only is of theoretical interest but also might find some practical applications in the design of spin-based electronic devices.[3]

In this paper, we study theoretically some interesting problems related to spin-dependent transports in multiterminal mesoscopic SO-coupled systems. We focus our study on some symmetry properties of spin currents and spin polarizations in such mesoscopic structures. As is well known, symmetry analysis is usually of great theoretical importance in the studies of many physical phenomena, including the studies of spin-dependent transports in SO-coupled systems.[32] Based on the analyses of some symmetry properties of spin currents and spin polarizations in multiterminal mesoscopic SO-coupled systems, some controversial issues related to spin-dependent transports in mesoscopic SO-coupled systems will be investigated in some detail in the present paper. Some results obtained in the present paper might also be valuable for the clarification of some controversial issues encountered in the studies of spin-dependent transports in macroscopic SO-coupled systems. The paper is organized as follows: In Sec. II, we will first give a brief introduction of the structure considered and the scattering wave function approach applied. In Sec. III, we will use the approach introduced in Sec. II to investigate whether there can exist nonvanishing equilibrium spin polarizations and/or nonvanishing equilibrium terminal spin currents in a multiterminal mesoscopic SO-coupled system. In Sec. IV, we will study some symmetry properties of nonequilibrium spin polarizations and nonequilibrium terminal spin currents in a typical two-terminal mesoscopic structure with both Rashba and Dresselhaus SO couplings. Finally in Sec. V, a brief summary of the main conclusions obtained in the paper will be given.

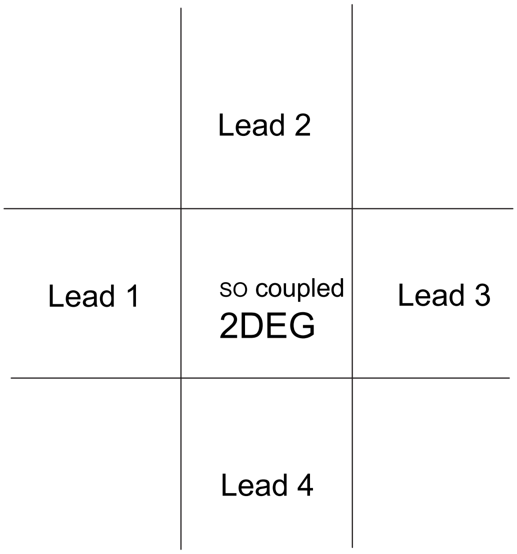

FIG. 1. Schematic geometry of a multiterminal mesoscopic SO-coupled system.

II. DESCRIPTION OF THE STRUCTURE AND THE SCATTERING WAVE FUNCTION APPROACH

We consider a general multiterminal mesoscopic structure as shown in Fig. 1, where a SO-coupled mesoscopic system is attached to several ideal leads. In a discrete representation, both the SO-coupled region and the ideal leads are described by a tight-binding (TB) Hamiltonian, and the total Hamiltonian for the entire structure reads

$$\hat{H} = H_{leads} + H_{sys} + H_{s-l}. \tag{1}$$

Here, $H_{leads}=\Sigma_p H_p$ and $H_p=-t_p\Sigma_{\langle \mathbf{p}_i,\mathbf{p}_j\rangle\sigma}(\hat{C}^\dagger_{\mathbf{p}_i\sigma}\hat{C}_{\mathbf{p}_j\sigma}+\text{H.c.})$ is the Hamiltonian for an isolated lead p, with $\hat{C}_{\mathbf{p}_i\sigma}$ and $\hat{C}^\dagger_{\mathbf{p}_i\sigma}$ denoting the annihilation and creation operators of electrons (with a spin index σ) at a lattice site $\mathbf{p}_i$ in lead p and t_p the hopping parameter between two nearest-neighbored lattice sites in the lead. We assume that the leads are ideal and nonmagnetic, i.e., no SO couplings (or other kinds of spin-flip processes rather than that induced by the scatterings from the central SO-coupled region) are present in the leads. In such ideal cases, the standard definition of spin current can be well applied without ambiguities in the leads. If not specified, we will choose the z axis (normal to the 2DEG plane) as the quantization axis of spin. $H_{sys}=H_0+H_{\text{SO}}$ is the Hamiltonian for the isolated SO-coupled region, in which H_{SO} describes the SO coupling of electrons and $H_0=-t\Sigma_{\langle \mathbf{r}_i,\mathbf{r}_j\rangle\sigma}(\hat{C}^\dagger_{\mathbf{r}_i\sigma}\hat{C}_{\mathbf{r}_j\sigma}+\text{H.c.})$ describes the spin-independent hopping of electrons between nearest-neighbored lattice sites. The discrete version of H_{SO} will depend on the actual form of the SO interaction, e.g., for the usual Rashba and k-linear Dresselhaus SO coupling, one has

$$H^R_{\text{SO}} = -t_R\sum_{\mathbf{r}_i}[i(\hat{\Psi}^\dagger_{\mathbf{r}_i}\sigma^x\hat{\Psi}_{\mathbf{r}_i+\boldsymbol{\Delta}_y} - \hat{\Psi}^\dagger_{\mathbf{r}_i}\sigma^y\hat{\Psi}_{\mathbf{r}_i+\boldsymbol{\Delta}_x}) + \text{H.c.}], \tag{2a}$$

$$H^D_{\text{SO}} = -t_D\sum_{\mathbf{r}_i}[i(\hat{\Psi}^\dagger_{\mathbf{r}_i}\sigma^y\hat{\Psi}_{\mathbf{r}_i+\boldsymbol{\Delta}_y} - \hat{\Psi}^\dagger_{\mathbf{r}_i}\sigma^x\hat{\Psi}_{\mathbf{r}_i+\boldsymbol{\Delta}_x}) + \text{H.c.}], \tag{2b}$$

where t_R and t_D are the Rashba and Dresselhaus SO coupling strengths, respectively, $\hat{\Psi}_{\mathbf{r}_i}=(\hat{C}_{\mathbf{r}_i\uparrow},\hat{C}_{\mathbf{r}_i\downarrow})$ denotes the spinor annihilation operator at the lattice site $\mathbf{r}_i$, and $\boldsymbol{\Delta}_x$ and $\boldsymbol{\Delta}_y$ denote the lattice vectors between two nearest-neighbored lattice sites along the x and y directions, respectively. The last term in the Hamiltonian (1) describes the coupling between the leads and the central SO-coupled region, $H_{s-l}=-\Sigma_p t_{ps}\Sigma_n(\hat{C}^\dagger_{\mathbf{p}_n\sigma}\hat{C}_{\mathbf{r}_n\sigma}+\text{H.c.})$, where $\mathbf{p}_n$ denotes a boundary lattice site in lead p connected directly to a boundary lattice site $\mathbf{r}_n$ in the SO-coupled region and t_{ps} the hopping parameter between lead p and the SO-coupled region. It should be noticed that, in general, the TB Hamiltonian will also contain an on-site energy term, which is not explicitly shown above.

The study carried out in the present paper will be based on the scattering wave function approach within the framework of the standard Landauer-Büttiker's formalism.[33] To this end, we will start by considering the scatterings of incident electrons from a given lead by the central SO-coupled region. For convenience of notation, we will use a double coordinate index (x_p,y_p) to denote a lattice site in lead p, where $x_p=1,2,\ldots,\infty$ (away from the border between the lead and the SO-coupled region) and $y_p=1,\ldots,N_p$ (N_p is the width of lead p). In the local coordinate frame, the spatial wave function of an electron incident on lead p will be given by $e^{-ik^p_m x_p}\chi^p_m(y_p)$, where k^p_m denotes the longitudinal wave vector, $\chi^p_m(y_p)$ the transverse spatial wave function, and m the label of the transverse mode. The longitudinal wave vector will be determined by the following dispersion relation: $-2t\cos(k^p_m)+\varepsilon^p_m=E$, where ε^p_m is the eigenenergy of the mth transverse mode and E the energy of the incident electron. It should be noted that due to the presence of SO coupling in the central scattering region, spin-flip processes (e.g., the spin-flip reflection) will be induced in the leads when an incident electron is scattered or reflected by the central SO-coupled region, even if the leads are ideal and nonmagnetic (which is the case assumed in the present paper). Due to this fact, for an incident electron in the mth transverse channel of lead p and with a *given* spin index σ, both the scattering wave function $|\psi^{pm\sigma}(\mathbf{r})\rangle$ in the central SO-coupled region and the scattering wave function $|\psi^{pm\sigma}(\mathbf{x}_{p'})\rangle$ $[\mathbf{x}_{p'}\equiv(x_{p'},y_{p'})]$ in a lead p' will be a superposition of a spin-up and a spin-down component,

$$|\psi^{pm\sigma}(\mathbf{r})\rangle = \sum_{\sigma'}\psi^{pm\sigma}_{\sigma'}(\mathbf{r})\hat{C}^\dagger_{\mathbf{r}_i,\sigma'}|0\rangle, \tag{3a}$$

$$|\psi^{pm\sigma}(\mathbf{x}_{p'})\rangle = \sum_{\sigma'}\psi^{pm\sigma}_{\sigma'}(\mathbf{x}_{p'})\hat{C}^\dagger_{\mathbf{x}_{p'},\sigma'}|0\rangle, \tag{3b}$$

where $|0\rangle$ stands for the vacuum state and $\psi^{pm\sigma}_{\sigma'}(\mathbf{r})$ and $\psi^{pm\sigma}_{\sigma'}(\mathbf{x}_{p'})$ the spin-resolved components. The spin-resolved components of the scattering wave function in lead p' can be expressed in the following general form:

$$\psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'}) = \delta_{pp'}\delta_{\sigma\sigma'}e^{-ik_m^p x_p}\chi_m^p(y_p) + \sum_{m'\in p'} \phi_{p'm'\sigma'}^{pm\sigma} e^{ik_{m'}^{p'}x_{p'}}\chi_{m'}^{p'}(y_{p'}), \qquad (4)$$

where $\phi_{p'm'\sigma'}^{pm\sigma}$ stands for the scattering amplitude from the $(m\sigma)$ channel of lead p (the *incident* mode) to the $(m'\sigma')$ channel of lead p' (the *outgoing* mode). If $p'=p$, the second term on the right-hand side of Eq. (4) will denote actually the spin-resolved reflected waves in lead p and $\phi_{p'm'\sigma'}^{pm\sigma}$ denote the spin-flip $(\sigma\neq\sigma')$ and non-spin-flip $(\sigma=\sigma')$ reflection amplitudes. The scattering amplitudes $\phi_{p'm'\sigma'}^{pm\sigma}$ can be obtained by solving the Schrödinger equation for the entire structure. Since Eq. (4) is just a linear combination of all outgoing modes with the same energy E in lead p', the Schrödinger equation is satisfied automatically in lead p' except at those boundary lattice sites in lead p' connected directly to the SO-coupled region. Due to the coupling between the leads and the SO-coupled region, the amplitudes of the wave function at these boundary lattice sites (which are determined by the scattering amplitudes $\{\phi_{qn\sigma'}^{pm\sigma}\}$) must be solved simultaneously with the wave function $\psi^{pm\sigma}(\mathbf{r})$ in the central SO-coupled region. From the discrete version of the Schrödinger equation, one can show that the amplitudes of the wave function in the SO-coupled region and at the boundary lattice sites of the leads connected directly to the SO-coupled region will satisfy the following coupled equations:

$$E\psi_{\sigma'}^{pm\sigma}(\mathbf{r}_s) = \sum_{\mathbf{r}_s',\sigma''} H_{sys}(\mathbf{r}_s\sigma',\mathbf{r}_s'\sigma'')\psi_{\sigma''}^{pm\sigma}(\mathbf{r}_s') - \sum_{p',y_{p'}} t_{p's}\delta_{\mathbf{r}_s,n_{p'y'}}\psi_{\sigma'}^{pm\sigma}(1,y_{p'}'), \qquad (5a)$$

$$E\psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'}') = \sum_{\mathbf{x}_{p'}''} H_{p'}(\mathbf{x}_{p'}',\sigma',\mathbf{x}_{p'}'',\sigma')\psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'}'') - \sum_{\mathbf{r}_s} t_{p's}\delta_{\mathbf{r}_s,n_{p'y'}}\psi_{\sigma'}^{pm\sigma}(\mathbf{r}_s), \qquad (5b)$$

where $(\mathbf{r}_s,\mathbf{r}_s')$ denote two nearest-neighbored lattice sites in the SO-coupled region and $(\mathbf{x}_{p'}',\mathbf{x}_{p'}'')$ two nearest-neighbored boundary lattice sites in lead p'. [For simplicity of notation, in the subscript of the Kronecker δ function we have used a simple symbol $n_{p'y'}$ to denote a boundary lattice site in the SO-coupled region which is connected directly to a boundary lattice site $\mathbf{x}_{p'}'=(1,y_{p'}')$ in lead p'.] The matrix elements $H_{sys}(\mathbf{r}_s\sigma',\mathbf{r}_s'\sigma'')$ and $H_{p'}(\mathbf{x}_{p'}',\sigma',\mathbf{x}_{p'}'',\sigma')$ can be written down directly from the Hamiltonian (1). Equations (5a) and (5b) are the match conditions of the scattering wave function on the borders between the SO-coupled region and the leads, from which both the scattering wave function in the entire structure and all scattering amplitudes can be obtained simultaneously. Some details of the derivations are given in the Appendix.

III. SOME RIGOROUS PROPERTIES OF EQUILIBRIUM STATES

A controversial issue encountered in the studies of spin-polarized transports in SO-coupled systems is that whether there can exist nonvanishing *equilibrium background spin currents* in such systems. It was shown in Ref. 8 that a finite equilibrium background spin current could be obtained if the conventional definition of spin current is applied to a two-dimensional electronic system with Rashba SO coupling.[8] If this background spin current does exist, nonvanishing equilibrium spin polarizations should also exist near the edges of such a system due to the flow of the background spin currents. It was argued in several recent papers that such equilibrium background spin currents are an artifact caused by the improper use of the conventional definition of spin current to a SO-coupled system, i.e., the conventional definition of spin current cannot be applied in the presence of SO coupling.[28–31] Since no consensus had been arrived on whether there is a unique definition for spin current in a SO-coupled system, in the present paper we will discuss this controversial issue in a somewhat different way. Based on the scattering wave function approach introduced in Sec. II, we will show rigorously that no finite equilibrium spin polarizations and/or finite equilibrium terminal spin currents can exist in a multiterminal mesoscopic SO-coupled system.

A. Absence of equilibrium spin polarizations

In the tight-binding representation, the operator for the local spin density at a lattice site i (either in the central SO-coupled region and in the external leads) will be given by

$$\hat{\vec{S}}(i) = \frac{\hbar}{2}\sum_{\alpha\beta} \hat{C}_{i\alpha}^{\dagger}\vec{\sigma}_{\alpha\beta}\hat{C}_{i\beta}. \qquad (6)$$

Under the time-reversal transformation, the local spin density operator will transform as $\hat{\vec{S}}(i)\to\hat{T}\hat{\vec{S}}(i)\hat{T}^{-1}=-\hat{\vec{S}}(i)$ and the spin operator will transform as $\vec{\sigma}_{\alpha\beta}\to\hat{T}\vec{\sigma}_{\alpha\beta}\hat{T}^{-1}=(-1)^{\alpha+\beta}\vec{\sigma}_{\bar{\alpha}\bar{\beta}}^{*}=-\vec{\sigma}_{\alpha\beta}$, where $\bar{\alpha}\equiv-\alpha$, $\bar{\beta}\equiv-\beta$, and $\hat{T}\equiv i\sigma_y\hat{K}$ denote the time-reversal transformation operator and $\hat{K}$ the conjugate operator.

Within the framework of the standard Landauer-Büttiker's formalism, a physical quantity of a mesoscopic system will be contributed by all scattering states of conduction electrons incident from all contacts, which are described by the scattering wave functions $\{\psi^{pm\sigma}\}$ introduced in Eqs. (3) and (4). In order that there is only one particle feeding into each incident channel, when we use the scattering wave functions $\{\psi^{pm\sigma}\}$ introduced in Eqs. (3) and (4) to calculate the expectation value of an operator, we should first normalize the scattering wave functions $\{\psi^{pm\sigma}\}$ by a factor of $1/\sqrt{L}$, where $L\to\infty$ is the length of the leads.[33] By use of the normalized scattering wave functions and noticing that the density of states for the mth transverse mode of lead p is given by $\frac{L}{2\pi}\frac{dk_m^p}{dE}=\frac{L}{2\pi\hbar v_{pm}}$, where $v_{pm}=\frac{2t_p}{\hbar}\sin(k_m^p)$ is the longitudinal velocity of the mth transverse mode of lead p, then one can see that the local spin density at a lattice site i (either in the central SO-coupled region or in the external leads) will be given by

$$\langle\hat{\vec{S}}(i)\rangle = \sum_{pm\sigma}\int\frac{dE}{2\pi}f(E,\mu_p)\frac{1}{\hbar v_{pm}}\sum_{\alpha,\beta}\left[\psi_\alpha^{pm\sigma*}(i)\left(\frac{\hbar}{2}\vec{\sigma}\right)_{\alpha\beta}\psi_\beta^{pm\sigma}(i)\right.$$
$$\left.+\,\text{H.c.}\right], \tag{7}$$

where $\psi^{pm\sigma}$ is the scattering wave function corresponding to an incident electron from the $(m\sigma)$ channel of lead p with a given energy E, μ_p is the chemical potential in lead p, and $f(E,\mu_p)$ is the usual Fermi distribution function. This formula is valid *both* in the equilibrium and in the nonequilibrium states. If the system is in an equilibrium state, the chemical potential μ_p will be independent of the lead label, i.e., $\mu_p\equiv\mu$ and $f(E,\mu_p)\equiv f(E,\mu)$. Then, in Eq. (7) the summation $\Sigma_{pm\sigma}[\cdots]$ can be performed first before carrying out the energy integration, and the summation can be transformed to the following form:

$$\sum_{\alpha\beta}\sum_{pm\sigma}\left[\psi_\alpha^{pm\sigma*}(i)\left(\frac{\hbar}{2}\vec{\sigma}\right)_{\alpha\beta}\psi_\beta^{pm\sigma}(i)/\hbar v_{pm}+\text{H.c.}\right]$$
$$=\sum_{\alpha\beta}\left[A_{\beta\alpha}(i,i;E)\left(\frac{\hbar}{2}\vec{\sigma}\right)_{\alpha\beta}+\text{H.c.}\right]$$
$$=i\sum_{\alpha\beta}\left\{[G^R(E)-G^A(E)]_{i\beta,i\alpha}\left(\frac{\hbar}{2}\vec{\sigma}\right)_{\alpha\beta}-\text{H.c.}\right\}, \tag{8}$$

where $G^{R,A}(E)=[E\mathbf{I}-\hat{H}\pm i0^+]^{-1}$ are the usual retarded and advanced Green's functions, whose explicit spin-resolved matrix forms are given by $G^{R,A}_{i\alpha,j\beta}(E)=\Sigma_{p'm'\sigma'}\psi_\alpha^{p'm'\sigma'*}(i)\psi_\beta^{p'm'\sigma'}(j)/[E-E'\pm i0^+]^{-1}$ (E' is the incident energy corresponding to the scattering wave function $\psi^{p'm'\sigma'}$), and $A_{\beta\alpha}(j,i;E)\equiv\sum_{p'm'\sigma'}\psi_\alpha^{p'm'\sigma'*}(i)\psi_\beta^{p'm'\sigma'}(j)/\hbar v_{p'm'}=i[G^R(E)-G^A(E)]_{j\beta,i\alpha}$ is the spin-resolved spectral function. If the total Hamiltonian $\hat{H}$ of the system is time-reversal invariant, the retarded and advanced Green's functions can be related by the time-reversal transformation as follows:

$$G^A_{i\alpha,j\beta}=(\hat{T}G^R\hat{T}^{-1})_{i\alpha,j\beta}=(-1)^{\alpha+\beta}G^{R*}_{i\bar{\alpha},j\bar{\beta}}. \tag{9}$$

By use of Eq. (9) and noticing that $\hat{T}\vec{\sigma}_{\alpha\beta}\hat{T}^{-1}=-\vec{\sigma}_{\alpha\beta}$, one can show readily that the right-hand side of Eq. (8) will vanish exactly; thus, the spin density given by Eq. (7) cannot survive in the equilibrium state. It should be noted that in arriving at this conclusion, we have only made use of the assumption that the total Hamiltonian $\hat{H}$ of the system is time-reversal invariant (which should be the case in the absence of external fields) and did not involve the actual form of the SO coupling in the system, so it is a much general conclusion.

B. Absence of equilibrium terminal spin currents

Next, we discuss whether there can exist nonvanishing equilibrium terminal spin currents in a multiterminal mesoscopic SO-coupled system. Since we have assumed that the leads are ideal and nonmagnetic [i.e., described by a simple Hamiltonian $\hat{H}_p=-t_p\Sigma_{\langle\mathbf{p}_i,\mathbf{p}_j\rangle\sigma}(\hat{C}^\dagger_{\mathbf{p}_i\sigma}\hat{C}_{\mathbf{p}_j\sigma}+\text{H.c.})$], the conventional definitions of charge and spin currents can be well applied in the leads without ambiguities. According to the conventional definitions and in the lattice representation, the charge current and spin current (with spin parallel to the α axis[34]) flowing from a lattice site $\mathbf{p}_i$ to a nearest-neighbored lattice site $\mathbf{p}_j$ in lead p can be given by the corresponding particle density current as follows:

$$\hat{I}_{p,\mathbf{p}_i\to\mathbf{p}_j}=e[\hat{J}^+_{\mathbf{p}_i\to\mathbf{p}_j}+\hat{J}^-_{\mathbf{p}_i\to\mathbf{p}_j}], \tag{10a}$$

$$\hat{I}^\alpha_{p,\mathbf{p}_i\to\mathbf{p}_j}=\frac{\hbar}{2}[\hat{J}^+_{\mathbf{p}_i\to\mathbf{p}_j}-\hat{J}^-_{\mathbf{p}_i\to\mathbf{p}_j}], \tag{10b}$$

where $\hat{I}_{p,\mathbf{p}_i\to\mathbf{p}_j}$ denotes the charge current operator, $\hat{I}^\alpha_{p,\mathbf{p}_i\to\mathbf{p}_j}$ the spin current operator with spin parallel to the α axis, and $\hat{J}^\sigma_{\mathbf{p}_i\to\mathbf{p}_j}$ the spin-resolved particle density current operator ($\sigma=\pm$ denotes the spin-up and spin-down states with respect to the α axis, respectively). From the Heisenberg equation of motion for the on-site particle density, $\frac{d}{dt}\hat{N}_{\mathbf{p}_i}=\frac{1}{i\hbar}[\hat{N}_{\mathbf{p}_i},\hat{H}_p]$, where $\hat{N}_{\mathbf{p}_i}=\hat{C}^\dagger_{\mathbf{p}_i\sigma}\hat{C}_{\mathbf{p}_i\sigma}$ is the on-site particle density operator in lead p, one can show easily that the spin-resolved particle density current flowing from a lattice site $\mathbf{p}_i$ to a nearest-neighbored lattice $\mathbf{p}_j$ in lead p will be given by

$$\hat{J}^\sigma_{\mathbf{p}_i\to\mathbf{p}_j}=\frac{it_p}{\hbar}(\hat{C}^\dagger_{\mathbf{p}_j\sigma}\hat{C}_{\mathbf{p}_i\sigma}-\hat{C}^\dagger_{\mathbf{p}_i\sigma}\hat{C}_{\mathbf{p}_j\sigma}). \tag{11}$$

Now, we calculate the terminal charge and spin currents flowing along the *longitudinal* direction of a lead q. Firstly, we consider the contribution of an incident electron from the $(m\sigma)$ channel of lead p to the longitudinal charge and spin currents (with spin parallel to the α axis) flowing through a transverse cross section (saying, for example, the cross section at $x=x_q$) of lead q, which (by definition) will be given by

$$\langle\hat{I}_q\rangle_{pm\sigma}=\frac{1}{L}\sum_{y_q}\langle\psi^{pm\sigma}(x_q+1,y_q)|\hat{I}_{q,(x_q,y_q)\to(x_q+1,y_q)}|\psi^{pm\sigma}(x_q,y_q)\rangle$$
$$=\frac{e}{L}\left\{\sum_{n,\sigma'}v_{qn}|\phi^{pm\sigma}_{qn\sigma'}|^2-v_{pm}\delta_{pq}\right\}, \tag{12a}$$

$$\langle\hat{I}^\alpha_q\rangle_{pm\sigma}=\frac{1}{L}\sum_{y_q}\langle\psi^{pm\sigma}(x_q+1,y_q)|\hat{I}^\alpha_{q,(x_q,y_q)\to(x_q+1,y_q)}|\psi^{pm\sigma}(x_q,y_q)\rangle$$
$$=\frac{h}{4\pi L}\left\{\sum_n v_{qn}[|\phi^{pm\sigma}_{qn+}|^2-|\phi^{pm\sigma}_{qn-}|^2]-\sigma v_{pm}\delta_{pq}\right\}, \tag{12b}$$

where (x_q,y_q) and (x_q+1,y_q) denote two nearest-neighbored lattice sites along the longitudinal direction of lead q, $\frac{1}{\sqrt{L}}|\psi^{pm\sigma}(x_q,y_q)\rangle$ denotes the *normalized* scattering wave function in lead q corresponding to the incident electron from the $(m\sigma)$ channel of lead p [which is given by Eqs. (3) and (4)], and $v_{qn}=\frac{2t_p}{\hbar}\sin(k^q_n)$ denotes the longitudinal velocity of the nth transverse mode in lead q and v_{pm}

$=\frac{2t_p}{\hbar}\sin(k_m^p)$ the longitudinal velocity of the mth transverse mode in lead p. The summation over the transverse coordinate y_q runs over from 1 to N_q (N_q is the width of lead q),[35] and the following orthogonality relation for transverse modes in lead q has been applied in obtaining the last lines of Eqs. (12a) and (12b): $\Sigma_{y_q}\chi_m^q(y_q)\chi_n^q(y_q)=\delta_{mn}$. It should be noted that if $p=q$, the results given by Eqs. (12a) and (12b) will denote actually the contribution of an incident electron from lead q to the charge and spin currents flowing in the same lead and $\phi_{qn\sigma'}^{pm\sigma}(\sigma'=\pm)$ denote actually the spin-flip ($\sigma\neq\sigma'$) and non-spin-flip ($\sigma=\sigma'$) reflection amplitudes. [See the explanations given to Eqs. (3) and (4) in Sec. II]. In such cases, the results given by Eqs. (12a) and (12b) can be expressed as the subtraction of the contributions due to the incident wave [i.e., the terms proportional to δ_{pq} in Eqs. (12a) and (12b)] and the contributions due to the spin-flip and non-spin-flip reflected waves.

The total terminal charge current I_q and the total terminal spin current I_q^α flowing in lead q will be obtained by summing the contributions of all incident electrons from all leads with the corresponding density of states [see the explanations given above Eq. (7)]. Then, we get

$$\begin{aligned}I_q&=\frac{e}{h}\sum_{pm\sigma}\int dEf(E,\mu_p)\left[\sum_{n,\sigma'}|\phi_{qn\sigma'}^{pm\sigma}|^2\frac{v_{qn}}{v_{pm}}-\delta_{pq}\right]\\&=\frac{e}{h}\sum_{p\sigma\sigma'}\int dEf(E,\mu_p)[T_{q\sigma'}^{p\sigma}(E)-\delta_{pq}\delta_{\sigma\sigma'}N_q(E)]\\&=\frac{e}{h}\sum_{p\sigma\sigma'}\int dE[f(E,\mu_p)T_{q\sigma'}^{p\sigma}(E)-f(E,\mu_q)T_{p\sigma'}^{q\sigma}(E)],\end{aligned}\tag{13a}$$

$$\begin{aligned}I_q^\alpha&=\sum_{pm\sigma}\int dEf(E,\mu_p)\left[\sum_n\frac{v_{qn}}{4\pi v_{pm}}(|\phi_{qn+}^{pm\sigma}|^2-|\phi_{qn-}^{pm\sigma}|^2)\right]\\&=\frac{1}{4\pi}\sum_{p\sigma}\int dEf(E,\mu_p)[T_{q+}^{p\sigma}(E)-T_{q-}^{p\sigma}(E)],\end{aligned}\tag{13b}$$

where $T_{q\sigma'}^{p\sigma}(E)\equiv\sum_{m,n}|\phi_{qn\sigma'}^{pm\sigma}|^2\frac{v_{qn}}{v_{pm}}$ denotes (by definition) the transmission probability from lead p with spin σ to lead q with spin σ' (see Ref. 33 and also the explanations given in the Appendix). In obtaining the last line of Eq. (13a) the following relation has been applied:[33]

$$\sum_{p\sigma}T_{q\sigma'}^{p\sigma}(E)=\sum_{p\sigma}T_{q\bar{\sigma}'}^{p\sigma}(E)=N_q(E),\tag{14}$$

where $N_q(E)$ is the total number of conducting transverse modes in lead q corresponding to a given energy E. As was discussed in detail in Ref. 33, this relation follows directly from the unitarity of the S matrix, which is essential for the particle number conservation.

Equation (13a) is just the usual Landauer-Büttiker formula for terminal charge currents in a multiterminal mesoscopic system.[33] The second line in Eq. (13a) indicates clearly that the terminal charge current flowing in lead q can be expressed as the subtraction of the contributions due to all incident modes (corresponding to the terms proportional to δ_{pq}) and the contributions due to all outgoing modes (corresponding to the terms proportional to $T_{q\sigma'}^{p\sigma}$), which include both the transmitted waves from other leads ($p\neq q$) and the reflected waves in lead q ($p=q$), noticing that $T_{q\sigma'}^{p\sigma}$ denotes actually the spin-flip or non-spin-flip reflection probabilities from lead q to lead q if $p=q$. Equation (13b) is somewhat different from Eq. (13a) in appearance, but the terminal spin current given by Eq. (13b) can still be divided into two different kinds of contributions, namely, the contributions due to the transmitted waves from other leads (corresponding to those terms with $p\neq q$ in the summation $\Sigma_{p\sigma}[\cdots]$) and the contributions due to the spin-flip and non-spin-flip reflected waves in lead q (corresponding to those terms with $p=q$ in the summation $\Sigma_{p\sigma}[\cdots]$).[36]

Equations (13a) and (13b) are valid both in the equilibrium and in the nonequilibrium states. In the equilibrium state, since $\mu_p\equiv\mu$ (independent of the lead label) and $f(E,\mu_p)\equiv f(E,\mu)$, in Eqs. (13a) and (13b) the summation $\Sigma_{p\sigma}[\cdots]$ can be performed first before carrying out the energy integration, and we get

$$I_q=\frac{e}{h}\int dEf(E,\mu)\sum_{p\sigma\sigma'}[T_{q\sigma'}^{p\sigma}(E)-T_{p\sigma'}^{q\sigma}(E)],\tag{15a}$$

$$I_q^\alpha=\frac{1}{4\pi}\int dEf(E,\mu)\sum_{p\sigma}[T_{q+}^{p\sigma}(E)-T_{q-}^{p\sigma}(E)].\tag{15b}$$

Then, by use of Eq. (14) one can clearly see that both terminal charge currents and terminal spin currents will vanish exactly in the equilibrium state. It should be stressed that in arriving at this conclusion, we did not involve the controversial issue of what is the correct definition of spin current in the central SO-coupled region at all, so those ambiguities that might be caused by the use of an improper definition of spin current to the SO-coupled region have been eliminated in our derivations. Though we cannot prove that the spin current also vanishes exactly inside the SO-coupled region based on the approach applied above, however, for a mesoscopic system only the terminal (charge or spin) currents are the real useful quantities from the *practical* point of view (i.e., one needs to add external contacts to induct the charge or spin currents out of a mesoscopic sample). We note that a similar conclusion as was obtained above has also been derived in Ref. 37 based on some somewhat different arguments. Compared with the derivations given in Ref. 37, the arguments given above seem to be more simple and more transparent, in principle. It also should be noted that, based on a similar Landauer-Büttiker formalism, it was argued in Ref. 38 that the equilibrium terminal spin currents should indeed take place in a three-terminal system with spin-orbit coupling,[38] in contradiction to the conclusion obtained in the present paper and in Ref. 37. To our understanding, this contradiction was caused by the fact that the contributions due to the spin-flip and non-spin-flip reflections in the leads (induced by the scatterings from the central SO-coupled region) was neglected in the calculations of terminal spin currents

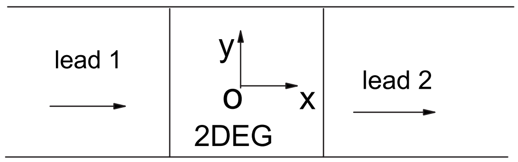

FIG. 2. Schematic geometry of a two-terminal mesoscopic SO-coupled system.

performed in Ref. 38. In contrast, in the calculations of terminal spin currents performed in the present paper, the contributions due to the spin-flip and non-spin-flip reflections in the leads induced by the scatterings from the central SO-coupled region have been treated in an accurate and strict way, assuming that the leads are ideal and nonmagnetic.

IV. SOME SYMMETRY PROPERTIES OF NONEQUILIBRIUM SPIN CURRENTS AND SPIN POLARIZATIONS IN A TYPICAL TWO-TERMINAL MESOSCOPIC SO-COUPLED SYSTEM

When a multiterminal mesoscopic SO-coupled system is driven into a nonequilibrium state, nonequilibrium spin polarizations and/or terminal spin currents may be induced by the charge current flow. In this section, we discuss some symmetry properties of such nonequilibrium spin currents and spin polarizations. For clarity, we take a typical two-terminal mesoscopic structure as shown in Fig. 2 as the example, where a ballistic two-dimensional electron gas (2DEG) with Rashba and/or *k*-linear Dresselhaus SO coupling is attached to two ideal leads.

A. Nonantisymmetric lateral edge spin accumulations

The study of nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of a two-dimensional electron gas with intrinsic SO coupling is of great theoretical interest since it was generally believed that the principal observable signature of the spin Hall effect in such a system is that, when a longitudinal charge current circulates through a thin strip of such a system, *antisymmetric* lateral edge spin accumulation (polarized perpendicular to the 2DEG plane) will be induced at the two lateral edges of the strip due to the flow of the transverse spin Hall current. Some numerical calculations had demonstrated that a longitudinal charge current circulating through a thin strip of a ballistic two-dimensional electron gas with Rashba SO coupling does lead to the generation of nonequilibrium lateral edge spin accumulation polarized perpendicular to the 2DEG plane, and the antisymmetric character of the lateral edge spin accumulation (i.e., $\langle S_z(x,y)\rangle = -\langle S_z(x,-y)\rangle$) had been argued to be a strong support of the existence of the spin Hall effect in such mesoscopic SO-coupled systems.[15] Here, we discuss this issue from a somewhat different point of view. We will show that when a longitudinal charge current circulates through a thin strip of a ballistic two-dimensional electron gas with both Rashba and Dresselhaus SO couplings, the induced nonequilibrium lateral edge spin accumulation may be nonantisymmetric in general. The nonantisymmetric character of the lateral edge spin accumulation is in contradiction to the usual physical pictures of the spin Hall effect, though according to some theoretical predictions, the spin Hall effect should also survive in the presence of both Rashba and Dresselhaus SO couplings.[12] The nonantisymmetric character of the lateral edge spin accumulation implies that, in addition to the spin Hall effect, there may exist some other physical reasons that might also lead to the generation of nonequilibrium lateral edge spin accumulation in a SO-coupled system (with a thin strip geometry) when a longitudinal charge current circulates through it.[23,24]

Firstly, let us look at what symmetry relations can be obtained for the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current based on the symmetry analysis of the Hamiltonian of the structure under study (sketched in Fig. 2). If only Rashba (or only Dresselhaus) SO coupling is present, based on the symmetry properties of the Hamiltonian of the structure under study, one can rigorously show that the nonequilibrium lateral edge spin accumulation does should be antisymmetric at the two lateral edges. Let us consider first the case in which only Rashba SO coupling is present (i.e., the Dresselhaus SO coupling strength is zero). If only Rashba SO coupling is present, from Eq. (2a) and Fig. 2 one can see that the Hamiltonian of the entire structure is invariant under the combined transformation of the real-space reflection $y \Rightarrow -y$ and the spin space rotation around the S_y axis (with an angle π). From this invariance, one can immediately get $\langle S_z(x,y)\rangle_I = -\langle S_z(x,-y)\rangle_I$, where $\langle S_z\rangle_I$ denotes the nonequilibrium spin density induced by a longitudinal charge current flowing from lead 1 to lead 2. It is interesting to note that this antisymmetric relation can be deduced directly from the symmetry of the structure under study but without need to resort to the concept of spin Hall effect at all. Next, let us consider the case in which only Dresselhaus SO coupling is present (i.e., the Rashba SO coupling strength is zero). If only Dresselhaus SO coupling is present, then from Eq. (2b) and Fig. 2 one can see that the Hamiltonian of the entire structure is invariant under the combined transformation of the real-space reflection $y \Rightarrow -y$ and the spin space rotation around the S_x axis (with an angle π). From this invariance, one also immediately gets $\langle S_z(x,y)\rangle_I = -\langle S_z(x,-y)\rangle_I$, i.e., the nonequilibrium lateral edge spin accumulation still should be antisymmetric at the two lateral edges if only Dresselhaus SO coupling is present.

If both Rashba and Dresselhaus SO couplings are present, then from Eqs. (2a) and (2b) and Fig. 2 one can see that the total Hamiltonian of the entire structure is invariant under the combined transformation of the real-space center inversion $\mathbf{r} \Rightarrow -\mathbf{r}$ and the spin space rotation around the S_z axis (with an angle π). From this invariance one can get $\langle S_z(x,y)\rangle_I = \langle S_z(-x,-y)\rangle_{-I}$, where $\langle S_z\rangle_{-I}$ denotes the nonequilibrium spin density induced by a longitudinal charge current flowing from lead 2 to lead 1. On the other hand, from Eq. (7) one can see that in the linear-response regime one has

$$\langle S_z(x,y)\rangle_I = -\langle S_z(x,y)\rangle_{-I}. \tag{16}$$

Combining the two relations $\langle S_z(x,y)\rangle_I = \langle S_z(-x,-y)\rangle_{-I}$ and $\langle S_z(x,y)\rangle_I = -\langle S_z(x,y)\rangle_{-I}$, then the following symmetry relation can be obtained for the nonequilibrium spin accumulation induced by a longitudinal charge current flowing from lead 1 to lead 2: $\langle S_z(x,y)\rangle_I = -\langle S_z(-x,-y)\rangle_I$. This symmetry relation implies that the transverse spatial distribution of the nonequilibrium lateral edge spin accumulation will be antisymmetric in the center cross section of the strip, i.e., $\langle S_z(0,y)\rangle_I = -\langle S_z(0,-y)\rangle_I$. Due to the existence of this symmetry relation, one can deduce that in an *infinite* strip (i.e., the length of the strip tends to infinity and hence the effects of the contacted leads can be neglected), the transverse spatial distribution of the nonequilibrium lateral edge spin accumulation will still be antisymmetric (i.e., $\langle S_z(x,y)\rangle_I = -\langle S_z(x,-y)\rangle_I$ for all x) in the presence of both Rashba and Dresselhaus SO couplings. However, unlike the case in which only Rashba (or only Dresselhaus) SO coupling is present, for a thin strip of finite length, in the presence of both Rashba and Dresselhaus SO couplings, one cannot deduce a general antisymmetric relation for the transverse spatial distribution of the nonequilibrium lateral edge spin accumulation based on the symmetry analysis of the total Hamiltonian of the entire structure under study. From the theoretical points of view, this is due to the fact that, in the presence of both Rashba and Dresselhaus SO couplings, the total Hamiltonian of the entire structure under study (i.e., a thin strip of finite length contacted to two ideal leads) is no longer invariant under the combined transformation of the real-space reflection $y \Rightarrow -y$ and the spin space rotation around the S_y (or S_x) axis with an angle π. Indeed, as will be shown below, this nonantisymmetric character can be verified by detailed numerical calculations.

One particular interesting case is that the Rashba and the Dresselhaus SO coupling strengths are equal (i.e., $t_R = t_D$ or $t_R = -t_D$). In this particular case, the total Hamiltonian is invariant under the following unitary transformation in spin space (while the real-space coordinate **r** remain unchanged): $\hat{U}_+\hat{H}\hat{U}_+ = \hat{H}$ (if $t_R = t_D$) or $\hat{U}_-\hat{H}\hat{U}_- = \hat{H}$ (if $t_R = -t_D$), where $\hat{U}_+ = (\hat{\sigma}_x + \hat{\sigma}_y)/\sqrt{2}$ and $\hat{U}_- = (\hat{\sigma}_x - \hat{\sigma}_y)/\sqrt{2}$. Under this unitary transformation, the spin operators will transform as follows: $\hat{\sigma}_z \to -\hat{\sigma}_z$, $\hat{\sigma}_x \rightleftharpoons \hat{\sigma}_y$ (if $t_R = t_D$) or $\hat{\sigma}_x \rightleftharpoons -\hat{\sigma}_y$ (if $t_R = -t_D$). Since the real-space coordinate **r** remains unchanged under this symmetry manipulation, from the above symmetry properties in spin space one immediately gets $\langle S_z(x,y)\rangle_I = -\langle S_z(x,y)\rangle_I$, suggesting that $\langle S_z(x,y)\rangle_I$ should vanish everywhere if the Rashba and the Dresselhaus SO coupling strengths are equal. This conclusion is in exact agreement with the corresponding numerical results obtained based on the scattering wave approach introduced in Secs. II and III, which shows that $\langle S_z(x,y)\rangle_I$ does vanish everywhere in the particular case of $t_R = t_D$ or $t_R = -t_D$.

To show more explicitly the nonantisymmetric character of the lateral edge spin accumulation induced by a longitudinal charge current in a 2DEG strip of finite length with both Rashba and Dresselhaus SO couplings, in Fig. 3 we plotted a typical pattern of the two-dimensional spatial distribution of the nonequilibrium spin density $\langle S_z\rangle$ in the strip obtained by numerical calculations with the scattering wave function approach introduced in Secs. II and III. In our numerical calculations, we take the typical values of the electron effective mass $m = 0.04 m_e$, the lattice constant $a = 3$ nm, and the 2DEG strip contains 100×40 lattice sizes. The chemical potentials in the two leads are set by fixing the longitudinal charge current to flow from lead 1 to lead 2 (as shown in Fig. 2) and fixing the longitudinal charge current density to 100 μA/1.5 μm. From Fig. 3, one can see clearly that the transverse spatial distribution of the nonequilibrium spin density $\langle S_z\rangle$ in the strip is nonantisymmetric in general (i.e., $\langle S_z(x,y)\rangle_I \neq -\langle S_z(x,-y)\rangle_I$ for general x), except in the center cross section (i.e., $x=0$) of the strip. The nonantisymmetric character of the transverse spatial distribution of the nonequilibrium spin density $\langle S_z\rangle$ can be more clearly seen from Fig. 4(a), where we plotted several typical patterns of the profiles of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z\rangle$ in a cross section of the strip at $x \neq 0$. [For comparison, the corresponding results obtained in the case that only Rashba or only Dresselhaus SO coupling is present were also plotted in Fig. 4(b).] The three typical patterns shown in Fig. 4(a) are obtained by fixing the Dresselhaus SO coupling strength to $t_D = 0.02t$ (t is the spin-independent hopping parameter) and varying the Rashba SO coupling strength t_R. From Fig. 4(a) one can see clearly that the transverse spatial distribution of the nonequilibrium spin density $\langle S_z(x,y)\rangle_I$ can have either the same signs or opposite signs at the two lateral edges of the strip, depending on the ratios of t_R/t_D. Even in the case that $\langle S_z\rangle$ has opposite signs at the two lateral edges, the transverse spatial distributions of $\langle S_z\rangle$ may still not be antisymmetric (i.e., $\langle S_z(x,y)\rangle \neq -\langle S_z(x,-y)\rangle$), in significant contradiction to the usual physical pictures of the spin Hall effect. The nonantisymmetric character of the lateral edge spin accumulation suggests that some cautions may need to be taken when attributing the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of a SO-coupled system to a spin Hall effect, especially in the mesoscopic regime.

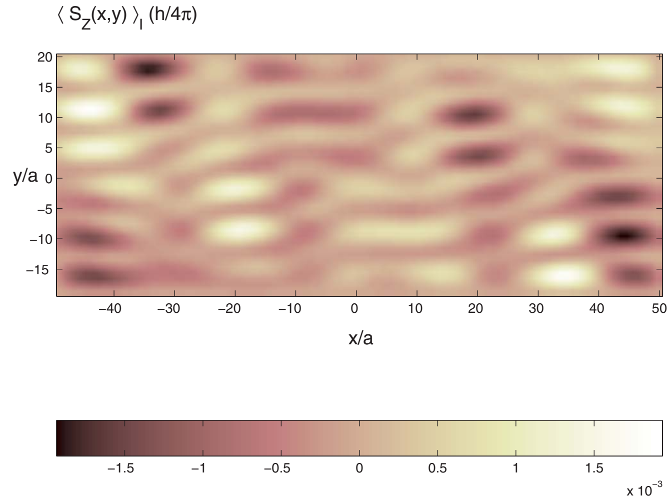

FIG. 3. (Color online) A typical pattern of the two-dimensional spatial distribution of the current induced nonequilibrium spin density $\langle S_z\rangle$ in a two-terminal structure (sketched in Fig. 2) in the presence of both Rashba and Dresselhaus SO couplings. The Rashba and Dresselhaus SO coupling strengths are set to $t_R/t = 0.08$ and $t_D/t = 0.02$.

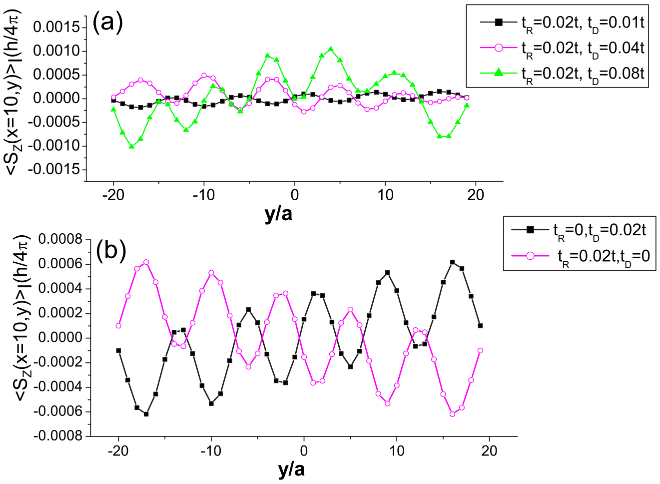

FIG. 4. (Color online) (a) Some typical profiles of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ induced by a longitudinal charge current in a 2DEG strip with both Rashba and Dresselhaus SO couplings, which show clearly that the transverse spatial distributions of $\langle S_z \rangle$ are nonantisymmetric in general at both edges of the strip in the presence of both Rashba and Dresselhaus SO couplings. $\langle S_z \rangle$ vanishes everywhere in the particular case of $t_R = t_D$ or $t_R = -t_D$ (not shown explicitly in the figure). (b) The corresponding results obtained in the case that only Rashba or only Dresselhaus SO coupling is present, which shows clearly that the transverse spatial distributions of $\langle S_z \rangle$ are antisymmetric at both edges of the strip if only Rashba or only Dresselhaus SO coupling is present.

The results shown in Figs. 3 and 4 are obtained in the absence of impurity scatterings. One can show that the symmetry properties shown in Figs. 3 and 4 are robust against spinless weak impurity scatterings. To model spinless weak disorder scatterings, we assume that the on-site energies at lattice sites in the 2DEG strip are randomly distributed in a narrow energy region $[-W, W]$, where W is the amplitude of the on-site energy fluctuations characterizing the disorder strength. (In the absence of disorder scatterings, the on-site energies at each lattice site are set simply to zero.) We calculate the spin density for a number of random impurity configurations and then do impurity average. In Fig. 5, we show the variations of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ in a cross section of the strip as the disorder strength increases, from which one can see that the symmetry properties of the transverse spatial distributions of the nonequilibrium spin density are robust against spinless weak disorder scatterings.

B. Nonconservative terminal spin currents

Another controversial issue encountered in the studies of spin-polarized transports in SO-coupled systems is that whether spin current should be a conserved quantity and what is the correct definition of spin current in such a system. Since spin current calculated based on the conventional definition is not a conserved quantity in the presence of SO coupling, significant modifications have to be made if one wants to define spin current a conserved quantity.[28,30]

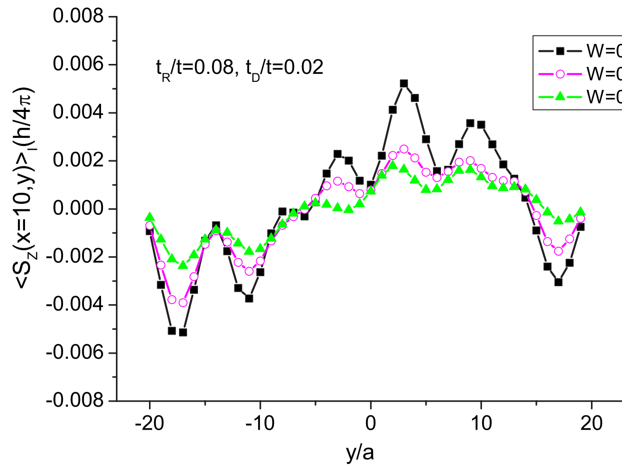

FIG. 5. (Color online) The profiles of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ in the presence of disorder. We have done impurity average over 1000 random impurity configurations for each case.

Though several alternative definitions were proposed recently, it remains a much controversial issue whether spin current could be defined uniquely for a SO-coupled system.[28–31] To avoid such ambiguities, below we will discuss this controversial issue from a different point of view, i.e., we will not consider the question of what is the correct definition of spin current in a SO-coupled system but focus our discussions on the question of whether the terminal spin currents in a multiterminal mesoscopic SO-coupled system are conservative. As mentioned earlier, for a mesoscopic SO-coupled system, only the terminal spin currents are the real useful quantities. By use of the two-probe mesoscopic structure shown in Fig. 2 as the example, we will show explicitly that the terminal spin currents in a multiterminal mesoscopic SO-coupled system may be nonconservative in general. To illustrate this point clearly, in Figs. 6(a) and 6(b) we plotted the terminal spin currents I_1^z and I_2^z (with spin parallel to the z axis) and the terminal spin currents I_1^y and I_2^y (with spin parallel to the y axis) in the two leads as a function of the Rashba SO coupling strength t_R, respectively, which were obtained by use of Eqs. (12b) and (13b). In our calculations, we fix the Dresselhaus SO coupling strength to $t_D = 0.02t$ and fix the longitudinal charge current density to $100\ \mu\text{A}/1.5\ \mu\text{m}$. The lattice constant $a = 3$ nm and the lattice size of the strip is taken to be 100×40. The positive direction of the spin current flow is defined to be from lead 1 to lead 2. From Fig. 6(a), one can see that the terminal spin currents I_1^z and I_2^z in the two leads have the same signs, which means that the spin current with spin parallel to the z axis will flow from lead 1 into the SO-coupled region and then flow out of the SO-coupled region into lead 2, similar to the usual charge current flow. From Fig. 6(b), one can see that the terminal spin currents I_1^y and I_2^y in the two leads have opposite signs, which means that the spin current with spin parallel to the y axis will flow out of the SO-coupled region in both leads and hence are nonconservative, i.e., the spin current flowing into the SO-coupled region does not equal to the spin current flowing out of the same region. Similarly, one can show that the terminal spin currents with spin par-

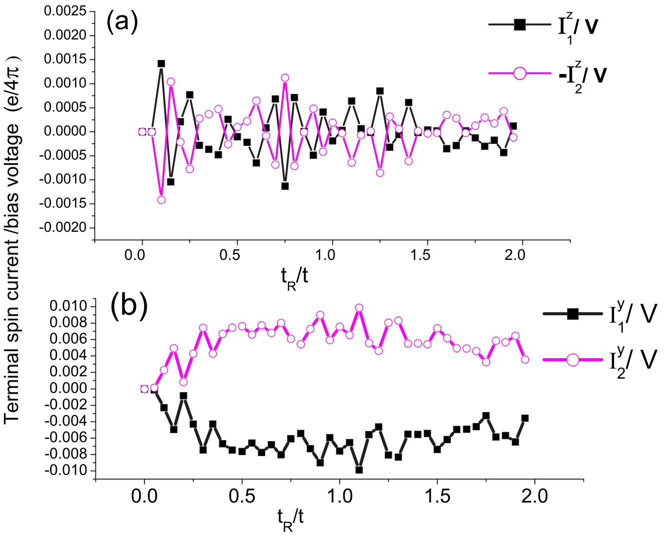

FIG. 6. (Color online) (a) The terminal spin currents I_1^z and $-I_2^z$ (divided by the voltage) as a function of the Rashba SO coupling strength t_R (in units of t). (b) The terminal spin currents I_1^y and I_2^y (divided by the voltage) as a function of the Rashba SO coupling strength t_R. The figures show that the terminal spin currents I_1^z and I_2^z in the two leads have the same signs and the terminal spin currents I_1^y and I_2^y in the two leads have opposite signs. [Note that for clarity a minus sign is added before I_2^z in (a).] The parameters used are given in the text or shown in the figures.

allel to the x axis have also opposite signs in the two leads, similar to the case shown in Fig. 6(b). This simple example illustrates explicitly that the terminal spin currents in a multiterminal mesoscopic SO-coupled system may be nonconservative in general. It should be stressed that this nonconservation of terminal spin currents is intrinsic but not caused by the use of an improper definition of spin current in the calculations. As a matter of fact, in our calculations we did not involve the definitions of spin current in the central SO-coupled region at all, so those ambiguities that might be caused by the use of an improper definition of spin current to the SO-coupled region have been eliminated in our calculations.

V. CONCLUSION

In summary, based on a scattering wave function approach, in this paper we have studied theoretically some symmetry properties of spin currents and spin polarizations in a multiterminal mesoscopic structure where a SO-coupled system is contacted to several ideal and nonmagnetic external leads. Some interesting results were obtained based on the symmetry analysis of spin currents and spin polarizations in such a multiterminal mesoscopic structure. First, we showed that in the equilibrium state no finite spin polarizations can exist both in the leads and in the central SO-coupled region and also no finite equilibrium terminal spin currents can survive. Second, we showed that the lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of a ballistic two-dimensional electron gas with both Rashba and Dresselhaus SO couplings may be nonantisymmetric in general, implying that some cautions may need to be taken when attributing the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of such a system to a spin Hall effect, especially in the mesoscopic regime. Finally, by use of a typical two-probe structure as the example, we showed explicitly that the nonequilibrium terminal spin currents may be nonconservative in general. Some symmetry properties obtained in the present paper might also be helpful for clarifying some controversial issues encountered in the study of spin-dependent transports in macroscopic SO-coupled systems.

ACKNOWLEDGMENTS

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APPENDIX: SOME DETAILS FOR THE DERIVATIONS OF THE SCATTERING AMPLITUDES AND THE TRANSMISSION PROBABILITIES

In this appendix, we give some details on how to derive the scattering amplitudes and the transmission probabilities from the scattering wave function approach introduced in Sec. II. For simplicity of notation, we arrange the scattering wave function $\psi^{pm\sigma}(\mathbf{r}_i)$ in the central SO-coupled region into a column vector Ψ_s whose dimension is $2N$ (N is the total number of lattice sites in the SO-coupled region) and arrange the scattering amplitudes $\phi^{pm\sigma}_{qn\sigma'}$ into a column vector Φ whose dimension is $2M$ ($M=\Sigma_p N_p$ and N_p is the width of lead p). Substituting Eqs. (3) and (4) into Eqs. (5a) and (5b) and making use of the orthogonality relations for the transverse modes in the leads, one can show readily that the two column vectors Ψ_s and Φ will satisfy the following relations:

$$\mathbf{A}\Psi_s = \mathbf{b} + \mathbf{B}\Phi, \quad \mathbf{C}\Phi = \mathbf{d} + \mathbf{D}\Psi_s, \tag{A1}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are four rectangular matrices with the dimensions of $2N\times 2N$, $2N\times 2M$, $2M\times 2M$, and $2M\times 2N$, respectively, and $\mathbf{b}$ and $\mathbf{d}$ are two column vectors with the dimensions of $2N$ and $2M$, respectively. The elements of these matrices and column vectors can be written down explicitly as

$$\mathbf{A} = E\mathbf{I} - \mathbf{H}_{sys},$$

$$\mathbf{B}(n_{p''y}\sigma'', p'm'\sigma') = -\delta_{p'',p'}\delta_{\sigma'',\sigma'}t_{p's}\chi^{p'}_{m'}(y_{p''})e^{ik^{p'}_{m'}},$$

$$\mathbf{D}(p'm'\sigma', n_{p''y}\sigma'') = -\delta_{p'',p'}\delta_{\sigma'',\sigma'}t_{p's}\chi^{p'}_{m'}(y_{p''}),$$

$$\mathbf{C}(p'm'\sigma', p''m''\sigma'') = -\delta_{p'',p'}\delta_{\sigma'',\sigma'}\delta_{m''m'}t_{p'},$$

$$\mathbf{b}(n_{p'y}\sigma') = -\delta_{pp'}\delta_{\sigma\sigma'}t_{ps}\chi^{p}_{m}(y_{p'})e^{-ik^{p}_{m}},$$

$$\mathbf{d}(p'm'\sigma') = \delta_{pp'}\delta_{mm'}\delta_{\sigma\sigma'}t_p, \tag{A2}$$

where $\mathbf{I}$ stands for the identity matrix. The indices for leads, transverse modes, lattice sites, and spins can take all possible values, and for simplicity of notation, we have used a simple symbol $n_{p'y'}$ to denote a boundary lattice site in the central SO-coupled region which is connected directly to a boundary lattice site $\mathbf{x}'_{p'}=(1,y'_{p'})$ in lead p'. Equation (A1) is just a compact form of the match conditions (5a) and (5b) on the borders between the leads and the central SO-coupled region, from which both the scattering amplitudes $\{\phi^{pm\sigma}_{qn\sigma'}\}$ and the transmission probabilities $\{T^{p\sigma}_{q\sigma'}\}$ can be obtained.

To derive a compact expression for the transmission probabilities, we define an auxiliary matrix $\Sigma^R \equiv \mathbf{B}\mathbf{C}^{-1}\mathbf{D}$. By use of Eq. (A2), the matrix elements of Σ^R can be written down readily as

$$\Sigma^R(n_{p'y_1}\sigma', n_{p'y_2}\sigma') = -\sum_{m'} \frac{t^2_{p's}}{t_{p'}} \chi^{p'}_{m'}(y_1)\chi^{p'}_{m'}(y_2)e^{ik^{p'}_{m'}}, \tag{A3}$$

and all other matrix elements that are not explicitly shown are zero. With the help of this auxiliary matrix, from Eq. (A1) one can get

$$\Psi_s = (\mathbf{A} - \Sigma^R)^{-1}(\mathbf{b} + \mathbf{B}\mathbf{C}^{-1}\mathbf{d}) = G^R\mathbf{g}, \tag{A4}$$

where $\mathbf{g}$ is a column vector defined by $\mathbf{g}\equiv\mathbf{b}+\mathbf{B}\mathbf{C}^{-1}\mathbf{d}$ and G^R a matrix defined by $G^R\equiv(\mathbf{A}-\Sigma^R)^{-1}=[E\mathbf{I}-H_{sys}-\Sigma^R]^{-1}$, which is just the usual retarded Green's function. By use of Eq. (A2), the elements of the column vector $\mathbf{g}$ can also be written down readily as follows:

$$\mathbf{g}(n_{p'y}\sigma') = 2i\delta_{pp'}\delta_{\sigma\sigma'}t_{ps}\sin(k^p_m)\chi^p_m(y). \tag{A5}$$

By substituting Eq. (A4) into Eq. (A1), one gets $\Phi=\mathbf{C}^{-1}\mathbf{d}+\mathbf{C}^{-1}\mathbf{D}G^R\mathbf{g}$. Inserting Eq. (A5) into this formula and making use of Eq. (A2), then one can show readily that the scattering amplitudes $\phi^{pm\sigma}_{qn\sigma'}$ can be given by

$$\phi^{pm\sigma}_{p'm'\sigma'} = -\delta_{pp'}\delta_{mm'}\delta_{\sigma\sigma'} + 2it^{-1}_{p'}t_{p's}\sum_{y_p,y_{p'}} t_{ps}\sin(k^p_m) \times\chi^{p'}_{m'}(y_{p'})G^R_{\sigma'\sigma}(n_{y_{p'}},n_{y_p})\chi_m(y_p). \tag{A6}$$

The total transmission probability of an incident electron from lead p (with spin index σ) to lead p' (with spin index σ') is defined by $T^{p\sigma}_{p'\sigma'}=\Sigma_{m,m'}|\phi^{pm\sigma}_{p'm'\sigma'}|^2\frac{v_{p'm'}}{v_{pm}}$, where $v_{p'm'}=\frac{1}{\hbar}2t_{p'}\sin(k^{p'}_{m'})$ is the longitudinal velocity of the transverse mode m' in lead p'. Substituting Eq. (A6) into this definition, then for $p\neq p'$ one can get

$$T^{p\sigma}_{p'\sigma'} = \mathrm{Tr}(\Gamma^p G^A_{\sigma\sigma'}\Gamma^{p'}G^R_{\sigma'\sigma}), \tag{A7}$$

where $G^R_{\sigma\sigma'}$ and $G^A_{\sigma\sigma'}(\equiv G^{R\dagger}_{\sigma'\sigma})$ are the spin-resolved retarded and advanced Green's functions, respectively, and $\Gamma^p(y_p,\bar{y}_p)$ is defined by

$$\Gamma^p(y_p,\bar{y}_p) = \sum_m \left(\frac{t_{ps}}{t_p}\right)^2 \chi_m(y_p)v_{pm}\chi_m(\bar{y}_p). \tag{A8}$$

The transmission probabilities given by Eq. (A7) have exactly the same form as was obtained by the usual Green's function approach.[33]

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[34] When we calculate the terminal spin currents with spin parallel to the α $(=x,y,z)$ axis, we will choose the α axis as the quantization axis of spins in the leads. This convention will be assumed throughout the paper.

[35] The expectation values of $\hat{I}_q$ and $\hat{I}_q^{\alpha}$ given by Eqs. (12a) and (12b) are independent of the longitudinal coordinate x_q in the steady state.

[36] Since we have assumed that the leads are ideal and nonmagnetic, the contributions due to incident waves from lead q with opposite spin indexes [given by the terms proportional to δ_{pq} in Eq. (12b)] cancel exactly with each other in Eq. (13b).

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