# CS 325 Project 2: Coin Change

*Your report must be typed and submitted online. Each team member's name must be listed as well as any resources used to finish the project.*

For this project, you will investigate the coin change problem:

Given coins of denominations (value) $1 = v_1 < v_2 < \ldots < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that $v_i$'s and A are integers. All values of A will have a solution since $v_1 = 1$.

Formally, an algorithm for this problem should take as input:

- An array V where V[i] is the value of the coin of the $i^{th}$ denomination.
- A value A which is the amount of change we are asked to make

The algorithm should return an array C where C[i] is the number of coins of value V[i] to return as change and m the minimum number of coins it took. You must return exact change so

$$\sum_{i=1}^{n} V[i] \cdot C[i] = A$$

The objective is to minimize the number of coins returned or:

$$\min \sum_{i=1}^{n} C[i]$$

## Implementation:

You may use any language you choose to implement your algorithms. You will implement three algorithms (three programs) for this problem. Your algorithms are to be based on these ideas:

1. **Brute Force or Divide and Conquer Algorithm:**

This implementation is called **changeslow**.

To make change for A cents:

- If there is a K-cent coin, then that one coin is the minimum
- Otherwise, for each value i < K,
    - Find the minimum number of coins needed to make i cents
    - Find the minimum number of coins needed to make K - i cents
- Choose the i that minimizes this sum

This algorithm can be viewed as divide-and-conquer, or as brute force. This solution is very recursive and runs in exponential time.

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**2. Greedy Algorithm:**

Another approach to coin change problem is the greedy approach. This implementation is called **changegreedy**. This is also "naive" since it may not be optimal.

- Use the largest value coin possible.
- Subtract the value of this coin from the amount of change to be made.
- Repeat.

**3. Dynamic Programming:**

One dynamic programming approach uses table $T$ indexed by values of change $0, 1, 2, \ldots, A$ where $T[v]$ is the minimum number of coins needed to make change for v.

$$T[v] = \min_{V[i] \le v} \{T[v - V[i]] + 1\}$$

We initialize $T[0] = 0$. How do you store and return the number of each type of coin to return? (That is, how do you build $C[i]$?) This implementation is called **changedp**. Note: there are other versions of the DP algorithm you may use one but need to explain in your report.

The execution of the program should be as follows:

- User runs the programs on the command-line, specifying a file in which the first line contains the array V, formatted as [1,5,10,25] without the quotes.
- Each subsequent line contains one integer value for which we must make change.

Program output should be to a file named [input filename]change.txt where [input filename].txt is the input filename, and should be formatted with one change result and the minimum number of coins m=3, per line. For example

Amount.txt:

[1, 2, 5]
10
[1, 3, 7, 26]
22

Amountchange.txt:

[0, 0, 2]
2
[1, 0, 3, 0]
4

**Testing for correctness.** Above all else your algorithm should be correct. You can test your algorithms on the following:

1. Suppose $V = [1, 10, 25, 50]$ and $A = 40$. The **changegreedy** should return $C = [5, 1, 1, 0]$ and **changedp** should return $C = [0, 4, 0, 0]$. The minimum number of coins m is 4.
2. If $A$ is changed to 76, all algorithms should return $C = [1, 0, 1, 1]$, with m = 3.

3. Suppose $V = [1, 10, 25, 50]$ and $A = 40$. The **changegreedy** should return $C = [5, 1, 1, 0]$ and **changedp** should return $C = [0, 4, 0, 0]$. The minimum number of coins m is 4.

You should also test your code on self-generated instances.

## Project Report

Your team's report should include the following:

1. Describe, in words, how you fill in the dynamic programming table in changedp. Justify why is this a valid way to fill the table?
2. Give pseudocode for each algorithm.
3. Prove that the dynamic programming approach is correct by induction. That is, prove that

$$T[v] = \min_{V[i] \le v}\{T[v - V[i]] + 1\},\ T[0] = 0 \text{ is}$$

   the minimum number of coins possible to make change for value *v*.
4. Suppose $V = [1, 10, 25, 50]$. For each integer value of *A* in [2000, 2100, 2200, …, 3000] determine the number of coins that changegreedy and changedp requires. You can attempt to run changeslow however if it takes too long you can select smaller values of A and also run the other algorithms on the values. Plot the number of coins as a function of *A* for each algorithm. How do the approaches compare?
5. Suppose $V = [1, 3, 4]$. For each integer value of *A* in [2000, 2100, 2200, …, 3000] determine the number of coins that changegreedy and changedp requires. You can attempt to run changeslow however if it takes too long you can select smaller values of A and also run all three algorithms on the values. Plot the number of coins as a function of *A* for each algorithm. How do the approaches compare?
6. For the above situations, determine (experimentally) the running times required by all algorithms, using the methods suggested for the first project. How do the running times of the algorithms compare? What is the *asymptotic* running time in terms of the number of denominations and the value *A* for each approach?
7. Suppose you are living in a country where coins have values that are powers of *p*, $V = [p^0, p^1, p^2, \cdots, p^n]$. How do you think the dynamic programming and greedy approaches would compare? Explain

## What to Submit

Your elected submitter must upload a ZIP file containing (1) Project Report PDF, (2) README, (3) CODE to Canvas and T.E.A.C.H.