21.1.

Method	General Charactristics
Jacobi	Converges when A is strictly row diagonally dominant.
Gauss-Seidel	Almost always superior to Jacobi because it uses new xi as soon as they are computed. Converges when A is strictly row diagonally dominant or positive definite.
SOR	With a good choice of ω, it is superior to Gauss-Seidel and Jacobi. It must be the case that $0 < \omega < 2$, and SOR converges if A is positive definite.
CG	CG is the method of choice for solving large sparse positive definite systems. If exact arithmetic is used, CG converges in n steps or less. Convergence is related to κ (A). Since κ depends on the largest and smallest eigenvalue of A, if the eigenvalues are clustered closely together, convergence will be good. If the eigenvalues of A are widely separated, CG convergence will be slower.
GMRES	Solves general sparse linear systems. If A is positive definite, then GMRES converges for any $m \geq 1$
MINRES	Solves symmetric indefinite sparse linear systems. It is difficult to find a preconditioner for a symmetric indefinite matrix, so if A is ill-conditioned GMRES is likely a better choice.

21.4.

i) $x^tAy = (x^tA)y = (Ax)^ty = y^t(Ax)$

$= x^t(Ay) = (Ay)^tx$

ii) $\frac{1}{2}(x-\bar{x})^tA(x-\bar{x}) - \frac{1}{2}\bar{x}^tA\bar{x}$

$= \frac{1}{2}x^tAx - \frac{1}{2}(\bar{x}^tAx + x^tA\bar{x})$

$= \frac{1}{2}(x^tAx) - \frac{1}{2}(x^t(A\bar{x}) + x^t(A\bar{x})) = \frac{1}{2}x^tAx - A^tb = \phi(x).$

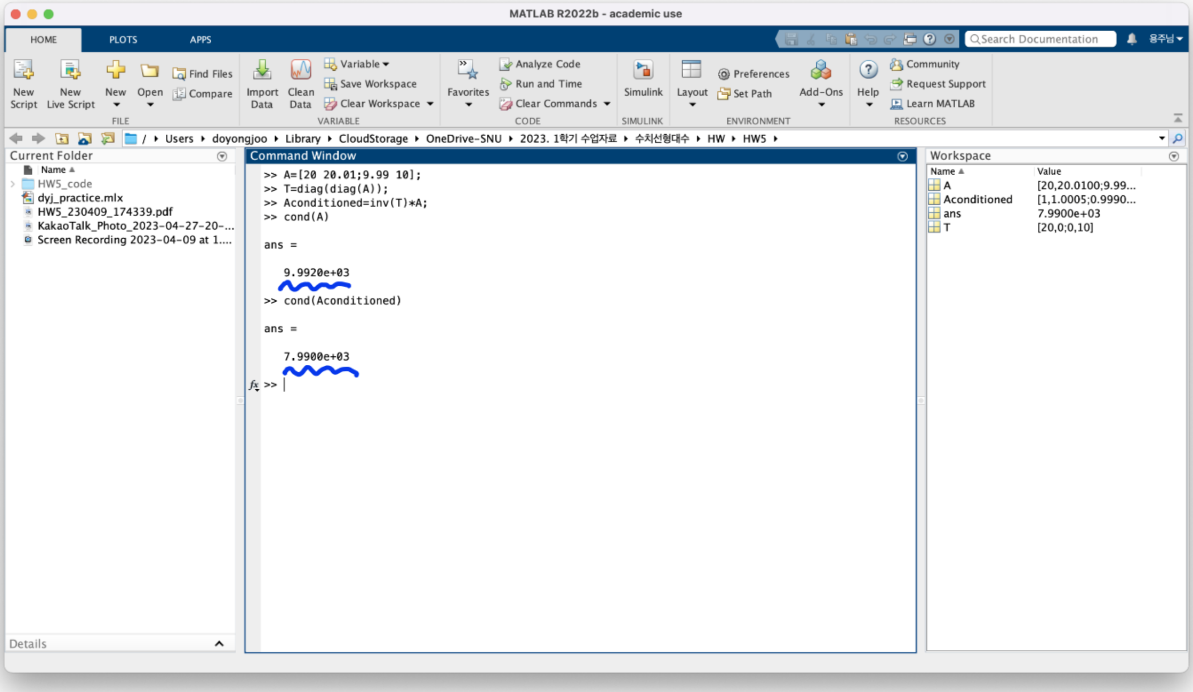
iii) If $A>0$, $\phi(x) = \frac{1}{2}(x-\bar{x})^tA(x-\bar{x}) - \frac{1}{2}\bar{x}^tA\bar{x} \geq -\frac{1}{2}\bar{x}^tA\bar{x}$

when $x = \bar{x}$.

21.9.

i) Divide row i of A and ith element of b by a_{ii}

ii)



iii) cond. num does not change.

21.10.

$$(D+U)^{-1}\cdot D\cdot (D+L)^{-1}\cdot Ax = (D+U)^{-1}\cdot D\cdot (D+L)^{-1}\cdot b$$

$$D\cdot (D+L)^{-1}\cdot Ax = D\cdot (D+L)^{-1}\cdot b$$

$$D\cdot (D+L)^{-1}\cdot A(D+U)^{-1}\cdot (D+U)x = D\cdot (D+L)^{-1}\cdot b$$

Let.

$$\begin{cases} \bar{A} = D(D+L)^{-1}A(D+U)^{-1} \\ \bar{b} = D(D+L)^{-1}b \\ \bar{x} = (D+U)x \end{cases}$$

For $\underbrace{(D+L)}\cdot D^{-1}\cdot D(D+L)^{-1} = I$

$\hookrightarrow = (I+LD^{-1}) \rightsquigarrow D(D+L)^{-1} = [I+LD^{-1}]^{-1}$

$$\bar{A} = (I + LD^{-1})^{-1} \cdot A \cdot (D+U)^{-1} = (I+LD^{-1})^{-1} \cdot (A/(D+U))$$

$$= (I + (L/D))^{-1} \cdot (A/(D+U))$$

$$= (I+(L/D)) \backslash (A/(D+U)).$$

$$\bar{b} = [I + LD^{-1}]^{-1} \cdot b = (I + L/D) \backslash b.$$

$\bar{A}\bar{x} = \bar{b} \rightarrow$ calculate by GMRES, then.

$$x = (D+U) \backslash \bar{x}$$

21.15.

$$[u_1 \cdots u_m]^t \cdot [w_1, \cdots w_m]$$

$$= \begin{bmatrix} u_1^t \\ \vdots \\ u_m^t \end{bmatrix} \cdot [w_1, \cdots w_m] = [u_i^t \cdot w_j]_{ij}$$

$$= [\delta_{ij}]_{ij} = I_m. \blacksquare$$

나머지 문제 → 전부 다 coding problem

* 21.23 이 2개가 존재하는데, python 상에서 mat 파일이 작동하지 않는 오류가 발생해서 컴파일은 하지 못한 코드입니다.

이에 차선책으로 python 코드로 짜서 첨부했습니다.