22.2

See $p_r(A) := \sum_{j=0}^{p} a_j A^j$.

$\rightsquigarrow$ For eigvec v_i of A, w.r.t to eigval λ_i, $p_r(A)v_i = \sum_{j=0}^{p} a_j A^j v_i$

$= \sum_{j=0}^{p} a_j \lambda_i^j v_i \quad (\because A^j v_i = \lambda_i A^{j-1} v_i = \cdots = \lambda_i^j v_i)$

$= p_r(\lambda_j) v_i$

$\therefore p_r(A) w = \sum_{i=1}^{n} c_i p_r(A) v_i = \sum_{i=1}^{n} c_i p_r(\lambda_i) v_i = \sum_{i=1}^{k} c_i p_r(\lambda_i) v_i + \sum_{i=k+1}^{n} c_i p(\lambda_i) v_i$

22.3.

$(A - \lambda_{u_2}^{(m)}) V_m^{(1)} = V_m^{(1)} (H_m^{(1)} - \lambda_{u_2}^{(m)} I) + h_{m+1,m} v_{m+1} (b_{m+1}^{(1)})^t$

$(A - \lambda_{u_2}^{(m)}) V_m^{(1)} Q_2 = [V_m^{(1)} (H_m^{(1)} - \lambda_{u_2}^{(m)} I) + h_{m+1,m} v_{m+1} (b_{m+1}^{(1)})^t] Q_2$

$H_m^{(1)} = Q_2 R_2 + \lambda_{u_2}^{(m)} I$

$\downarrow$

$(A - \lambda_{u_2}^{(m)}) V_m^{(1)} Q_2 = [V_m^{(1)} (Q_2 R_2 + \cancel{\lambda_{u_2}^{(m)} I} - \cancel{\lambda_{u_2}^{(m)} I}) + h_{m+1,m} v_{m+1} (b_{m+1}^{(1)})^t] Q_2$

$\therefore A V_m^{(1)} Q_2 = \lambda_{u_2}^{(m)} V_m^{(1)} Q_2 + [V_m^{(1)} (Q_2 R_2) + h_{m+1,m} v_{m+1} (b_{m+1}^{(1)})^t] Q_2$

setting $V_m^{(2)} = V_m^{(1)} Q_2$, $H_m^{(2)} = R_2 Q_2 + \lambda_{u_2}^{(m)} I$, $b_{m+1}^{(2)} = Q_2^t \cdot b_{m+1}^{(1)}$

$A V_m^{(2)} = V_m^{(2)} H_m^{(2)} + h_{m+1,m} v_{m+1} (b_{m+1}^{(2)})^t$ ∎

22.5.

i) $(A + E) v = r + \lambda v - r v^t v = \lambda v + r - r = \lambda v$.

$E = -r v^t \rightsquigarrow \|E\|_2 = ?$

$\rightsquigarrow E^t E = v r^t r v^t = \underline{(r^t r) \cdot v v^t}$

$\rightsquigarrow$ vv^t is mtx of rk 1 $\Rightarrow$ $I\epsilon$ has only one non-zero egval.

(If not, $\dim(Im(vv^t)) \geq 2$ (*))

$\Rightarrow$ For v is eternal egvec of vv^t $\rightsquigarrow$ $vv^t \cdot v = v \cdot 1$

$\therefore$ non-zero egval of vv^t is 1.

$\therefore$ non-zero egval of $r^t r \cdot vv^t$ is ε^2.

$\rightsquigarrow$ $\|E\|_2 = \sqrt{\varepsilon^2} = \varepsilon$

ii) If the norm of residual is small, $\|E\|_2$: small

$\therefore$ (λ, v) : exact eigenpair of the mtx $A+E$, close to A.

iii)

$$\begin{aligned} eig(A+E) &= \boxed{(\lambda, v)} \\ eig(A) &= \boxed{(\lambda_0, v_0)} \end{aligned}$$

$\rightarrow$ Approximated egval, egvec if $\|E\|_2$ are small

$\rightsquigarrow$ good approximate to backword error ■