A Projection Concentration Maximization Method for Event-based Imaging Velocimetry

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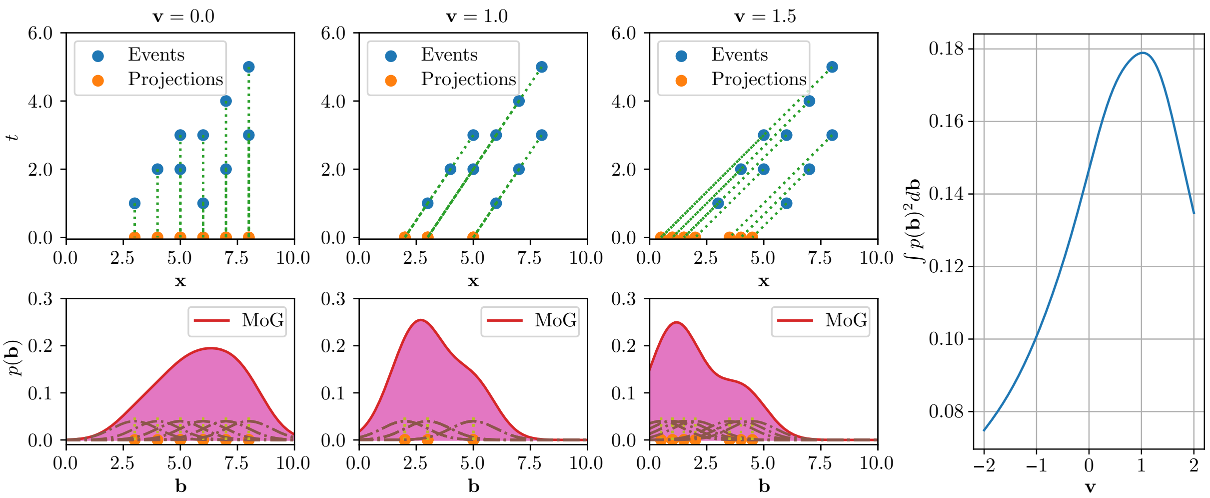
Abstract

Event cameras can capture changes in brightness with microsecond-level temporal precision, making them highly suitable for imaging velocimetry (Willert and Klinner, 2022). Given the unique characteristics of event data, such as its asynchronous nature, sparse representation, and high temporal resolution, accurately extracting velocity values from event data is a critical challenge (Willert and Klinner, 2022; Arja et al., 2023). This work proposes a Projection Concentration Maximization (PCM) method, in which the velocity estimation is formulated as a maximization problem for projection concentration. Specifically, each event is projected as a Gaussian function, resulting in a projection landscape that forms a mixture-of-Gaussians (MoG). And the landscape is encouraged to be concentrated, as measured by continuous Simpson index. Given the non-convex nature of the objective function, we employ a grid search method to determine an initial solution. Subsequently, a Newton-Raphson iterative algorithm is applied in the vicinity of the initial solution to estimate a better velocity vector. Our PCM objective directly links the latent velocity vector to the raw event data, bypassing intermediate representations such as synchronous frame representations (Gu et al., 2022) or images of warped events (Gallego et al., 2018). Additionally, the optimization strategy ensures faster convergence to a globally optimal solution. Our PCM method opens up a new door for velocity estimation with events.

1 Projection Concentration Maximization Method



(a) The MoG distribution of projections

(b) The objective curve

Figure 1: The motivation of our PCM method with a toy case.

Given the events $\{\mathbf{x}_i, t_i, p_i\}_{i=1}^{n}$ in a sampled spatiotemporal volume, we assume constant (linear) motion, $\mathbf{x}_i = t_i\mathbf{v} + \mathbf{b}_i$, i.e.,

$$\mathbf{b}_i = \mathbf{x}_i - t_i\mathbf{v} \tag{1}$$

where $\mathbf{x}_i$ denotes the spatial coordinates of the i^{th} event in the image plane, t_i represents the timestamp of the event, and $p_i \in \{1, -1\}$ indicates the polarity. The projections $\mathbf{b}_i$ are the initial positions at a reference time $t = 0$. The distribution $p(\mathbf{b})$ of these projections reflect the concentration, described as an image of warped events (IWE) (Willert and Klinner, 2022). Our insight is that the distribution can also be approximately parameterized via a mixture-of-Gaussians (Dasgupta, 1999), as demonstrated in Fig. 1.

$$p(\mathbf{b}) = \sum_{i=1}^{n} \frac{1}{\sqrt{(2\pi)^2|\mathbf{S}|}} e^{-\frac{1}{2}(\mathbf{b}-\mathbf{b}_i)^T \mathbf{S}^{-1}(\mathbf{b}-\mathbf{b}_i)} \tag{2}$$

where $\mathbf{S}$ is the covariance matrix. The MoG trick thus enables direct concentration computation without requiring intermediate image reconstruction, compared to contrast maximization methods (Gallego et al., 2018).

To measure the concentration of distribution, we adopt the continuous Simpson index, similar to the best intensity variance of IWE (Gallego et al., 2019),

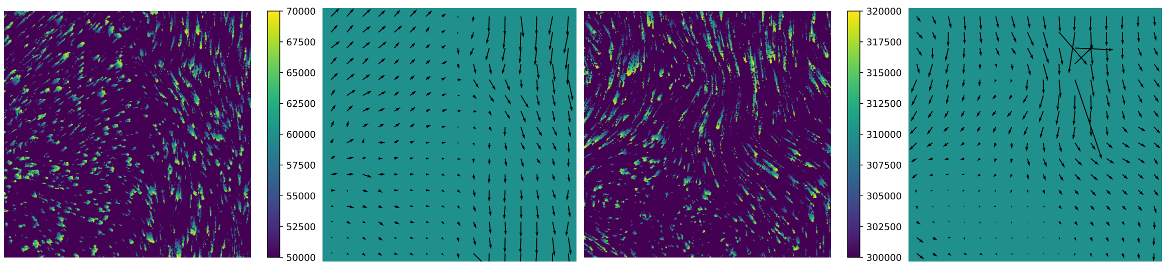
$$\begin{aligned} \mathcal{J}(\mathbf{v}) &= \int p^2(\mathbf{b}) d\mathbf{b} = \int p(\mathbf{b}) \times p(\mathbf{b}) d\mathbf{b} \\ &\approx \sum_{j=1}^{n} p(\mathbf{b}_j) = \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{1}{\sqrt{(2\pi)^2|\mathbf{S}|}} e^{-\frac{1}{2}(\mathbf{b}_j-\mathbf{b}_i)^T \mathbf{S}^{-1}(\mathbf{b}_j-\mathbf{b}_i)} \end{aligned} \tag{3}$$

where the objective $\mathcal{J}(\cdot)$ quantifies the quality of an estimation, as shown in Fig. 1. However, the objective $\mathcal{J}(\cdot)$ is inherently non-convex, making the optimization process more challenging. We employ a fast grid search method to determine an initial solution $\mathbf{v}_0$. Subsequently, a Newton-Raphson iterative algorithm is applied in the vicinity of the initial solution to estimate a better velocity vector.

$$\mathbf{v}_{k+1} = \mathbf{v}_k - \mathbf{H}^{-1}(\mathbf{v}_k)\mathbf{J}(\mathbf{v}_k) \tag{4}$$

where $\mathbf{J}$ and $\mathbf{H}$ represent the Jacobian vector and Hessian matrix of $\mathcal{J}(\cdot)$ evaluated at the previous solution $\mathbf{v}_k$, respectively.

2 Results and Discussion



(a) Case 1 (50ms-70ms)

(b) Case 2 (300ms-320ms)

Figure 2: Event data (Willert and Klinner, 2022) and velocimetry results of our PCM.

Fig. 2 presents two measurement snapshots, with the event data sourced from the official repository of EBIV (Willert and Klinner, 2022). Each vector is computed using a fixed sample size of $32 \times 32 pixels$ on a time-slab of $20ms$. Despite several outliers, the results overall align well with the reported findings (Willert and Klinner, 2022), demonstrating the feasibility of our PCM method. That says,

- The concentration maximization assumption, modeled as the objective in Eq. (3), is also well-suited for velocity estimation from event data. Note that the MoG trick plays a crucial role in simplifying the approximation of the concentration.

- Due to the explicit modeling, many efficient optimization algorithms can be applied. The Newton-Raphson optimization method, in particular, converges to the optimal solution quickly (5 iterations), and guarantees the accuracy of the velocity estimation.

Along with motion compensation and frame-based methods, our PCM paves a new way to compute the velocity field from events. Further performance comparisons and evaluations will be comprehensively investigated and detailed in the full conference paper.

Acknowledgements

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