Computer lab 4: Markov Chain Monte Carlo

Learning objectives

The main objective of this computer lab is to make the student familiar with major tools for Markov Chain Monte Carlo simulation in R.

After completing the lab the student shall be able to:

(i) Evaluate properties of the Metropolis-Hastings algorithm in R.

(ii) Implement and apply the Gibbs sampling techniques in R.

Recommended reading

Chapter 7 - 8 in Givens and Hoeting (2013)

Assignment 1: Different versions of the Metropolis-Hastings algorithm

a.) The function `normm` is simulating from a normal with zero mean and unit variance using a Metropolis algorithm with uniform proposal distribution. The exercise is to try the following alpha = 0.1,1 and 200 for 2000 iterations. Present trace plots of line type and histograms with interpretations. Tune the sampler by finding a value of alpha that gives an acceptance probability of 0.3. You have to modify the code in order to save the acceptance rate. Explain the code with comments.

```
normm<-function (Nsim, a)
{
        vec <- vector("numeric", Nsim)
        x <- 0
        vec[1] <- x
        for (i in 2:Nsim) {
                innov <- runif(1, -a, a)
                Xstar <- x + innov
                aprob <- min(1, dnorm(Xstar)/dnorm(x))
                u <- runif(1)
                if (u < aprob)
                        x <- Xstar
                vec[i] <- x
        }
        vec
```

```
}
```

b.) Function `gammh` is a Metropolis-Hastings independence sampling algorithm with normal proposal distribution with the same mean and variance as the desired gamma. Try a = 0.1, 2 and b = 0.01, 2. Present trace plots and histograms with interpretations. Explain the code with comments.

```
gammh<-function (Nsim, a, b)
{
        mu <- a/b
        sig <- sqrt(a/(b * b))
        vec <- vector("numeric", Nsim)
        x <- a/b
        vec[1] <- x
        for (i in 2:Nsim) {
             can <- rnorm(1, mu, sig)
             aprob <- min(1, (dgamma(can, a, b)/dgamma(x,
             a, b))/(dnorm(can, mu, sig)/dnorm(x,mu, sig)))
                u <- runif(1)
                if (u < aprob)
                        x <- can
                vec[i] <- x
        }
        vec
}
```

Assignment 2: The Gibbs sampling algorithm for the one-way random effects model

a). Implement R-code for the normal one-way random effects model with *k* levels. One way of parameterizing this model is:

$$Y_{ij} \sim N(\theta_i, \sigma^2), \quad i = 1, \ldots, k, \;\; j = 1, \ldots, n_i$$

$$\theta_i \sim N(\mu, \tau^2), \quad i = 1, \ldots, k$$

$$\mu \sim N(\mu_0, \sigma_u^2),$$

$$\sigma^2 \sim IG(a_1, b_1), \qquad \tau^2 \sim IG(a_2, b_2), \qquad \sigma_u^2 \sim IG(a_3, b_3)$$

The sampling should be performed from the full conditional distributions

$$\theta_i \sim N\left(\frac{\sigma^2}{\sigma^2 + n_i\tau^2}\mu + \frac{n_i\tau^2}{\sigma^2 + n_i\tau^2}\bar{Y}_i, \frac{\sigma^2\tau^2}{\sigma^2 + n_i\tau^2}\right), \quad i = 1, \ldots, k$$

$$\mu \sim N\left(\frac{\tau^2}{\tau^2 + k\sigma_u^2}\mu_0 + \frac{k\sigma_u^2}{\tau^2 + k\sigma_u^2}\bar{\theta}, \frac{\sigma_u^2\tau^2}{\tau^2 + k\sigma_u^2}\right)$$

$$\sigma^2 \sim IG\left(n/2 + a_1, (1/2)\sum_{ij}\left(Y_{ij} - \theta_i\right)^2 + b_1\right)$$

$$\tau^2 \sim IG\left(k/2 + a_2, (1/2)\sum_{ij}(\theta_i - \mu)^2 + b_2\right)$$

$$\sigma_u^2 \sim IG(1/2 + a_3, (1/2)(\mu - \mu_0)^2 + b_3)$$

where a_i and b_i can be set to 0.001 and $\mu_0 = 0$. Compare your code with the results obtained with the Gibbs sampler implemented in the `MCMChregress()` function in the `MCMCpack` library. Provide results in Tables with mean, standard deviations and 95% credible intervals for the mean, the random effects and all variances for both your own code and the `MCMChregress()` analysis.

The data comes from the WinBUGS example Dyes where Yield is the response variable and random effect Batch has 6 levels:

Batch	Yield (in grams)				
1	1545	1440	1440	1520	1580
2	1540	1555	1490	1560	1495
3	1595	1550	1605	1510	1560
4	1445	1440	1595	1465	1545
5	1595	1630	1515	1635	1625
6	1520	1455	1450	1480	1445

To hand in

A written report (preferably a Word or .pdf document) where you summarize your main findings in the assignments. Submit your report via Lisam before the deadline.