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Fixed-time Consensus Tracking of Multi-agent Systems under a Directed Communication Topology*

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Abstract— **This paper considers the fixed-time consensus tracking problem for a class of first-order multi-agent systems with directed topology, where the agents may be subject to unknown disturbances. First, some consensus tracking protocols for the nominal multi-agent systems are designed to make the states of followers track those of the leader within fixed time. Then, a new class of consensus tracking protocols with augmented term is further designed to ensure fixed-time robust consensus tracking of multi-agent systems in the presence of disturbance. Finally, some simulations are performed to verify the theoretical results proposed in this paper.**

Index Terms— **Multi-agent system, fixed-time consensus, directed network, sliding mode control.**

I. INTRODUCTION

In recent years, the topic of cooperative control for multi-agent systems has gained much attention due to its wide applications in the real world, such as flocking of networked systems, formation control of autonomous underwater vehicles, robotic teams [1], [2], to name only a few. Particularly, distributed cooperative control here means designing appropriate control protocols for achieving typical cooperative behaviors by merely using local information among neighboring agents [3].

Being one of the most essential research topics in cooperative control of multi-agent systems, consensus problem has received an increasing attention recently. For an arbitrarily given initial condition, the consensus problem aims to design the communication protocols based on the relative local information such that the states of all agents could achieve an agreement [2], [4], [5], [6]. Obviously, convergence rate is an critical performance indicator to evaluate the effectiveness of the consensus protocols. In [2], it was pointed out that a faster asymptotic convergence rate can be yielded by enlarging the algebraic connectivity of an undirected network topology for a first-order multi-agent system. Furthermore, it was shown in [6] that the general algebraic connectivity for a strongly connected topology also plays a key role in achieving state consensus. Note that most of the aforementioned results on consensus of multi-agent systems may only guarantee an asymptotic convergence rate. Although these results are remarkable in theory, they may not be qualified for practical applications. Motivated by this observation, finite-time control protocols for various multi-agent systems have been designed and utilized [7], [8], [9], [10], [11], [12]. Finite-time consensus for first-order multi-agent system was addressed in [7], [8]. Then, in [9], [10], [11], [12], finite-time leaderless consensus and consensus tracking problems for second-order multi-agent systems were respectively studied. However, the guaranteed convergence time derived from most existing finite-time consensus protocols depends on the initial states of the agents, which is unfavorable in practical applications when the initial conditions are unavailable.

Motivated by the new concept of fixed-time stability in [13], fixed-time consensus has been recently addressed in [14], [15], [16] where the convergence time of consensus is independent of the initial conditions. In particular, robust fixed-time consensus for first-order multi-agent systems with external disturbances has been addressed in [16]. It can be observed that the underlying communication topology of the multi-agent systems considered in [16] is assumed to be undirected, where the symmetry property is essential for reaching consensus as well as dealing with the disturbances. Thus, the results provided in [16] is inapplicable to multi-agent systems with directed topologies. Furthermore, the robust control protocol proposed in [16] is discontinuous, which may cause the chattering problem. In view of the demands in solving fixed-time consensus of multi-agent systems with directed topologies and avoiding the chattering behaviors, it is interesting yet important to design some continuous fixed-time consensus protocols based on local information. Motivated by the above discussions and by using tools from sliding mode control theory, two new kinds of continuous fixed-time consensus tracking protocols for multi-agent systems with directed topologies are proposed in this paper, where the dynamics can be in the presence of disturbances or nonlinear terms.

In this paper, a fully distributed consensus protocol is proposed to solve the fixed-time consensus problem for first-order multi-agent systems with directed topologies. Note that the underlying topologies of most practical multi-agent systems are directed as the sensors embedded in agents may have limited yet heterogeneous communication ranges. Furthermore, a two-stage design idea borrowed from sliding mode control (SMC) is introduced to design robust control to ensure the achievement of consensus in multi-agent system with disturbances.

The rest of this paper is organized as follows. Some preliminaries are given in Section II. In Section III, fixed-

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time consensus for nominal multi-agent system with directed topologies is addressed. Then, the robust fixed-time consensus for multi-agent system in the presence of disturbances is studied in Section IV. In Section V, numerical examples are given to verify the effectiveness of the theoretical analysis. Finally, Section VI concludes this paper.

II. PRELIMINARIES

In this section, some basic concepts and preliminary results about graph theory are introduced. briefly in the following.

A directed network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ involves a set of nodes which is defined by $\mathcal{V} = \{v_1, v_2, ..., v_N\}$, a set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and a weighted adjacency matrix $G = (G_{ij})_{N\times N}$. Here, a directed edge $\mathcal{E}_{ij}$ in $\mathcal{G}$ means an ordered pair of nodes (v_i, v_j) representing that the node v_i can receive information from the node v_j. G records the coupling configuration information of all the directed edges in $\mathcal{G}$. According to the definition of adjacency matrices [11], weights $G_{ij} > 0$ is positive if and only if there is a directed edge (v_i, v_j) in $\mathcal{G}$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is denoted by:

$$l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}; l_{ij} = -G_{ij}, \ \ i \neq j.$$

Definition 1: [6] A network $\mathcal{G}$ is *directed* if there is an edge from node v_j to v_i in $\mathcal{G}$, then $G_{ij} > 0$; otherwise, $G_{ij} = 0$ $(i \neq j;\ i, j = 1, 2, ..., N)$.

Definition 2: [6] A *directed path* from node v_j to v_i is a sequence of directed edges (v_i,v_{i_1}), (v_{i_1},v_{i_2}), ..., (v_{i_l},v_j) in the directed network with distinct nodes v_{i_k}, $k = 1, 2, ..., l$. A directed network $\mathcal{G}$ is *strongly connected* if between any pair of distinct nodes v_i and v_j in $\mathcal{G}$, there exists a directed path from v_i to v_j, $i, j = 1, 2, ..., N$. Furthermore, a directed network is called a *directed tree* if the underlying network is a tree when the direction of the network is ignored. A *directed rooted tree* is a directed network with at least one root r having the property that, for each node v different from r, there is a unique directed path from r to v. A *directed spanning tree* of a network $\mathcal{G}$ is a directed rooted tree, which contains all the nodes and some edges in $\mathcal{G}$.

Lemma 1: [23] A network $\mathcal{G}$ is strongly connected if and only if its corresponding adjacency matrix G is irreducible.

Lemma 2: [24] The Laplacian matrix L has a simple eigenvalue zero, and all the other eigenvalues have positive real parts if and only if the directed network has a directed spanning tree.

Lemma 3: [6] Suppose that L is irreducible. Then, $L1_N = 0$, and there is a positive vector $\xi = (\xi_1, \xi_2, ..., \xi_N)^T$ such that $\xi^T L = 0$. Denote $\Xi = diag(\xi_1, \xi_2, ..., \xi_N)$. Then, $\Xi > 0$ and $\Xi^{-1} > 0$.

The dynamics of the leader has the following form:

$$\dot{x}_0 = h_0(x_0, t), \tag{1}$$

and the followers are described by

$$\dot{x}_i = u_i + h_i(x_i, t), \quad i = 1, ..., N, \tag{2}$$

where x_0 and x_i represent the positions of leader and the ith agent, and $u_i \in R$ is the control input. Furthermore, $h_i(\cdot, \cdot),\ i = 0, 1, ..., N$ represents disturbance in this paper. Denote $x = [x_1, x_2, ..., x_N]^T$. In the following, L corresponds to the subgraph describing the communication topology only among followers. Define a nonnegative diagonal matrix $B = diag(b_1, b_2, ..., b_N)$, where $b_i > 0$ means that x_0 is accessible by the ith agent, and $b_i = 0$ otherwise.

Lemma 4: [15] Let $x_1, x_2, ..., x_N \geq 0$, $0 < p \leq 1$ and $q > 1$. Then,

$$\sum_{i=1}^{N} x_i^p \geq (\sum_{i=1}^{N} x_i)^p, \ \ \sum_{i=1}^{N} x_i^q \geq N^{1-q}(\sum_{i=1}^{N} x_i)^q. \tag{3}$$

Lemma 5: [25] Consider the following system of differential equation:

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0, \tag{4}$$

where $x = [x_1, x_2, ..., x_N]^T \in R^N, f(x) : R^N \to R^N$ is continuous on R^N, and $f(0) = 0$. Suppose there exits a continuous positive definite function $V(x) : R^N \to R$ such that there exist real numbers $a > 0$, $b > 0$, $p > 1$ and $q \in (0, 1)$ such that

$$\dot{V}(x) + a(V(x))^p + b(V(x))^q \leq 0, \quad x \in R^N \backslash \{0\}.$$

Then, the origin is a globally fixed-time stable equilibrium, i.e., any solution of (4) satisfies $x(t, x_0) = 0,\ \forall t \geq T$, where T satisfies

$$T \leq \frac{1}{a(p-1)} + \frac{1}{b(1-q)}.$$

Definition 3: With given protocols $u_i,\ i = 1, ..., N$, the leader-follower system (1) and (2) is said to achieve *fixed-time consensus tracking*, if for any initial value $x_0(0)$ and $x(0)$, there exists a positive value T which is related to the initial value and a fixed constant $T_{\max} > 0$ such that $T < T_{\max}$ and

$$\begin{cases} \lim_{t\to T} |x_i(t) - x_0(t)| = 0, \\ \quad x_i(t) = x_0(t), \ \forall t \geq T. \end{cases}$$

III. TRACKING CONSENSUS WITHOUT DISTURBANCE

In this part, the tracking consensus problem under the directed networks is considered. Firstly, the nominal dynamics is investigated:

$$h_i(x_i, t) = 0, \ i = 0, 1, ..., N. \tag{5}$$

For simplicity, denote L_B as $L+B$, $\lambda_{\min}(A)$ as the minimum eigenvalue of a symmetric matrix A, respectively. Furthermore, define

$$s^{[k]} = \text{sign}(s)|s|^k, \tag{6}$$

where $s \in R$, $\text{sign}(s)$ is the sign function and $k > 0$ is a constant. Now, consider the control input:

$$\begin{aligned} u_i = &-\alpha[\sum_{j=1}^{N} a_{ij}(x_i - x_j) + b_i(x_i - x_0)]^{[\mu]} \\ &-\alpha[\sum_{j=1}^{N} a_{ij}(x_i - x_j) + b_i(x_i - x_0)]^{[\nu]}, \end{aligned} \tag{7}$$

where $\mu > 1 > \nu > 0$ and α is a positive parameter. Set $e_i = x_i - x_0$. Substitute (5) and (7) into (1) and (2), and then obtain the following error system:

$$\begin{aligned}\dot{e}_i = &-\alpha[\sum_{j=1}^{N} a_{ij}(e_i - e_j) + b_i e_i]^{[\mu]} \\ &- \alpha[\sum_{j=1}^{N} a_{ij}(e_i - e_j) + b_i e_i]^{[\nu]}.\end{aligned} \tag{8}$$

One can also write the error system into a more compact form as follows:

$$\dot{e} = -\alpha(L_B e)^{[\mu]} - \alpha(L_B e)^{[\nu]}. \tag{9}$$

Multiply both sides by L_B and denote $y = [y_1, ..., y_N]^T = L_B e$, then the above equation becomes

$$\dot{y} = -\alpha L_B(y^{[\mu]} + y^{[\nu]}). \tag{10}$$

To derive the main results, the following assumption is required.

Assumption 1: Suppose that the underlying topology of leader and followers contains a directed spanning tree and the subgraph describing the communication topology among followers is strongly connected.

Motivated by [17] and [18], one can obtain the following Lemma:

Lemma 6: Suppose that Assumption 1 holds. Then L_B is invertible and $\Xi L_B + L_B^T \Xi$ is positive definite.

Theorem 1: Suppose that Assumption 1 holds. Then, fixed-time consensus tracking problem in multi-agent systems (1) and (2) with dynamics (5) can be solved by the protocol (7) for any given $\alpha > 0$.

Proof. Consider the following Lyapunov function:

$$V = \frac{1}{\mu+1}\sum_{i=1}^{N} \xi_i |y_i|^{\mu+1} + \frac{1}{\nu+1}\sum_{i=1}^{N} \xi_i |y_i|^{\nu+1}. \tag{11}$$

Take the derivative of V along the trajectory (10), it follows that:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N} \xi_i |y_i|^{\mu} \mathrm{sign}(y_i)\dot{y}_i + \sum_{i=1}^{N} \xi_i |y_i|^{\nu} \mathrm{sign}(y_i)\dot{y}_i \\ &= (y^{[\mu]})^T \Xi \dot{y} + (y^{[\nu]})^T \Xi \dot{y} \\ &= -\alpha(y^{[\mu]} + y^{[\nu]})^T \Xi L_B (y^{[\mu]} + y^{[\nu]}) \\ &= -\frac{1}{2}\alpha(y^{[\mu]} + y^{[\nu]})^T(\Xi L_B + L_B^T \Xi)(y^{[\mu]} + y^{[\nu]}) \\ &\le -\frac{1}{2}\alpha\bar{\lambda}(y^{[\mu]} + y^{[\nu]})^T(y^{[\mu]} + y^{[\nu]}) \\ &= -\frac{1}{2}\alpha\bar{\lambda}\sum_{i=1}^{N}(|y_i|^{2\mu} + 2|y_i|^{\mu+\nu} + |y_i|^{2\nu}),\end{aligned} \tag{12}$$

where $\bar{\lambda} = \lambda_{\min}(\Xi L_B + L_B^T \Xi) > 0$. On the other hand, one has:

$$V \le \frac{\xi_{\max}}{\nu+1}(\sum_{i=1}^{N}|y_i|^{\mu+1} + \sum_{i=1}^{N}|y_i|^{\nu+1}). \tag{13}$$

Consider $\bar{V} = \sum_{i=1}^{N}|y_i|^{\mu+1} + \sum_{i=1}^{N}|y_i|^{\nu+1}$, and according to Lemma 4, with $\frac{\mu+\nu}{\mu+1} < 1$, it follows that:

$$\begin{aligned}&(\sum_{i=1}^{N}|y_i|^{\mu+1} + \sum_{i=1}^{N}|y_i|^{\nu+1})^{\frac{\mu+\nu}{\mu+1}} \\ \le\ &\sum_{i=1}^{N}|y_i|^{\mu+\nu} + \sum_{i=1}^{N}|y_i|^{\frac{\nu+1}{\mu+1}(\mu+\nu)}.\end{aligned}$$

What's more, it's easy to check that $\frac{\nu+1}{\mu+1}(\mu+\nu) - 2\mu < 0$ and $\frac{\nu+1}{\mu+1}(\mu+\nu) - 2\nu > 0$, which leads to:

$$|y_i|^{\frac{\nu+1}{\mu+1}(\mu+\nu)} \le |y_i|^{2\mu} + |y_i|^{2\nu}. \tag{14}$$

Combining the above inequalities, one gets that:

$$\begin{aligned}(\bar{V})^{\frac{\mu+\nu}{\mu+1}} &\le \sum_{i=1}^{N}|y_i|^{\mu+\nu} + \sum_{i=1}^{N}|y_i|^{2\mu} + \sum_{i=1}^{N}|y_i|^{2\nu} \\ &\le \sum_{i=1}^{N}(|y_i|^{2\mu} + 2|y_i|^{\mu+\nu} + |y_i|^{2\nu}).\end{aligned} \tag{15}$$

Similarly, notice that $\frac{2\mu}{\mu+1} > 1$, one can estimate $\bar{V}$ in another way according to Lemma 4:

$$\begin{aligned}&(2N)^{1-\frac{2\mu}{\mu+1}}(\sum_{i=1}^{N}|y_i|^{\mu+1} + \sum_{i=1}^{N}|y_i|^{\nu+1})^{\frac{2\mu}{\mu+1}} \\ &\qquad \le \sum_{i=1}^{N}|y_i|^{2\mu} + \sum_{i=1}^{N}|y_i|^{2\mu\frac{\nu+1}{\mu+1}}.\end{aligned}$$

Then, one has:

$$|y_i|^{2\mu\frac{\nu+1}{\mu+1}} \le |y_i|^{2\nu} + |y_i|^{\mu+\nu}, \tag{16}$$

because of $2\mu\frac{\nu+1}{\mu+1} - 2\nu > 0$ and $2\mu\frac{\nu+1}{\mu+1} - (\mu+\nu) < 0$. Then, it follows that:

$$\begin{aligned}&(2N)^{1-\frac{2\mu}{\mu+1}}(\bar{V})^{\frac{2\mu}{\mu+1}} \\ &\le \sum_{i=1}^{N}|y_i|^{2\mu} + \sum_{i=1}^{N}|y_i|^{2\nu} + \sum_{i=1}^{N}|y_i|^{\mu+\nu} \\ &\le \sum_{i=1}^{N}(|y_i|^{2\mu} + 2|y_i|^{\mu+\nu} + |y_i|^{2\nu}).\end{aligned} \tag{17}$$

Combining (12) (13) (15) (17), it follows:

$$\begin{aligned}\dot{V} &\le -\frac{1}{4}\alpha\bar{\lambda}[(2N)^{1-\frac{2\mu}{\mu+1}}(\bar{V})^{\frac{2\mu}{\mu+1}} + (\bar{V})^{\frac{\mu+\nu}{\mu+1}}] \\ &= -L_1 V^{\frac{2\mu}{\mu+1}} - L_2 V^{\frac{\mu+\nu}{\mu+1}},\end{aligned} \tag{18}$$

where $L_1 = \frac{N}{2}\alpha\bar{\lambda}(\frac{\nu+1}{2N\xi_{\max}})^{\frac{2\mu}{\mu+1}}$ and $L_2 = \frac{1}{4}\alpha\bar{\lambda}(\frac{\nu+1}{\xi_{\max}})^{\frac{\mu+\nu}{\mu+1}}$. Using Lemma 5, y reaches to 0 in fixed time T_0, which satisfies:

$$T_0 \le \frac{\mu+1}{L_1(\mu-1)} + \frac{\mu+1}{L_2(1-\nu)}. \tag{19}$$

Notice that $e = L_B^{-1}y$. Thus e also reaches to 0 in fixed time T_0, which means that the designed control (6) solves the fixed-time tracking consensus problem. □

Remark 1: In Theorem 1, fixed-time consensus for multi-agent systems with directed topologies is finally solved, which utilizes a transformation and some inequality approaches. In the literature, the analysis for finite-time consensus in multi-agent systems with directed topologies is still a challenging problem, not even to say about fixed-time consensus. Consider (19), it is easy to see that the upper bound of the convergence time is no longer related to initial states but merely to the communication topology, the group volume and the designed parameters. Therefore, the convergence time can be estimated or even designed in advance whatever initial states are, which is more practical in reality than asymptotic or finite-time tracking consensus.

IV. TRACKING CONSENSUS WITH DISTURBANCES

In this part, we consider the leader-follower consensus problem with disturbance. The following situation is considered here:

$$h_0(x_0,t)=0,\ h_i(x_i,t)=d_i(t),\ i=1,...,N, \tag{20}$$

where d_i is the uncertain disturbance for agent i, $i = 1,2,\dots,N$.

Assumption 2: The disturbance is assumed to satisfy the following condition:

$$|d_i(t)| \le \beta_i, \tag{21}$$

where β_i is a known constant, $i=1,2,\dots,N$.
Introduce an auxiliary variable z_i, $i=1,...,N$, which is generated by an integrator:

$$\begin{aligned}\dot{z}_i =& \alpha[\sum_{j=1}^{N} a_{ij}(x_i-x_j)+b_i(x_i-x_0)]^{[\mu]}\\ &+\alpha[\sum_{j=1}^{N} a_{ij}(x_i-x_j)+b_i(x_i-x_0)]^{[\nu]}.\end{aligned} \tag{22}$$

Define the sliding mode variable $s_i=z_i+x_i$ and the control input is designed as follows:

$$u_i=u_{eqi}+u_{di}, \tag{23}$$

where u_{eqi} plays a part on sliding mode surface $s_i=0$, while u_{di} deals with the disturbance in the reaching phase. Design the following disturbance rejection control protocol:

$$\begin{aligned}u_{eqi} =& -\alpha[\sum_{j=1}^{N} a_{ij}(x_i-x_j)+b_i(x_i-x_0)]^{[\mu]}\\ &-\alpha[\sum_{j=1}^{N} a_{ij}(x_i-x_j)+b_i(x_i-x_0)]^{[\nu]},\end{aligned} \tag{24}$$

$$u_{di}=-(a+K_i)s_i^{[\alpha_1]}-(b+K_i)s_i^{[\alpha_2]}, \tag{25}$$

where a, b are any positive constants, $K_i \ge \beta_i$, $i=1,2,\dots,N$, $0<\alpha_1<1$ and $\alpha_2>1$. Based on the proposed protocol, one can reject the disturbance and drive the system with disturbance into the nominal one.

Theorem 2: Suppose that Assumption 1 and 2 hold. Then, fixed-time consensus tracking problem in multi-agent systems (1) and (2) with dynamics (20) can be solved by the protocol (23) for any given $\alpha>0$.

Proof. Take the derivative of s_i along the trajectory of z_i, one gets that

$$\begin{aligned}\dot{s}_i &= \dot{x}_i+\dot{z}_i\\ &= u_{eqi}+u_{di}+d_i\\ &\quad+\alpha[\sum_{j=1}^{N} a_{ij}(x_i-x_j)+b_i(x_i-x_0)]^{[\mu]}\\ &\quad+\alpha[\sum_{j=1}^{N} a_{ij}(x_i-x_j)+b_i(x_i-x_0)]^{[\nu]}\\ &= -(a+K_i)s_i^{[\alpha_1]}-(b+K_i)s_i^{[\alpha_2]}+d_i.\end{aligned} \tag{26}$$

Consider the following Lyapunov function $V=s^Ts$ where $s=[s_1,s_2,\dots,s_N]$. Take the derivative of V, one has

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N} s_i\dot{s}_i\\ &= -\sum_{j=1}^{N}[(a+K_i)|s_i|^{\alpha_1+1}+(b+K_i)|s_i|^{\alpha_2+1}]\\ &\quad+\sum_{i=1}^{N} d_is_i\\ &\le -\sum_{i=1}^{N}(a|s_i|^{\alpha_1+1}+b|s_i|^{\alpha_2+1})\\ &\quad-\sum_{i=1}^{N}K_i(|s_i|^{\alpha_1+1}+|s_i|^{\alpha_2+1})+\sum_{i=1}^{N}\beta_i|s_i|\\ &\le -\sum_{i=1}^{N}(a|s_i|^{\alpha_1+1}+b|s_i|^{\alpha_2+1})\\ &\le -aV^{\frac{\alpha_1+1}{2}}-bV^{\frac{\alpha_2+1}{2}},\end{aligned} \tag{27}$$

which follows that the sliding surface $s_i=0$ is reached in fixed-time. After that, the system states stay on the surface, and then, the fixed-time consensus tracking is fulfilled according to Theorem 1. □

Remark 2: In this section, sliding mode control idea is applied where there are two-step process. The first step is to drive the states of the agents with external disturbances into the sliding mode surface where $s_i=0$ as in Theorem 2. Then, the second step solves the fixed-time consensus for multi-agent systems with directed topologies according to Theorem 1.

V. SIMULATION EXAMPLES

In this section, some numerical simulations are provided to verify the effectiveness of the theoretical analysis.

Consider a multi-agent system consisting of 5 followers and 1 leader. The communication topology is shown in Fig. 1, where the communication weights are indicated on the edges. The adjacency matrix of the subgraph describing

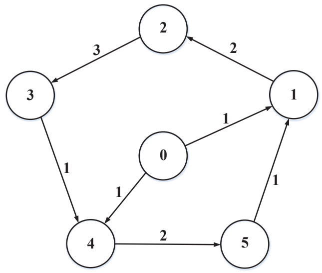

Fig. 1: Directed network with 5 followers and 1 leader.

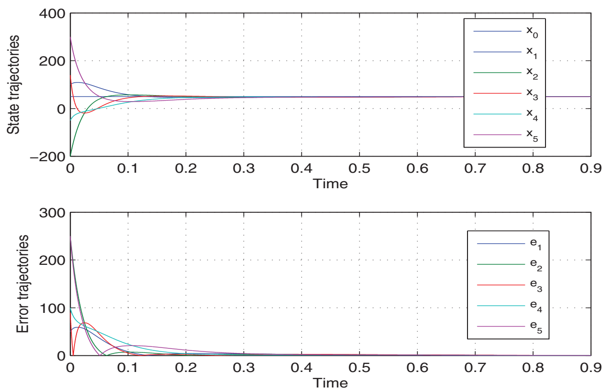

Fig. 2: State trajectories of leader and followers and error trajectories of followers without disturbance.

the communication topology among the followers is $A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}$, and the accessible matrix is $B = diag(1, 0, 0, 1, 0)$.

A. Example 1

Consider the nominal system (1) and (2) with dynamics (5), the consensus protocol takes the form in (7). The designed parameters are chosen as $\alpha = 3$, $\mu = 1.3$, $\nu = 0.4$, and a_{ij} is defined in adjacency matrix A. By simple calculation, the theoretical upper bound given by Theorem 1 is no more than 19.19 s. Consider the following initial condition: $x_0(0) = 100$ for the leader, and $[x_1(0),\ x_2(0),\ x_3(0),\ x_4(0),\ x_5(0)] = [250, -200, 140, -50, 300]$ for the followers. The numerical result described in Figs. 2 shows that the convergence time of the proposed control law (7) under different initial situations is about 0.7 s. In the figure, the errors are defined as $e_i = |x_i - x_0|$, $i = 1, 2, 3, 4, 5$. This result coincides with the upper bound given in Theorem 1.

B. Example 2

Fixed time robust consensus is considered in this example. It is assumed that the leader has the following dynamics:

$$\dot{x}_0(t) = 0. \tag{28}$$

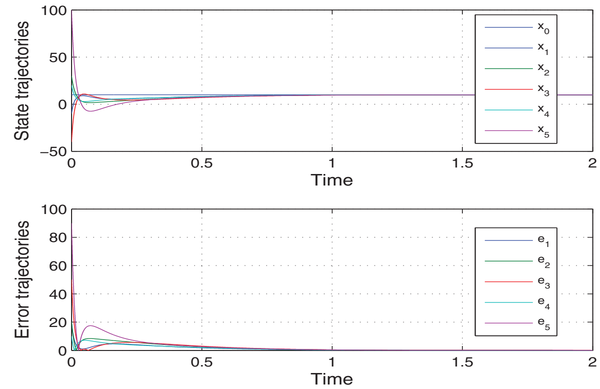

Fig. 3: State trajectories of leader and followers and error trajectories of followers with disturbance.

The dynamics of followers are assumed to be subject to disturbances, given as

$$\dot{x}_i = u_i + 7\sin(2t), i = 1, 2, 3, 4, 5. \tag{29}$$

The parameters β_i in Assumption 2 are given as $\beta_i = 7$, $i = 1, 2, 3, 4, 5$ here. The consensus protocol takes the form of (22), (23) and (24). The designed parameters are chosen as $\alpha = 3$, $\mu = 1.3$, $\nu = 0.4$, $a = 1.2$, $b = 1.3$, $\alpha_1 = 0.8$, $\alpha_2 = 1.4$ and $K_i = \beta_i = 7$, $i = 1, 2, 3, 4, 5$. By simple calculation, the theoretical upper bound for consensus tracking given by Theorem 2 is 28.7 s. Also consider the initial condition: $x_0(0) = 10$ for the leader, and $[x_1(0),\ x_2(0),\ x_3(0),\ x_4(0),\ x_5(0)] = [-10, 30, -40, 20, 100]$ for the followers. The simulation result is shown in Fig. 3. The convergence time for consensus tracking under the given initial condition is about 1.5 s, which verifies the theoretical analysis given in Theorem 2.

VI. CONCLUSIONS

In this paper, a fully distributed nonsmooth protocol has been proposed for achieving fixed-time consensus tracking in a class of first-order multi-agent systems, where the communication topology is directed and agent dynamics are subject to unknown disturbances. A consensus tracking protocol for the nominal multi-agent system has been first designed to make the states of the followers track those of the leader in fixed time. Based on which, a class of consensus tracking protocols with augmented term has been further designed to ensure fixed-time robust consensus tracking for multi-agent systems.

In the future, fully distributed control design for second-order dynamics with undirected or directed topologies will be considered.

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