

Accurate fitting of measured reflectances using a Shifted Gamma micro-facet distribution - Supplemental

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This document is supplemental to the paper titled *Accurate fitting of measured reflectances using a Shifted Gamma micro-facet distribution*. In the following section we list the equations for the Cook-Torrance micro-facets model with the SGD probability distribution function. Next, we explain how to use importance sampling with the SGD distribution.

This supplemental material contains complete fitting results for 100 isotropic measured materials from the MERL database, together with sample images, and comparisons with both the ground truth and Cook-Torrance with Beckmann distribution, as well as BRDF plots in the incidence plane in three separate red, green, and blue channels, and Fresnel plots¹.

¹The Beckmann images and it's corresponding BRDF lobes are not available for some of the materials.

The reflectance function

The Cook-Torrance micro-facets reflectance model is:

$$\begin{aligned}\rho(\mathbf{i}, \mathbf{o}) &= \rho_{\text{diffuse}}(\mathbf{i}, \mathbf{o}) + \rho_{\text{specular}}(\mathbf{i}, \mathbf{o}) \\ &= \frac{\rho_d}{\pi} + \frac{\rho_s}{\pi} \frac{F(\mathbf{i} \cdot \mathbf{h}) D(\theta_h) G(\mathbf{i}, \mathbf{o})}{\cos(\theta_i) \cos(\theta_o)}\end{aligned}$$

where \mathbf{i} is the incoming direction, \mathbf{o} is the outgoing direction, \mathbf{h} is the half-vector ($\mathbf{h} = \mathbf{i} + \mathbf{o}$, normalized). θ_i , θ_o and θ_h are the respective angles between these vectors and the normal \mathbf{n} . Also ρ_d and ρ_s are the diffuse and specular coefficients in RGB, respectively.

The SGD normal distribution function is:

$$\begin{aligned}D(\theta_m) &= \frac{\chi_{[0, \frac{\pi}{2}]}(\theta_m)}{\pi \cos^4 \theta_m} P_{22}(\tan^2 \theta_m) \\ P_{22}(x) &= K_{\alpha, p} \frac{e^{-(\alpha + \frac{x}{\alpha})}}{\left(\alpha + \frac{x}{\alpha}\right)^p} \\ K_{\alpha, p} &= \frac{1}{\alpha \Gamma(1-p, \alpha)}\end{aligned}$$

Using this distribution, the shadowing and masking function G is:

$$\begin{aligned}G(\mathbf{i}, \mathbf{o}) &= G_1(\mathbf{i}) G_1(\mathbf{o}) \\ G_1(\theta) &= \frac{\chi_{[0, \frac{\pi}{2}]}(\theta)}{1 + \Lambda(\theta)} \\ \Lambda(\theta) &= \int_{\frac{1}{\tan \theta}}^{\infty} (r \tan \theta - 1) P_2(r) dr \\ P_2(r) &= \frac{1}{\sqrt{\pi \alpha} \Gamma(1-p, \alpha)} U\left(p, p + \frac{1}{2}, \alpha + \frac{r^2}{\alpha}\right) e^{-\alpha - \frac{r^2}{\alpha}}\end{aligned}$$

where U is the confluent hyper-geometric function of the second kind.

Alternatively, the following approximation can be used:

$$\begin{aligned}G_1(\theta) &\approx \begin{cases} 1 + \lambda (1 - e^{c(\theta - \theta_0)^k}) & \text{if } \theta > \theta_0 \\ 1 & \text{if } \theta \leq \theta_0 \end{cases} \\ \theta_0 &= \frac{\pi}{2} - \left(\frac{\log(1 + \frac{1}{\lambda})}{c} \right)^{\frac{1}{k}}\end{aligned}$$

The Fresnel term is:

$$F(\theta_d) = F_0 + (1 - F_0)(1 - \cos \theta_d)^5 - F_1 \cos \theta_d$$

Importance sampling

For importance sampling according to the GGX distribution, use the following equation to sample $D_{GGX}(\mathbf{m}) \cos \theta_m$:

$$\mathbf{m} = (\cos \phi_m \sin \theta_m, \sin \phi_m \sin \theta_m, \cos \theta_m)$$

where,

$$\begin{aligned}\theta_m &= \arctan\left(\frac{\alpha \sqrt{u_2}}{\sqrt{1-u_2}}\right) \\ \phi_m &= 2\pi u_1\end{aligned}$$

where u_1 and u_2 are realizations of a uniform random variable in $[0, 1]$.

Given the incoming direction, \mathbf{i} , the outgoing direction, \mathbf{o} , can be found by:

$$\mathbf{o} = 2|\mathbf{i} \cdot \mathbf{m}| \mathbf{m} - \mathbf{i}$$

Since the diffuse and specular components of the BRDF require different importance sampling weights, we sample the specular component with probability $p_{specular}$:

$$p_{specular} = \frac{\rho_s}{\rho_d + \rho_s}$$

and sample the diffuse component with probability $1 - p_{specular}$.

The importance sampling weight must be multiplied by the inverse of this probability depending on whether the specular or diffuse component is selected for each individual sample.

Therefore, the importance sampling weight for the specular component is:

$$w_{specular} = \frac{4\pi}{p_{specular}} \rho_{specular}(\mathbf{i}, \mathbf{o}) \cos \theta_i \cos \theta_o \cos^3 \theta_m \frac{(\alpha^2 + \tan^2 \theta)^2}{\alpha^2}$$

where,

$$\rho_{specular}(\mathbf{i}, \mathbf{o}) = \frac{\rho_s}{\pi} \frac{F(\mathbf{i} \cdot \mathbf{h}) D(\theta_h) G(\mathbf{i}, \mathbf{o})}{\cos \theta_i \cos \theta_o}$$

It simplifies to:

$$w_{specular} = \frac{4\rho_s}{p_{specular}} F(\mathbf{i} \cdot \mathbf{h}) D(\theta_h) G(\mathbf{i}, \mathbf{o}) \cos^3 \theta_m \frac{(\alpha^2 + \tan^2 \theta)^2}{\alpha^2}$$

, and the importance sampling weight for the diffuse component is:

$$w_{diffuse} = \frac{\rho_d}{1 - p_{specular}}$$

The integrated illumination will be:

$$I(\theta_o) = \frac{1}{N} \sum_{k=0}^N w L(i_k)$$

Material Name: *alum-bronze*

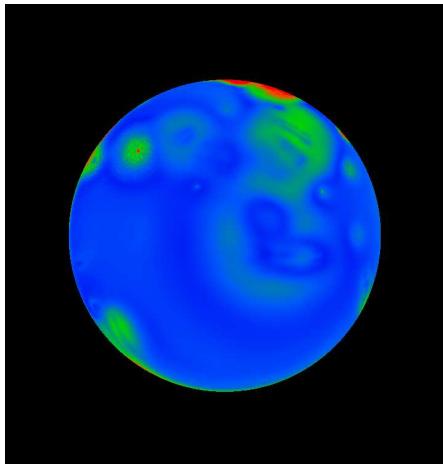
Rendered Images



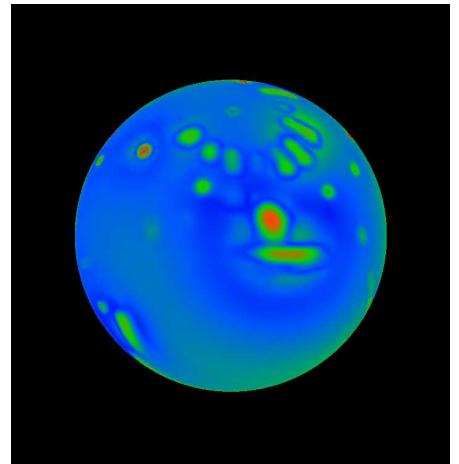
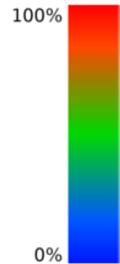
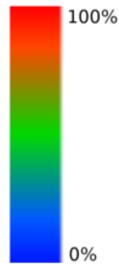
SGD

Reference

Beckmann



SGD (Lab difference)



Beckmann (Lab difference)

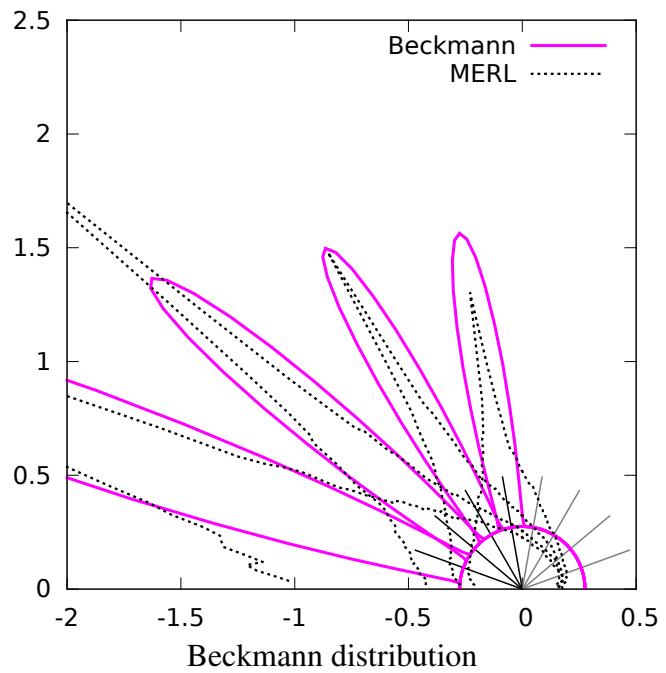
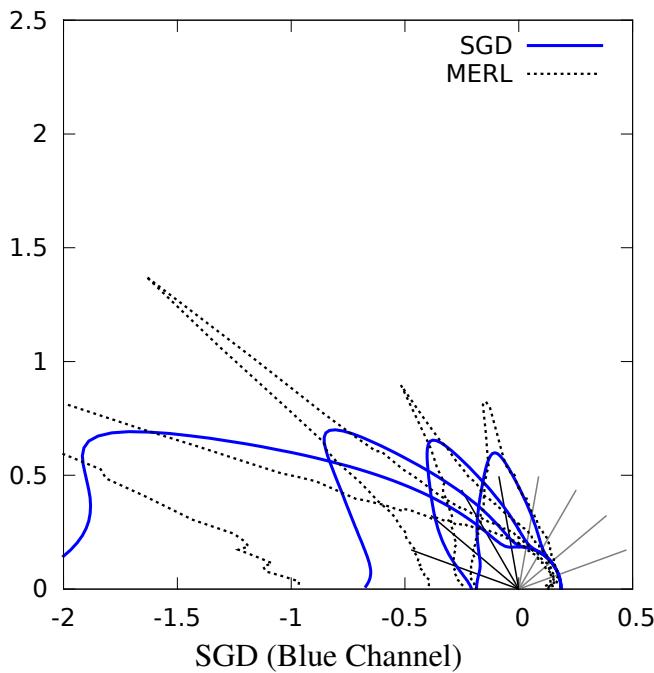
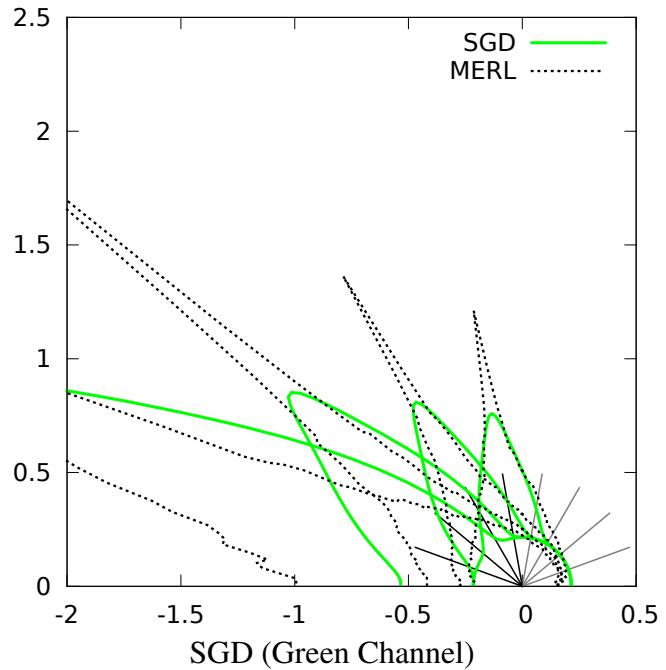
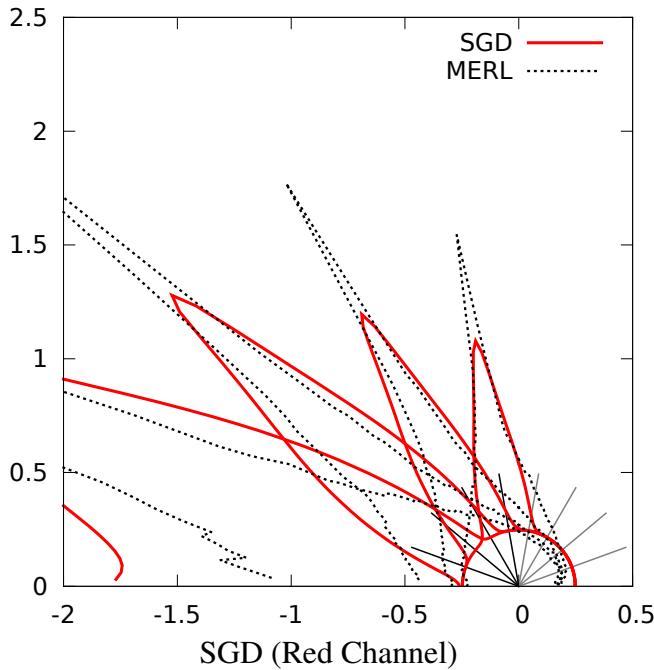
SGD fitted parameters and absolute fitting error

Channel	ρ_d	ρ_s	α	p	F_0	F_1	$K_{\alpha,p}$	error
Red	0.0478786	0.0364976	0.014832	0.459076	6.05524	5.05524	46.3841	0.100735
Green	0.0313514	0.664975	0.0300126	0.450056	0.235756	0.182842	24.5961	0.0878706
Blue	0.0200638	0.268836	0.0490339	0.529272	0.580647	0.476088	14.8261	0.0861907

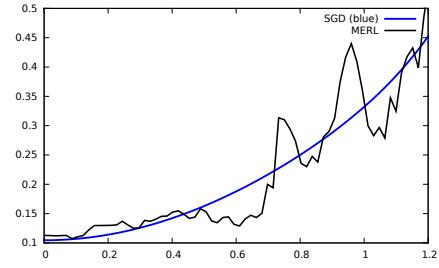
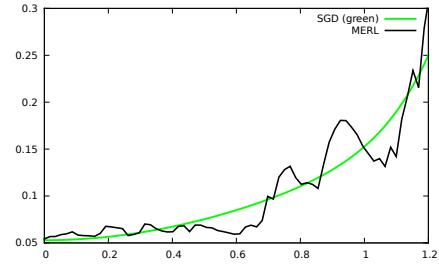
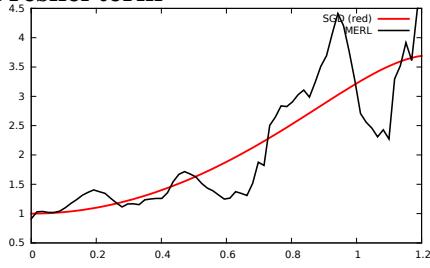
Shadowing / Masking fitted parameters

Channel	λ	c	k	θ_0
Red	2.60672	1.12717e-07	47.783	0.205635
Green	2.97371	1.06401e-07	36.2767	0.066289
Blue	2.7827	5.27952e-08	31.6066	-0.0661091

BRDF lobes in the incidence plane (Cubic root applied)



Fresnel term



Material Name: *blue-acrylic*

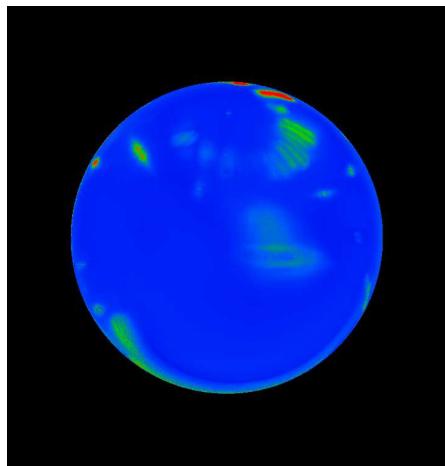
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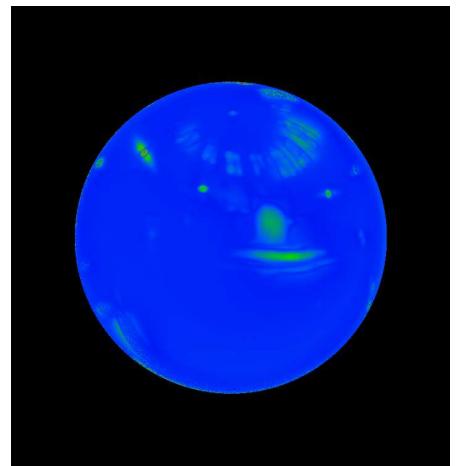
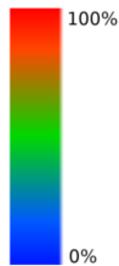
SGD

Reference

Beckmann



SGD (Lab difference)



Beckmann (Lab difference)



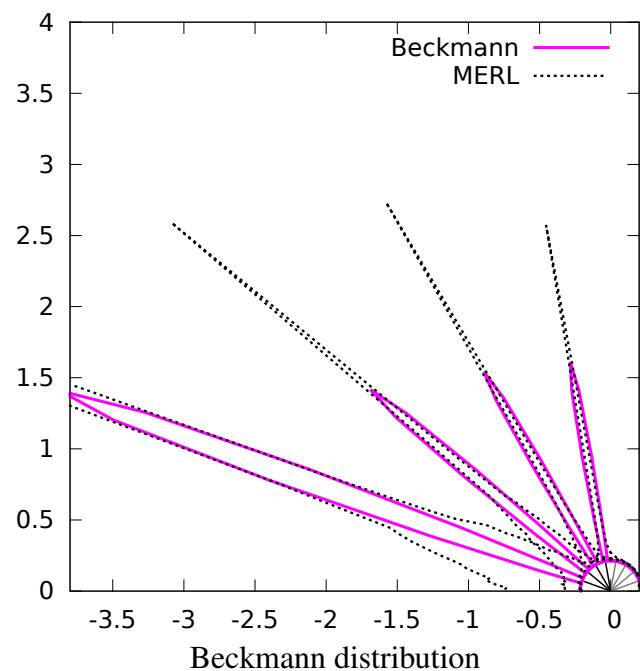
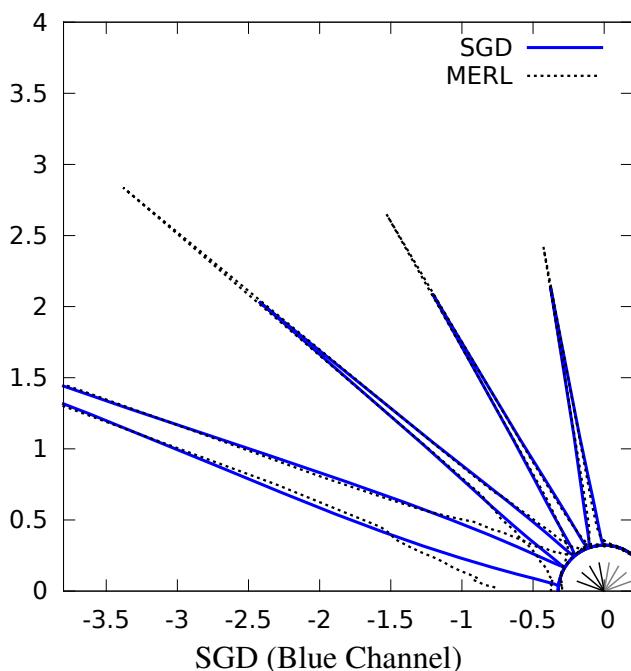
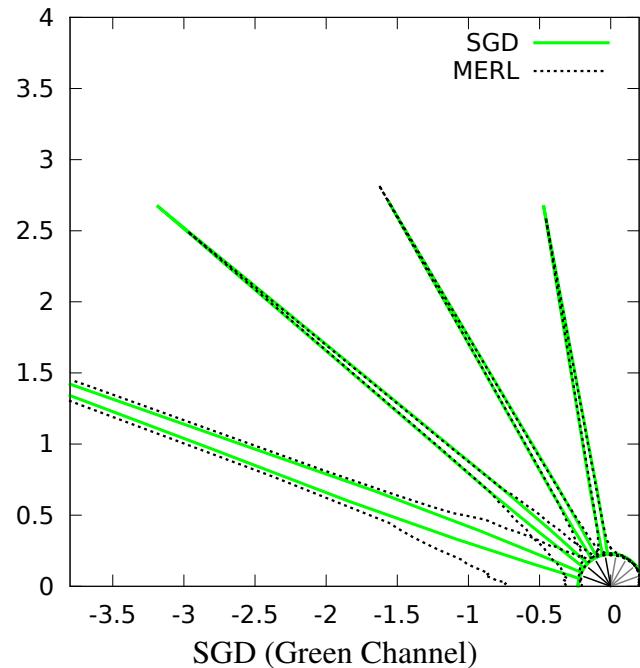
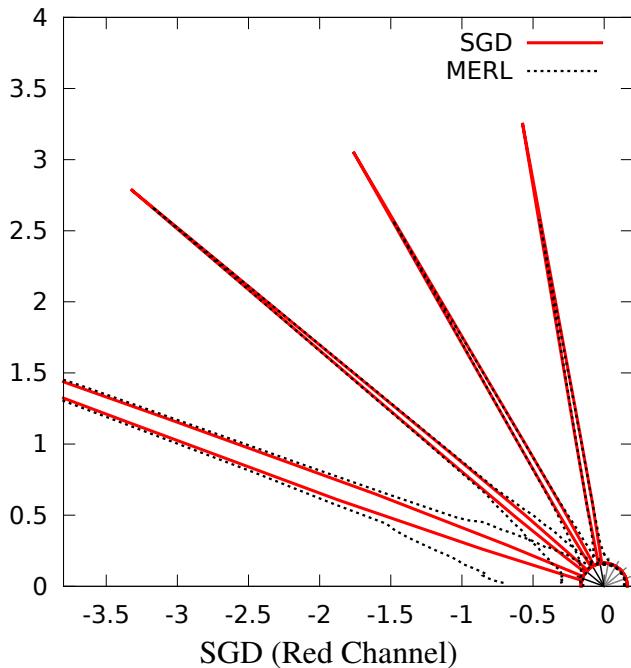
SGD fitted parameters and absolute fitting error

Channel	ρ_d	ρ_s	α	p	F_0	F_1	$K_{\alpha,p}$	error
Red	0.0134885	0.0864901	0.000174482	0.373948	0.0185562	-0.0209713	4021.24	0.0612235
Green	0.0373766	0.0228191	0.000269795	0.362425	0.399982	0.241543	2646.36	0.0789826
Blue	0.10539	0.204042	0.0015211	0.563636	0.0525861	0.0169474	346.898	0.0461093

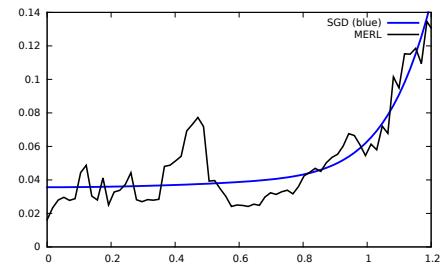
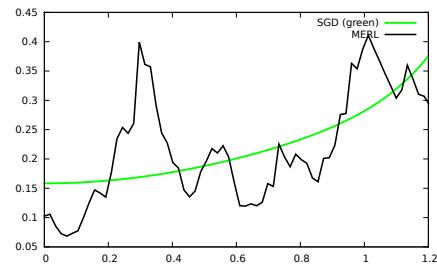
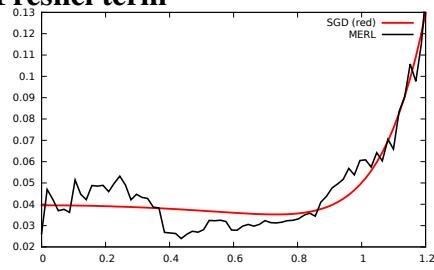
Shadowing / Masking fitted parameters

Channel	λ	c	k	θ_0
Red	3.38722	9.64334e-08	338.073	0.526039
Green	3.62885	9.96105e-08	272.828	0.515404
Blue	1.83684e-06	3.61787e-07	23.5039	-0.526935

BRDF lobes in the incidence plane (Cubic root applied)

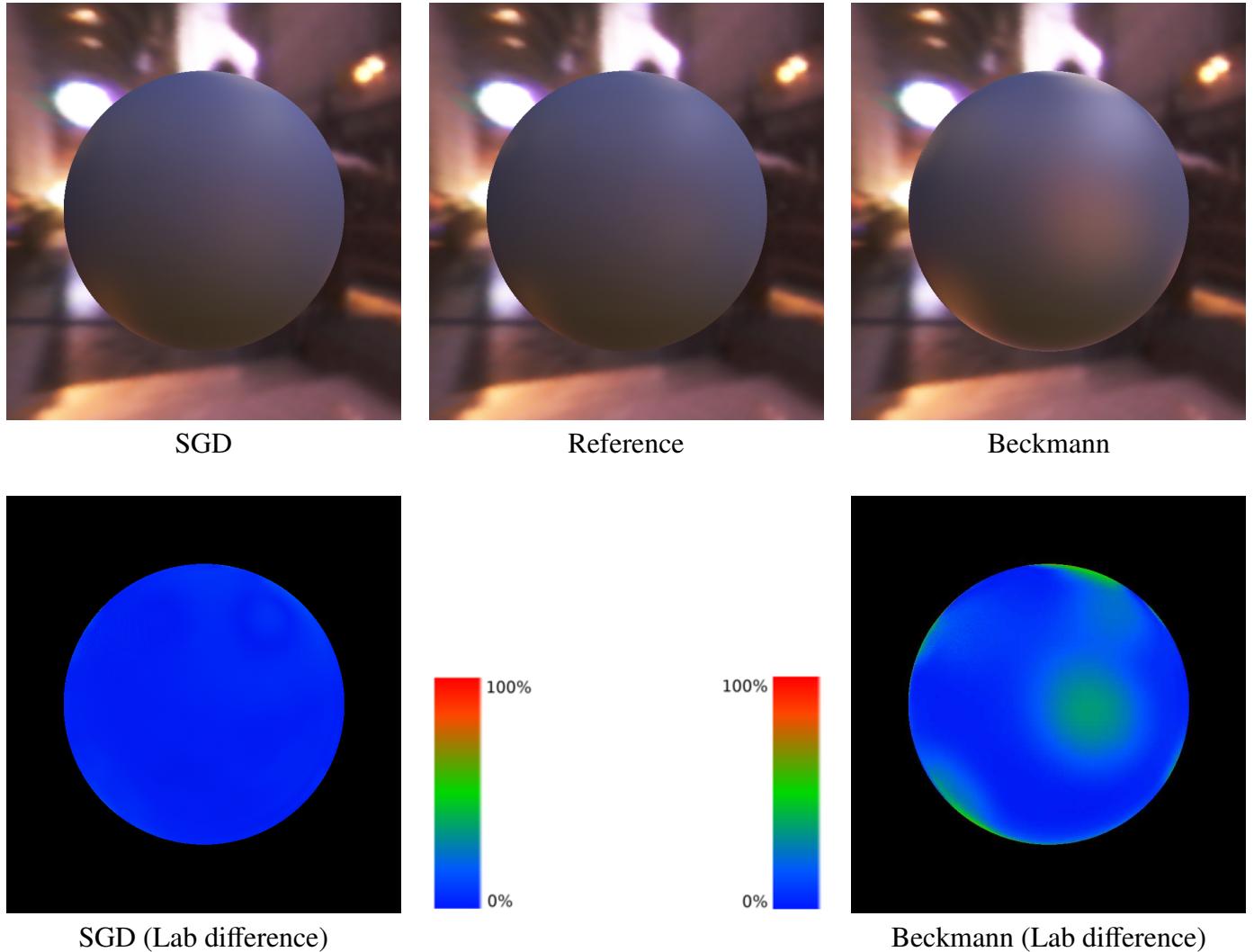


Fresnel term



Material Name: blue-rubber

Rendered Images



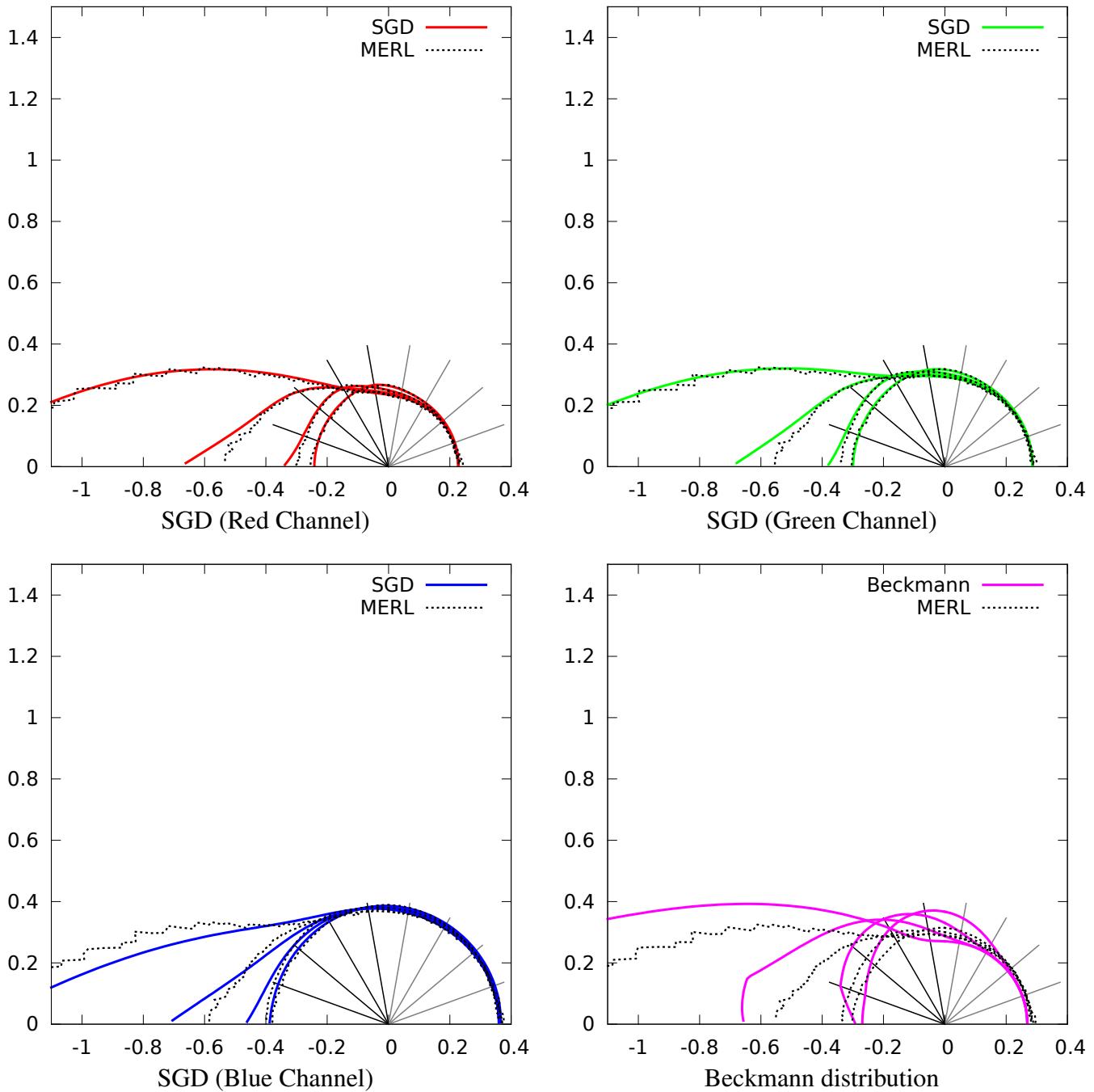
SGD fitted parameters and absolute fitting error

Channel	ρ_d	ρ_s	α	p	F_0	F_1	$K_{\alpha,p}$	error
Red	0.0371302	0.384232	0.218197	0.815054	0.0631713	0.0478254	4.41586	0.000630461
Green	0.0732915	0.412357	0.2668	1.00146	0.0622636	0.0422186	3.76795	0.000733287
Blue	0.146637	0.612608	0.478375	1.24995	0.0399196	0.007015	3.46276	0.00171092

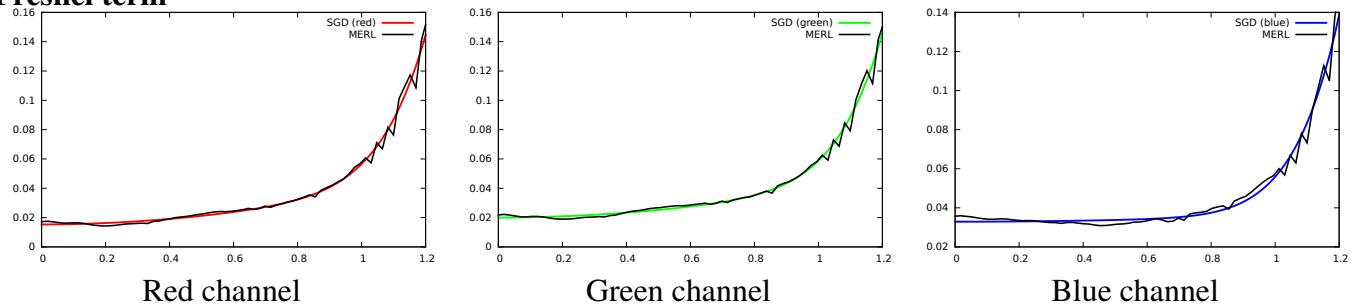
Shadowing / Masking fitted parameters

Channel	λ	c	k	θ_0
Red	3.77807	1.2941e-07	19.8046	-0.499472
Green	3.82679	1.07194e-07	19.3115	-0.557706
Blue	3.33186	0.00045665	11.4364	-0.172177

BRDF lobes in the incidence plane (Cubic root applied)

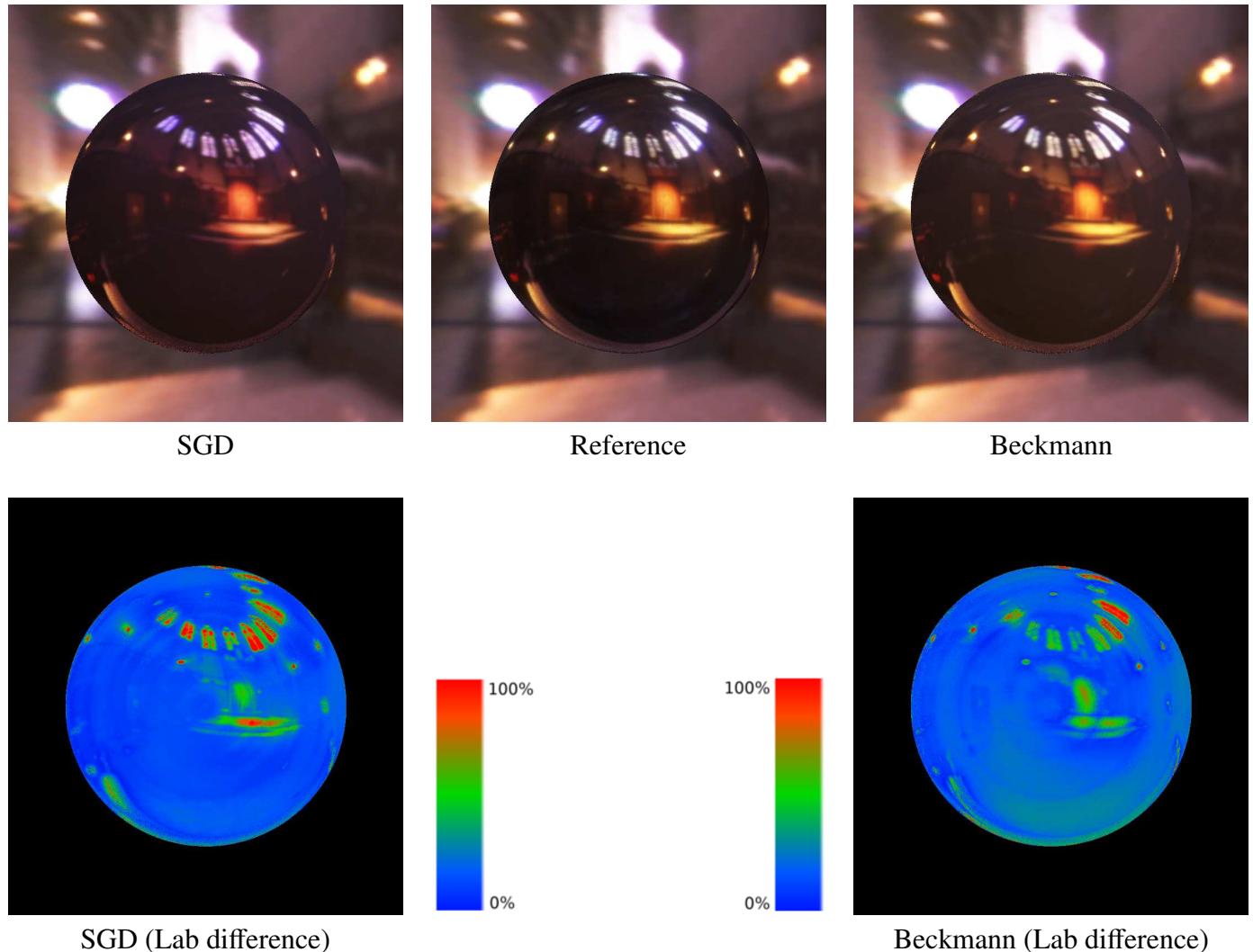


Fresnel term



Material Name: *chrome*

Rendered Images



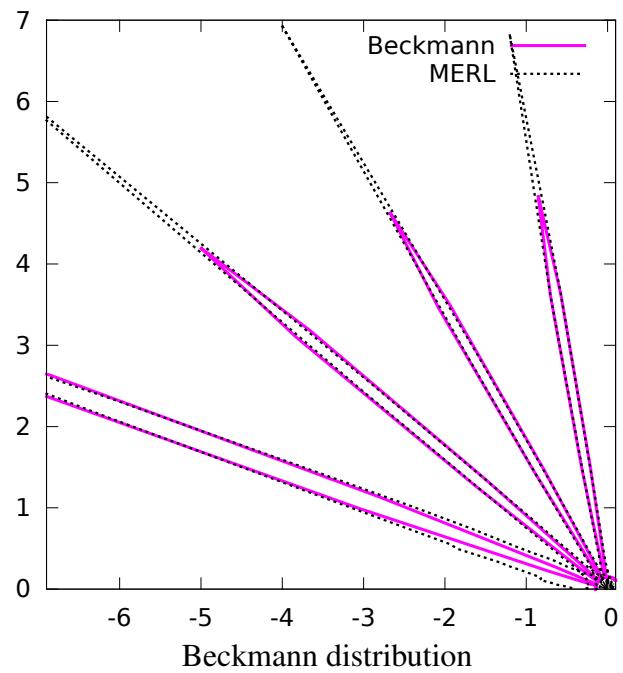
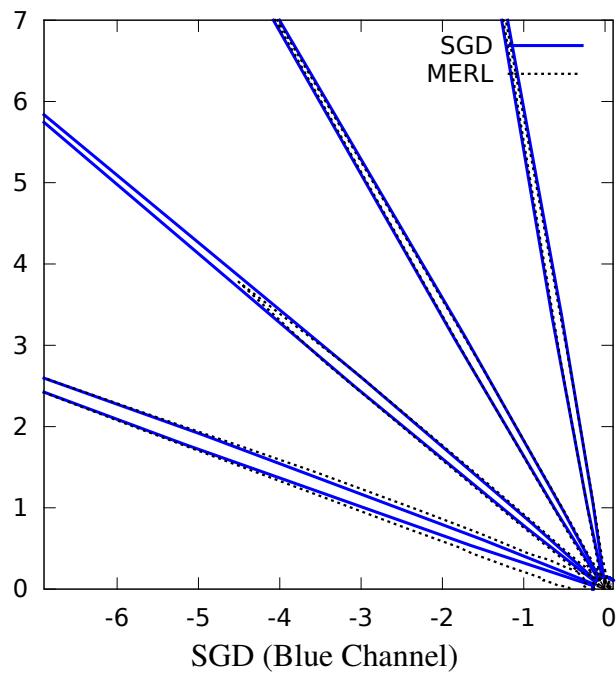
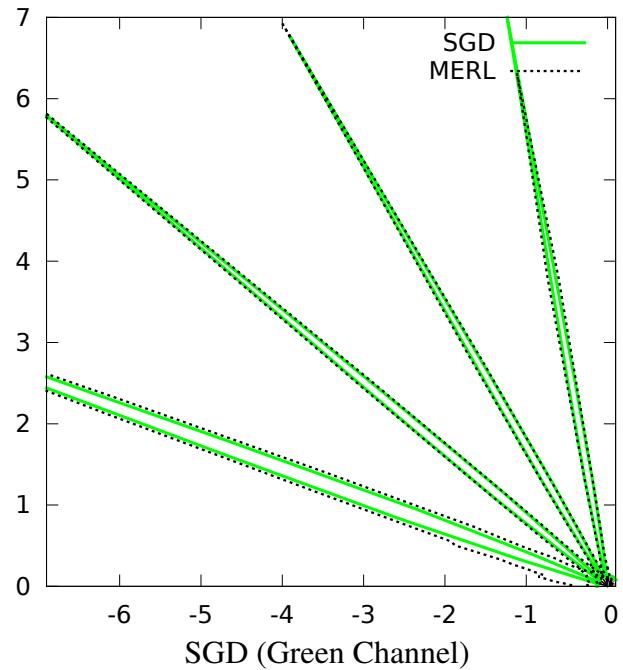
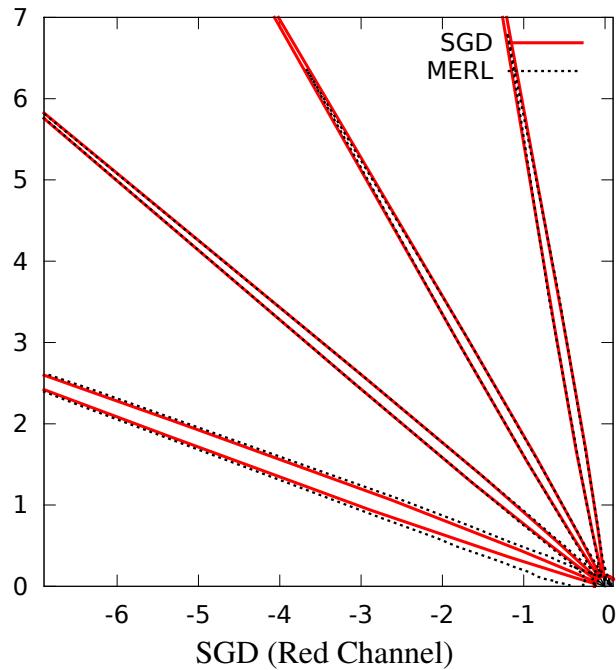
SGD fitted parameters and absolute fitting error

Channel	ρ_d	ρ_s	α	p	F_0	F_1	$K_{\alpha,p}$	error
Red	0.00697189	0.0930656	0.000155335	0.353854	0.256314	-0.743686	4642.1	0.802138
Green	0.00655268	0.041946	0.000156872	0.3327	0.819565	-0.180435	4726.66	1.29121
Blue	0.0101854	0.104558	7.39851e-05	0.300437	3.22085e-13	-1	10421.9	1.06148

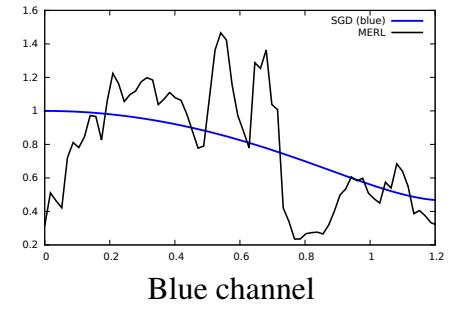
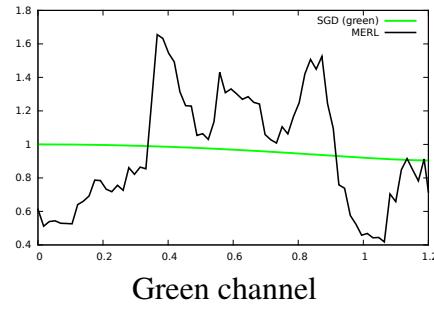
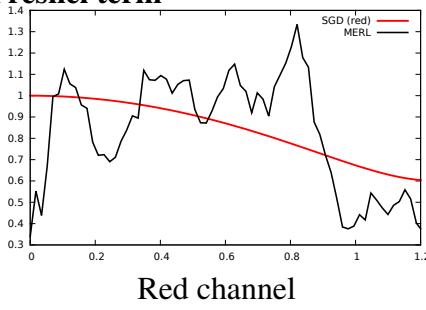
Shadowing / Masking fitted parameters

Channel	λ	c	k	θ_0
Red	3.8545	5.30781e-08	354.965	0.526796
Green	4.44817	1.04045e-07	349.356	0.528469
Blue	5.40959	1.79834e+66	272.736	1.00293

BRDF lobes in the incidence plane (Cubic root applied)



Fresnel term



Material Name: *gold-metallic-paint2*

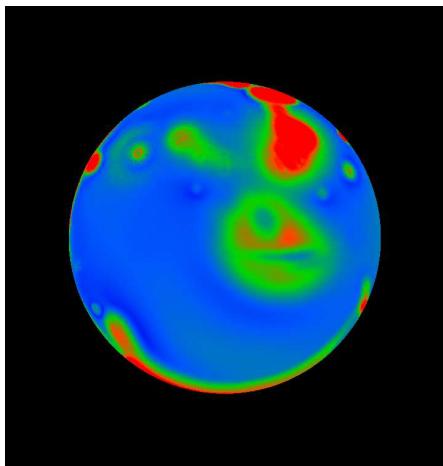
Rendered Images



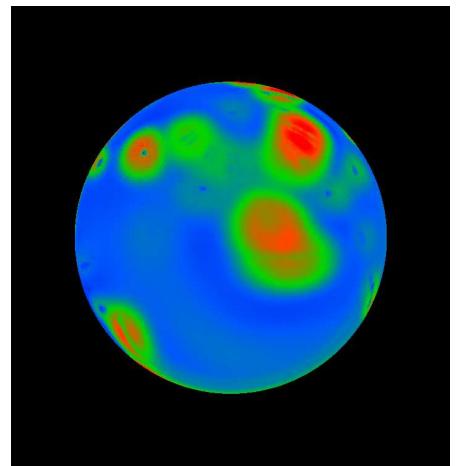
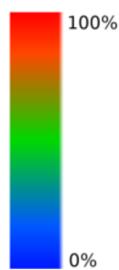
SGD

Reference

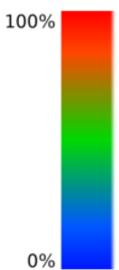
Beckmann



SGD (Lab difference)



Beckmann (Lab difference)



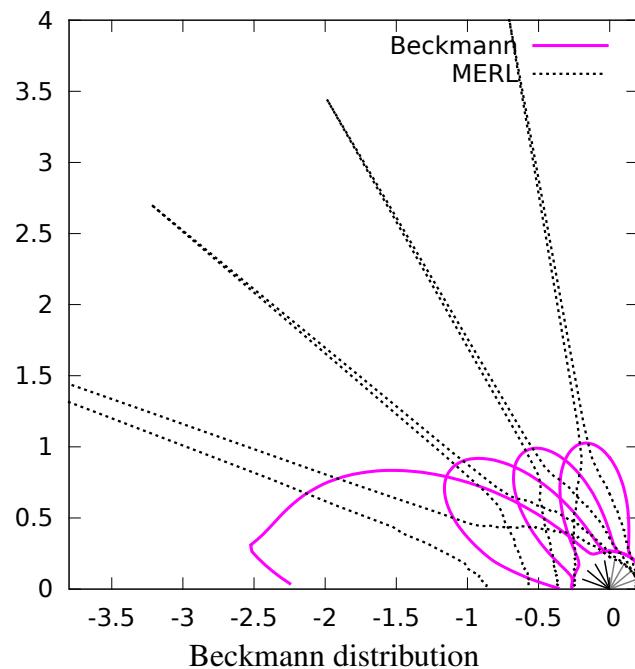
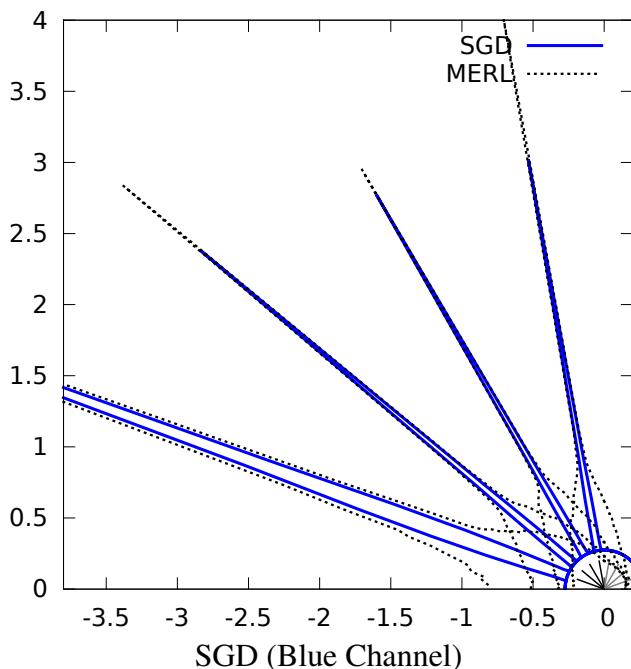
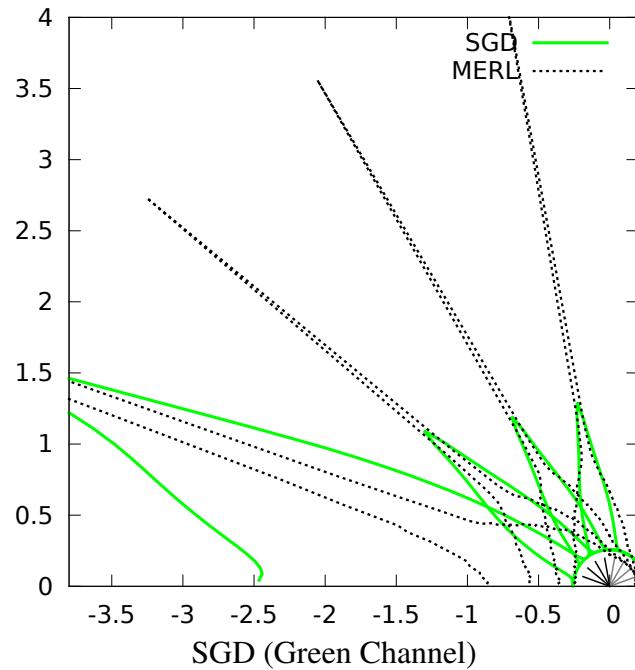
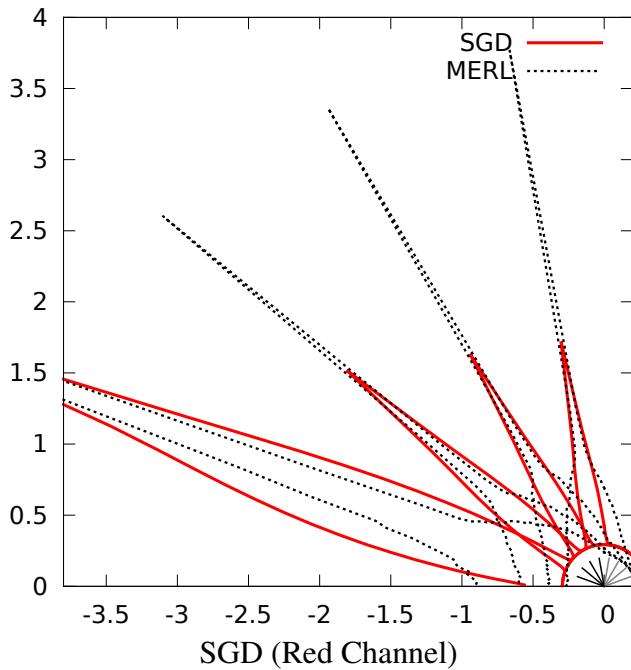
SGD fitted parameters and absolute fitting error

Channel	ρ_d	ρ_s	α	p	F_0	F_1	$K_{\alpha,p}$	error
Red	0.0796008	0.633627	0.00376608	0.415684	0.0343265	-0.00929705	181.769	0.117145
Green	0.0538361	1.77116	0.00871206	0.368424	0.00330259	-0.0219437	85.527	0.17457
Blue	0.0649523	0.0564028	0.000572055	0.623038	4.15759e-12	-0.0596944	795.042	0.0754722

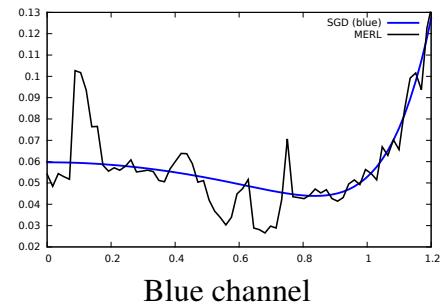
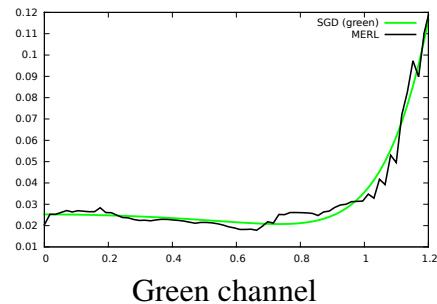
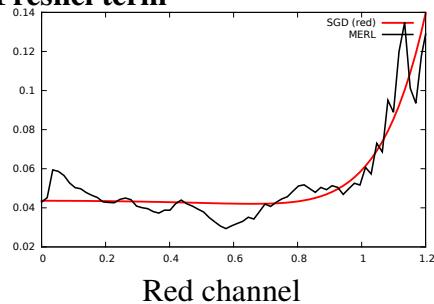
Shadowing / Masking fitted parameters

Channel	λ	c	k	θ_0
Red	2.77279	4.78294e-08	83.871	0.365256
Green	3.50114	9.8399e-08	57.2291	0.276734
Blue	0.985212	4.45358e-08	216.537	0.491264

BRDF lobes in the incidence plane (Cubic root applied)



Fresnel term



Material Name: *gold-paint*

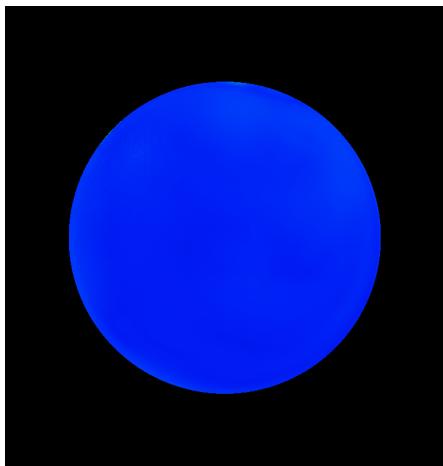
Rendered Images



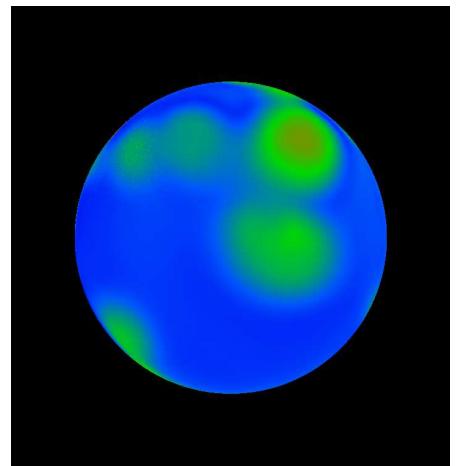
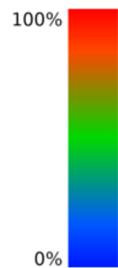
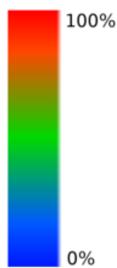
SGD

Reference

Beckmann



SGD (Lab difference)



Beckmann (Lab difference)

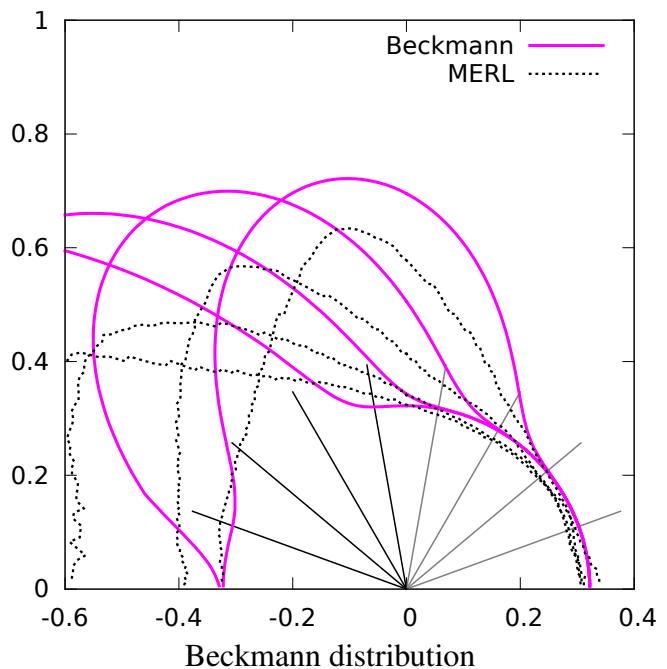
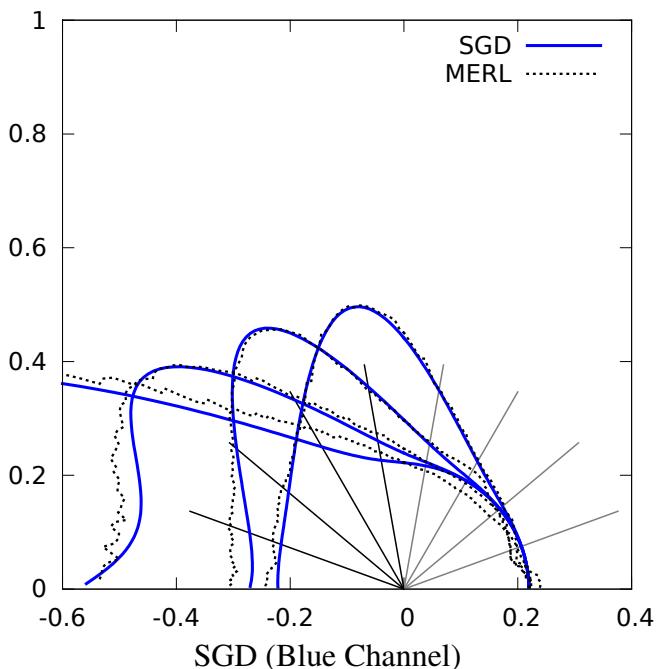
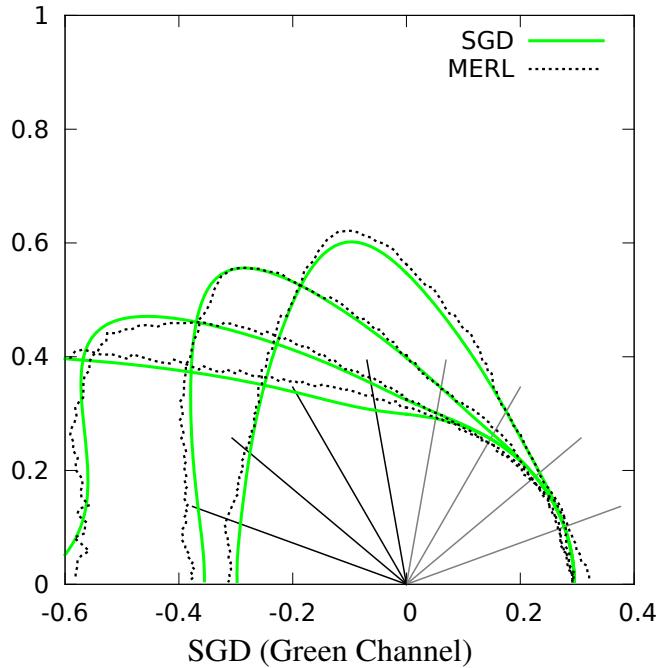
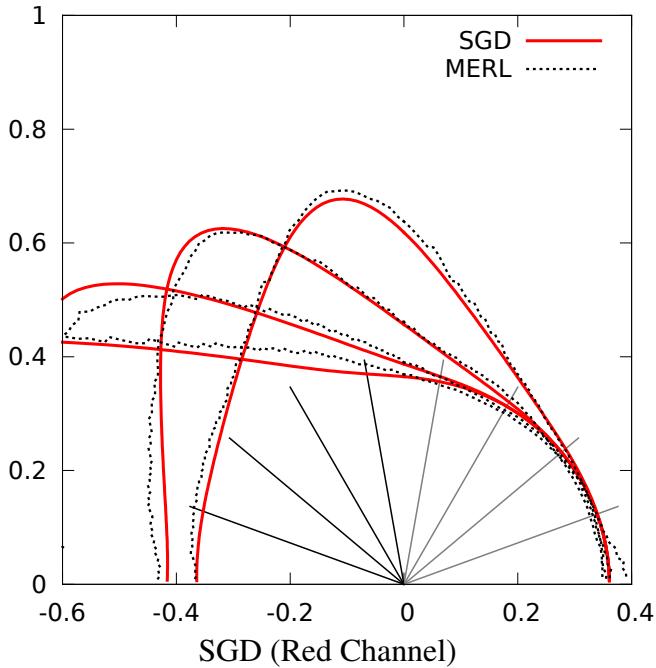
SGD fitted parameters and absolute fitting error

Channel	ρ_d	ρ_s	α	p	F_0	F_1	$K_{\alpha,p}$	error
Red	0.147708	0.160592	0.122506	0.795078	9.16095e-12	-0.596835	5.98176	0.00313716
Green	0.0806975	0.217282	0.108069	0.637578	1.81225e-12	-0.331147	7.35539	0.00203922
Blue	0.033172	0.236425	0.12187	0.936117	0.0024589	-0.140729	5.29722	0.00165683

Shadowing / Masking fitted parameters

Channel	λ	c	k	θ_0
Red	2.64832	9.3111e-08	24.3593	-0.284195
Green	3.04253	8.80143e-08	24.4037	-0.277297
Blue	2.3013	9.65288e-08	25.3623	-0.245352

BRDF lobes in the incidence plane (Cubic root applied)



Fresnel term

