Online Technical Interview Bootcamp at Stanford

Session 4

Yongwhan Lim

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Yongwhan Lim

Education

Stanford University

Part-time Jobs

MIT EECS

Full-time Job

Google Research

TWO SIGMA

Workshops

Stanford ENGINEERING | Stanford Computer Forum

Coach/Judge

icpc International Collegiate Programming Contest

icpc.foundation

https://www.yongwhan.io

Yongwhan Lim

- Currently:
  - **CEO** (Co-Founder) in a Stealth Mode Startup;
  - **Co-Founder** in Christian and Grace Consulting;
  - ICPC **Internship Manager**;
  - ICPC **North America Leadership** Team;
  - Columbia ICPC **Head Coach**;
  - ICPC **Judge** for NAQ and Regionals;
  - **Lecturer** at MIT;
  - **Adjunct** (Associate in CS) at Columbia;

https://www.yongwhan.io

Session 4: Overview

- **Part I**
  - Catalan number; Bell Number; Bernoulli number; Stirling numbers of the first kind; Stirling numbers of the second kind;
  - Generating Function; Bernoulli polynomials; Bernoulli polynomials of the second kind; Stirling polynomials;
- **Part II: Problem Walkthroughs**
  - LeetCode Weekly 343
  - AtCoder Beginner Contest 300
  - Codeforces Round 869 (Div. 2)
- **Important Reminders**

I. Catalan Numbers: Motivating Examples

- Number of correct bracket sequence consisting of n opening and n closing brackets.
- The number of rooted full binary trees with $n + 1$ leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.
- The number of ways to completely parenthesize $n + 1$ factors.
- The number of ways to connect the $2n$ points on a circle to form n disjoint chords.
- The number of non-isomorphic full binary trees with n internal nodes (i.e. nodes having at least one son).
- ...

I. Catalan Numbers (A000108)

- 1
- 1
- 2
- 5
- 14
- 42
- 132
- 429
- 1430
- …

I. Catalan Numbers: Recursive Formula

$$C_0 = C_1 = 1$$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, n \geq 2$$

I. Catalan Numbers: Implementation

```
int catalan[MAX];
void init() {
    catalan[0] = catalan[1] = 1;
    for (int i=2; i<=n; i++) {
        catalan[i] = 0;
        for (int j=0; j < i; j++) {
            catalan[i] += (catalan[j] *
                           catalan[i-j-1]) % MOD;
            if (catalan[i] >= MOD) catalan[i] -= MOD;
        }
    }
}
```

I. Catalan Numbers: Analytical Formula

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

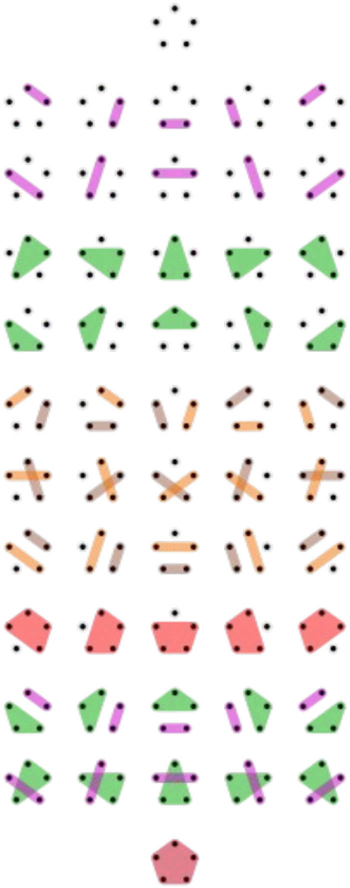
$$C_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}, n \geq 0$$

II. Bell Numbers

- Bell numbers count **the possible partitions of a set**.
- For example, when n=3 (e.g., {a,b,c}), we have:
  - {{a},{b},{c}};
  - {{a},{b,c}};
  - {{b},{a,c}};
  - {{c},{a,b}};
  - {{a,b,c}};

II. Bell Numbers (A000110)

- 1
- 1
- 2
- 5
- 15
- 52
- 203
- 877
- 4140
- …



II. Bell Numbers: Recurrence & Explicit

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$$

Binomial coefficient

$$B_n = \sum_{k=0}^{n} \left\{ {n \atop k} \right\}$$

Stirling number of second kind

number of ways to partition a set of cardinality n into exactly k nonempty subsets

III. Bernoulli Numbers

- Taylor series expansions of the tangent and hyperbolic tangent functions;
- Faulhaber's formula for the sum of m-th powers of the first n positive integers;
- the Euler–Maclaurin formula;
- Certain values of the Riemann zeta function;

III. Bernoulli Numbers: Numerator (A027641)

- 1
- -1
- 1
- 0
- -1
- 0
- 1
- 0
- -1
- ...

III. Bernoulli Numbers: Denominator (A027642)

- 1
- 2
- 6
- 1
- 30
- 1
- 42
- 1
- 30
- …

III. Bernoulli Numbers: Recurrence

$$B_m^- = \delta_{m,0} - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k^-}{m-k+1}$$

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases}$$

Kronecker delta

$B_n^+ = (-1)^n B_n^-$, or for integer n = 2 or greater

III. Bernoulli Numbers: Explicit

$$B_m^- = \sum_{k=0}^{m} \sum_{v=0}^{k} (-1)^v \binom{k}{v} \frac{v^m}{k+1}$$

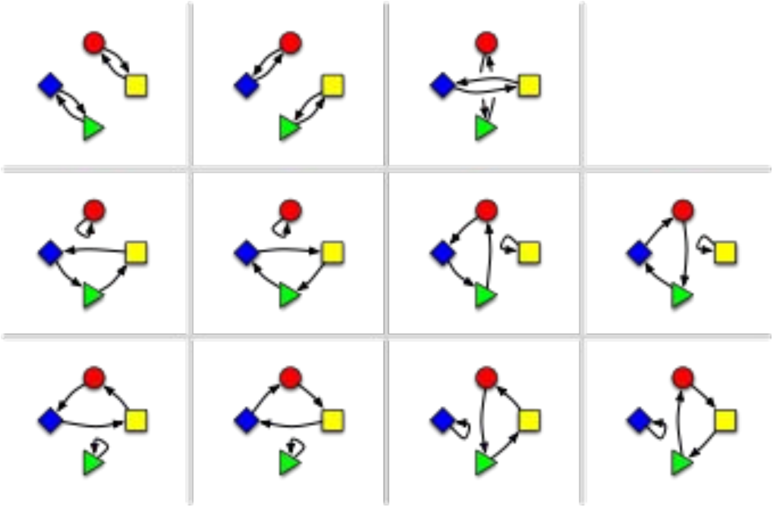
III. Bernoulli Numbers: Riemann Zeta Function

$$B_n^+ = -n\zeta(1-n) \qquad \text{for } n \geq 1$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

IV. Stirling numbers of the first kind

- Count permutations according to their number of cycles (counting fixed points as cycles of length one)



$$\begin{bmatrix} 4 \\ 2 \end{bmatrix} = 11$$

IV. Stirling numbers of the first kind: Recurrence

$$\begin{bmatrix} n+1 \\ k \end{bmatrix} = n \begin{bmatrix} n \\ k \end{bmatrix} + \begin{bmatrix} n \\ k-1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 1 \quad \begin{bmatrix} n \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ n \end{bmatrix} = 0$$

IV. Stirling numbers of the first kind: Explicit

$$s(n, n-p) = \frac{1}{(n-p-1)!} \sum_{0 \leq k_1, \ldots, k_p : \sum_1^p m k_m = p} (-1)^K \frac{(n+K-1)!}{k_1! k_2! \cdots k_p! \, 2!^{k_1} \, 3!^{k_2} \, \cdots (p+1)!^{k_p}}$$

IV. Stirling numbers of the second kind

- the number of ways to partition a set of n objects into k non-empty subsets

IV. Stirling numbers of the second kind: Recurrence

$$\left\{ {n+1 \atop k} \right\} = k \left\{ {n \atop k} \right\} + \left\{ {n \atop k-1} \right\} \quad \text{for } 0 < k < n$$

$$\left\{ {n \atop n} \right\} = 1 \quad \text{for } n \geq 0 \quad \text{and} \quad \left\{ {n \atop 0} \right\} = \left\{ {0 \atop n} \right\} = 0 \quad \text{for } n > 0.$$

IV. Stirling numbers of the second kind: Explicit

$$\left\{ {n \atop k} \right\} = \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^n$$

V. Generating Function

- a way of encoding an infinite sequence of numbers (a_n) by treating them as the **coefficients** of a formal **power series**.

V. Ordinary Generating Function (OGF)

$$G(a_n; x) = \sum_{n=0}^{\infty} a_n x^n.$$

$$G(a_{m,n}; x, y) = \sum_{m,n=0}^{\infty} a_{m,n} x^m y^n$$

V. Exponential Generating Function (EGF)

$$\mathrm{EG}(a_n; x) = \sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$$

V. Generating Function: Example: Geometric Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

1, 1, 1, 1, 1, ...

$$\sum_{n=0}^{\infty} (ax)^n = \frac{1}{1-ax}$$

1, a, a^2, a^3, a^4, a^5, ...

VI. Bernoulli Polynomials

$$\frac{te^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \frac{t^n}{n!}$$

$$B_n(x) = \sum_{k=0}^{n} \binom{n}{k} B_{n-k} x^k$$

VI. Bernoulli Polynomials of the second kind

$$\frac{z(1+z)^x}{\ln(1+z)} = \sum_{n=0}^{\infty} z^n \psi_n(x), \qquad |z| < 1$$

VII. Stirling polynomials

$$\left(\frac{t}{1-e^{-t}}\right)^{x+1} = \sum_{k=0}^{\infty} S_k(x)\frac{t^k}{k!}$$


BREAK

Problem Walkthroughs

- LeetCode Weekly 343
- AtCoder Beginner Contest 300
- Codeforces Round 869 (Div. 2)

Request 1:1 Meeting, through Calendly

- Use calendly.com/yongwhan/one-on-one to request 1:1 meeting:
  - Mock Interview
  - Career Planning
  - Resume Critique
  - Practice Strategy
  - Volunteering Opportunity
  - ...
- I am always **inspired** by driven students like yourself!
- Since I'd feel honored/thrilled to talk to you, do not feel shy to sign up!!!

Terse Guide Google Drive

- Browse through Terse Guides, which include:
  - Behavioral interview preparation
  - System design interview preparation
  - ICPC preparation
  - Live contests
  - Useful resources

Discord Server Invitations

- Some discord server invitations:
  - **[Online Technical Interview Bootcamp at Stanford]** https://discord.gg/aJwHBccg3n
  - **[ICPC CodeForces Zealots]** https://discord.gg/QC9ss6WJPy

Contact Information

- **Email**: yongwhan@yongwhan.io

- **Personal Website**: https://www.yongwhan.io/

- **LinkedIn Profile**: https://www.linkedin.com/in/yongwhan/
  - Feel free to send me a connection request!
  - Always happy to make connections with promising students!

Q&A's