Spring 2024

UCF Training Camp Contest #2

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Friday, March 29, 2024

Difficulty Order, by CodeForces rating

- F: **1300** (7/7, 14 minutes +)
- D: **1600** (6/7, 17 minutes +)
- C: **1900** (6/7, 16 minutes +2)
- B: **2300** (5/7, 80 minutes +)
- E: **2800** (3/7, 90 minutes +4)
- A: **3500** (0/7)

[3500] A: Game of AI (hard version)

- Given b, the conclusion of possible array a remains.
  - If we fix array a, we will find that for each i, b_i will only be j such that $a_j=i$ or $j=i$.
  - For any pair (x,y), if $a_x=y$, $b_x=x$ and $b_y=y$ at the same time is impossible.
  - All arrays b which satisfies the above conditions are valid.
- So for a fixed array a, and we fix the position that $b_i=i$ (It can be shown that the chosen nodes form a independent set), then the number of possible b will be product of each in-degree of each x that $b_x \neq x$.

[3500] A: Game of AI (hard version)

- If we build a graph according to a, the answer will be **the sum of product over all independent sets**.

- We need to solve the problem of three parts:
  - the nodes that form a tree;
  - some trees' roots form a ring;
  - some trees based on rings form a graph;

[3500] A: Game of AI (hard version)

- For the tree part, let $P(x)$ be generating function of the sum of products if the root is not included in the independent set and $Q(x)$ otherwise. Then,
  - $P(x)=x(P(x)+Q(x))e^{P(x)+Q(x)}$
  - $Q(x)=xe^{P(x)}$

- Use **Newton's method** to calculate $P(x)$ and $Q(x)$.

[3500] A: Game of AI (hard version)

- How to merge trees? If one root is not in the independent set, it can either be occupied by one of its children or the previous node on the ring, so its generating function can be represented as $\boldsymbol{A(x)}$.
  - Compared with $P(x)$, $A(x)$ adds the contribution from previous node on the ring.

- For those which are in the independent set, their previous nodes must not be in. So, after combining them together, their generating functions can be represented as $\boldsymbol{B(x)}$.

[3500] A: Game of AI (hard version)

- The final answer is $\Sigma((A(x)+B(x))^i/i)$ - $A(x)$ = $-\ln(1-A(x)-B(x))-A(x)$.

- Note that when every node on the ring is occupied by previous node, it is impossible. So we need to subtract this answer.

- At last, we need to combine these rings into a graph, which can be done with exp operation.