Delft University of Technology

Thermodynamics

Extra Assignment: Radiation, Absorption and Emission

Due date: July 15, 2024 (23:59)

Extra Assignment: Radiative Equilibrium

In this exercise you will be exploring how the vertical temperature profile of the Earth's atmosphere behaves under the condition of Radiative Equilibrium.

Consider an atmosphere that is transparant to shortwave (solar) radiation. The average flux of energy entering the Earth's atmosphere is $S_0 = 1361\,\mathrm{Wm^{-2}}$. This flux is entering the projected disk of the Earth so if we spread this out evenly on the whole sphere of the Earth this means that the annually averaged radiative shortwave flux is a factor 4 smaller, i.e $S_0/4$ (see Figure 1). Assume further that the Earth's surface has an albedo $\alpha = 0.3$ implying that 30 percent of the incoming solar radiation will be reflected back into space.

The Earth's surface behaves as black body, i.e. it radiates longwave radiation with a flux equal to $\sigma T - g^4$ where σ is the Stefan-Boltzmann constant (see table for numerical value) and T_g denotes the temperature of the Earth's surface (ground temperature) in units of Kelvin.

symbol	value	unit
S	1361	W m^{-2}
σ	$5.67 \cdot 10^{-8}$	W $\mathrm{m}^{-2}\mathrm{K}^{-4}$
α	0.3	-
a_1	0.8	-
ϵ_1	0.8	-

Table 1: Parameter values of the simple one layer atmosphere model.

We start with representing the whole atmosphere as one-layer with an absorptivity a_1 which is defined as the fraction of longwave radiation that is being absorbed and an emissivity ϵ_1 defined as the ratio between the actual emitted radiation and the radiation flux that the atmosphere would emit if it would act as a black body. In practice this means that the atmosphere emits radiative energy equal to $\epsilon_1 T_a^4$ in both the upward and downward direction where T_a is the temperature of the atmospheric layer (see Figure 1). We will further assume in general that the absorptivity of a layer is always equal to its emissivity.

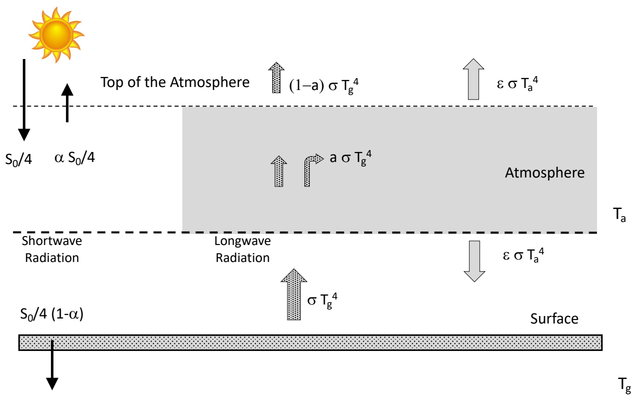

Figure 1

a. Calculate the equilibrium surface temperature T_g and the equilibrium temperature of the atmosphere T_a. (Tip: In radiative equilibrium the sum of energy leaving and entering the atmosphere layer adds up to zero. Furthermore the energy leaving and entering the atmosphere also adds up to zero in equilibrium.)

The typical thickness of the atmosphere is 10 km so the one layer atmospheric temperature is characteristic for the whole atmosphere. If we want to know how the temperature changes with height we need to introduce multiple atmospheric layers. Start with two layers of each 5 km thick. The 2 thinner layers need to have the same transmissivity for longwave radiaton as for the one-layer case. The transmissivity of the one-layer problem is $1 - \epsilon_1$.

b. If the two layers each have an equal absorptivity/emissivity of ϵ_2, show that they have the same total transmissivity as in the one-layer atmosphere model if and only if each of the two atmospheric layers have each an emissivity of

$$\epsilon_2 = 1 - (1 - \epsilon_1)^{\frac{1}{2}} \tag{1}$$

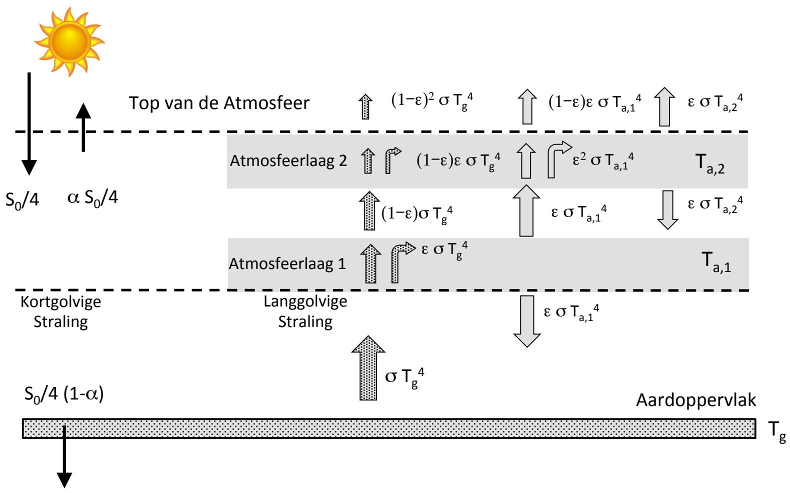

Figure 2

c. Calculate again the surface temperature, the equilibrium atmosphere temperature of layer 1 and of layer 2. Make a plot of temperature on the horizontal axis and height on the vertical axis. Discuss the difference with your results for a one layer atmosphere.

Now generalise the procedure to n layers where each layer has the same emissivity (and absorptivity)

d. Show that the n-layer model has the same transmissivity as the 1-layer atmosphere model only and only if they all have an emissivity of

$$\epsilon_n = 1 - (1 - \epsilon_1)^{\frac{1}{n}} \tag{2}$$

e. Write a python program in a jupyter notebook that solves the atmospheric temperatures for n layers. (Realise that you have to solve a set of n coupled equations) Make for n=10 a plot with the height of the atmosphere on the vertical axis and the corresponding temperature and the horizontal axis. Compare your results with the n=2 case. How does the surface temperature change with n. Explain your results.

f. Will the profile converge if n goes to infinity? (Try for n is a large number , i.e. n = 1000) and plot again. use for the height of the ith level $z = (1 - 0.5) \times \frac{10^4}{n} m$