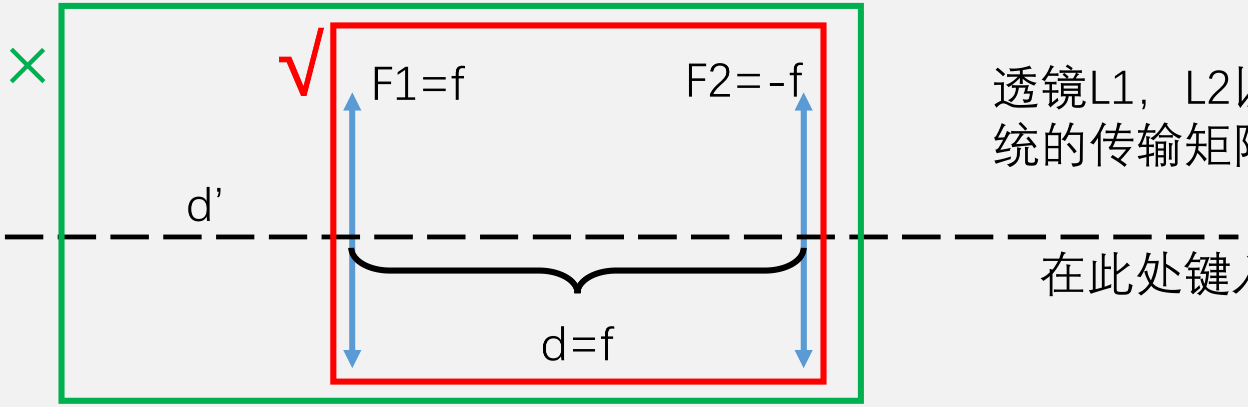
第一次作业 题一

Ray-Transfer Matrix of a Lens System. Determine the ray-transfer matrix for an optical system made of a thin convex lens of focal length f and a thin concave lens of focal length -f separated by a distance f. Discuss the imaging properties of this composite lens.



透镜L1，L2以及其间隔d决定该系统的传输矩阵，与物的位置无关

在此处键入公式。

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{-f} & 1 \end{bmatrix} \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 0 & f \\ -\frac{1}{f} & 2 \end{bmatrix}$$

第一次作业 题二

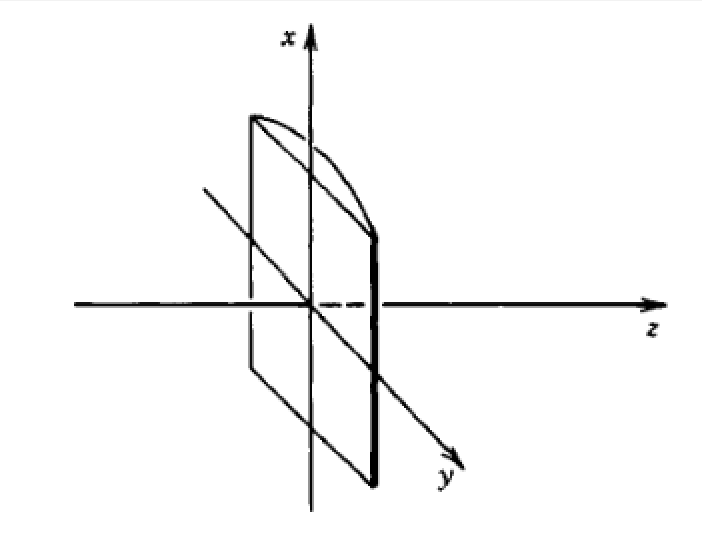
4 X 4 Ray-Transfer Matrix for Skewed Rays. Matrix methods may be generalized to describe skewed paraxial rays in circularly symmetric systems, and to astigmatic (non-circularly symmetric) systems. A ray crossing the plane z = 0 is generally characterized by four variables-the coordinates (x, y) of its position in the plane, and the angles (e,, ey) that its projections in the x-z and y-z planes make with the z axis. The emerging ray is also characterized by four variables linearly related to the initial four variables. The optical system may then be characterized completely, within the paraxial approximation, by a 4 X 4 matrix.

(a) Determine the 4 x 4 ray-transfer matrix of a distance d in free space.

(b) Determine the 4 X 4 ray-transfer matrix of a thin cylindrical lens with focal length f oriented in the y direction. The cylindrical lens has focal length f for rays in the y-z plane, and no focusing power for rays in the x-z plane.

(a)

$$\begin{cases} x' = x + \theta_x d \\ \theta'_x = \theta_x \\ y' = y + \theta_y d \\ \theta'_y = \theta_y \end{cases} \qquad \begin{bmatrix} x' \\ \theta'_x \\ y' \\ \theta'_y \end{bmatrix} = \begin{bmatrix} 1 & d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}$$



(b)

$$\begin{cases} x' = x \\ \theta'_x = \theta_x \\ y' = y \\ \theta'_y = \theta_y - \dfrac{1}{f} y \end{cases} \qquad \begin{bmatrix} x' \\ \theta'_x \\ y' \\ \theta'_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & -1/f & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}$$

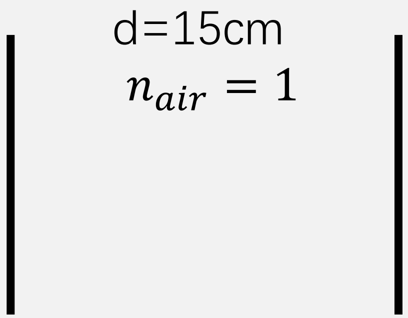
第二次作业 补充题一

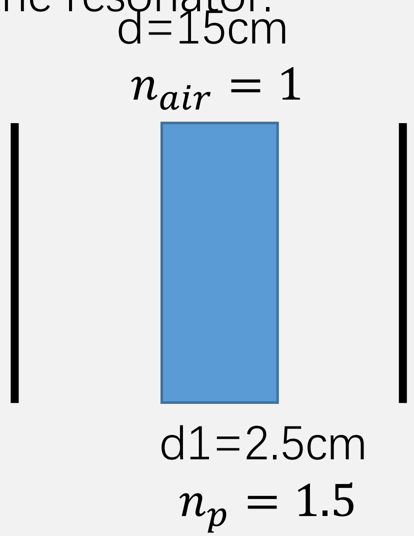
Resonance Frequencies of a Resonator with an Etalon.

(a) Determine the spacing between adjacent resonance frequencies in a resonator constructed of two parallel planar mirrors separated by a distance d = 15 cm in air (n = 1).

(b) A transparent plate of thickness d1= 2.5 cm and refractive index n = 1.5 is placed inside the resonator and is tilted slightly to prevent light reflected from the plate from reaching the mirrors.

Determine the spacing between the resonance frequencies of the resonator.



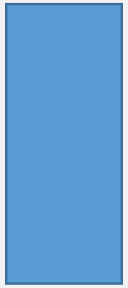


(a)$\nu_F = \frac{c}{2n_{air}d} = 1GHz$

(b)$\nu_F = \frac{c}{2n_{air}(d-d_1)+2n_p d_1} = 0.923GHz$

第二次作业 补充题二

Semiconductor lasers are often fabricated from crystals whose surfaces are cleaved long crystal planes. These surfaces act as reflectors and therefore serve as the resonator mirrors. Consider a crystal with refractive index n = 3.6 placed in air (n = 1). The light reflects between two parallel surfaces separated by the distance d = 0.2 mm. Determine the spacing between resonance frequencies v_f,the overall distributed loss coefficient a_r,the finesse , and the spectral widthΔv. Assume that the loss coefficient $a_s = \ 1\ cm^{-1}$.



d=2.5cm

$n_c = 3.6$

$$v_F = \frac{c}{2n_c d} = 2.0833 * 10^{11} Hz$$

$$\mathrm{R}_1 = R_2 = \left(\frac{n-1}{n+1}\right)^2 = 0.32$$

$$a_r = a_s + \frac{1}{2\mathrm{d}} \ln\left(\frac{1}{R_1 R_2}\right) = 58 cm^{-1}$$

$$F = \frac{\pi \exp(-a_r d/2)}{1 - \exp(a_r d)} = 2.56$$

$$\mathrm{I} = \frac{I_{max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\frac{\pi v}{v_F})} = \frac{1}{2} I_{max}$$

在一个周期内的解为$v1, v2$

则$\Delta v = |v1 - v2| = 8.76 * 10^{10} Hz$

2.1

Nd^{3+}:YAG激光器（波长为1.06um）发出功率为1W的高斯光束，其发散角$2\theta_0 = 1\text{mrad}$，求束腰半径，焦深，最大光强，以及轴上距离束腰位置100cm处的光强。

束腰半径：$\theta_0 = \dfrac{\lambda}{\pi W_0}$ ➡ $W_0 = 0.675mm$

焦深：$z_0 = \dfrac{W_0}{\theta_0}$ ➡ $2z_0 = 2.7m$

最大光强：$P = 0.5I_0(\pi W_0^2)$ ➡ $I_0 = 1.4 * 10^6 W/m^2$

距束腰100cm处光强：$I(z) = I_0 * \left[\dfrac{W_0}{W(z)}\right]^2$ ➡ $I(100cm) = 9.02 * 10^5 W/m^2$

$$W(z) = W_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2}$$

2.2

波长为10.6um的CO2激光器的激光光束为高斯光束，再相距d=10cm的两位置上，光束半径分别为$W_1 = 1.669\mathrm{mm}$与$W_2 = 3.38\mathrm{mm}$，求该光束的束腰位置与束腰半径。

$$\mathrm{W}_0 = \left(\frac{\lambda z_0}{\pi}\right)^{1/2} \Rightarrow z_0 = \frac{W_0^2\pi}{\lambda}$$

$$\begin{cases} W(z_1) = \mathrm{W}_0\left[1+\left(\frac{z_1}{z_0}\right)^2\right]^{1/2} = W_1 \quad ① \\ W(z_2) = \mathrm{W}_0\left[1+\left(\frac{z_2}{z_0}\right)^2\right]^{1/2} = W_2 \quad ② \\ z_2 - z_1 = d \quad ③ \\ z_0 = \frac{W_0^2\pi}{\lambda} \quad ④ \end{cases}$$

①^2 − ②^2，得$z_1 + z_2 = \frac{\pi(W_1^2 - W_1^2)}{\lambda(z_1 - z_2)} z_0 = \frac{\pi(W_2^2 - W_1^2)}{\lambda d} z_0$ ⑤

由③⑤，得

$$z_1 = \frac{\pi(W_2^2 - W_1^2)}{2\lambda d} z_0 - \frac{d}{2} \overset{\text{def}}{=} mz_0 - d/2 \quad ⑥$$

$$z_2 = \frac{\pi(W_2^2 - W_1^2)}{2\lambda d} z_0 + \frac{d}{2} \overset{\text{def}}{=} mz_0 + d/2 \quad ⑦$$

其中，$m = \frac{\pi(W_2^2 - W_1^2)}{2\lambda d}$

将④⑥代入①，得到$(m^2+1)z_0^2 - \left(Ad + \frac{\pi W_1^2}{\lambda}\right) z_0 + \frac{d^2}{4} = 0$

$\Rightarrow z_0 = 1.3246mm$或$11.4464\mathrm{mm}$ $\Rightarrow$

$$\begin{aligned} z_0 &= 1.3246mm \\ W_0 &= 0.06685mm \\ z_1 &= -33.04mm \\ z_2 &= 66.96m \end{aligned}$$

或

$$\begin{aligned} z_0 &= 11.4464\mathrm{mm} \\ W_0 &= 0.19652mm \\ z_1 &= 96.53mm \\ z_2 &= 196.53mm \end{aligned}$$

2.4

一氩离子激光器能发出波长为488nm的高斯光束，束腰 $W_0 = 0.5mm$。 请设计一个单透镜光学系统，是的光斑聚焦到100um，最短的聚焦长度是多少？

$$z_0 = \frac{W_0^2 \pi}{\lambda} \Rightarrow z_0 = 1.61\text{m}$$

$$M = \frac{100um}{2\mathrm{W}_0} = 0.1$$

$$M = \frac{M_r}{(1-r^2)^{1/2}} = \frac{\left|\frac{f}{z-f}\right|}{1-\frac{z_0^2}{(z-f)^2}} \Rightarrow (1-M^2)f^2 + 2M^2 zf - M^2(z^2 - z_0^2) = 0$$

$$\Rightarrow f = -\frac{M^2 z}{1-M^2} + \frac{M}{1-M^2}\sqrt{z^2 + (1-M^2)z_0^2} = -\frac{\mathrm{z}}{99} + \frac{10}{99}\sqrt{z^2 + \frac{99}{100}z_0^2}$$

f关于z的导数$f'(z) = -\frac{1}{99} + \frac{10}{99} * \frac{z}{\sqrt{z^2+\frac{99}{100}z_0^2}}$

从而可知$f_{min} = f(0.1z_0) = 0.1z_0 = 0.161m$

束腰在透镜上时 $f = 161.75\text{m}$，
此时f不是最小

2.11

用一个可调谐单色光源测试对称F-P腔的透射率。透射光谱呈现周期脉冲式的光谱分布，脉冲周期为150MHz，每个光谱脉冲的宽度（FWHM）为5MHz。设腔内介质折射率为1，腔的损耗均为腔镜损耗，求谐振长度与谐振光谱的细度。

（1） $\mathrm{v}_F = \frac{c}{2d}$ ➡ $d = 1m$

（2） $\sigma_F = v_F/F$ ➡ $\mathrm{F} = 30$

2.12

一个谐振细度F=100，腔长d=50cm，折射率n=1，求当腔内储能降到初始值一半时所需要的时间

$$\mathrm{F} = \frac{\pi * \exp(-\frac{\alpha_r d}{2})}{1 - \exp(-\alpha_r d)} \quad \Rightarrow \quad \alpha_r = 0.0628 m^{-1}$$

$$\exp(-\alpha_r c t) = 0.5 \quad \Rightarrow \quad t = 3.68 * 10^{-8} s$$

2.13

设光波长为1.06um，光谱带宽σv为120GHz，在下列腔（n=1）中有多少个模式?
（a）一维谐振腔，腔长10cm;
（b）二维谐振腔，腔长10*10cm^2;
（c）三维谐振腔，腔长10*10*10cm^2;

（b） $\frac{4}{c} * \sigma v * L = 160$

（b） $\frac{4\pi v}{c^2} * \sigma v * L^2 = 4.74 * 10^7$

（c） $\frac{8\pi v^2}{c^3} * \sigma v * L^3 = 8.95 * 10^{12}$

2.16

假设一个有两个半径为R的凹面镜组成的对称谐振腔，其腔长为d=3|R|/2。请问光线在腔内传播时要几个来回才能重复开始的路径?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -2 & 3R/2 \\ -2/R & D1 \end{bmatrix} \longrightarrow \begin{cases} b = \dfrac{A+D}{2} = -\dfrac{1}{2} \\ F^2 = AD - BC = 1 \end{cases} \longrightarrow \varphi = \arccos\left(\frac{b}{F^2}\right) = \frac{2\pi}{3}$$

$3 * \varphi = 2\pi * 1$，所以$T = 3$

2.17

证明，在非稳定腔中传播m个来回的光线高度满足：$y_m = \alpha_1 h_1^m + \alpha_2 h_2^m$,其中$\alpha_1\alpha_2$为常数，$h_1 = b + \sqrt{b^2-1}$，$h_2 = b + \sqrt{b^2-1}$，且$b = 2\left(1+\frac{d}{R_1}\right)\left(1+\frac{d}{R_2}\right)-1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{2d}{R_2} & 2d(1+\frac{d}{R_2}) \\ \frac{2}{R_1}+\frac{2}{R_2}+\frac{4d}{R_1R_2} & \frac{2d}{R_1}+(\frac{2d}{R_1}+1)(\frac{2d}{R_2}+1) \end{bmatrix}$$

$$y_{m+2} = 2b' y_{m+1} - F^2 y_m = 0$$

$$b' = \frac{A+D}{2} = 2\left(1+\frac{d}{R_1}\right)\left(1+\frac{d}{R_2}\right)-1 = b \qquad F^2 = AD - BC = 1$$

b是题目中的常数记号，b'是传输矩阵的参数，没证明数值相等（b=b'）前，这两者没有联系

因为是非稳定腔，所以|b|>1

所以特征方程$h^2 - 2bh + F^2 = 0$的解$h = b \pm \sqrt{b^2 - F^2} = b \pm \sqrt{b^2-1}$
特征解属于实数，其通解为$y_m = \alpha_1 h_1^m + \alpha_2 h_2^m$

2.18

在证明，由一对间距d为65cm，曲率半径R=-30cm的凹面镜组成的对称谐振为非稳腔。假设腔镜的直径为5cm，初始点为腔的中心的光线（$y_0=0$，$\theta_0=0.1°$）在腔中传播几个来回才能传出腔外?

根据$\begin{cases} y_0 = 0 \\ \theta_0 = 0.1° \end{cases}$与传输矩阵，

可得$y_1 = -0.26471cm$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -\frac{10}{3} & -\frac{455}{3} \\ \frac{7}{45} & \frac{61}{9} \end{bmatrix}\text{(单位为cm)}$$

$$\Rightarrow y_{m+2} = 2by_{m+1} - F^2 y_m = 0$$

其中$\begin{cases} b = \frac{A+D}{2} = \frac{31}{18} \\ F^2 = AD - BC = 1 \end{cases}$

$$\Rightarrow y_m = \alpha_1 h_1^m + \alpha_2 h_2^m$$

其中$h_1 = \frac{31+7\sqrt{13}}{18}$, $h_2 = \frac{31-7\sqrt{13}}{18}$

将y_0, y_1代入，可得$y_m = -0.0944(h_1^m - h_2^m)$

$y_2 = -0.912cm$

$y_3 = -2.876cm$

第三次逸出

4.1

在一个初速度为零的电子上加多少电压，可以使其具有与波长为870nm的光子一样的能量？
一个1.06um波长的光子与一个波长为10.6mm的光子结合，产生一个光子，其能量为两者之和，求新光子的波长

（1） $\mathrm{U}e = hv = hc/\lambda \;\Rightarrow\; \mathrm{U} = 1.43\mathrm{V}$

(2) $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2} \;\Rightarrow\; \lambda = 1.059894\mathrm{um}$

4.3

比较一个能量为10J的激光脉冲的光子总动量与一个质量为1g以1cm/s运动的物体的动量，以及以$c_0/10$速度运动的电子的动量

（1）光子：

$$\mathrm{N} * hv = 10J$$

$$\mathrm{P_{photon}} = N * \frac{hv}{c} \quad \Rightarrow \quad \mathrm{P_{photon}} = 3.33 * 10^{-8}\mathrm{kg} \cdot \mathrm{m}/s$$

（2）物体：

$$\mathrm{P_{obj}} = mv \quad \Rightarrow \quad \mathrm{P_{obj}} = 1.00 * 10^{-5}\mathrm{kg} \cdot \mathrm{m}/s$$

（3）电子：

$$\mathrm{m} = \mathrm{m_0/sqrt}\left(1 - \left(\frac{\frac{\mathrm{c_0}}{10}}{\mathrm{c_0}}\right)^2\right) \quad \Rightarrow \quad \mathrm{P_{electron}} = 2.75 * 10^{-23}\mathrm{kg} \cdot \mathrm{m}/s$$

$$\mathrm{P_{electron}} = \mathrm{m} * \mathrm{c_0}/10$$

4.4

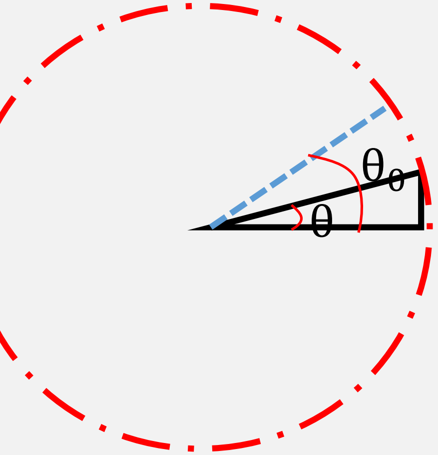
求一个束腰为W_0的高斯光束，动量矢在发散角θ_0内对应的光子的的几率，此时$p = E/c_0$还成立吗？

$$\mathrm{U}(\rho, z) = A_0 * \frac{W_0}{W(z)} \exp\left(-\frac{2\rho^2}{W^2(z)}\right) \exp\left(-ikz - \frac{ik\rho^2}{2R(z)} + i\zeta(z)\right)$$

$$\mathrm{U}(\mathrm{k}_\rho, \mathrm{k}_z) = \iint U(\rho, z) * \exp(-i(\mathrm{k}_\rho \rho + \mathrm{k}_z z)) d\mathrm{u}_\rho \mathrm{du}_z$$

$$= \iint U^2(\rho, z) * \exp(-i(k sin(\theta)\rho + k cos(\theta) z))\, dk sin(\theta)\, \mathrm{d}k cos(\theta)$$

$$= \iint_{\theta, \mathrm{k}} U^2(\rho, z) * \exp(-i(k sin(\theta)\rho + k cos(\theta) z)) * \cos(2\theta)\, d\theta\, \mathrm{d}k$$



$$\left|\overrightarrow{k_z} + \overrightarrow{k_\rho}\right| = \mathrm{k}$$

动量矢在发散角θ_0内的光子满足 $|\tan(\theta)| = \left|\frac{k_\rho}{k_z}\right| < \tan(\theta_0)$

4.4

求一个束腰为W_0的高斯光束，动量矢在发散角θ_0内对应的光子的的几率，此时$p = E/c_0$还成立吗?

$$P = \frac{\underset{k\ \theta}{\iint_{-\theta_0}^{\theta_0}} U^2(\rho, z) * \exp(-i(k\sin(\theta)\rho + k\cos(\theta)z)) * \cos(2\theta)\, d\theta\, dk}{\underset{k\ \theta}{\iint_{-\pi}^{\pi}} U^2(\rho, z) * \exp(-i(k\sin(\theta)\rho + k\cos(\theta)z)) * \cos(2\theta)\, d\theta\, dk}$$

或者

$$P = \frac{\int dk_z \int_{-k_z \tan(\theta_0)}^{k_z \tan(\theta_0)} U^2(k_\rho, k_z) dk_\rho}{\int dk_z \int U^2(k_\rho, k_z) dk_\rho}$$

4.6

一个腔长d=1cm的FP谐振腔，腔内充满折射率n=1.5的无吸收介质，两个反射镜均是理想反射镜。

假设对应于驻波$\sin(\frac{10^5\pi x}{d})$模式只有一个光子，求

（1）光子的波长与能量

（2）估计光子位置与动量的不确定性，并与$\sigma_p\sigma_x \approx h/4\pi$关系比较

（1）$\frac{2\pi n}{\lambda} = \frac{10^5\pi}{d}$ ➡ $\lambda = 300nm$

$$E = \frac{hc}{\lambda}$$ ➡ $E = 6.62 * 10^{-19}J$

（2）$\Delta_p = 2 * p = \frac{2h}{\lambda}$

$\Delta_x = d$ ➡ $\Delta_p\Delta_x = \frac{2hd}{\lambda} > \frac{h}{4\pi}$

4.8

一平面波模式的光子，照射到一个无吸收的分束镜上，当光子照射分束镜之前，求光子的动量矢量，透过分束镜后保持这样的动量矢量的概率是多少？

$$\vec{p} = h\vec{k}/2\pi$$

透过后保持这样动量的概率等于分束镜的能量透过率

4.11

假设一个100pW的He-Ne单模激光器输出633nm的$\mathrm{TEM}_{0,0}$模式的高斯光。求:

（a）在100ns时间内，该光束在截面为束腰大小的圆面积内的平均光子数;

（b）（a）中的均方根光子数

（c）在（a）中没有记录到光子的概率

（a）束腰内能量占总能量的86%

$$\mathrm{N}\frac{\mathrm{hc}}{\lambda} = 86\% * \mathrm{P} * \mathrm{t} \Longrightarrow N = 27.4$$

（b）

$$\begin{aligned}\sigma^2 &= \sum_n p(n) * (n - \bar{n})^2 \\ &= \sum_n p(n) * n^2 - \sum_n 2 * p(n) * n * \bar{n} + \sum_n p(n) * \bar{n}^2 \\ &= -\bar{n}^2 + \sum_n p(n) * n^2\end{aligned}$$

且泊松分布的$\sigma^2 = \bar{n}$

$$\Longrightarrow \sum_n p(n) * n^2 = \bar{n} + \bar{n}^2$$

所以$RMS = \sqrt{\sum_n p(n) * n^2} = 27.9$

（c）$\mathrm{p}(0) = \exp(-N) = 1.26 * 10^{-12}$

4.13

某原子有两个能级，对应的跃迁参数为$\lambda_0 = 700\text{nm}$，$t_{sp} = 3\text{ms}$，$\Delta v = 50\text{GHz}$洛伦兹线形，将其充满一个体积为$V = 100\text{cm}^3$，折射率为1的谐振腔。两个辐射模式（一个位于中心频率v_0，一个位于$v_0 + \Delta v$）分别由1000个光子激发。

（a）求受激辐射的概率

（b）如果N_2个原子被激发到2能级，请确定由受激辐射与自发辐射引起的2能级原子数衰减时间常数；

（c）要使受激辐射与自发辐射的衰减率一样，需要多少光子数？

(a) $P_{st} = n_1 * \frac{c}{V}\sigma(v_0) + n_2 * \frac{c}{V}\sigma(v_0 + \Delta v)$

$$v_0 = \frac{c}{\lambda} = 4.286 * 10^{14}Hz$$

$$\text{n}_1 = n_2 = 1000$$

$$\sigma(v) = S * g(v) = \frac{\lambda^2}{8\pi t_{sp}} * \frac{\Delta v/2\pi}{(v - v_0)^2 + \left(\frac{\Delta v}{2}\right)^2}$$

$$S = 6.5 * 10^{-12}m^2/s$$

$$P_{st} = 2.98 * 10^{-7}s^{-1}$$

4.13

(b)

$$P = P_{st} + P_{sp} = P_{st} + \frac{1}{t_{sp}} = 0.333\mathrm{ms}^{-1}$$

$$t = \frac{1}{P} = 3ms$$

(c)

设 $n_1 = n_2 = n'$时

$Pst = Psp$

则 $\frac{1}{\mathrm{t}_{sp}} = n' * \frac{c}{V}\sigma(v_0) + n' * \frac{c}{V}\sigma(v_0 + \Delta v)$ ➡ $n' = 1.12 * 10^{12}$

4.15

单位体积谐振腔的腔内原子有两个能级，用能级1,2表示，对应跃迁谐振频率为v_0，线宽为Δv。在1,2能级上的原子数分别是N_1, N_2。在v_0频率附近的各模式中有$\bar{n}$个光子。谐振腔因为腔镜的透射光子数减少率为$1/t_p$，假设在2和1能级之间没有非辐射跃迁，请写出N_2与$\bar{n}$的速率方程。

$$\Delta v = \frac{1}{2\pi t_{sp}} \Rightarrow \boxed{t_{sp} = \frac{1}{2\pi\Delta v}}$$

$N_2 \to N_1$自发辐射：$P_{sp} = \frac{1}{t_{sp}}$

$N_2 \to N_1$受激辐射：$P_{st} = \bar{n}\frac{1}{t_{sp}}$

$$\begin{cases} \frac{dN_2}{dt} = -(P_{sp} + P_{st})N_2 \\ \frac{dn}{dt} = -\frac{dN_2}{dt} = (P_{sp} + P_{st})N_2 \end{cases}$$

$N_1 \to N_2$吸收：$P_{ab} = \bar{n}\frac{1}{t_{sp}}$ $\Rightarrow$

$$\begin{cases} \frac{dN_2}{dt} = -\frac{dN_1}{dt} = -(P_{ab})N_1 \\ \frac{dn}{dt} = -\frac{dN_2}{dt} = (P_{ab})N_1 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{dN_2}{dt} = -(1+\bar{n})\frac{1}{t_{sp}}N_2 + \bar{n}\frac{1}{t_{sp}}N_1 \\ \frac{dn}{dt} = -\frac{dN_2}{dt} = (1+\bar{n})\frac{1}{t_{sp}}N_2 - \bar{n}\frac{1}{t_{sp}}N_1 \end{cases}$$

光子透射衰减$\bar{n}$：$\frac{d\bar{n}}{dt} = \bar{n}\frac{1}{\mathrm{t_p}}$

在v_0附近光子模式密度

$$\boxed{M(v) = M(v_0) = \frac{8\pi v_0^2}{c^3}}$$

平均光子数$\bar{n} = n/M(v)$

$$\frac{dn}{dt} = \bar{n}\frac{1}{\mathrm{t_p}}M(v)$$

4.15

$$\Delta v=\frac{1}{2\pi t_{sp}} \Rightarrow \boxed{t_{sp}=\frac{1}{2\pi\Delta v}}$$

$$\frac{dN_2}{dt}=-(1+\bar{n})\frac{1}{t_{sp}}N_2+\bar{n}\frac{1}{t_{sp}}N_1$$

$$\frac{dN_2}{dt}=[-(1+\bar{n})N_2+\bar{n}N_1]*2\pi\Delta v$$

$$\frac{dn}{dt}=-\frac{dN_2}{dt}=(1+\bar{n})\frac{1}{t_{sp}}N_2-\bar{n}\frac{1}{t_{sp}}N_1$$

在v_0附近光子模式密度

$$\boxed{M(v)=M(v_0)=\frac{8\pi v_0^2}{c^3}}$$

平均光子数$\bar{n}=n/M(v)$

辐射引起的光子变化

$$\frac{d\bar{n}}{dt}=\frac{dn}{M(v)\,dt}=\frac{1}{M(v)}\left[(1+\bar{n})\frac{1}{t_{sp}}N_2-\bar{n}\frac{1}{t_{sp}}N_1\right]$$

光子透射衰减$\bar{n}$: $\frac{d\bar{n}}{dt}=-\bar{n}\frac{1}{\mathrm{t_p}}$

光子平均数: $\frac{d\bar{n}}{dt}=\frac{1}{M(v)}\left[(1+\bar{n})\frac{1}{t_{sp}}N_2-\bar{n}\frac{1}{t_{sp}}N_1\right]-\bar{n}\frac{1}{\mathrm{t_p}}$

$$=\frac{c^3\Delta v}{4v_0^2}[(1+\bar{n})N_2-\bar{n}N_1]-\bar{n}\frac{1}{\mathrm{t_p}}$$

5.1

一个腔长为100cm的氩离子激光器，其折射率n=1

（a）求谐振腔腔模的频率间隔

（b）如果多普勒增宽线宽的半高全宽$\Delta v_D = 3.5GHz$，损耗系数为小信号增益系数峰值的一半，求此激光器可以允许的纵模数。

（c）如果此激光器要以单纵模运转，则谐振腔的腔长d应该为多少？CO2激光器的多普勒线宽$\Delta v_D = 60MHz$，远小于氩离子激光器，在其他条件相同的情况下，CO2激光器的腔长为多少时才能单纵模运转。

（a）$v_F = \frac{c}{2nd} = 0.15GHz$

(b)

因为损耗系数是小信号增益系数峰值的一半，

所以满足增益大于损耗的带宽$\Delta v = \Delta v_D$

纵模模式数$M = \frac{\Delta v}{v_F} = 23$

(c)

在（b）的条件下，单纵模运转的条件为

$$v_F = c/2nd > \Delta v \quad \Rightarrow \quad d < \frac{c}{2n\Delta v} = 4.286\text{cm}$$

对于CO2激光器，$\Delta v = \Delta v_D = 60\text{MHz}$

$$v_F = c/2nd > \Delta v \quad \Rightarrow \quad d < \frac{c}{2n\Delta v} = 2.5\text{m}$$

5.2

一He-Ne激光器参数如下：
（a）谐振腔的量腔镜反射率分别为97%和100%，忽略腔内损耗；
（b）原子跃迁多普勒增宽的线宽$\Delta v_D = 1.5GHz$
（c）小信号增益系数的峰值$\gamma_0(v_0) = 2.5 * 10^{-3} cm^{-1}$
激光器运行时，热效应会导致谐振腔腔长抖动，因此纵模的频率会随时间变化而产生漂移。要求纵模数保持为1或者2（但是不超过2），求此时腔长的允许范围。

$$a = \frac{1}{2d} \ln\left(\frac{1}{R_1 R_2}\right)$$

$$\gamma_0(v) = \gamma_0(v_0) * \frac{\left(\frac{\Delta v_D}{2}\right)^2}{(v - v_0)^2 + \left(\frac{\Delta v_D}{2}\right)^2}$$

令$\gamma_0(v) = a$，则$v = v_0 \pm \frac{\Delta v_D}{2}\sqrt{\frac{2d\gamma_0(v_0)}{\ln\left(\frac{1}{R_1R_2}\right)} - 1}$ ➡ $\Delta v = \Delta v_D \sqrt{\frac{2d\gamma_0(v_0)}{\ln\left(\frac{1}{R_1R_2}\right)} - 1}$

$$v_F < \Delta v < 2v_F$$

$$v_F = \frac{c}{2d}$$

➡ $0.111m < d < 0.158m$

5.3

一多普勒增宽的气体激光器运行波长为515nm，其谐振腔两腔镜之间距离为50cm，光子寿命为0.33ns，可以谐振的频率窗口带宽B=1.5GHz，折射率n=1。为了选择某一纵模，要使光穿过标准具（无源FP谐振器），标准具两腔镜之间距离为d，锐度为F，标准具相当于一个滤波器。请给出合适的d与F，标准具放在谐振腔内好还是谐振腔外好?

谐振腔的$v_F = \frac{c}{2D} = 0.3GHz$

标准具的$v_F' = \frac{c}{2d} = \mathrm{B}$ ➡ $d = 0.1m$

标准具的$\Delta v' = \frac{v_F'}{F} < v_\mathrm{F}$ ➡ $F > 5$

放在谐振腔内好，这样的话，只有被标准具选出的频率才会饱和，而其他模式不饱和

5.4

一频率为$\lambda_0 = 632.8nm$的He-Ne激光器多模输出为50mW。此激光器的非均匀谱宽，多普勒线宽$\Delta v_D = 1.5GHz$，折射率n=1，谐振腔长度为30cm。

（a）如果小信号增益系数的最大值为损耗系数的两倍，求此激光器的纵模数

（b）如果调整了激光器的腔镜使最强模式的功率最大，计算其功率

(a)

$$\left.\begin{aligned} B &= \Delta v_D = 1.5GHz \\ v_F &= \frac{c}{2nd} = 0.5GHz \end{aligned}\right\} \quad M = \frac{B}{v_F} = 3$$

(b)

$$\text{增益系数}\gamma(\nu) = \gamma_0(\nu_0) * \exp\left[-\left(\frac{\nu-\nu_0}{\sqrt{2}\sigma_D}\right)^2\right]$$

$$\gamma_0(v_0 + v_F) = \gamma_0(v_0 - v_F) = 0.7349\gamma_0(v_0)$$

$$P = \frac{1}{0.7349 * 2 + 1} P_0 = 20.2446mW$$

5.5

一谐振腔长为10cm的气体激光器，以单横模和单纵模运转在600nm。两腔镜反射率分别为$R_1=99\%$，$R_2=100\%$。折射率n=1，输出光束的截面有效面积为$1mm^2$。小信号增益系数$\gamma_0(v_0)=0.1cm^{-1}$，饱和光子流密度$\varphi_s=1.43*10^{19}cm^{-2}s^{-1}$。

（a）分别求出两腔镜的损耗系数α_{m1}，α_{m2},假设α_s =0，求谐振腔的损耗系数α_r

（b）求光子的寿命t_p

（c）求输出光子流密度φ_0及输出功率P_0

(a)

$$\left.\begin{aligned}\alpha_{m1}&=\frac{1}{2d}\ln\left(\frac{1}{R_1}\right)=0.05m^{-1}\\ \alpha_{m2}&=\frac{1}{2d}\ln\left(\frac{1}{R_2}\right)=0\end{aligned}\right\}\quad \alpha_r=\alpha_{m1}+\alpha_{m2}+\alpha_s=0.05m^{-1}$$

(b)

$$t_p=1/(c\alpha_r)=6.67*10^{-8}s$$

(c)

$$\varphi=\varphi_s\left(\frac{\gamma_0(v_0)}{\alpha_r}-1\right)=2.8457*10^{21}cm^{-2}s^{-1}$$

$$\varphi_0=(1-R_1)*\frac{\varphi}{2}=1.423*10^{19}cm^{-2}s^{-1}$$

$$P_0=\varphi_0*hc/\lambda=0.047W$$

5.6

氩离子激光器腔长1m,反射率分别为98%， 100%，跃迁中心波长为515nm， 自发辐射寿命 tsp=10ns， 线宽△λ=0.003nm， 谐振模直径为1mm。求光子寿命和产生激光所需的粒子数差 阈值

$$(1)\begin{cases} \alpha_r = \frac{1}{2d}\ln(\frac{1}{R_1 R_2}) \\ d = 1 \\ R_1 = 98\%, R_2 = 100\% \end{cases} \qquad \therefore \alpha_r = 0.01m^{-1}$$

$$\tau_p = \frac{1}{c\alpha_r} = 3.3\times 10^{-7}s$$

$$(2)\ \Delta\nu = \frac{c}{\lambda^2}\Delta\lambda = 3.4\times 10^9 Hz$$

$$\text{粒子数差阈值}N_t = \frac{2\pi}{\lambda^2 c}\cdot\frac{2\pi\Delta\nu t_{sp}}{\tau_p} = 5.11\times 10^{13}$$

- P165 5.7

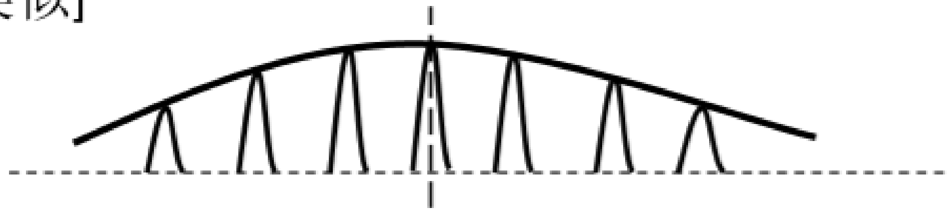
光透过未泵浦的气体激光器的谐振腔——代公式

$$(1)\ \mathrm{v}_F = 200\mathrm{MHz},\ \text{故腔长}d = \frac{c}{2n\mathrm{v}_F} = 75cm$$

$$\delta\mathrm{v} = 2\mathrm{MHz},\ \text{故光子寿命}\tau_p = \frac{1}{2\pi\delta\mathrm{v}} = 79.6ns$$

$$\text{阈值时的增益系数}\alpha_r = \frac{1}{c\tau_p} = 0.0418m^{-1}$$

当气体激光器加上一个小于阈值的泵浦且中心波长为5x10^{14}Hz时，则谐振腔内不同纵模的光在谐振腔内的损耗不同，由于中心波长在5x10^{14}Hz，故示意图如下图所示，虚竖线处对应中心频率[和增益分布类似]



- **P165 5.8**

4能级系统的速率方程

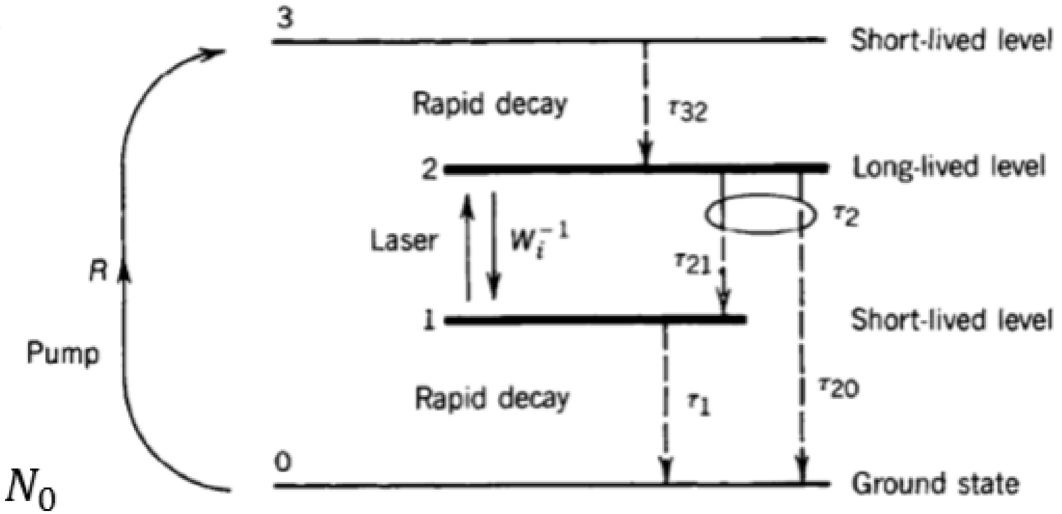
题意：求四能级系统的激光N_2，N_1，反转粒子数N和光子数n的速率方程并求稳态下的反转粒子数与光子数的值，已知量为无受激吸收和辐射时的反转粒子数N_0，自发辐射寿命t_{sp}，能级寿命τ_1,τ_2,τ_{21},光子寿命τ_p等等

方法：列速率方程，以已知量替代未知量即可,这里假设泵浦较弱

无辐射跃迁时，只考虑能级寿命，且 $R_1 \approx 0$ ：

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1}$$



稳定时 $\frac{dN_2}{dt}$=0，$\frac{dN_1}{dt}$=0

N_0=N_2-N_1 $= R_2\tau_2\left(1-\frac{\tau_1}{\tau_{21}}\right)$ ⇒ $R_2 = \dfrac{N_0}{\tau_2\left(1-\frac{\tau_1}{\tau_{21}}\right)}$

有辐射跃迁时，考虑受激辐射概率W_i，：

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2W_i + N_1W_i$$

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1} + N_2W_i - N_1W_i$$

反转粒子数的速率方程为: $\frac{dN}{dt} = \frac{d(N_2-N_1)}{dt} = R_2 - N_2(\frac{1}{\tau_{21}} + \frac{1}{\tau_2} + 2W_i) + N_1(\frac{1}{\tau_1} + 2W_i)$

光子数的速率方程 $\frac{dn}{dt} = N_2W_i - N_1W_i - \frac{n}{\tau_p}$, $\frac{n}{\tau_p}$为光子密度损失率

稳态时，利用$N = \frac{N_0}{1+t_{sp}W_i}$, $N = \frac{n}{\tau_p W_i} = \frac{1}{c\tau_p\sigma(\nu)}$

故有$n = \frac{\tau_p N_0}{t_{sp}} - \frac{1}{c\sigma(\nu)t_{sp}}$

附加题：一个三能级系统，E1是基态，泵浦光频率与E1和E3之能级跃迁相对应，其跃迁几率W13=W31=Wp。能级E3的寿命较长t3，E2能级寿命较短t2，E3到E2的跃迁几率为1/t32，求：

E3，E2之间形成粒子数反转的条件

E3，E2之间粒子数反转密度与跃迁几率Wp的关系

泵浦极强时，E3，E2之间的粒子数反转密度（E3，E2之间的受激辐射可以忽略）

写出速率方程

(a) $$\frac{dN_3}{dt} = (N_1 - N_3)W_p - \frac{N_3}{t_3} = 0 \qquad (1)$$

$$\frac{dN_2}{dt} = \frac{N_3}{t_{32}} - \frac{N_2}{t_2} = 0 \qquad (2)$$

$$N_1 + N_2 + N_3 = N \qquad (3)$$

由(2)得 $$\Delta N = N_3 - N_2 = N_3\left(1 - \frac{t_2}{t_{32}}\right) > 0$$

$$t_{32} > t_2$$

(b) 由(1)(2)(3)整理可得

$$N_3 = \frac{NW_p t_3}{1 + W_p t_3 (2 + \frac{t_2}{t_{32}})}$$

$$\Delta N = N_3 \left(1 - \frac{t_2}{t_{32}}\right) = \frac{NW_p t_3 \left(1 - \frac{t_2}{t_{32}}\right)}{1 + W_p t_3 (2 + \frac{t_2}{t_{32}})}$$

(c) 泵浦极强时， $W_p t_3 \gg 1$

$$\Delta N = \frac{N(t_{32} - t_2)}{2t_{32} + t_2}$$