一、麦克斯韦方程组

微分形式:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{B} = 0$$

积分形式:

$$\oint_C \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}$$

$$\oint_C \vec{H}\cdot d\vec{l} = I_c + \int_S \frac{\partial\vec{D}}{\partial t}\cdot d\vec{S}$$

$$\oint_S \vec{D}\cdot d\vec{S} = Q$$

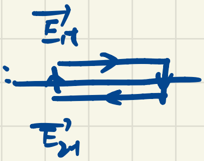
$$\oint_S \vec{B}\cdot d\vec{S} = 0$$

二、边界条件:

1. 基础

$$\begin{cases} \vec{E_{1t}} = \vec{E_{2t}} & ① \\ \vec{B_{1n}} = \vec{B_{2n}} & ② \\ \vec{a_{n2}}\cdot(\vec{D_{1n}} - \vec{D_{2n}}) = \rho_s & ③ \\ \vec{a_{n2}}\times(\vec{H_{1t}} - \vec{H_{2t}}) = \vec{J_s} & ④ \end{cases}$$

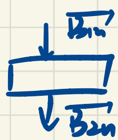
推导: ①:



$$\int \vec{E}\, d\vec{l} = 0 \Rightarrow \vec{E_{1t}}\cdot\Delta l - \vec{E_{2t}}\cdot\Delta l = 0$$

$$\vec{E_{1t}} = \vec{E_{2t}}$$

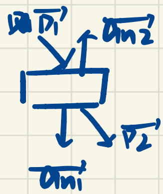
②:



$$\oint \vec{B}\cdot d\vec{S} = 0$$

$$\vec{B_{1n}} = \vec{B_{2n}}$$

③:



$$\oint \vec{D}\cdot d\vec{S} = Q$$

$$\Rightarrow (\vec{D_1}\cdot\vec{a_{n1}} + \vec{D_2}\cdot\vec{a_{n2}})\Delta S = \rho_s\cdot\Delta S$$

2. 两种无损线性材料分界面

$$\begin{cases} \vec{E_{1t}} = \vec{E_{2t}} \\ \vec{B_{1n}} = \vec{B_{2n}} \\ \vec{D_{1n}} = \vec{D_{2n}} \\ \vec{H_{1t}} = \vec{H_{2t}} \end{cases}$$

(无表面 ρ_s, $\vec{J_s}$)

3. 绝缘材料和理想导体分界面 $\to\sigma\to\infty$

$\vec{E_2}=\vec{0}$ （防止 $\vec{J}\to\infty$）

$$\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}\Rightarrow\vec{H}=\vec{0}$$

$$\therefore\begin{cases}\vec{E_{1t}}=\vec{0}\\ \vec{B_{1n}}=0\\ \vec{a_{2n}}\cdot\vec{D_{1n}}=\rho_s\\ \vec{a_{2n}}\times\vec{H_{1n}}=\vec{J_s}\end{cases}$$

$$\begin{cases}\vec{B}=\nabla\times\vec{A}\\ \vec{E}=-\nabla\vec{V}-\underbrace{\frac{\partial\vec{A}}{\partial t}}_{\text{稳恒场中无这一项}}\end{cases}$$

二、势函数

1、波动方程导出

对于时变场而言

由 $\vec{B}=\nabla\times\vec{A}$

有 $\nabla\times\vec{E}=-\frac{\partial}{\partial t}(\nabla\times\vec{A})$

$\Rightarrow\nabla\times(\vec{E}+\frac{\partial}{\partial t}\vec{A})=\vec{0}$

$\because\nabla\times\nabla V\equiv 0$

且在静电场中 $-\nabla V=\vec{E}$

$\therefore\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\nabla V$

$\because\vec{E}=-\nabla V-\frac{\partial\vec{A}}{\partial t}$

在静态、准静态中

$$V=\frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho}{R}dV$$

$$A=\frac{\mu_0}{4\pi}\int_V\frac{1}{R}dV$$

时变场中，$\vec{E}$与V、A两种势同时相关

势⇒场

接下来得出 V, A 满足方程:

$$\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

$$\nabla\times(\nabla\times\vec{A})=\mu\vec{J}+\mu\varepsilon\frac{\partial\vec{E}}{\partial t}$$

左边: $\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$

右边: $\mu\vec{J}+\mu\varepsilon\frac{\partial}{\partial t}\left(-\nabla V-\frac{\partial\vec{A}}{\partial t}\right)$

$$\Rightarrow\quad \nabla^2\vec{A}+\mu\vec{J}-\mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=\nabla\left(\nabla\cdot\vec{A}+\mu\varepsilon\frac{\partial V}{\partial t}\right)$$

洛仑兹规范: $\nabla\cdot\vec{A}+\mu\varepsilon\frac{\partial V}{\partial t}=0$

$\therefore\ \nabla^2\vec{A}-\mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=-\mu\vec{J}$

类似地有:

$$\nabla^2 V-\mu\varepsilon\frac{\partial^2 V}{\partial t^2}=-\frac{\rho}{\varepsilon}$$

至此, 波动方程已导出

$$\begin{cases}\nabla^2 V-\mu\varepsilon\frac{\partial^2 V}{\partial t^2}=-\frac{\rho}{\varepsilon}\\ \nabla^2\vec{A}-\mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=-\mu\vec{J}\end{cases}$$

2. 波动方程的解

考虑 $\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$

若在球坐标系下，且脱离有源区

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial V}{\partial R}\right) - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = 0$$

令 $V(R,t) = \frac{1}{R} U(R,t)$

$$\frac{\partial^2 U}{\partial R^2} - \mu\varepsilon \frac{\partial^2 U}{\partial t^2} = 0$$

$$U = f(t - R\sqrt{\mu\varepsilon})$$

$\because V = \frac{1}{R} f(t - R\sqrt{\mu\varepsilon})$ $\quad u = \frac{1}{\sqrt{\mu\varepsilon}}$

$\therefore V = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho(t - R/u)}{R} dV$

类似地

$$A = \frac{\mu_0}{4\pi}\int_V \frac{J(t - R/u)}{R} dV$$

四、无源波方程

无源：$\rho=0$，$J=0$

考虑一般的 $\sigma=0$ 材料.

则 Maxwell 方程组化为

$$\nabla\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t} \quad ①$$

$$\nabla\times\vec{H}=\varepsilon\frac{\partial\vec{E}}{\partial t} \quad ②$$

$$\nabla\cdot\vec{D}=0 \quad ③$$

$$\nabla\cdot\vec{H}=0 \quad ④$$

对①式两边求旋度

$$\Rightarrow \nabla\times\nabla\times\vec{E}=-\mu\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$=-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}=-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

代入③

$$\Rightarrow \nabla^2\vec{E}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=0$$

令 $u=\frac{1}{\sqrt{\mu\varepsilon}}$ $\Rightarrow$ $\boxed{\nabla^2\vec{E}-\frac{1}{u^2}\frac{\partial^2\vec{E}}{\partial t^2}=0}$ 会考推导!!

同理：$\boxed{\nabla^2\vec{H}-\frac{1}{u^2}\frac{\partial^2\vec{H}}{\partial t^2}=0}$

五：时谐场

$$E(x,y,z,t) = E(x,y,z)e^{j\omega t}$$

$$H(x,y,z,t) = H(x,y,z)e^{j\omega t}$$

$\Rightarrow$

Maxwell Eq:

$$\begin{cases} \nabla\times\vec{E} = -j\omega\mu H \\ \nabla\times\vec{H} = \vec{J} + j\omega\varepsilon\vec{E} \\ \nabla\cdot\vec{E} = \frac{\rho}{\varepsilon} \\ \nabla\cdot\vec{H} = 0 \end{cases}$$

若 $\sigma=0$, $\rho=0$, $J=0$

则 $\nabla\times\vec{H} = j\omega\varepsilon\vec{E}$

波动方程：

$$\nabla^2 V + k^2 V = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 A + k^2 A = -\mu J$$

$k = \omega\sqrt{\mu\varepsilon}$

$\varepsilon'' = \frac{\sigma}{\omega}$　$\varepsilon' = \varepsilon$

损耗正切：$\varepsilon_c = \varepsilon' - j\varepsilon''$

$$\tan\delta_c = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$$

反应传导电流与位移电流之比

$\sigma \gg \omega\varepsilon \leftrightarrow$ 良好导体

$\sigma \ll \omega\varepsilon \leftrightarrow$ 良好绝缘体

可据此改写 Maxwell 方程组