MATH1103 FALL 2022
EXAM 1

WEDNESDAY, OCTOBER 19, 2022

Name: _____

This exam is open notes, but calculators are not allowed. There are 50 points total in this exam.

Problem 1. Integrals. If you are using a result from class/homework/discussion, make sure you state it clearly.

(a) (2 points) Calculate $\int_{-\pi}^{\pi} \sin x \, dx$.

(b) (2 points) Calculate the indefinite integral $\int \frac{e^x}{e^x + 1} \, dx$.

(c) (2 points) Calculate $\int_1^3 \ln x \, dx$.

(d) (2 points) Calculate $\int_{-1}^{1} (x^5 + \sin(x^3)) \, dx$.

(e) (2 points) For which values of x is $\int_1^x \left(\frac{1}{|t|} + e^{t^2} \right) dt$ a well-defined number?

Problem 2. Let P be the paraboloid formed by rotating the region bounded by $y = \frac{1}{2}x^2$, $x = 0$, and $y = 2$ around the y-axis.

(a) (5 points) What is the volume of P?

Hint: I found the disk method the easiest here.

(b) (5 points) Show that the lateral surface area of P is $\frac{2}{3}\pi(5\sqrt{5} - 1)$. (Lateral just means not including the top lid portion.)

Problem 3. A dartboard has the shape of a circle of radius 1. A dart hits a random point in the circle, where by random we mean that the probability that the dart lands in any region R inside the circle is equal to the area of R divided by the area of the circle.

(a) (5 points) Let X be the random variable representing the dart's distance from the center. For any r between 0 and 1, what is the probability that $X \leq r$?

(b) (5 points) What is the expected distance of the dart from the center?

Problem 4 (10 points)**.** Find the variance of the exponential distribution given by $p(x) = ae^{-ax}$ for $x \geq 0$ (and 0 for $x < 0$). You may use the fact that $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ for any random variable X, and the fact the mean of this exponential distribution is $1/a$.

Problem 5 (10 points)**.** Let f be a continuous function and suppose that $f(x) = f(-x)$ for all x. Prove algebraically (not graphically) that

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

for all a.