MATH1103 FALL 2022
DISCUSSION SHEET 9

Problem 1. Hard problems on Problem Set 07.

(1) **2(c) on PS07**: Use the result that $r^n \to 0$, for $0 < r < 1$ to prove that $r^n \to 0$, for $-1 < r < 0$.

(2) **3(a) on PS07**: Use the fact that $r^n \to 0$ for any $0 < r < 1$ to prove that for any $\epsilon > 0$, there exists a positive integer n such that $10^{-n} < \epsilon$.

Problem 2. *(Application of geometric series).* Prove that the harmonic series given by

$$s = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$$

diverges.

Problem 3. Walk yourself through the proof of $1 = .999\cdots$ with both the $\epsilon-$lemma method(Problem 3 on PS07) and the geometric series method(Problem 5 on PS08).