MATH1103 FALL 2022
PROBLEM SET 1

This problem set is due on Wednesday, September 7 at 11:59 pm. Each problem part is worth 3 points. Collaboration is encouraged. In all cases, you must write your own solutions, and and you must cite collaborators and resources used.

Problem 1. Algebra, functions, and differential calculus review and practice.

(a) Find a clever and simple way to evaluate $1002 \cdot 998$.

(b) Derive a formula for $1 + 2 + \cdots + n$, the sum of the first n positive integers. Try to think of as short a derivation as possible.

(c) What is $\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$?

You do not have to prove this continued fraction converges; just use algebra to find its value.

(d) What is the coefficient of xyz in $(x + y + z)^3$?

(e) Find three different real-valued functions f such that $(f(x))^2 = x^2$ for all $x \in \mathbb{R}$.

(f) State the intermediate value theorem as precisely as possible, but in your own words.

(g) Find a formula, in terms of the positive integer n, for the nth derivative of $\ln(x)$.

Problem 2. Using only Euclidean geometry (no trigonometry), determine the area of a regular 12-sided polygon inscribed in a unit circle (i.e. a circle of radius 1). You should be able to get an exact answer. How close is the area to π?

Problem 3. Use the method of Riemann sums with 20 equal divisions to approximate π, using the function $f(x) = \sqrt{1 - x^2}$. Use a calculator or computer to help you with all the additions! (Excel or Google Sheets should be helpful here.)

Problem 4. Find a general (possibly piecewise) expression for

$$\int_a^b |x| \, dx$$

in terms of the two real numbers a, b (which can each be positive, negative, or zero!).

Problem 5. For any sequence $\underline{a} = (a_1, a_2, a_3, \dots)$ of real numbers, define the *finite difference operator* Δ by

$$\Delta(\underline{a}) = (a_2 - a_1, a_3 - a_2, a_4 - a_3, \dots).$$

For example, $\Delta(1, 2, 3, 4, \dots) = (1, 1, 1, 1, \dots)$. Also define the *cumulative sum operator* Σ by

$$\Sigma(\underline{a}) = (a_1, a_1 + a_2, a_1 + a_2 + a_3, \dots).$$

For example, $\Sigma(1, 2, 3, 4, \dots) = (1, 3, 6, 10, \dots)$. In general, what is $\Delta(\Sigma(\underline{a}))$? (If you aren't sure, try on a few examples of your own.) Once you find a result, give a proof that your result holds.

Remark: This is the discrete analog of the fundamental theorem of calculus! Finite differences are the discrete version of derivatives, and cumulative sums are the discrete version of integrals.