MATH1103 FALL 2022
DISCUSSION SHEET 8

Problem 1. Recall that a sequence $\{a_n\}$ converges to L if for any $\epsilon > 0$, there exists a positive integer N such that

$$|a_n - L| < \epsilon, \text{ for all } n \geq N$$

Then show that $a_n = \frac{1}{n^2} \to 0$ by the $\epsilon-$definition of the convergence of a sequence.

Problem 2. During the class yesterday, you learned that if $\{x_n\} \to x$ and $\{y_n\} \to y$, then $\{x_n + y_n\} \to x + y$ and $\{x_n y_n\} \to xy$. Similarly, we can ask questions as follows: If $\{x_n\}$ and $\{y_n\}$ both diverge, then must $\{x_n + y_n\}$ and $\{x_n y_n\}$ diverge as well? If yes, prove it! If not, give counterexamples!
Hint: Can you give some examples of divergent sequences?

Problem 3. *(Geometric series)* Let's work with our old friend again! Let a be any real number, and r be a positive real number.

(1) Consider the finite series as follows.

$$s_n = a + ar + ar^2 + ar^3 + \cdots + ar^n$$

How can you calculate such a sum?

(2) Then let's consider the infinite series as follows.

$$s = a + ar + ar^2 + ar^3 + \cdots$$

For which $r's$ is this series convergent? For the convergent case, how can you calculate s? Can you use some similar argument to the first part?

(3) Prove that $1 = .999\cdots$ following the idea of geometric series.