

MATH2211 SPRING 2022
EXAM 1

WEDNESDAY, MARCH 2 2022

Name: _____

This exam is open notes. There are 50 points total in this exam.

Problem 1.

- (a) (5 points) Find, with proof, the dimension of

$$V := \text{span}\{(1, 1, 1), (2, 2, 2), (3, 3, 3)\} \subseteq \mathbb{R}^3.$$

- (b) (5 points) Let $P_2(\mathbb{R})$ be the space of polynomials of degree at most 2. Write down the matrix of the derivative operator $\frac{d}{dx}$ from $P_2(\mathbb{R})$ to $P_2(\mathbb{R})$ with respect to the ordered basis $(1, x, x^2)$ for both the source and target.

Problem 2. (10 points) Find, with proof, all complex numbers $z \in \mathbb{C}$ such that z and z^2 are linearly independent over \mathbb{R} .

Problem 3. (10 points) Prove or disprove: Let v_1, v_2, v_3 be a basis of a real vector space V . Then the set $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$ is also a basis of V .

Problem 4. (10 points) Let $T: V \rightarrow W$ be a linear transformation between two finite-dimensional \mathbb{R} -vector spaces. Prove that if T is injective, then $\dim V \leq \dim W$.

Problem 5. (10 points) Let \mathbb{R}^∞ denote the space of infinite sequences of real numbers. For each $n \in \mathbb{Z}^+$, let v_n be the element $(n, n+1, n+2, n+3, \dots) \in \mathbb{R}^\infty$. Prove that $\text{span}(v_1, v_2, v_3, \dots)$ is finite dimensional and determine its dimension.