MATH2211 SPRING 2022
EXAM 2

WEDNESDAY, APRIL 20 2022

Name: ______________________________

This exam is open notes. There are 50 points total in this exam.

Problem 1. Let $A = \begin{pmatrix} -1 & 2 \\ 3 & -1 \end{pmatrix}$.

(a) (5 points) Find an elementary matrix E such that EA is upper triangular.

(b) (5 points) Give a basis of $\bigwedge^2 \mathbb{R}^2$ and write the matrix of the linear operator

$$\bigwedge^2 A \colon \bigwedge^2 \mathbb{R}^2 \to \bigwedge^2 \mathbb{R}^2$$

in this basis.

Problem 2. (10 points) The *right shift* operator $T\colon \mathbb{R}^\infty \to \mathbb{R}^\infty$ is defined by

$$T(a_0, a_1, a_2, \dots) = (0, a_0, a_1, a_2, \dots).$$

What are the eigenvalues and eigenvectors of T?

Problem 3. (10 points) Prove that if $T\colon V \to V$ is diagonalizable, then $T^2 + T + I_V$ is also diagonalizable.

Hint: Eigenvectors.

Problem 4. Let A be a 50×100 real matrix whose kernel is 74-dimensional.

(a) (5 points) What is the dimension of the space of row vectors y such that $yA = 0$?

(b) (5 points) Suppose you know that the vector $\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{R}^{100}$ is in the kernel of A. This implies that every row vector $(y_1\ y_2\ \cdots\ y_{100})$ in the image of ${}^tA$ satisfies some condition. What is this condition? Prove your answer.

Problem 5. (10 points) Let $M = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}$. For $n \geq 0$, let a_n be the top left entry of M^n. Prove that

$$\lim_{n\to\infty} \frac{a_n}{(1+\sqrt{3})^n} = \frac{1}{2}.$$