MATH2211 SPRING 2022
EXAM 1

WEDNESDAY, MARCH 2 2022

Name: ______________________________

This exam is open notes. There are 50 points total in this exam.

High 49.5/50, low 20/50, mean 35.2/50, median 36.5/50. Good exam. Problems 2 and 5 were the most difficult, with average points obtained on them being 59% and 58% respectively. Every problem was approachable, and everyone had enough time to attempt every problem.

Problem 1.

(a) (5 points) Find, with proof, the dimension of

$$V \coloneqq \operatorname{span}\{(1,1,1),(2,2,2),(3,3,3)\} \subseteq \mathbb{R}^3.$$

(b) (5 points) Let $P_2(\mathbb{R})$ be the space of polynomials of degree at most 2. Write down the matrix of the derivative operator $\frac{d}{dx}$ from $P_2(\mathbb{R})$ to $P_2(\mathbb{R})$ with respect to the ordered basis $(1, x, x^2)$ for both the source and target.

Problem 2. (10 points) Find, with proof, all complex numbers $z \in \mathbb{C}$ such that z and z^2 are linearly independent over $\mathbb{R}$.

Problem 3. (10 points) Prove or disprove: Let v_1, v_2, v_3 be a basis of a real vector space V. Then the set $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$ is also a basis of V.

Problem 4. (10 points) Let $T\colon V \to W$ be a linear transformation between two finite-dimensional $\mathbb{R}$-vector spaces. Prove that if T is injective, then $\dim V \leq \dim W$.

Problem 5. (10 points) Let $\mathbb{R}^\infty$ denote the space of infinite sequences of real numbers. For each $n \in \mathbb{Z}^+$, let v_n be the element $(n, n+1, n+2, n+3, \dots) \in \mathbb{R}^\infty$. Prove that $\operatorname{span}(v_1, v_2, v_3, \dots)$ is finite dimensional and determine its dimension.