MATH2211 SPRING 2022
PROBLEM SET 1

This problem set is due on Wednesday, January 26 at 11:59 pm. Each problem part is worth 3 points.

Problem 1. Let S be a set and let $A, B, C \subseteq S$. Prove that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

Problem 2. Show that the set of real numbers of the form $a + b\sqrt{2}$, where $a, b \in \mathbb{Q}$, with addition and multiplication as in $\mathbb{R}$, is a field.

Problem 3. Define the Fibonacci numbers $F_0, F_1, F_2, \ldots$ by $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

(a) Use induction to prove that $F_1 + \cdots + F_n = F_{n-2} - 1$.

(b) Use induction to prove that $F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$.

Problem 4. Prove that for every $n \in \mathbb{Z}^+$

$$\frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}.$$

Problem 5. Suppose that $z_1, z_1 \in \mathbb{C}$. Prove that

(a) $|z_1 z_2| = |z_1| \cdot |z_2|$.

(b) $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$.

Problem 6.

(a) Find all complex solutions to $z^4 = -1$.

(b) Find all complex solutions to $z^3 = i$.

Express your answers both in the form $re^{i\theta}$ and by giving the real and imaginary parts.

Problem 7. Suppose $z_1, z_2 \in \mathbb{C}$. Prove the *triangle inequality*

$$|z_1 + z_2| \leq |z_1| + |z_2|.$$