MATH4460 SPRING 2023
PROBLEM SET 3

This problem set is due on Wednesday, February 15 at 11:59 pm. Each problem part is worth 3 points. Collaboration is encouraged. In all cases, you must write your own solutions, and and you must cite collaborators and resources used.

Problem 1. If $f(z) = \sum_{n\geq 0} a_n z^n$, what is $\sum_{n\geq 0} n^3 a_n z^n$?

Problem 2. Prove that

$$\sum_{n=0}^{\infty} \frac{\cos n}{2^n} = \frac{2\cos(1) - 4}{4\cos(1) - 5}.$$

Problem 3.

(a) A famous "result" you may have seen on the web is that $i^i = e^{-\pi/2}$, which is remarkable because the right hand side is a real number approximately equal to $0.20787957635\ldots$. There is something inaccurate about that equation, however. Correct it.

(b) What are all the values of 2^i? $(-1)^{2i}$?

(c) Determine the real and imaginary parts of z^z.

(d) Determine the real and imaginary parts of $\exp(e^z) = e^{e^z}$.

Problem 4. For which values of z is e^z equal to 2? What about $1 + 2i$?

Problem 5. Express $\arctan w$ in terms of the logarithm.[1]

Problem 6. Prove that the nth roots of unity are indeed the vertices of a regular polygon. Do this by showing that the side lengths are equal and that the angles are equal.

A hint about side lengths: the distance between two complex numbers z and w is $|z - w|$.

Problem 7. This is a space to reflect on something about this problem set. You can mention if you found any problems particularly difficult, or particularly easy. You can also mention problems you liked, or problems that took a long time, etc. (Please write something here to get credit!)

[1] The following may be useful: $\tan z = \sin(z)/\cos(z)$ and there are exponential expressions for $\sin(z)$ and $\cos(z)$.