ECES-352
Winter 2019
Homework #4

Reading: Chapter 3 on Spectrum Representation and beginning of Chapter 4 on Sampling.

PROBLEM 4.1:

A periodic signal is represented by the Fourier Series synthesis formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j30\pi kt} \qquad \text{where} \quad a_k = \begin{cases} \dfrac{1}{4+j2k} & \text{for } k = -3,-2,-1,0,1,2,3 \\ 0 & \text{for } |k| > 3 \end{cases}$$

(a) Sketch the two-sided spectrum of this signal. Label all complex amplitudes in **polar form.**

(b) Determine the fundamental frequency (in Hz) and the fundamental period (in secs.) of this signal.

PROBLEM 4.2*:

Let $x(t)$ be the periodic signal

$$x(t) = \Big[3 + 5\cos(250\pi t - 0.25\pi)\Big] \cos(400\pi t)$$

(a) What is the fundamental frequency of $x(t)$?

(b) A periodic signal may be expanded in a Fourier series expansion as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Find the Fourier series coefficients a_k for the signal above.

(c) Plot the coefficients a_k versus k. Note that you should be able to do this without evaluting any integrals.

PROBLEM 4.3*:

A signal $x(t)$ is periodic with period $T_0 = 12$. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/12)kt}.$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{12} \int_{-6}^{0} (4+t) e^{-j(2\pi/12)kt} dt. \tag{1}$$

(a) In the expression for a_k in Equation (1) above, the integral and its limits define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-12 \leq t \leq 12$ seconds. Label it carefully.

(c) Determine a_0, the DC value of $x(t)$.

PROBLEM 4.4*:

A periodic signal $x(t)$ is described over one period $0 \leq t \leq 4$ by the equation

$$x(t) = e^{-2t} \qquad 0 \leq t < 4.$$

The period of this signal is $T_0 = 4$ sec. We have seen that such a periodic signal can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt$$

with $\omega_0 = 2\pi/T_0$.

(a) Sketch the periodic function $x(t)$ for $-5 < t < 5$.

(b) Determine a_0, the D.C. coefficient for the Fourier series.

(c) Set up the Fourier $\boxed{\textit{analysis}}$ integral for determining a_k for $k \neq 0$. (Insert proper limits and integrand.)

(d) Evaluate the integral in part (c) and obtain an expression for a_k that is valid for all $k \neq 0$.

(e) Make a plot of the spectrum over the range $-3f_0 \leq f \leq 3f_0$ where f_0 is the fundamental frequency of the signal. Use your calculator to determine the complex numerical values (polar form) for each of the Fourier coefficients corresponding to this range of frequencies.

PROBLEM 4.5

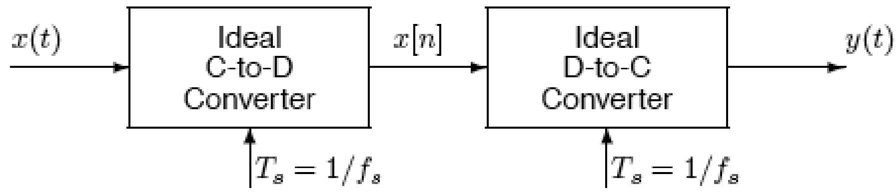


Figure 1: Ideal sampling and reconstruction system.

Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

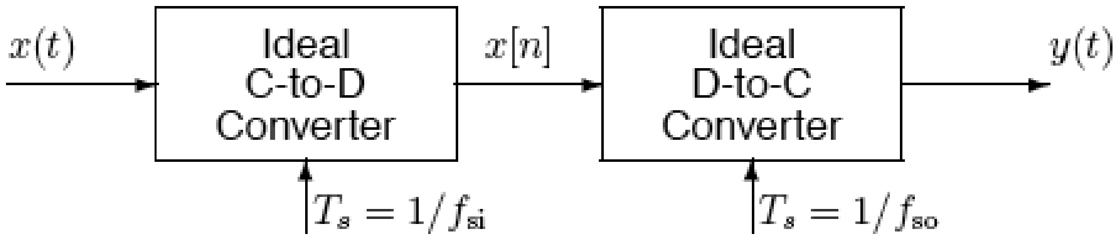
$$x(t) = [5 + 15\cos(400\pi t + \pi/2)]\cos(2000\pi t - \pi/4).$$

(a) Use Euler's formulas for the cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the *minimum* sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 4000$. Your plot should include labels on the frequency (on the $\hat{\omega}$ scale), amplitude and phase of each spectrum component.

PROBLEM 4.6



(a) Suppose that the input $x(t)$ is given by

$$x(t) = 15 + 8\cos(2\pi(2000)t - \pi) + 4\cos(2\pi(7000)t + 3\pi/4)$$

Determine the spectrum for $x[n]$ when $f_{\text{si}} = 10000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, making sure to label the frequency, amplitude and phase of each spectral component.

(b) Using the discrete-time spectrum from part (a), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{\text{so}} = 10000$ Hz.

(c) Again using the discrete-time spectrum from part (b), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{\text{so}} = 20000$ Hz. In other words, the sampling rates of the two converters are different.