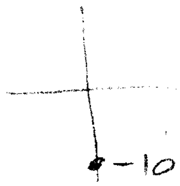
ECE HW #1 Solutions

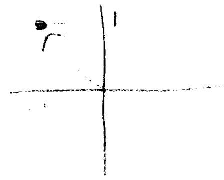
1.1) a)



$z = -j10 = \boxed{10\angle -\pi}$

or $\boxed{10\angle \frac{3\pi}{2}}$

d)

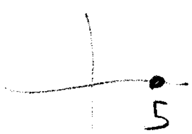


$z = \boxed{\sqrt{2}\angle \frac{3\pi}{4}}$

c)

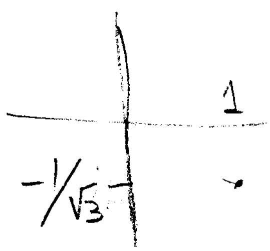
$z = \boxed{5\sqrt{2}\angle \frac{3\pi}{4}}$

b)



$z = 5\angle 0$

e)



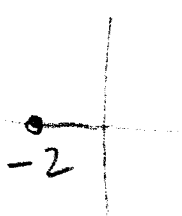
$|z| = \sqrt{1^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$

$= \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

$\arg z = \operatorname{atan}\left(\frac{-1/\sqrt{3}}{1}\right) = -0.5236$

$z = \boxed{\frac{2}{\sqrt{3}}\angle -0.5236}$

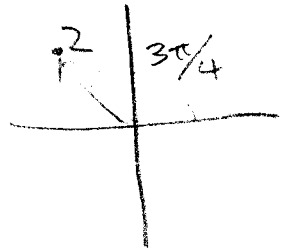
f)



$z = \boxed{2\angle \pi}$ or $\boxed{2\angle -\pi}$

adl

1.2) a)

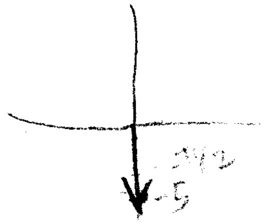


$Re\{z\} = 2\left(\cos\frac{3\pi}{4}\right) = -2\frac{\sqrt{2}}{2} = -\sqrt{2}$

$Im\{z\} = 2\left(\sin\frac{3\pi}{4}\right) = 2\frac{\sqrt{2}}{2} = \sqrt{2}$

$z = \sqrt{2} + j\sqrt{2} \cong \boxed{-1.4142 + j1.4142}$

b)



$\boxed{z = -5j}$ $Re\{z\} = 0$

$Im\{z\} = -5$

c) $Re\{z\} = 4\cos\left(\frac{\pi}{6}\right) = 3.4641$

$Im\{z\} = 4\sin\left(\frac{\pi}{6}\right) = 2$

$z = \boxed{3.4641 + j2}$

d) $Re\{z\} = 3\cos(-4.5\pi) = 3\cos(-0.5\pi) = 0$

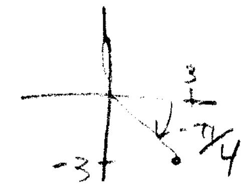
$Im\{z\} = 3\sin(-4.5\pi) = 3\sin(-0.5\pi) = -3$

$z = \boxed{-j3}$

adl

1.3) a) $z_1^* = \boxed{3+j3}$

$$z_1 = \sqrt{3^2+(-3)^2}\,\angle\, \operatorname{atan}\left(\frac{-3}{3}\right)$$



$$= 3\sqrt{2}\,\angle -\pi/4$$

$$z_1^* = \boxed{3\sqrt{2}\,\angle\, \pi/4}$$

b) $jz_2 = e^{j\frac{\pi}{2}}e^{j\frac{3\pi}{4}} = e^{j\frac{5\pi}{4}} = \boxed{e^{-j\frac{3\pi}{4}} = 1\angle -3\pi/4} = \boxed{-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j}$

↖ rotates counterclockwise by 90°

c) $$\frac{z_2}{z_1} = \frac{e^{j\frac{3\pi}{4}}}{3\sqrt{2}\,e^{j(-\pi/4)}} = \frac{1}{3\sqrt{2}}e^{j\frac{4\pi}{4}} = \boxed{-\frac{1}{3\sqrt{2}}}$$

$$= \boxed{\frac{1}{3\sqrt{2}}\angle\,\pi}$$

d) $$z_2^2 = \left[e^{j\left(\frac{3\pi}{4}\right)}\right]^2 = e^{j\frac{6\pi}{4}} = e^{j\frac{3\pi}{2}}$$

$$= \boxed{1\angle\frac{3\pi}{2} = 1\angle -\frac{\pi}{2}}$$

$$= \boxed{-j}$$

This is interesting; it says $\sqrt{-j} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$. Also note $\sqrt{j} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$, so square roots of complex #s are meaningful.

1.3 cont'

adl

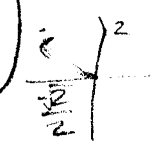
e) $z_1^{-1} = \frac{1}{z_1} = \frac{1}{3\sqrt{2}\angle -\pi/4} = \boxed{\frac{1}{3\sqrt{2}}\angle \pi/4}$

$= \frac{1}{3\sqrt{2}}\cos\left(\frac{\pi}{4}\right) + j\frac{1}{3\sqrt{2}}\sin\left(\frac{\pi}{4}\right)$

$= \frac{1}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{2} + j\frac{1}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{1}{6} + j\frac{1}{6}}$

f) $z_1 z_2 = \left(3\sqrt{2}\angle -\frac{\pi}{4}\right)\left(\angle \frac{3\pi}{4}\right)$

$= 3\sqrt{2}\angle \frac{2\pi}{4} = \boxed{3\sqrt{2}\angle \frac{\pi}{2}} = \boxed{3\sqrt{2}j}$



g) $z_1 + z_2^* = (3 - j3) + \left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$ $z_2 = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$\simeq (+3 - 0.707) + j(-3 - 0.707)$

$= \boxed{2.2929 - j3.707}$

$|z_1 + z_2^*| = \sqrt{(2.2929^2 + 3.707^2)}$

$= 4.3589$

$\arg(z_1 + z_2^*) = \mathrm{atan}\left(\frac{-3.707}{2.293}\right) = -1.0168$

$z_1 + z_2^* = \boxed{4.3589\angle -1.0168}$

1.3 con't

odd

h) $|z_2|^2 = z_2 z_2^* = e^{j\left(\frac{3\pi}{4}\right)} e^{-j\left(\frac{3\pi}{4}\right)} = e^{j0} = \boxed{1}$

or $\boxed{1 \angle 0}$

i) $z_2 + z_2^* = 2\,\mathrm{Re}\{z_2\} = 2\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\sqrt{2}}$

$= \boxed{\sqrt{2} \angle \pi}$

1.4) a) $z^* = Ae^{-j\pi/6}$

$\mathrm{Re}\{z^*\} = A\cos\left(-\frac{\pi}{6}\right) = \boxed{A\cos\left(\frac{\pi}{6}\right)}$

$\mathrm{Im}\{z^*\} = A\sin\left(-\frac{\pi}{6}\right) = \boxed{-A\sin\left(\frac{\pi}{6}\right)}$

b) $z - z^* = \mathrm{Re}\{z\} + \mathrm{Im}\{z\} - (\mathrm{Re}\{z\} - \mathrm{Im}\{z\})$

$= 2\,\mathrm{Im}\{z\}$

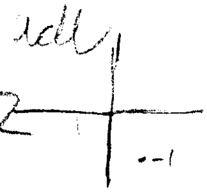
$= 2A\sin\left(-\frac{2\pi}{3}\right)$

$= \boxed{-2A\sin\left(\frac{2\pi}{3}\right)}$

c) $-jz = 3e^{-j\frac{\pi}{2}} e^{j\phi} = 3e^{j\left(\phi - \frac{\pi}{2}\right)}$

$\mathrm{Im}\{-jz\} = \boxed{3\sin\left(\phi - \frac{\pi}{2}\right) = -3\cos\phi}$

1.4 cont'd

d) $z = \propto 2 \angle \operatorname{atan}\left(\frac{-1}{\sqrt{3}}\right)$ $\quad \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$ 

$= \boxed{\propto 2 \angle -0.5236}$

e) $\frac{|z|}{z^*} = \frac{\cancel{A}}{\cancel{A}e^{-j2\pi/3}} = \boxed{e^{j2\pi/3} = 1 \angle 2\pi/3}$

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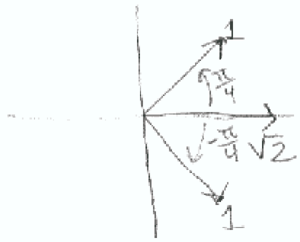
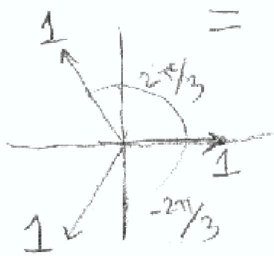
1.5) a) $z_a = e^{-j\pi/4} + e^{+j\pi/4}$

(Euler's formula)



$= 2\cos\left(\frac{\pi}{4}\right) = 2\frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$

b) $z_b = 1 + e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}}$ (Euler's Again)



$= 1 + 2\cos\left(\frac{2\pi}{3}\right) = 1 + 2\left(-\frac{1}{2}\right) = 1 - 1$

$= \boxed{0}$

c)

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za= exp(-j*pi/4)+exp(+j*pi/4)
zb=1+exp(-j*2*pi/3)+exp(+j*2*pi/3)
abs(za)
angle(za)
abs(zb)
angle(zb)
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$$\phi = -\omega_0 t_d = -(-10^{-3} \times 500\pi) = 0.5\pi$$

1.6) $\boxed{A = 75}$ $\boxed{T = 4 \times 10^{-3} \text{ sec}}$

$$w_0 = \frac{2\pi}{T} = \frac{2\pi}{4\times 10^{-3}} = \boxed{500\pi \ \frac{\text{rad}}{\text{sec}}}$$

$t_d = \boxed{-1 \text{ msec}}$ or $\boxed{-10^{-3} \text{ sec}}$

(can just as easily see $\frac{\pi}{2}$ right off the graph)

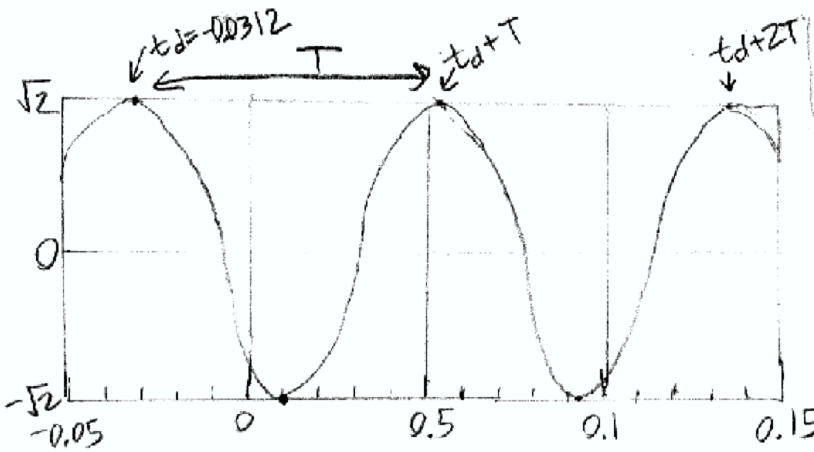
1.7) $Z = -1 + j = \sqrt{2}\, e^{j\frac{3\pi}{4}}$

$\boxed{A = \sqrt{2}}$ $\boxed{\phi = \frac{3\pi}{4}}$

$$T = \frac{1}{F_0} = \frac{1}{12} = 0.0833$$

$$t_d = -\frac{\phi}{2\pi F_0} = \frac{-3\pi/4}{2\pi \cdot 12} = -0.0312$$

$$\cos\left(\frac{3\pi}{4}\right) \approx -0.707$$



$t_d + T = -0.0312 + 0.0833 = 0.0521$

$t_d + 2T = 0.0521 + 0.0833 = 0.1354$

$t_d + \frac{T}{2} = -0.0312 + 0.0416 = 0.0104$

$t_d + \frac{T}{2} + T = 0.0104 + 0.083\ldots = 0.0937$