ECES-352
Winter 2019
Homework #7

Reading: Chapter 6 on FIR Filter Frequency Response

PROBLEM 7.1:

Suppose that a discrete-time system is described by the input-output relation

$$y[n] = (x[n])^3$$

(a) Determine the output when the input is the complex exponential signal

$$x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$$

(b) Is the output of the form

$$y[n] = \mathcal{H}(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n}$$

If not, why not?

PROBLEM 7.2*:

A discrete-time system is defined by the input/output relation

$$y[n] = x[n+1] - 2x[n] + x[n-1]. \tag{1}$$

(a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal. Explain your answers.

(b) Obtain an expression for the frequency response of this system.

(c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency. *Hint: Use symmetry to simplify your expression before determining the magnitude and phase.*

(d) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.25\pi n) = e^{j0.25\pi n} + e^{-j0.25\pi n}.$$

Express your answer in terms of cosine functions.

(e) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 1 + \cos(0.25\pi(n-1)).$$

Hint: use the linearity and time-invariance properties.

PROBLEM 7.3:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

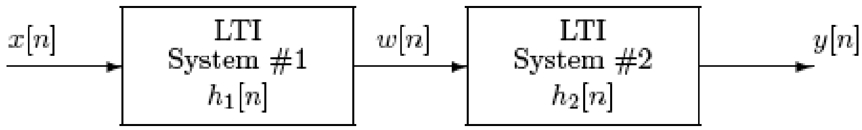

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that the two LTI systems are described by the impulse responses

$$h_1[n] = \delta[n] - \delta[n-1] \qquad \text{and} \qquad h_2[n] = u[n] - u[n-10].$$

(b) Determine $H_1(\hat{\omega})$, the frequency response of the first system.

(c) Determine $H_2(\hat{\omega})$, the frequency response of the second system.

(d) By using numerical convolution, show that $h[n] = h_1[n] * h_2[n] = \delta[n] - \delta[n-10]$.

(e) From $h[n]$ determine $H(\hat{\omega})$ the frequency response of the overall system (from $x[n]$ to $y[n]$).

(f) Show that your result in part (d) is the product of the results in parts (a) and (b); i.e., $H_1(\hat{\omega})H_2(\hat{\omega}) = H(\hat{\omega})$.

PROBLEM 7.4*:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\begin{aligned} \mathcal{S}_1: &\quad \mathcal{H}_1(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} \\ \mathcal{S}_2: &\quad y_2[n] = x_2[n] + x_2[n-2] \\ \mathcal{S}_3: &\quad y_3[n] = 2x_3[n-1] + 2x_3[n-2] \end{aligned}$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the difference equation for $\mathcal{S}_1$, i.e., express $y_1[n]$ in terms of $x_1[n]$, $x_1[n-1]$, $x_1[n-2]$, etc.

(b) Determine the frequency response of the other two systems: $\mathcal{H}_i(\hat{\omega})$ for $i = 2, 3$.

(c) Determine the frequency response of the overall cascaded system.

(d) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

Problem 7.5

The frequency response of a linear time-invariant filter is given by the formula

$$H(\hat{\omega}) = (1 + e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}}). \tag{2}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. *Hint:* Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.

(b) What is the impulse response of this system?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n?

(d) Use superposition to determine the output of this system when the input is

$$x[n] = 2 - 3\delta[n-4] + 7\cos(\pi/3n) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results.