ECES-352
Winter 2019
Homework #1

Reading: In DSP First, Appendix A on Complex Numbers; and Ch. 2 on Sinusoids.

Your homework is due at the beginning of class each Tuesday. See syllabus for late policy.

A complex number is just an ordered pair of real numbers. Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$\begin{aligned} z &= (x, y) \\ &= x + jy \qquad\qquad \text{where } j = \sqrt{-1} \\ &= \Re e\{z\} + j\Im m\{z\} \end{aligned}$$

Note that $i = \sqrt{-1}$ in most math courses. The pair (x, y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate in a two-dimensional space. Addition of complex numbers is the same as vector addition; i.e., add the real parts and add the imaginary parts.

In *polar form* we will use these notations:

$$\begin{aligned} z &= |z|e^{j \arg z} \\ &= re^{j\theta} \\ &= r\angle\theta \end{aligned}$$

where $|z| = r = \sqrt{x^2 + y^2}$ and $\arg z = \theta = \arctan(y/x)$. In a vector drawing, r is the length and θ the direction of the vector.

Euler's Formula:

$$re^{j\theta} = r\cos\theta + j\,r\sin\theta$$

can be used to convert between Cartesian and polar forms.

Problems 1.1–1.4 should be review problems. In these problems you will manipulate some complex numbers. A calculator will be useful for this purpose, especially if it is one with complex arithmetic capability. It is convenient to learn how to use this feature. However, it is also worthwhile to be able to do the calculations by hand; i.e., it is important to *understand* what your calculator is doing!

PROBLEM 1.1:
Convert the following to polar form:

(a) $z = -j10$	(b) $z = 5$	(c) $z = (-5, 5)$
(d) $z = -1 + j$	(e) $z = 1 - j\frac{1}{\sqrt{3}}$	(f) $z = -2$

Give numerical values for the magnitude, and the angle (or phase) in radians.

PROBLEM 1.2:
Convert the following to rectangular form:

(a) $z = 2\,e^{j(3\pi/4)}$	(c) $z = 4\,\angle\,(\pi/6)$
(b) $z = 5e^{-j(\pi/2)}$	(d) $z = 3\,\angle\,(-4.5\pi)$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3:
Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = 3 - j3$ and $z_2 = e^{j(3\pi/4)}$.

(a) Conjugate: z_1^*	(d) z_2^2	(g) $z_1 + z_2^*$
(b) jz_2	(e) $z_1^{-1} = 1/z_1$	(h) $\lvert z_2\rvert^2 = z_2 z_2^*$
(c) z_2/z_1	(f) $z_1 z_2$	(i) $z_2 + z_2^*$

Note: z^* means the "conjugate" of z. Part (h) is the *magnitude-squared.*

PROBLEM 1.4:
Simplify the following complex-valued expressions. Give your answer in either rectangular or polar form, whichever is most convenient. In parts (a)-(e) assume that A, α, and ϕ are positive real numbers. Your answers may be in terms of these quantities.

(a) For $z = Ae^{j\pi/6}$, determine a simple expression for $\Re m\{z^*\}$.

(b) For $z = Ae^{-j2\pi/3}$, determine a simple expression for $z - z^*$.

(c) For $z = 3e^{j\phi}$, determine a simple expression for $\Im m\{-jz\}$.

(d) For $z = -\alpha\sqrt{3} + j\alpha$, determine a simple expression for z in polar form.

(e) For $z = Ae^{j2\pi/3}$, determine a simple expression for $|z|/z^*$ in polar form.

PROBLEM 1.5*:

Simplify the following and give the answer in polar form. Make a plot of all the vectors involved in the complex addition.

(a) $z_a = e^{-j\pi/4} + e^{j\pi/4}$

(b) $z_b = 1 + e^{-j2\pi/3} + e^{j2\pi/3}$

(c) In addition, write the MATLAB statements that will perform the addition and also display the magnitude and phase of the result. Consult `help` on the MATLAB functions `abs` and `angle`, and also the DSP-First Toolbox functions: `zprint`, `zvect`, etc. Use these to check your hand calculations.

PROBLEM 1.6*:

The waveform in the following figure can be expressed as

$$x(t) = A\cos[\omega_0(t - t_d)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, f_0, t_d, and ϕ. Choose the value of ϕ such that $-\pi < \phi \leq \pi$.