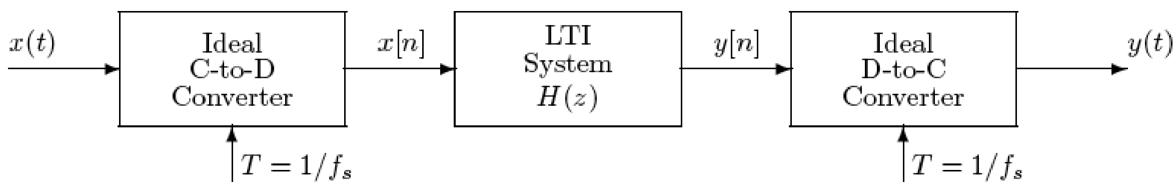
ECES-352
Winter 2019
Homework #8

Reading: Chapter 9 on Z-transform

Problem 8.1

A system for filtering continuous-time signals is shown in the following figure



The input to the C-to-D converter in this system is

$$x(t) = 5 + 4\cos(200\pi t) + 3\cos(500\pi t + \pi/4)$$

The system function of the LTI system is

$$H(z) = (1 - z^{-2})$$

If $f_s = 1000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.

Problem 8.2

We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. You need to be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving these problems. Indeed for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

> *Given the input sequence $x[n]$ find the output sequence $y[n]$ of a 5-point running average filter for all values of n.*

The following is a partial list of possible approaches to solving this problem:

1. Use the difference equation representation of the system to compute (e.g., using MATLAB) the output $y[n]$ for all required values of n.

2. Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.

3. Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

4. Some combination of the above methods. Remember, when the input is a sum of two or more signals and the system is linear, we can solve the problem separately for each of the input components and then superimpose the outputs. We can therefore use the method that is most appropriate for each of the components of the input.

In each of these solution methods you would use one or more of the basic representations of the 5-point running average filter. Which method is easiest will have a lot to do with the nature of the input signal. This may require that you convert a given representation of the system into one of the other forms. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1 Find $X(z)$, the z-transform of $x[n]$.

Step 2 Find $H(z)$, the system function of the 5-point running averager.

Step 3 Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4 Find the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the approaches above (#1, #2, #3, or #4) you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Outline your approach to solving the problem of finding the output of the 5-point moving averager. **You do not have to actually find the output—just tell how you would solve it in a step-by-step procedure described as illustrated above.**

(a) $x[n]$ is a sampled audio signal. It is represented by a vector of 100000 numbers.

(b) $x[n] = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

(c) $x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

(d) $x[n] = 10\delta[n-50]$.

(e) $x[n] = 10\delta[n-50] + 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

Problem 8.3

Consider the following MATLAB program:

```
nn = 0:16000;
xx = 3 + 2*cos(0.75*pi*nn-pi/4) + 11*cos(1.5*pi*nn-pi/3);
yy = conv([1,0,0,0,-1]/4,xx);
soundsc(yy,8000)
```

(a) What is the system function $H(z)$ of the system that is implemented by the `conv( )` statement?

(b) What is the frequency response of the system?

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.

Problem 8.4

We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = (x[n] - 2x[n-2] + x[n-4])$.

(b) $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] + \delta[n-4]$.

(c) $H(e^{j\hat{\omega}}) = 2[1 + \cos(2\hat{\omega})]e^{-j\hat{\omega}2}$. *Hint: Expand the cosine using Euler's formula.*

(d) $H(z) = z^2 - z^{-2} + 6z^{-6} + z^{-7}$.

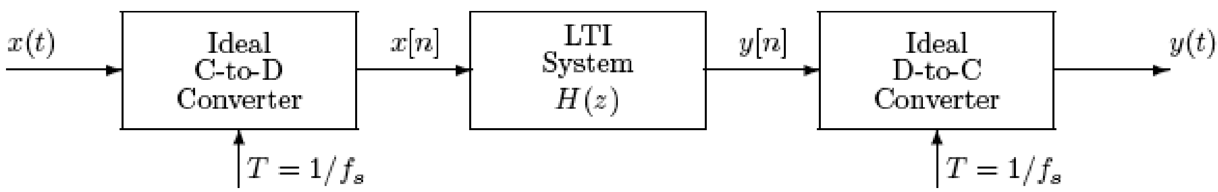
Problem 8.5

The input to the C-to-D converter in the figure below is

$$x(t) = 1 + 3\cos(3000\pi t - \pi/8) + 2\cos(8000\pi t + \pi/3)$$

The system function of the LTI system is

$$H(z) = (1 + z^{-2})$$



(a) If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.

(b) If $f_s = 5000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter. *Note that even when aliasing distortion occurs, we can still determine the effect of the system on the input $x[n]$ and therefore we can determine $y(t)$ from $y[n]$.*

Problem 8.6

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5] = \sum_{k=0}^{5} x[n-k]$$

(a) What is the impulse response, $h[n]$, of this system?

(b) Show that $H(z)$ for this system can be expressed as follows:

$$H(z) = \frac{1 - z^{-6}}{1 - z^{-1}}$$

.

(c) Plot the zeros of $H(z)$ in the complex z-plane. *Hint: Remember the N-th roots of unity.*

Problem 8.7

Consider the linear time-invariant system given in the previous problem.

(a) Find an expression for the frequency response $H(\hat{\omega})$ of the system.

(b) Show that your answer in (a) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(3\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-j5\hat{\omega}/2}.$$

(c) Sketch the frequency response (magnitude and phase) as a function of frequency from the formula above (or plot it using `freqz( )`).

(d) Suppose that the input is

$$x[n] = 2 + 8\cos(\hat{\omega}_0 n) \text{ for } -\infty < n < \infty$$

Find a non-zero frequency $0 < \hat{\omega}_0 < \pi$ for which the ouput $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

Problem 8.8

The system function of a linear time-invariant filter is given by the formula

$$H(z) = z^{-1}(1 - z^{-2})(1 - jz^{-1})(1 + jz^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. (*Hint: First multiply the factors to get a polynomial.*)

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-3] - \delta[n-4].$$

(d) If the input to the system is of the form

$$x[n] = e^{j\hat{\omega}n} \qquad -\infty < n < \infty,$$

for what values of $\hat{\omega}$ will the output be zero for all n? We cannot use z-transforms directly to solve this problem, but we can find the frequency response from $H(z)$ and then solve the problem. Note that the factored form will tell you the answer and so will a plot of the zeros of $H(z)$.