ECES-352
Summer 2014
Homework #3
Solutions

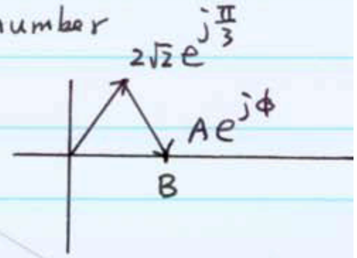
Reading: In Signal Processing First, Chapter 3 on Spectrum Representation.

Your homework is due at the beginning of class each Tuesday. See syllabus for late policy.

Problem 1

(a) Using phasor addition yields

$$2\sqrt{2}\, e^{j\frac{\pi}{3}} + A e^{j\phi} = B = \text{a positive number}$$



$$\Rightarrow 2\sqrt{2}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + A(\cos\phi + j\sin\phi) = B$$

$$\Rightarrow \quad \sqrt{6} + A\sin\phi = 0 \quad \&$$

$$-\frac{2}{3}\pi < \phi < 0$$

(b) $2\sqrt{2}\, e^{j\frac{\pi}{3}} + A e^{j\phi} = 25$

$$25 - 2\sqrt{2}\, e^{j\frac{\pi}{3}} = 23.5858 - j\,2.4495$$

$$= 23.7126\, e^{-j0.1035}$$

$$\Rightarrow \quad A = 23.7126 \qquad \phi = -0.1035$$

(c) Since A is the magnitude of $Ae^{j\phi}$, the smallest A is achieved when the "vector" $Ae^{j\phi}$ is perpenticul[ar] to the real axis (see Fig. 1)

Thus, $A = \sqrt{6}$, $\phi = -\frac{\pi}{2}$ and $B = \sqrt{2}$ (see Fig. 2)

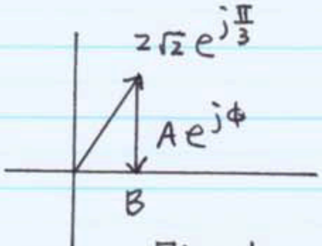


Fig. 1

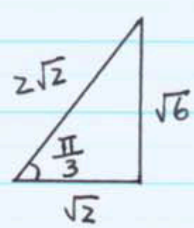
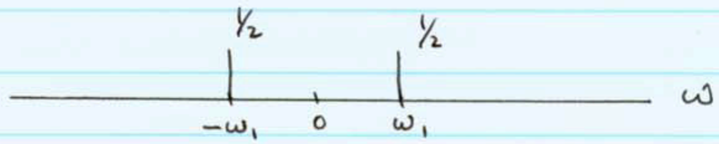


Fig. 2

Problem 2

Let $\omega_1 = 2\pi \times 1000$ and $\omega_2 = 2\pi \times 750 \times 10^3$

(a) $v(t) = \cos(\omega_1 t)$

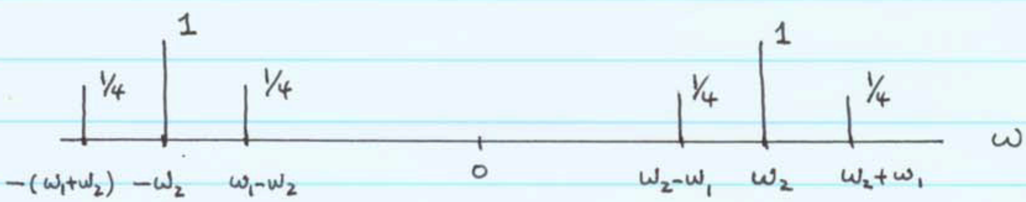


(b) $x(t) = (\cos(\omega_1 t) + 2)\cos(\omega_2 t)$

$= 2\cos(\omega_2 t) + \cos(\omega_1 t)\cos(\omega_2 t)$

$= 2\cos(\omega_2 t) + \frac{1}{2}\left(\cos((\omega_2+\omega_1)t) + \cos((\omega_2-\omega_1)t)\right)$

(using $\cos\theta\cos\phi = \frac{1}{2}\left(\cos(\theta+\phi) + \cos(\theta-\phi)\right)$)



(c) Same except ω_2 is changed to

$2\pi \times 680 \times 10^3$

Problem 2

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = [v(t) + A]\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 1 kHz sinusoid, $v(t) = \cos(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a) and assume that $A = 2$. *Hint: Substitute for* $v(t)$ *and expand* $x(t)$ *into a sum of cosine terms of three different frequencies. Note that the product of two cosines is equivalent to a sum of sinusoids. If you do not recall this, substitute Euler's relation for each of the cosine terms, and expand the product - tedius, but it works.*

(c) How would the spectrum of the AM radio signal change if the carrier frequency is changed to 680 kHz (WCNN) and $v(t)$ and A are the same as defined in parts (a) and (b).

Problem 3

a) The frequency of the DC component is always zero. The period of the signal is 2 msec., so:

$$f = \frac{1}{2\times 10^{-3}} = 500\ Hz$$

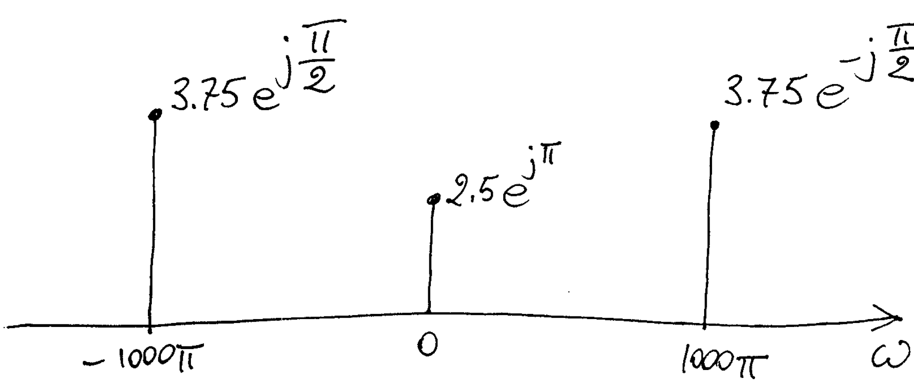
b) $$x(t) = -2.5 + 7.5\cos\left[1000\pi\left(t - 0.5\times 10^{-3}\right)\right] =$$

$$= -2.5 + 7.5\cos\left(1000\pi t - \frac{\pi}{2}\right)$$

c) $$x(t) = -2.5 + 7.5\,\frac{e^{j(1000\pi t - \frac{\pi}{2})} + e^{-j(1000\pi t - \frac{\pi}{2})}}{2} =$$

$$= -2.5 + 3.75\,e^{-j\frac{\pi}{2}}e^{j1000\pi t} + 3.75\,e^{j\frac{\pi}{2}}e^{-j1000\pi t}$$

d)



Problem 4

2

a) $$x(t) = 8\,\frac{e^{j100\pi t} + e^{-j100\pi t}}{2} \cdot \frac{e^{j200\pi t} - e^{-j200\pi t}}{2j} =$$

$$= -2j\left(e^{j300\pi t} - e^{-j100\pi t} + e^{j100\pi t} - e^{-j300\pi t}\right) =$$

$$= -2j\,e^{j300\pi t} + 2j\,e^{-j100\pi t} - 2j\,e^{j100\pi t} + 2j\,e^{-j300\pi t}$$

b) $$x(t) = 2e^{-j\frac{\pi}{2}}e^{j300\pi t} + 2e^{j\frac{\pi}{2}}e^{-j300\pi t} +$$

$$+ 2e^{-j\frac{\pi}{2}}e^{j100\pi t} + 2e^{j\frac{\pi}{2}}e^{-j100\pi t} =$$

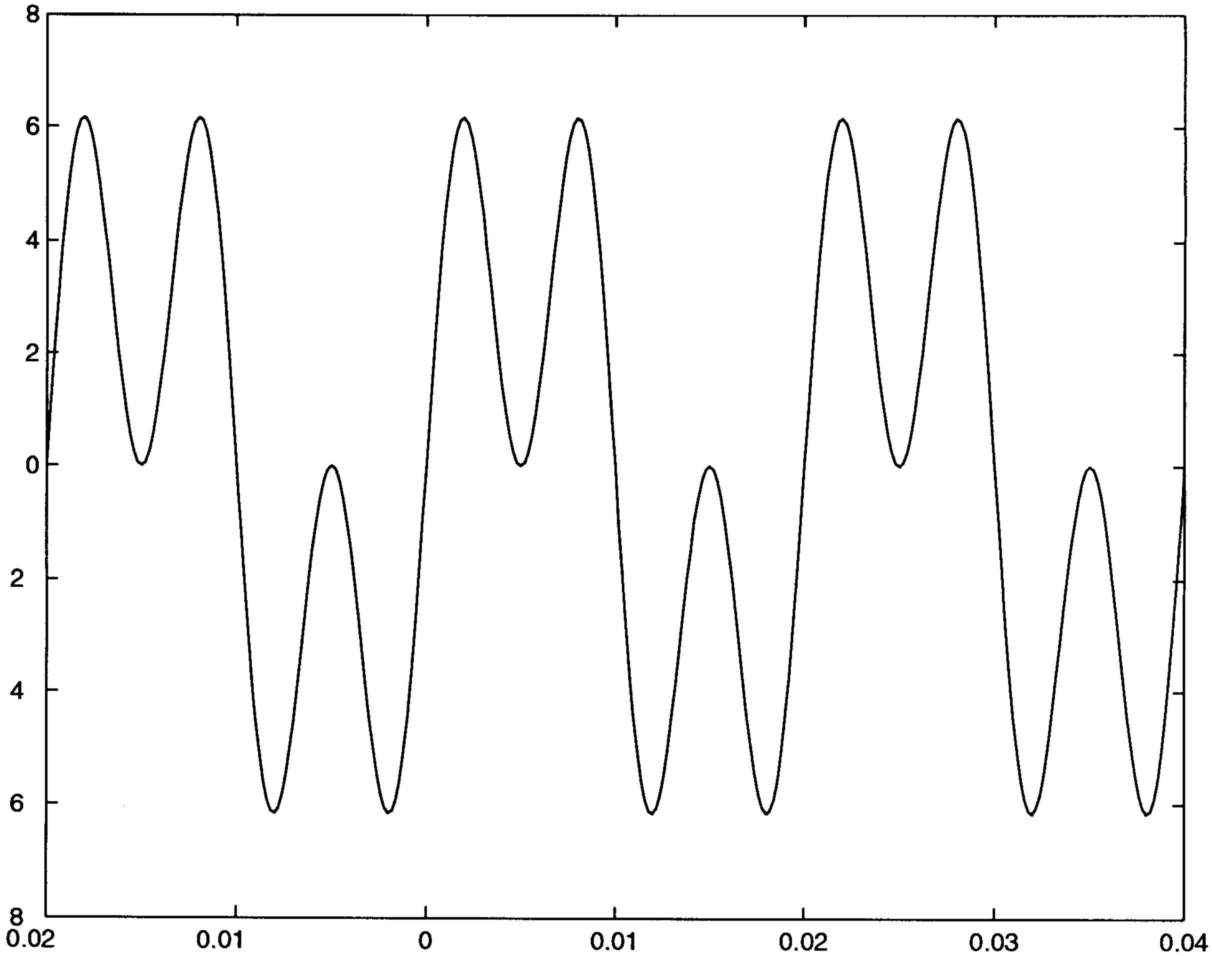
$$= 2\,e^{j(300\pi t - \frac{\pi}{2})} + 2\,e^{-j(300\pi t - \frac{\pi}{2})} +$$

$$+ 2e^{j(100\pi t - \frac{\pi}{2})} + 2e^{-j(100\pi t - \frac{\pi}{2})} =$$

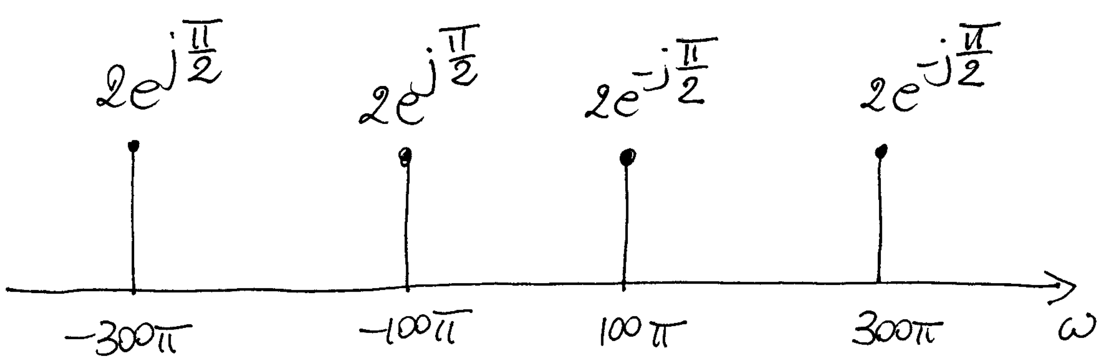
$$= 4\cos\left(300\pi t - \frac{\pi}{2}\right) + 4\cos\left(100\pi t - \frac{\pi}{2}\right)$$

c) Fundamental freq. $\frac{100\pi}{2\pi} = 50\,Hz$

$\Rightarrow T_0 = \frac{1}{50} = 20$ msec.



8
6
4
2
0
2
4
6
8
0.02
0.01
0
0.01
0.02
0.03
0.04



$2e^{j\frac{\pi}{2}}$
$2e^{j\frac{\pi}{2}}$
$2e^{-j\frac{\pi}{2}}$
$2e^{-j\frac{\pi}{2}}$
-300π
-100π
100π
300π
ω

Problem 5

4

a) $x(t) = 0.5\, e^{-j\frac{\pi}{2}} e^{j300\pi t} + 0.5\, e^{j\frac{\pi}{2}} e^{-j300\pi t} +$

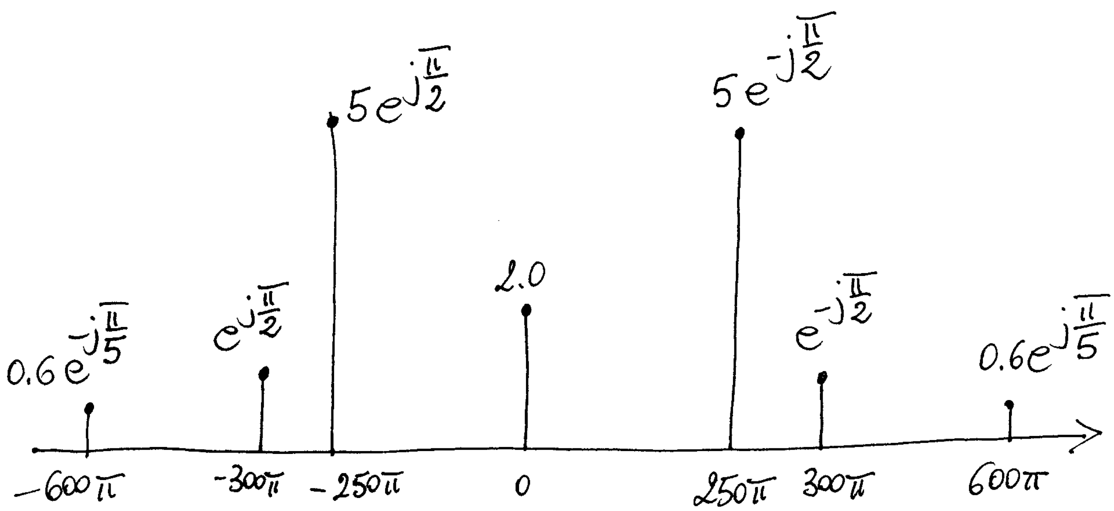
$+ 1 + 0.3\, e^{j\frac{\pi}{5}} e^{j600\pi t} + 0.3\, e^{-j\frac{\pi}{5}} e^{-j600\pi t} =$

$= 1 + 0.5 e^{+j(300\pi t - \frac{\pi}{2})} + 0.5\, e^{-j(300\pi t - \frac{\pi}{2})} +$

$+ 0.3\, e^{j(600\pi t + \frac{\pi}{5})} + 0.3 e^{-j(600\pi t + \frac{\pi}{5})} =$

$= 1 + \cos\left(300\pi t - \frac{\pi}{2}\right) + 0.6 \cos\left(600\pi t + \frac{\pi}{5}\right)$

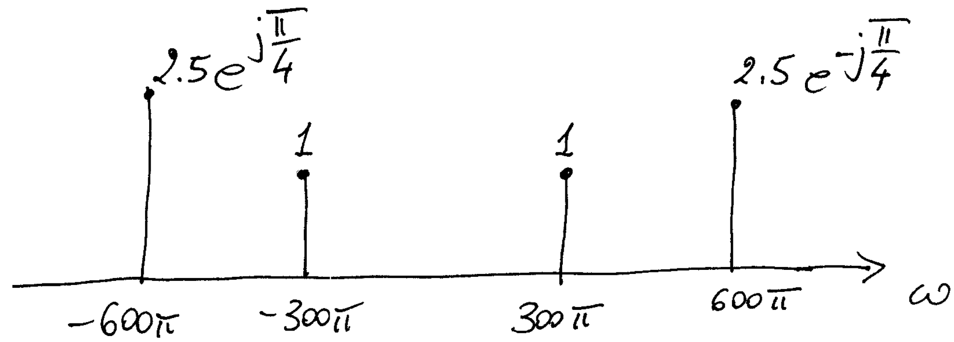
b)



Problem 6

5

a)

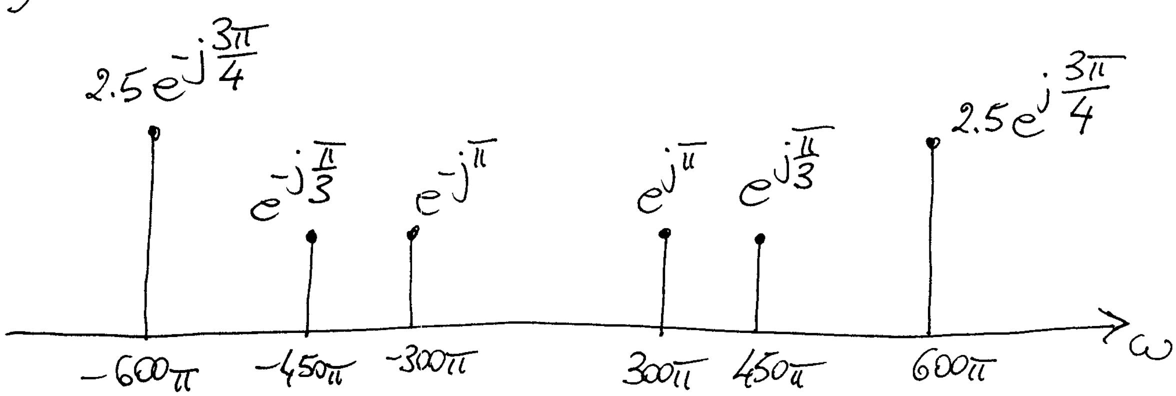


b) Fundamental freq. $f_0 = \frac{300\pi}{2\pi} = 150\,Hz$

$\Rightarrow T_0 = \frac{1}{150} \cong 6.67$ msec.

First and second harmonics are present.

c)



Fundamental freq. $f_0 = \frac{150\pi}{2\pi} = 75\,Hz$

$\Rightarrow T_0 = \frac{1}{75} \cong 13.3$ msec.

Problem 7

a)

A : 440 Hz

B^b : 466 Hz

B : 494 Hz

C : 523 Hz

$C^\#$: 554 Hz

D : 587 Hz

E^b : 622 Hz

E : 659 Hz

F : 698 Hz

$F^\#$: 740 Hz

G : 784 Hz

$G^\#$: 831 Hz

A : 880 Hz.

b) The ratio between the frequencies of two consecutive notes is constant. Let f_n be the frequency of the n-th note. Then:

$$f_{n+1} = k f_n$$

$$f_{n+2} = k f_{n+1} = k^2 f_n$$

$$\vdots$$

$$f_{n+12} = k f_{n+11} = k^{12} f_n$$

But: $f_{n+12} = 2 f_n$, so:

$$k^{12} = 2 \Rightarrow k = 2^{1/12}$$

So: $f_{49+m} = 2^{m/12} f_{49}$

$$f_n = 2^{\frac{n-49}{12}} f_{49} =$$

$$= 2^{\frac{n-49}{12}} \cdot 440 \quad [Hz]$$