ECES-352
Summer 2014
Homework #4
Solutions

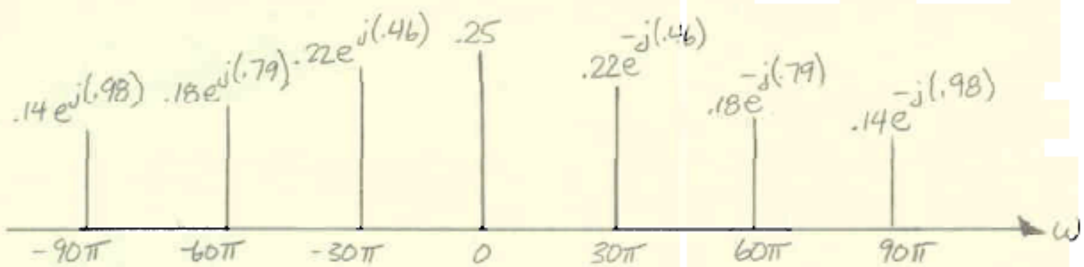
Reading: Chapter 3 on Spectrum Representation and beginning of Chapter 4 on Sampling.

PROBLEM 4.1:

THE FOLLOWING LINES OF MATLAB CODE WILL TAKE CARE OF THE MATH

```
k = -3:3;
ak = 1./(4+j*2*k);
abs(ak), angle(ak)
```

(a)



(b) $\omega_0 = 30\pi$ RAD./S $\Longrightarrow f_0 = 15$ Hz.

$$T_0 = \frac{1}{f_0} = \frac{1}{15} \text{ SEC} = .0667 \text{ SEC} = 66.7 \text{ mSEC}$$

PROBLEM 4.2*

USING THE INVERSE EULER RELATION, WE CAN EXPAND $x(t)$ INTO A SUM OF COMPLEX EXPONENTIALS.

$$x(t) = \left(3 + \frac{5}{2}e^{-j\frac{\pi}{4}}e^{j250\pi t} + \frac{5}{2}e^{j\frac{\pi}{4}}e^{-j250\pi t}\right) \cdot \left(\frac{1}{2}e^{j400\pi t} + \frac{1}{2}e^{-j400\pi t}\right)$$

$$= \frac{3}{2} e^{j400\pi t} + \frac{3}{2} e^{-j400\pi t} + \frac{5}{4} e^{-j\frac{\pi}{4}} e^{j650\pi t} + \frac{5}{4} e^{j\frac{\pi}{4}} e^{-j650\pi t}$$
$$+ \frac{5}{4} e^{j\frac{\pi}{4}} e^{j150\pi t} + \frac{5}{4} e^{-j\frac{\pi}{4}} e^{-j150\pi t}$$

(a) THE ARE COMPONENTS OF $x(t)$ AT 150π RAD/S., 400π RAD/S., AND 650π RAD/S. THE GREATEST COMMON DIVISOR OF THESE THREE FREQUENCIES IS 50π RAD/S.

$$\therefore \quad \omega_0 = 50\pi \text{ RAD/S.}$$

(b) THE EXPANSION OF $x(t)$ AT THE TOP OF THIS PAGE IS THE FOURIER SERIES EXPANSION, WHICH CONTAINS ONLY SIX NON-ZERO FOURIER COEFFICIENTS. SINCE $\omega_0 = 50\pi$ THE NON-ZERO COEFFICIENTS CORRESPOND TO

$$k = \pm 3, \pm 8, \pm 13$$

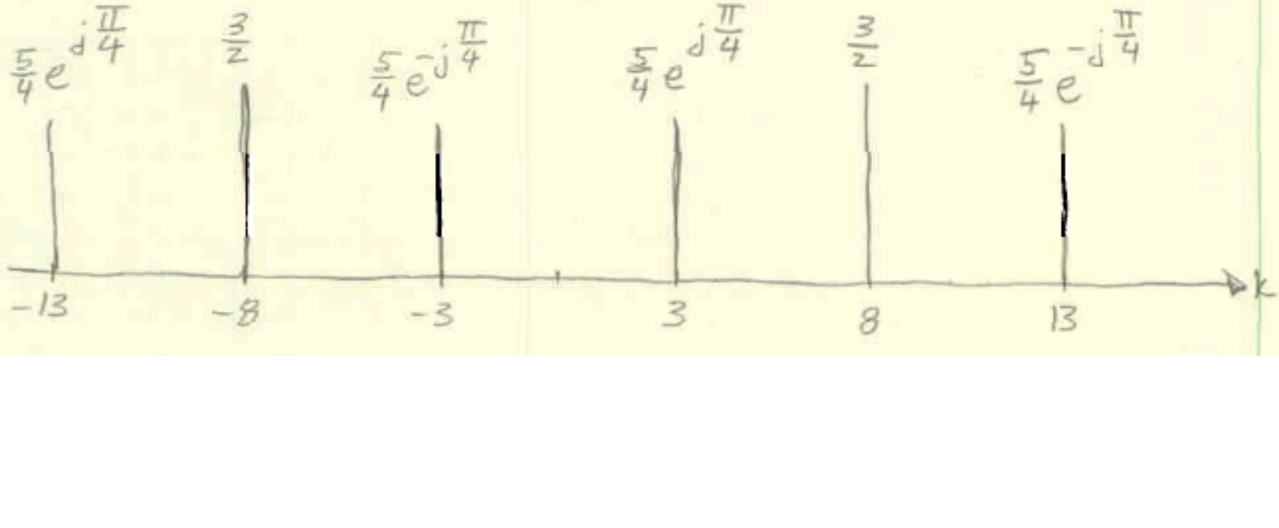
$$a_3 = \frac{5}{4} e^{j\frac{\pi}{4}} \qquad a_{-3} = \frac{5}{4} e^{-j\frac{\pi}{4}}$$

$$a_8 = \frac{3}{2} = a_{-8}$$

$$a_{13} = \frac{5}{4} e^{-j\frac{\pi}{4}} \qquad a_{-13} = \frac{5}{4} e^{j\frac{\pi}{4}}$$

FOR ALL OTHER k, $a_k = 0$.

(c)

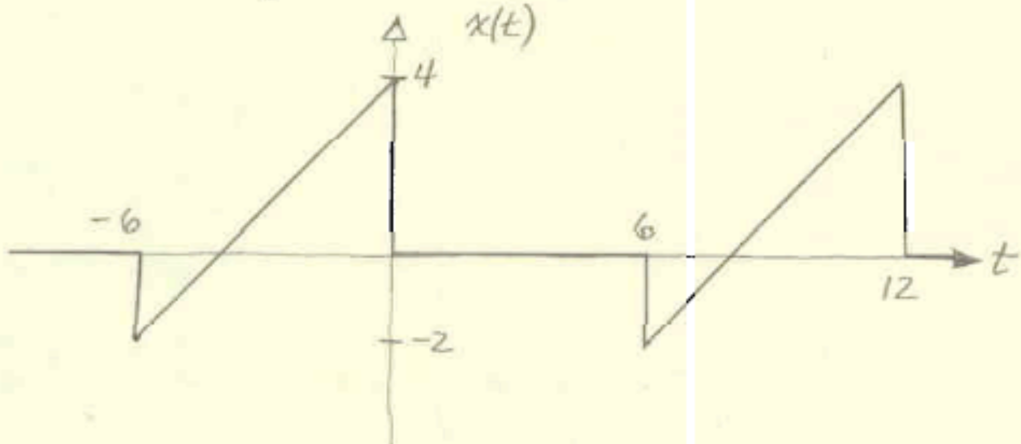


PROBLEM 4.3*:

(a) FOR THE PERIOD THAT EXTENDS FROM $-6<t<6$

$$x(t) = \begin{cases} 4+t & , -6<t<0 \\ 0 & , 0<t<6 \end{cases}$$

(b)



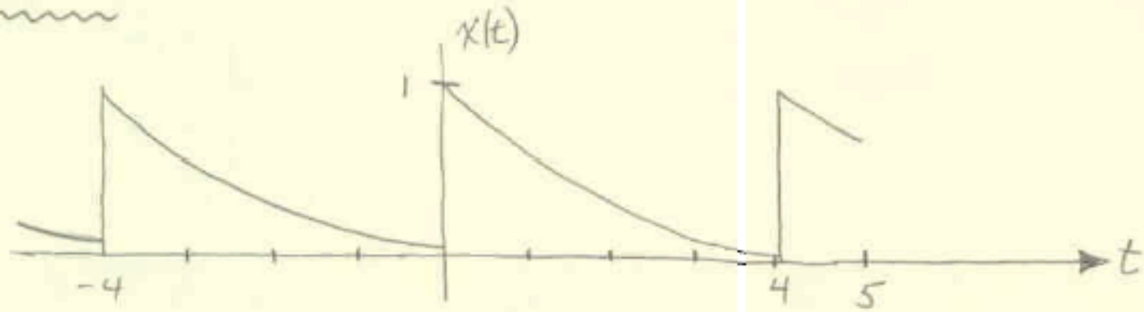
(c)

$$a_0 = \frac{1}{12}\int_{-6}^{0}(4+t)\,dt = \frac{1}{12}\left(4t+\frac{t^2}{2}\right)\Bigg|_{-6}^{0}$$

$$= -\frac{1}{12}\left(-24+\frac{36}{2}\right) = \frac{6}{12} = \frac{1}{2}$$

PROBLEM 4.4*:

(a)



(b) $$a_0 = \frac{1}{4}\int_{0}^{4} e^{-2t}\,dt = \frac{1}{4}\cdot\left(-\frac{1}{2}\right)e^{-2t}\Bigg|_{t=0}^{4}$$

(c)

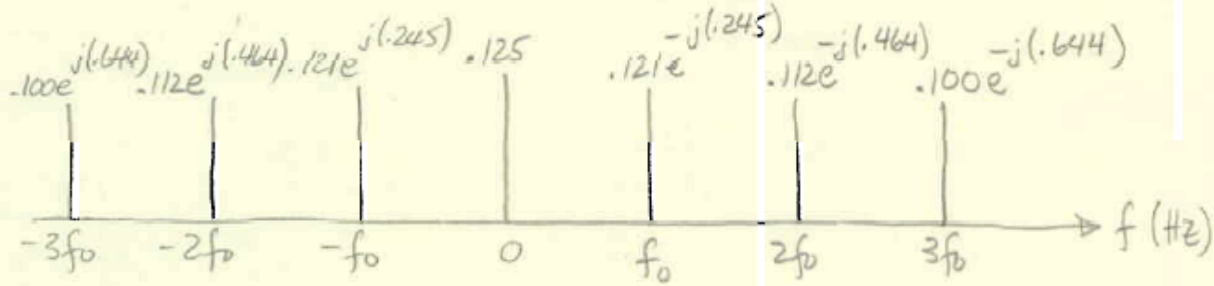
$$a_k = \frac{1}{4}\int_0^4 e^{-2t} e^{-jk\cdot\frac{2\pi}{4}t}\,dt = \frac{1}{4}\int_0^4 e^{-(2+jk\frac{\pi}{2})t}\,dt$$

(d)

$$a_k = \frac{-1}{4(2+jk\frac{\pi}{2})}\, e^{-(2+jk\frac{\pi}{2})t}\Big|_{t=0}^{4}$$

$$= \frac{1}{4(2+jk\frac{\pi}{2})}\left[1 - e^{-8(2+jk\frac{\pi}{2})}\right] \approx \frac{1}{8+j2\pi k}$$

(e) COMPARE THIS RESULT WITH PROBLEM 4.1. A SLIGHT MODIFICATION TO THE MATLAB CODE GIVES THE FOLLOWING SPECTRUM.



* PROBLEM 4.5 *:

(a) $$y(t) = Ax(t) = A\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty}(Aa_k)e^{jk\omega_0 t}$$

$$\boxed{\therefore\ b_k = Aa_k}$$

(b) $$y(t) = x(t-t_d) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0(t-t_d)}$$

$$= \sum_{k=-\infty}^{\infty} \underbrace{\left[a_k e^{-jk\omega_0 t_d}\right]}_{b_k} e^{jk\omega_0 t}$$

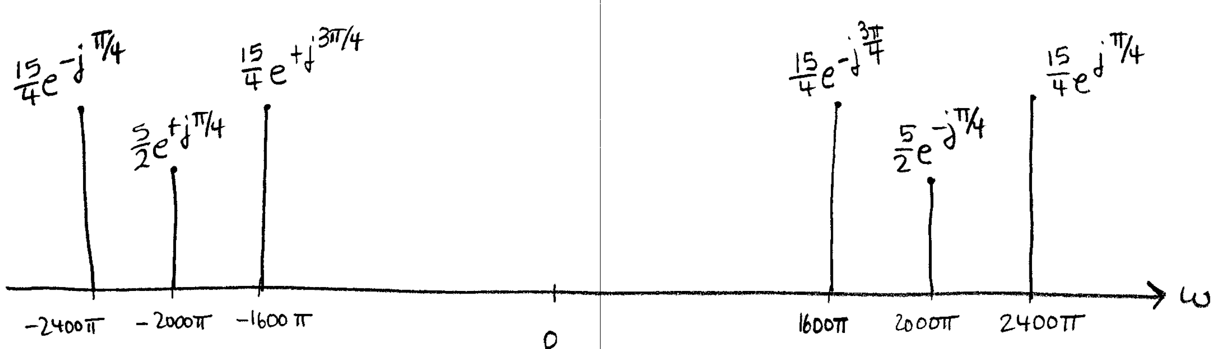
$$\boxed{\therefore\ b_k = a_k e^{-jk\omega_0 t_d}}$$

PROBLEM 4.6

5.1(a) $x(t) = [5 + 15\cos(400\pi t + \pi/2)]\cos(2000\pi t - \pi/4)$

$$= \left[5 + \tfrac{15}{2}e^{j(400\pi t + \pi/2)} + \tfrac{15}{2}e^{-j(400\pi t + \frac{\pi}{2})}\right]\left[\tfrac{1}{2}e^{j(2000\pi t - \pi/4)} + \tfrac{1}{2}e^{-j(2000\pi t - \pi/4)}\right]$$

$$= \tfrac{5}{2}e^{j(2000\pi t - \pi/4)} + \tfrac{5}{2}e^{-j(2000\pi t - \pi/4)} + \tfrac{15}{4}e^{j(2400\pi t + \pi/4)}$$
$$+ \tfrac{15}{4}e^{-j(1600\pi t - 3\pi/4)} + \tfrac{15}{4}e^{j(1600\pi t - 3\pi/4)} + \tfrac{15}{4}e^{-j(2400\pi t + \pi/4)}$$



The waveform is periodic, $\omega_0 = 400\pi$, $f_0 = 200$ Hz, $\boxed{T_0 = 5 \text{ msec.}}$

(b) $f_{max} = \dfrac{2400\pi}{2\pi} = 1200$ Hz

Minimum $f_s = 2 \cdot f_{max} = \boxed{2400 \text{ Hz}}$

(c) $f_s = 4000$ Hz, $\hat{\omega}_1 = \dfrac{1600\pi}{4000} = 0.4\pi$, $\hat{\omega}_2 = \dfrac{2000\pi}{4000} = 0.5\pi$, $\hat{\omega}_3 = \dfrac{2400\pi}{4000} = 0.6\pi$