Yonatan Carver HW 4
ECES 352 2.5.2019

Problem 4.1

a) Sketch the two-sided spectrum of this signal. Label all complex amplitudes in polar form

$$k = -3, \frac{1}{4-j6}e^{-j90\pi t} = \frac{1}{7.2e^{-j0.983}}e^{-j90\pi t}$$
$$k = -2, \frac{1}{4-j4}e^{-j60\pi t} = \frac{1}{5.66e^{-j0.785}}e^{-j60\pi t}$$
$$k = -1, \frac{1}{4-j2}e^{-j30\pi t} = \frac{1}{4.47e^{-j0.464}}e^{-j30\pi t}$$
$$k = 0, \frac{1}{4}$$
$$k = 1, \frac{1}{4+j2}e^{j30\pi t} = \frac{1}{4.47e^{j0.464}}e^{j30\pi t}$$
$$k = 2, \frac{1}{4+j4}e^{j60\pi t} = \frac{1}{5.66e^{j0.785}}e^{j60\pi t}$$
$$k = 3, \frac{1}{4+j6}e^{j90\pi t} = \frac{1}{7.2e^{j0.983}}e^{j90\pi t}$$

b) Determine the fundamental frequency (in Hz) and the fundamental period (in secs. Of this signal.

$$f_0 = 30\pi$$
$$T_0 = \frac{1}{30\pi}$$

Problem 4.2

a) What is the fundamental frequency of x(t)?

$$x(t) = 3\cos(8(50\pi)t) + 5\cos(5(50\pi)t - 0.25\pi)\cos(8(50\pi)t)$$
$$\omega_0 = 50\pi$$
$$\omega_0 = \frac{2\pi}{T_0}, T_0 = \frac{2\pi}{50\pi} = 0.04$$

b) A periodic signal may be expanded in a Fourier series expansion as $\boldsymbol{x(t)} = \sum_{k=-\infty}^{\infty} \boldsymbol{a_k e^{jk\omega_0 t}}$. Find the Fourier series coefficients a_k for the signal above.

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}dt$$
$$a_k = \frac{1}{0.04}\int_0^{0.04} x(t)e^{-jk\omega_0 t}dt$$
$$a_k = \frac{1}{2}A_k e^{j\phi_k}$$

c) Plot the coefficients a_k versus k. Note that you should be able to do this without evaluating any integrals

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Problem 4.3

a) In the expression for a_k in Equation (1) above, the integral and its limits define the signal x(t). Determine an equation for x(t) that is valid over one period.

$$a_k = \frac{1}{12}\int_0^{12} (4+t)e^{-j\left(\frac{2\pi}{12}\right)kt}\,dt$$

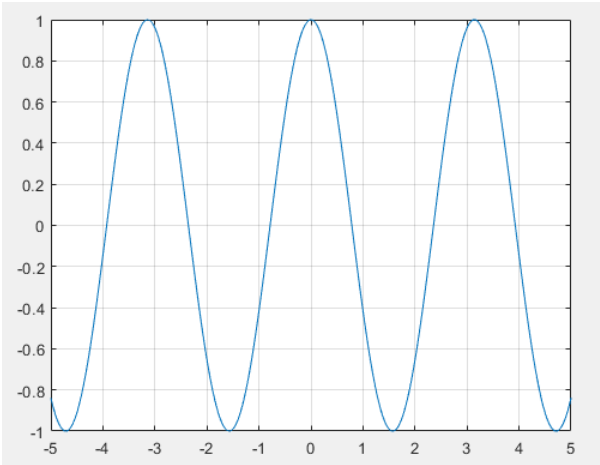
b) Using your result from part (a), draw a plot 9f x(t) over the range -12 <= t <= 12 seconds. Label it carefully.

c) Determine a_0, the DC value of x(t).

$$a_0 = \frac{1}{12}\int_0^{T_0} (4+t)\,dt$$

Problem 4.4

a) Sketch the periodic function x(t) for -5 <= t <= 5



b) Determine a_0, the DC coefficient for the Fourier series.

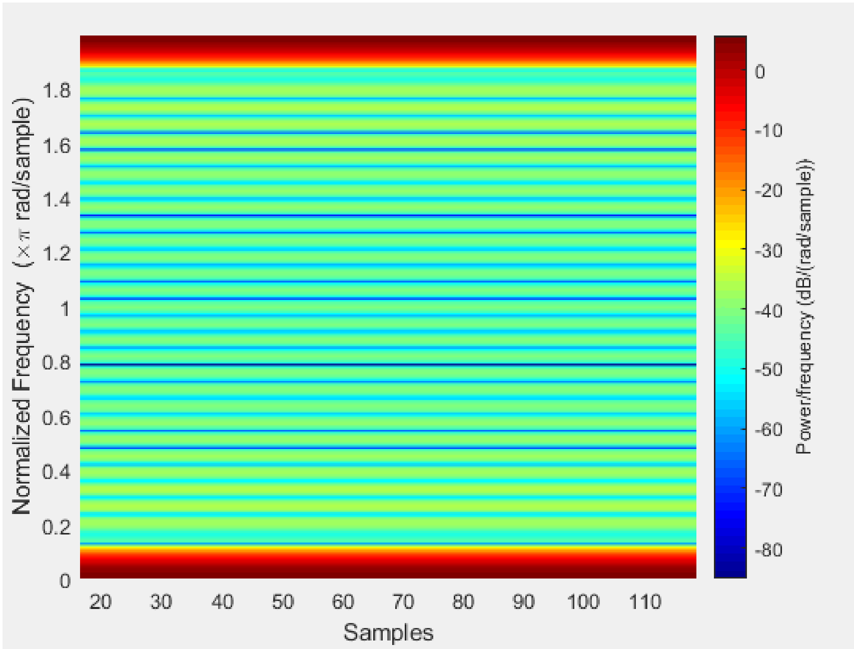
$$a_0 = \frac{1}{4}\int_0^{4} (e^{-2t})\,dt$$

c) Set up the Fourier analysis integral for determining a_k for k != 0

$$a_k = \frac{1}{4}\int_0^{4} (e^{-2t})e^{-jk\frac{\pi}{2}t}\,dt$$

d) Evaluate the integral in part (c) and obtain an expression for a_k that is valid for all k != 0

e) Make a plot of the spectrum over the range -3f$_0$ <= f <= 3f$_0$ where f$_0$ is the fundamental frequency of the signal.



Problem 4.5

a) Use Euler's formulas for the cosine functions to expand x(t) in terms of complex exponential signals so that you can sketch the two-sided spectrum of the signal. Is the waveform periodic? What is the period?

$$x(t) = 5\cos\left(2000\pi t - \frac{\pi}{4}\right) + 15\cos\left(400\pi t + \frac{\pi}{2}\right)\cos\left(2000\pi t - \frac{\pi}{4}\right)$$

$$= 5\left(\frac{1}{2}\left(e^{j2000\pi t - \frac{\pi}{4}} + e^{-j2000\pi t - \frac{\pi}{4}}\right)\right) + 15\left(\frac{1}{2}\left(e^{j400\pi t + \frac{\pi}{2}} + e^{-j400\pi t + \frac{\pi}{2}}\right)\left(e^{j2000\pi t - \frac{\pi}{4}} + e^{-j2000\pi t - \frac{\pi}{4}}\right)\right)$$

$$= 5 + 15\left(\frac{1}{2}e^{j\frac{\pi}{2}}\left(e^{j400\pi t} + e^{-j400\pi t}\right)\right)\left(\frac{1}{2}e^{-j\frac{\pi}{4}}\left(e^{j2000\pi t} + e^{-j2000\pi t}\right)\right)$$

$$\omega_0 = 400\pi = \frac{2\pi}{T_0}$$

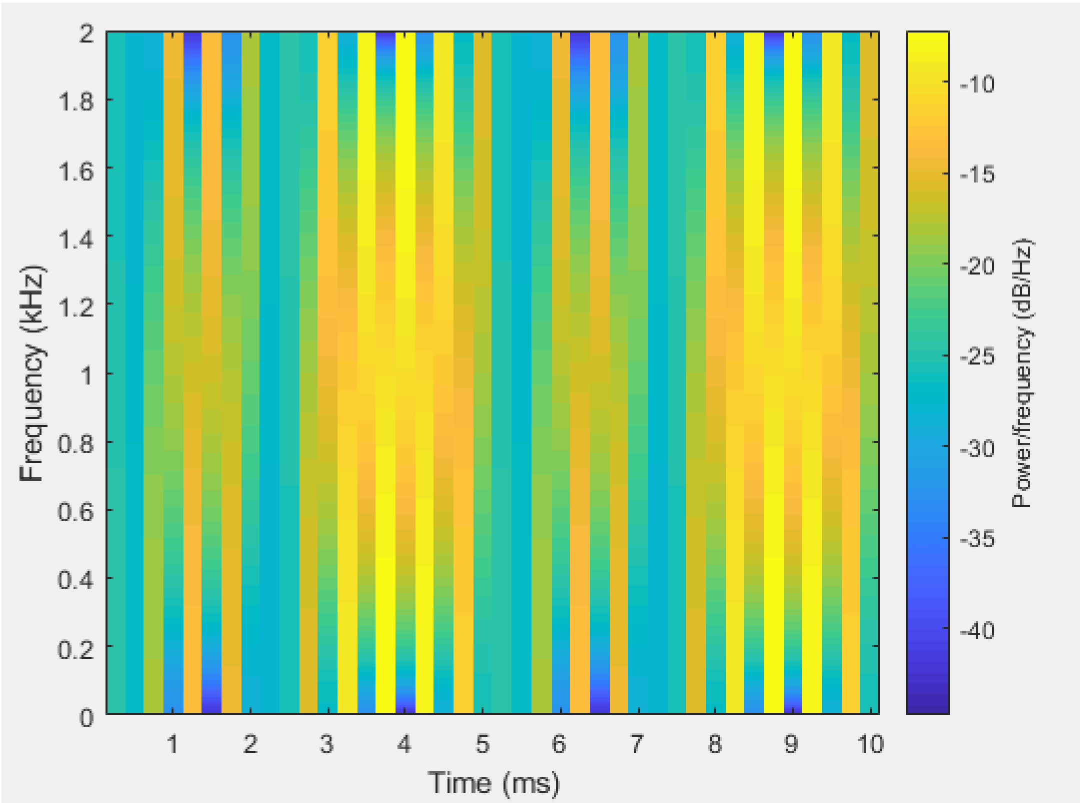
$$T_0 = \frac{2\pi}{400\pi} = 0.005$$

$$f_0 = \frac{1}{0.005} = 200$$

b) What is the minimum sampling rate f$_s$ that can be used in the above system so that y(t) = x(t)?

$$f_s \text{ must be at least } = 2 * f_0 + (a\ little) \text{to avoid aliasing } (Nyquist\ rate)$$

c) Plot the spectrum of the sampled signal x[n] for the case when f_s = 4000.



Problem 4.6
a) Draw the spectrum @ f_{si} = 10000 samples/sec