ECES-352
Winter 2019
Homework #6

Reading: Chapter 5 on FIR Filters

PROBLEM 6.1:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = 3x[n-1] + x[n] + 3x[n+1]$

(b) $y[n] = x[n]\cos(.3\pi n)$

(c) $y[n] = |x[-n]|$

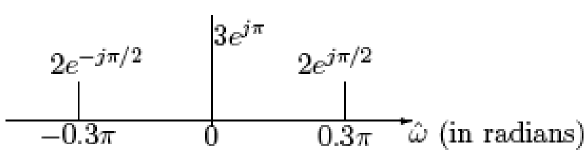
(d) $y[n] = x[n-2] + 2x[n] + x[n+2]$

(e) $y[n] = nx[n]$

(f) $y[n] = (x[-n])^2$

PROBLEM 6.2

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

PROBLEM 6.3:

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^3. \tag{1}$$

(a) Determine whether or not the system defined by (1) is (i) linear; (ii) time-invariant; (iii) causal.

(b) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.6\pi n) = e^{j0.6\pi n} + e^{-j0.6\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any powers of cosine functions in your answers. Note that this system produces output contain frequencies that are not present in the input signal. Explain how this system might cause "aliasing" of sinusoidal components of the input.

PROBLEM 6.4

The *unit step* sequence, denoted by $u[n]$, is defined as

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

(a) Make a plot of $u[n]$ for $-5 \leq n \leq 10$. Describe the plot of $u[n]$ outside this range.

(b) We can use the unit step sequence as a convenient representation for sequences that are given by formulas over a range of values. Make a plot of the sequence

$$x[n] = u[n] - u[n-5])$$

for $-5 \leq n \leq 10$.

(c) Now make a plot of the sequence

$$x[n] = (.9)^n(u[n] - u[n-5])$$

for $-5 \leq n \leq 10$.

(d) Suppose that $x[n]$ in part (c) is the input to a 4-point running average system. Compute and plot $y[n]$, the output of the system for $-5 \leq n \leq 10$.

PROBLEM 6.5

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4} (2-k)x[n-k]$$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use the above difference equation to compute the output $y[n]$ when the input is

$$x[n] = \begin{cases} 0 & n < 0 \\ 5 & 0 \le n \le 5 \\ 1 & 6 \le n \le 10 \\ 0 & n \ge 11. \end{cases}$$

Make a plot of both $x[n]$ and $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv()` function.)

PROBLEM 6.6

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

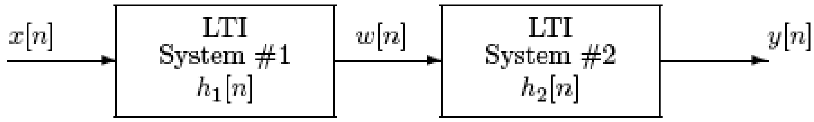


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that LTI System #1 is described by the difference equation

$$w[n] = x[n] - 0.2x[n-1].$$

Determine the impulse response $h_1[n]$ of the first system.

(b) The LTI System #2 is described by the impulse response

$$h_2[n] = (0.2)^n(u[n] - u[n-L]) = \sum_{k=0}^{L-1}(0.2)^k\delta[n-k] = \begin{cases} (0.2)^n & n = 0, 1, \ldots, L-1 \\ 0 & \text{otherwise.} \end{cases}$$

For the special case of $L = 10$, use convolution to show that the impulse response sequence of the overall cascade system is

$$h[n] = h_1[n] * h_2[n] = \delta[n] - (0.2)^{10}\delta[n-10].$$

(c) Generalize your result in part (b) for the general case of L any integer value.

(d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.

(e) How would you choose L so that $y[n] = x[n]$ in Figure 1; i.e., how would you choose L so that the second system "undoes" the effect of the first system?

PROBLEM 6.7

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] - 2x[n-1] + x[n-2] + 3x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the text.

(b) Determine the impulse response $h[n]$ for this system. Express your answer as a sum of scaled and shifted unit impulse sequences.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] + \delta[n-1] - \delta[n-2]$$

Plot the output sequence $y[n]$ for $-2 \leq n \leq 10$.

(d) Now consider another LTI system whose impulse response is

$$h_d[n] = \delta[n] + \delta[n-1] - \delta[n-2].$$

Use convolution again to determine $y_d[n] = x_d[n] * h_d[n]$, the output of this system when the input is

$$x_d[n] = \delta[n] - 2\delta[n-1] + \delta[n-2] + 3\delta[n-4].$$

How does your answer compare to the answer in part (c)? This example illustrates the general commutative property of convolution; i.e., $x[n] * h[n] = h[n] * x[n]$.