ECES-352
Winter 2019
Homework #9

Reading: Chapter 10 on IIR Filters

PROBLEM 9.1*:

Determine the z-transforms of the following. *Express your answer as the ratio of polynomials in z^{-1} by placing all terms over a common denominator.*

(a) $x_a[n] = 2(-0.8)^n u[n]$.

(b) $x_b[n] = 3(\frac{1}{3})^n u[n] + 3(-\frac{1}{3})^n u[n]$.

(c) $x_c[n] = -\delta[n] + u[n-1]$.

PROBLEM 9.2*:

Determine the inverse z-transforms of the following:

(a) $H_a(z) = \dfrac{1 + z^{-2}}{1 - 0.8z^{-1}}$.

(b) $H_b(z) = \dfrac{2 - 0.8z^{-1}}{1 - 0.8z^{-1} + 0.64z^{-2}} = \dfrac{1}{1 - 0.8e^{j\pi/3}z^{-1}} + \dfrac{1}{1 - 0.8e^{-j\pi/3}z^{-1}}$.

(c) $H_c(z) = \dfrac{0.6 + z^{-1}}{1 + 0.6z^{-1}}$.

PROBLEM 9.3*:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. Plot the poles (**X**) and zeros (**O**) in the complex z-plane.

$$\mathcal{S}_1: \quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$$

$$\mathcal{S}_2: \quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$$

$$\mathcal{S}_3: \quad y[n] = -0.25y[n-2] + x[n] + x[n-2]$$

$$\mathcal{S}_4: \quad y[n] = x[n] + \tfrac{3}{4}x[n-1] - \tfrac{1}{4}x[n-2]$$

PROBLEM 9.4*:

The system function $H(z)$ and the impulse response $h[n]$ are two ways to define a LTI system. Use z-transforms to answer the following:[1]

(a) Find the system function for $h_a[n] = u[n] - u[n-6]$.
Use the z-transform of $u[n]$ to express your answer as a ratio of polynomials in z^{-1}. Then simplify to get a polynomial in z^{-1} (i.e., no denominator). Is this an FIR or IIR filter?

(b) Find the system function for $h_b[n] = (1/3)^n u[n] + (-1/3)^n u[n]$.
Express your answer as: (1) a sum of two first-order rational functions; and (2) a ratio of polynomials in z^{-1} (one numerator over one denominator).

(c) Determine the impulse response when the system function is: $H_c(z) = \dfrac{1 - z^{-2}}{1 + 0.5z^{-1}}$.

(d) Determine the impulse response when $H_d(z) = \dfrac{1 - z^{-1}}{(1 + \frac{1}{2}z^{-1})(1 + 0.7z^{-1})}$.
Hint: write $H_d(z)$ as a sum of first-order rational functions.

(e) Determine the impulse response when $H_e(z) = 1 - 2z^{-2} + 3z^{-4} - 4z^{-6} - 5z^{-8}$.

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[1] A *rational* function is the ratio of two polynomials. For example, $H(z) = \dfrac{1 - z^{-1} + z^{-2}}{1 + 0.3z^{-1} + 0.4z^{-2}}$.

PROBLEM 9.5*:

An LTI system has the following system function:

$$H(z) = \frac{1 + z^{-2}}{1 + 0.5z^{-1}}.$$

The following questions cover most of the ways available for analyzing IIR discrete-time systems.

(a) Plot the poles and zeros of $H(z)$ in the z-plane.

(b) Determine the difference equation that is satisfied by the general input $x[n]$ and the corresponding output $y[n]$ of the system.

(c) Use z-transforms to determine the impulse response $h[n]$ of the system; i.e., the output of the system when the input is $x[n] = \delta[n]$.

(d) Determine an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(e) Use the frequency response function to determine the output $y_1[n]$ of the system when the input is

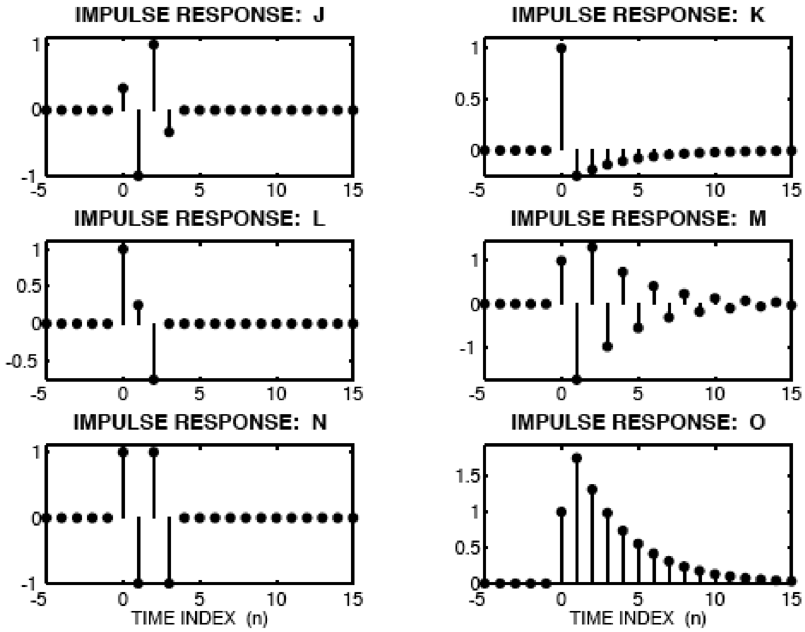
$$x_1[n] = 2\cos(0.5\pi n) \qquad -\infty < n < \infty.$$

(f) Use the z-transform to determine the output $y_2[n]$ when the input is

$$x_2[n] = 2\cos(0.5\pi n)u[n].$$

PROBLEM 9.6:

This problem has been given before on exams. It is a good review.



For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems[2] (specified by either an $H(z)$ or a difference equation) matches the impulse response. In addition, derive a formula for the impulse response, $h[n]$, for $\mathcal{S}_1$ and $\mathcal{S}_4$.

$\mathcal{S}_1$: $\quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2$: $\quad H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3$: $\quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4$: $\quad H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5$: $\quad y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6$: $\quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7$: $\quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8$: $\quad H(z) = \frac{1}{3}(1-z^{-1})^3$

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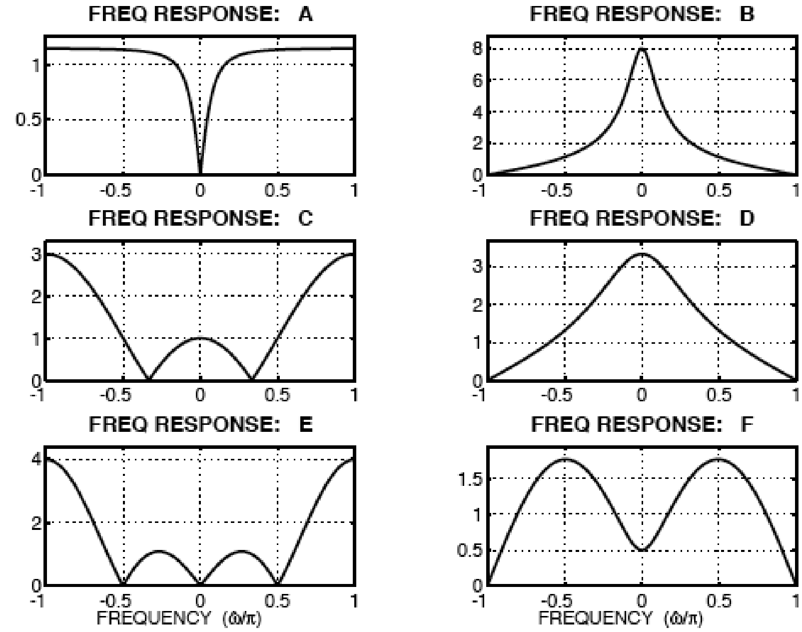
[2] These 8 systems are exactly the same as for the next problem.

PROBLEM 9.7:

This problem has also been given before on exams. It is a good review.



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only) and write an expression for the magnitude of the frequency response. NOTE: the frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1:\quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

$\mathcal{S}_2:\quad H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_3:\quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$\mathcal{S}_4:\quad H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

$\mathcal{S}_5:\quad y[n] = x[n] - x[n-1] + x[n-2]$

$\mathcal{S}_6:\quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$\mathcal{S}_7:\quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$\mathcal{S}_8:\quad H(z) = \frac{1}{3}(1-z^{-1})^3$