ECES-352

Winter 2019

Lab #7: FIR Filtering of Images

(Lab Report Du e at beginning of next lab)

This is the official Lab #7 description. You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.

Formal Lab Report: You must write a formal lab report that describes your approach to music synthesis (Section 4). **You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.**

Important: When it instructs you to get an updated matlab file (like specgram.m), download http://dspfirst.gatech.edu/matlab/toolbox/

Introduction

The goal of this lab is to learn how to implement FIR filters in MATLAB, and then study the response of FIR filters to various signals, including images. As a result, you should learn how filters can create effects such as blurring and ghosts. In addition, we will use FIR filters to study the convolution operation and properties such as linearity and time-invariance.

In the experiments of this lab, you will use firfilt( ), or conv(), to implement 1-D filters and conv2() to implement two-dimensional (2-D) filters. In this lab the 2-D filtering operation actually consists of 1-D filters applied to all the rows of the image and then to all of the columns.

1.1 Two GUIs

This lab involves the use of two MATLAB GUIs: one for sampling and aliasing and one for convolution.

1. **con2dis:** GUI for sampling and aliasing. An input sinusoid and its spectrum is tracked through A/D and D/A converters.

2. **dconvdemo:** GUI for discrete-time convolution. This is exactly the same as the MATLAB functions conv() and firfilt() used to implement FIR filters.

Both of these demos can be downloaded from WebCT for ECE2025. From the Homepage, click "Extra M-files for Labs". Then click "Educational Matlab GUIs". Then go to the two demos (*continuous-discrete sampling demo* and *discrete convolution demo*) and click on the download statement.

1.2 Overview of Filtering

For this lab, we will define an FIR *filter* as a discrete-time system that converts an input signal $x[n]$ into an output signal $y[n]$ by means of the weighted summation:

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k] \tag{1}$$

Equation (1) gives a rule for computing the n^{th} value of the output sequence from certain values of the input sequence. The filter coefficients $\{b_k\}$ are constants that define the filter's behavior. As an example, consider the system for which the output values are given by

$$\begin{aligned} y[n] &= \tfrac{1}{3}x[n] + \tfrac{1}{3}x[n-1] + \tfrac{1}{3}x[n-2] \\ &= \tfrac{1}{3}\left\{x[n] + x[n-1] + x[n-2]\right\} \end{aligned} \tag{2}$$

This equation states that the n^{th} value of the output sequence is the average of the n^{th} value of the input sequence $x[n]$ and the two preceding values, $x[n-1]$ and $x[n-2]$. For this example the b_k's are $b_0 = \frac{1}{3}$, $b_1 = \frac{1}{3}$, and $b_2 = \frac{1}{3}$.

MATLAB has built-in functions, `conv( )` and `filter( )`, for implementing this operation in (1), but we have also supplied another M-file `firfilt( )` for the special case of FIR filtering. The function `filter` implements a wider class of filters than just the FIR case. Technically speaking, the both the `conv` and the `firfilt` function implement the operation called *convolution*. The following MATLAB statements implement the three-point averaging system of (2):

firfilt.m

```
nn = 0:99;                  %<--Time indices
xx = cos( 0.08*pi*nn );     %<--Input signal
bb = [1/3 1/3 1/3];         %<--Filter coefficients
yy = firfilt(bb, xx);       %<--Compute the output
```

In this case, the input signal `xx` is a vector containing a cosine function. In general, the vector `bb` contains the filter coefficients $\{b_k\}$ needed in (1). These are loaded into the `bb` vector in the following way:

```
bb = [b0, b1, b2, ...  , bM].
```

In MATLAB, all sequences have finite length because they are stored in vectors. If the input signal has, for example, L samples, we would normally only store the L samples in a vector, and would assume that $x[n] = 0$ for n outside the interval of L samples; i.e., we do not have to store any zero samples unless it suits our purposes. If we process a finite-length signal through (1), then the output sequence $y[n]$ will be longer than $x[n]$ by M samples. Whenever `firfilt( )` implements (1), we will find that

```
length(yy) = length(xx)+length(bb)-1
```

In the experiments of this lab, you will use `firfilt( )` to implement FIR filters and begin to understand how the filter coefficients define a digital filtering algorithm. In addition, this lab will introduce examples to show how a filter reacts to different frequency components in the input.

1.3 Pre-Lab: Run the GUIs

The first objective of this lab is to demonstrate that you understand the concepts behind the two GUIs. First of all, you must download the ZIP files for each and install them. Each ZIP file installs as a directory containing a number of files. You can put the GUIs on the `matlabpath`, or you can run the GUIs from their home directories.

1.4 Sampling and Aliasing Demo

In this demo, you can change the frequency of an input signal that is a sinusoid, and you can change the sampling frequency. The GUI will show the sampled signal, $x[n]$, its spectrum, and also the reconstructed output signal, $y(t)$ with its spectrum. Figure 1 shows the interface for the `con2dis` GUI. In order to see the entire GUI, you must select `Show All Plots` under the `Plot Options` menu.

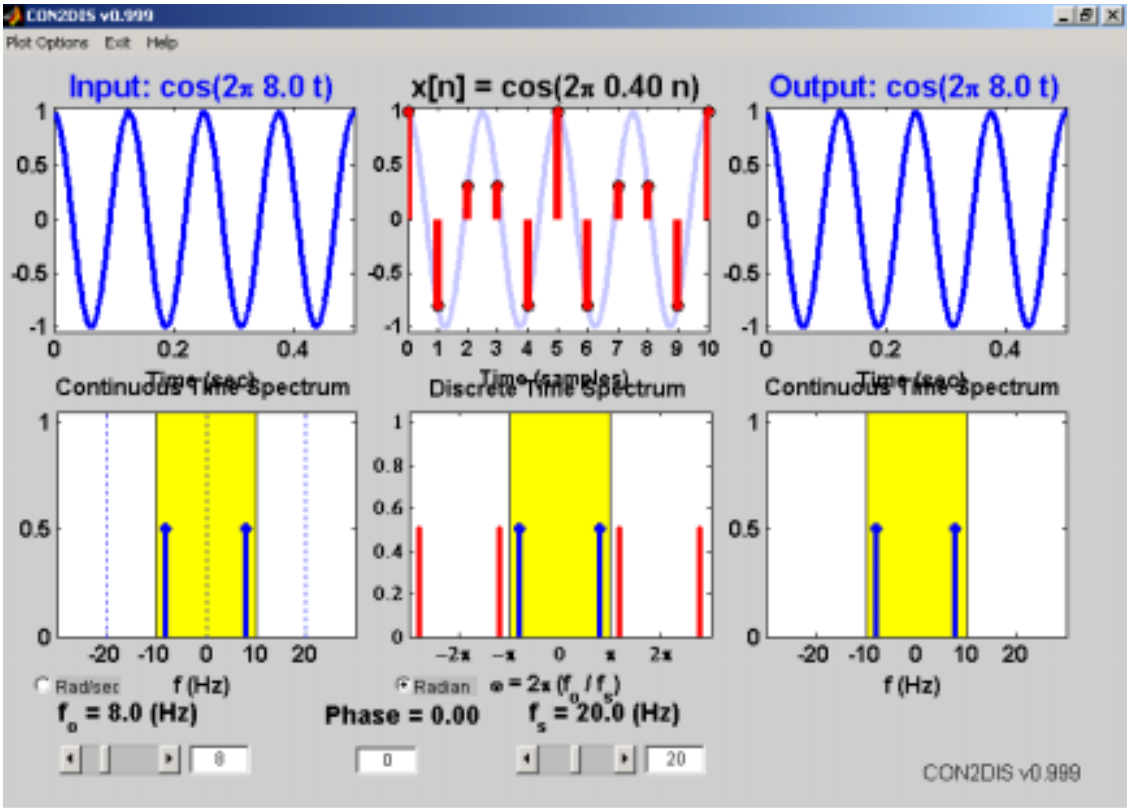


Figure 1: Sampling/reconstruction demo interface.

In the pre-Lab, you should perform the following steps with the `con2dis` GUI:

(a) Set the input to $x(t) = \cos(40\pi t)$

(b) Set the sampling rate to $f_s = 24$ samples/sec.

(c) Determine the locations of the spectrum lines for the discrete-time signal, $x[n]$, found in the middle panels. Click the `Radian` button to change the axis to from $\hat{f}$ to $\hat{\omega}$.

(d) Determine the formula for the output signal, $y(t)$ shown in the rightmost panels. What is the output frequency in Hz?

1.5 Discrete-Time Convolution Demo

In this demo, you can select an input signal $x[n]$, as well as the impulse response of the filter $h[n]$. Then the demo shows the "flipping and shifting" used when a convolution is computed. This corresponds to the sliding window of the FIR filter. Figure 2 shows the interface for the `dconvdemo` GUI.

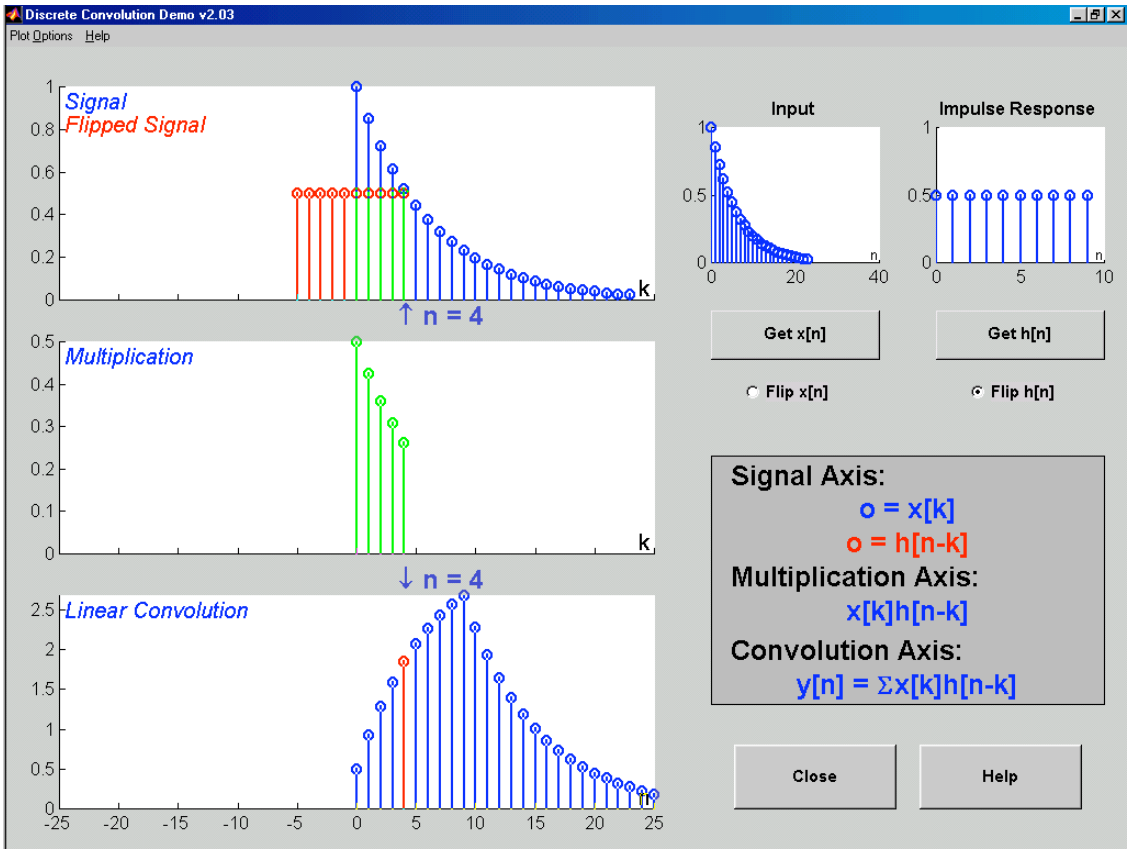


Figure 2: Interface for discrete-time convolution GUI.

In the pre-lab, you should perform the following steps with the `dconvdemo` GUI.

(a) Click on the `Get x[n]` button and set the input to a finite-length pulse: $x[n] = (u[n] - u[n-10])$.

(b) Set the filter to a three-point averager by using the `Get h[n]` button to create the correct impulse response for the three-point averager. Remember that the impulse response is identical to the b_k's for an FIR filter. Also, the GUI allows you to modify the length and values of the pulse.

(c) Use the GUI to produce the output signal.

(d) When you move the mouse pointer over the index "n" below the signal plot and do a click-hold, you will get a *hand tool* that allows you to move the "n"-pointer. By moving the pointer horizontally you can observe the sliding window action of convolution. You can even move the index beyond the limits of the window and the plot will scroll over to align with "n."

1.6 Filtering via Convolution

You can perform the same convolution as done by the `dconvdemo` GUI by using the MATLAB function `firfilt`, or `conv`. For ECE-2025, the preferred function is `firfilt`.

(a) For the Pre-Lab, you should do the filtering with a 3-point averager. The filter coefficient vector for the 3-point averager is defined via:

```
bb = 1/3*ones(1,3);
```

Use `firfilt` to process an input signal that is a length-10 pulse:

$$x[n] = \begin{cases} 1 & \text{for } n = 0,1,2,3,4,5,6,7,8,9 \\ 0 & \text{elsewhere} \end{cases}$$

NOTE: in MATLAB indexing can be confusing. Our mathematical signal definitions start at $n = 0$, but MATLAB starts its indexing at "1". Nevertheless, we can ignore the difference and pretend that MATLAB is indexing from zero, as long as we don't try to write `x[0]` in MATLAB. For this experiment, generate the length-10 pulse and put it inside of a longer vector with the statment `xx = [ones(1,10),zeros(1,5)].` This produces a vector of length 15, which has 5 extra zero samples appended.

(b) To illustrate the filtering action of the 3-point averager, it is informative to make a plot of the input signal and output signals together. Since $x[n]$ and $y[n]$ are discrete-time signals, a `stem` plot is needed. One way to put the plots together is to use `subplot(2,1,*)` to make a two-panel display:

```
nn = first:last;      %--- use first=1 and last=length(xx)
subplot(2,1,1);
stem(nn-1,xx(nn))
subplot(2,1,2);
stem(nn-1,yy(nn),'filled')   %--Make black dots
xlabel('Time Index (n)')
```

This code assumes that the output from `firfilt` is called `yy`. Try the plot with `first` equal to the beginning index of the input signal, and `last` chosen to be the last index of the input. In other words, the plotting range for both signals will be equal to the length of the input signal, even though the output signal is longer. Notice that using `nn-1` in the call to `stem( )` causes the x-axis to start at zero in the plot.

(c) Explain the filtering action of the 3-point averager by comparing the plots in the previous part. This filter might be called a "smoothing" filter. Note how the transitions in $x[n]$ from zero to one, and from one back to zero, have been "smoothed."

2 Warm-up

2.1 Sampling and Aliasing

Use the `con2dis` GUI to do the following problem:

(a) Input frequency is 12 Hz.

(b) Sampling frequency is 15 Hz.

(c) Determine the frequency of the reconstructed output signal

(d) Determine the locations in $\hat{\omega}$ of the lines in the spectrum of the discrete-time signal. Give numerical values.

Instructor Verification (separate page)

(e) Change the sampling frequency to 12 Hz, and explain the appearance of the output signal.

2.2 Discrete-Time Convolution

In this section, you will generate filtering results needed in a later section. Use the discrete-time convolution GUI, `dconvdemo`, to do the following:

(a) Set the input signal to be $x[n] = (0.92)^n (u[n] - u[n-10])$. Use the "Exponential" signal type within `Get x[n]`.

(b) Set the impulse response to be $h[n] = \delta[n] - 0.92\delta[n-1]$. Once again, use the "Exponential" signal type within `Get h[n]`.

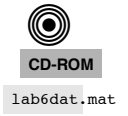
(c) Illustrate the output signal $y[n]$ and explain why it is zero for almost all points. Compute the numerical value of the last point in $y[n]$, i.e., the one that is negative and non-zero.

Instructor Verification (separate page)

2.3 Loading Data

In order to exercise the basic filtering function `firfilt`, we will use some "real" data. In MATLAB you can load data from a file called `lab6dat.mat` file by using the `load` command as follows:

```
load lab6dat
```



The data file `lab6dat.mat` contains two filters and three signals, stored as separate MATLAB variables:

- `x1:` a stair-step signal such as one might find in one sampled scan line from a TV test pattern image.
- `xtv:` an actual scan line from a digital image.
- `x2:` a speech waveform ("oak is strong") sampled at $f_s = 8000$ samples/second.
- `h1:` the coefficients for a FIR discrete-time filter of the form of (1).
- `h2:` coefficients for a second FIR filter.

For this lab we will only use the signal `x2`. After loading the data, use the `whos` function to verify that all five vectors are in your MATLAB workspace.

2.4 Filtering Images: 2-D Convolution

One-dimensional FIR filters, such as running averagers and first-difference filters, can be applied to one-dimensional signals such as speech or music. These same filters can be applied to images if we regard each row (or column) of the image as a one-dimensional signal. For example, the 50^{th} row of an image is the N-point sequence `xx[50,n]` for $1 \leq n \leq N$, so we can filter this sequence with a 1-D filter using the `conv` or `firfilt` operator.

One objective of this lab is to show how simple 2-D filtering can be accomplished with 1-D row and column filters. It might be tempting to use a `for` loop to write an M-file that would filter all the rows. For a *first-difference filter*, this would create a new image made up of the filtered rows:

$$y_1[m,n] = x[m,n] - x[m,n-1].$$

However, this image $y_1[m,n]$ would only be filtered in the horizontal direction. Filtering the columns would require another `for` loop, and finally you would have the completely filtered image:

$$y_2[m,n] = y_1[m,n] - y_1[m-1,n]$$

In this case, the image $y_2[m, n]$ has been filtered in both directions by a first-difference filter.

These filtering operations involve a lot of `conv` calculations, so the process can be slow. Fortunately, MATLAB has a built-in function `conv2( )` that will do this with a single call. It performs a more general filtering operation than row/column filtering, but since it can do these simple 1-D operations it will be very helpful in this lab.

(a) Load in the image `echart.mat` with the `load` command (it will create the variable `echart` whose size is 257×256). We can filter all the rows of the image at once with the `conv2( )` function. To filter the image in the horizontal direction using a second-difference filter, we form a *row* vector of filter coefficients and use the following MATLAB statements:

```
bdiffh = [0.25, -0.5, 0.25];
yy1 = conv2(echart, bdiffh);
```

In other words, the filter coefficients `bdiffh` for the first-difference filter are stored in a *row* vector and will cause `conv2( )` to filter all rows in the *horizontal* direction. Display the input image `echart` and the output image `yy1` on the screen at the same time. Compare the two images and give a qualitative description of what you see.

(b) Now filter the "eye-chart" image `echart` in the *vertical* direction with a second-difference filter to produce the image `yy2`. This is done by calling `yy2 = conv2(echart,bdiffh')` with a column vector of filter coefficients. Display the image `yy2` on the screen and describe in words how the output image compares to the input.

Instructor Verification (separate page)

3 Lab: FIR Filters

In the following sections we will study how a filter can produce the following special effects:

1. *Echo:* FIR filters can produce echoes and reverberations because the filtering formula (1) contains delay terms. In an image, such phenomena would be called "ghosts."
2. *Deconvolution:* One FIR filter can (approximately) undo the effects of another—we will investigate a cascade of two FIR filters that distort and then restore an image. This process is called *deconvolution*.

3.1 Deconvolution Experiment for 1-D Filters

Use the function `firfilt( )` to implement the following FIR filter

$$w[n] = x[n] - 0.92x[n-1] \tag{3}$$

on the input signal $x[n]$ defined via the MATLAB statement: `xx = 256*(rem(0:100,30)>10);` In MATLAB you must define the vector of filter coefficients `bb` needed in `firfilt`.

(a) Plot both the input and output waveforms $x[n]$ and $w[n]$ on the same figure, using `subplot`. Make the discrete-time signal plots with MATLAB's `stem` function, but restrict the horizontal axis to the range $0 \leq n \leq 90$. Explain why the output appears the way it does by figuring out (mathematically) the effect of the filter coefficients in (3).

(b) Note that $w[n]$ and $x[n]$ are not the same length. Determine the length of the filtered signal $w[n]$, and explain how its length is related to the length of $x[n]$ and the length of the FIR filter. (If you need a hint refer to Section 1.2.)

3.1.1 Restoration Filter

The following FIR filter

$$y[n] = \sum_{\ell=0}^{M} r^{\ell} w[n-\ell] \qquad \text{(FIR FILTER-2)}$$

can be use to undo the effects of the FIR filter in the previous section (see the block diagram in Fig. 3). It performs restoration, but it only does this approximately. Use the following steps to show how well it works when $r = 0.92$ and $M = 15$.

(a) Process the signal $w[n]$ from (3) with FILTER-2 to obtain the output signal $y[n]$.

(b) Make stem plots of $w[n]$ and $y[n]$ using a time-index axis n that is the same for both signals. Put the stem plots in the same window for comparison—using a two-panel subplot.

(c) Since the objective of the restoration filter is to produce a $y[n]$ that is almost identical to $x[n]$, make a plot of the error between $x[n]$ and $y[n]$ over the range $0 \leq n \leq 90$.

3.1.2 Worst-Case Error

(a) Evaluate the *worst-case error* by doing the following: find the maximum of the difference between $y[n]$ and $x[n]$ in the range $0 \leq n \leq 90$.

(b) What does the error plot and worst case error tell you about the quality of the restoration of $x[n]$? What parameter of FILTER-2 would you change to improve the quality of the restoration? How small do you think the worst case error has to be so that it cannot be seen on a plot?

3.1.3 An Echo Filter

The following FIR filter can be interpreted as an echo filter.

$$y_1[n] = x_1[n] + r\, x_1[n-P] \tag{4}$$

Explain why this is a valid interpretation by working out the following:

(a) You have an audio signal sampled at $f_s = 8000$ Hz and you would like to add a delayed version of the signal to simulate an echo. The time delay of the echo should be 0.2 seconds, and the strength of the echo should be 90% percent of the original. Determine the values of r and P in (4); make P an integer.

(b) Describe the filter coefficients of this FIR filter, and determine its length.

(c) Implement the echo filter in (4) with the values of r and P determined in part (a). Use the speech signal in the vector `x2` found in the file `lab6dat.mat`. Listen to the result to verify that you have produced an audible echo.

3.2 Cascading Two Systems

More complicated systems are often made up from simple building blocks. In the system of Fig. 3 two FIR filters are connected "in cascade." For this section, assume that the the filters in Fig. 3 are described by the two equations:

$$w[n] = x[n] - q\,x[n-1] \qquad \text{(FIR FILTER-1)}$$

$$y[n] = \sum_{\ell=0}^{M} r^{\ell} w[n-\ell] \qquad \text{(FIR FILTER-2)}$$

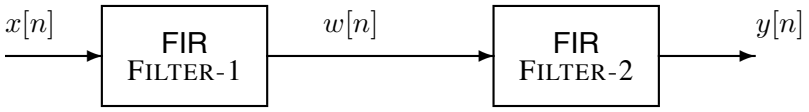


Figure 3: Cascading two FIR filters: the second filter attempts to "deconvolve" the distortion introduced by the first.

3.2.1 Overall Impulse Response

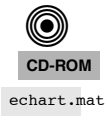
(a) Implement the system in Fig. 3 using MATLAB to get the impulse response of the overall cascaded system for the case where $q = 0.92$, $r = 0.92$ and $M = 15$. Use two calls to `firfilt().` Plot the impulse response of the overall cascaded system.

(b) Work out the impulse response $h[n]$ of the cascaded system by hand to verify that your MATLAB result in part (a) is correct. (Hint: consult Problem 6.7 of Problem Set #6.)

(c) In a *deconvolution* application, the second system (FIR FILTER-2) tries to undo the convolutional effect of the first. Perfect deconvolution would require that the cascade combination of the two systems be equivalent to the identity system: $y[n] = x[n]$. If the impulse responses of the two systems are $h_1[n]$ and $h_2[n]$, state the condition on $h_1[n] * h_2[n]$ to achieve perfect deconvolution.[1]

3.2.2 Distorting and Restoring Images

If we pick q to be a little less than 1.0, then the first system (FIR FILTER-1) will cause distortion when applied to the rows and columns of an image. The objective in this section is to show that we can use the second system (FIR FILTER-2) to undo this distortion (more or less). Since FIR FILTER-2 will try to undo the convolutional effect of the first, it acts as a *deconvolution* operator.



(a) Load in the image `echart.mat` with the `load` command. It creates a matrix called `echart`.

(b) Pick $q = 0.92$ in FILTER-1 and filter the image `echart` in both directions: apply FILTER-1 along the horizontal direction and then filter the resulting image along the vertical direction also with FILTER-1. Call the result `ech92`.

[1] Note: the cascade of FIR FILTER-1 and FILTER-2 does not perform *perfect* deconvolution.

(c) Deconvolve `ech92` with FIR FILTER-2, choosing $M = 15$ and $r = 0.92$. Describe the visual appearance of the output, and explain its features by invoking your mathematical understanding of the cascade filtering process. Explain why you see "ghosts" in the output image, and use some previous calculations to determine how big the ghosts (or echoes) are, and where they are located. Evaluate the *worst-case error* in order to say how big the ghosts are relative to "black-white" transitions which are 0 to 255.

3.2.3 A Second Restoration Experiment

(a) Now try to deconvolve `ech92` with several different FIR filters for FILTER-2. You should set $r = 0.92$ and try several values for M such as 10, 15 and 30. Pick the best result and explain why it is the best. Describe the visual appearance of the output, and explain its features by invoking your mathematical understanding of the cascade filtering process. HINT: determine the impulse response of the cascaded system and relate it to the visual appearance of the output image.
Hint: you can use `dconvdemo` to generate the impulse responses of the cascaded systems, like you did in the Warm-up.

(b) Furthermore, when you consider that a gray-scale display has 256 levels, how large is the worst-case error (from the previous part) in terms of number of gray levels? Do this calculation for each of the three filters in part (a). Think about the following question: "Can your eyes perceive a gray scale change of one level, i.e., one part in 256?"

Include all images and plots for the previous two parts to support your discussions in the lab report.

Lab #7
ECES-352
Instructor Verification Sheet

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name:_____ Date:_____

Part 2.1: Demonstrate that you can run the **`con2dis`** GUI. Calculate the locations of the spectrum lines for the discrete-time signal. Write the values of $\hat{\omega}$ below.

Verified:_____ Date/Time:_____

Part 2.2: Demonstrate that you can run the **`dconvdemo`** GUI. Explain why the output is zero for most points.

Verified:_____ Date/Time:_____

Part 2.3 (a),(b) Process the input image **`echart`** 2-D filter that filters in both the horizontal and vertical directions with a first difference filter. Explain how the filter changes the "image signal."