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ECES 352

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HW 7

Problem 7.1

a)

$$y[n] = \left(Ae^{j\phi}e^{j\omega n}\right)^3 = A^3e^{j3\phi}e^{j3\omega n}$$

b) The angular frequency is tripled so we cannot convert the output, y[n], into the form $y[n] = H(\omega)Ae^{j\phi}e^{j\omega n}$

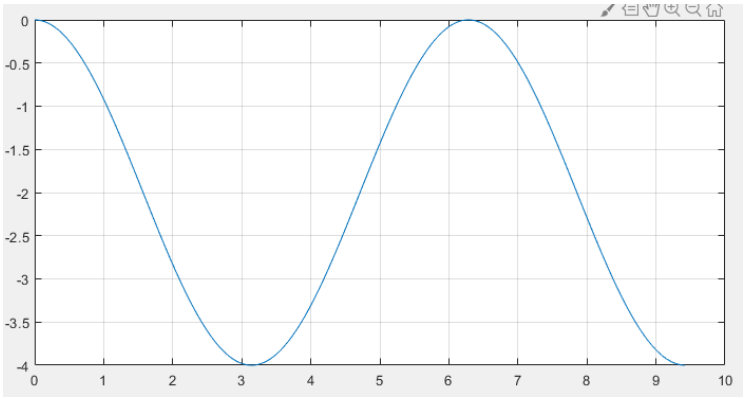
Problem 7.2

a) time invariant

b)

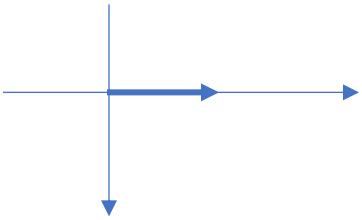
$$y[\omega] = x[\omega]e^{j\omega} - 2x[\omega] + x[\omega]e^{-j\omega}$$
$$\frac{y[\omega]}{x[\omega]} = e^{j\omega} + e^{-j\omega} - 2$$
$$H[\omega] = 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) - 2 = 2(\cos(\omega) - 1)$$

c)



Magnitude response

Phase response



d)

$$y[n] \text{ @ } e^{j0.25\pi} = 2\left(\cos\left(e^{j0.25\pi}\right) - 1\right)e^{j0.25\pi}$$
$$y[n] \text{ @ } e^{-j0.25\pi} = 2\left(\cos\left(e^{-j0.25\pi}\right) - 1\right)e^{-j0.25\pi}$$
$$y_1[n] = -0.585e^{j0.25\pi} - 0.585e^{-j0.25\pi} = -1.17\cos(0.25\pi)$$

e)

$$y_2[n] = 2\left(\cos\left(1 + cos\left(0.25\pi(n-1)\right)\right) - 1\right)$$

Problem 7.3

b) $H_1(\omega) = 1 - e^{j\omega}$

c)

$$H_2(\omega) = \frac{1}{1 - e^{-j\omega}}\left(1 - e^{-j\omega 10}\right)$$

d)

$$h_1[n] * h_2[n] = (\delta[n] - \delta[n-1]) * h_2[n]$$
$$= h_2[n] = h_2[n-1] = u[n] - u[n-10] - u[n-1] + u[n-11]$$
$$(u[n] - u[n-1]) - (u[n-10) - u[n-11]) = \delta[n] - \delta[n-10]$$

e)

$$H(z) = 1 - z^{-10} = 1 - e^{-j\omega 10}$$

f)

$$H_1(\omega)H_2(\omega) = \left(1 - e^{-j\omega}\right)\left(1 - e^{-j\omega 10}\right) = 1 - e^{-j\omega 10} = H(\omega)$$

Problem 7.4

a)

$$H\left(e^{j\omega}\right) = \frac{Y_1\left(e^{j\omega}\right)}{X_1\left(e^{j\omega}\right)} = e^{-j\omega} - e^{-j2\omega}$$
$$Y_1\left(e^{j\omega}\right) = X_1\left(e^{j\omega}\right)e^{-j\omega} - e^{-j2\omega}\left(e^{j\omega}\right)$$
$$Y_1(n) = x_1(n+1) - x_1(n+2)$$

b)

$$H_2\left(e^{j\omega}\right) = 1 + e^{-j2\omega}$$
$$H_3\left(e^{j\omega}\right) = 2e^{-j\omega} + 2e^{-j2\omega}$$

c)

$$H\left(e^{j\omega}\right) = H_1\left(e^{j\omega}\right) \times H_2\left(e^{j\omega}\right) \times H_3\left(e^{j\omega}\right)$$
$$= \left(e^{-j\omega} - e^{-j2\omega}\right)\left(1 + e^{-j2\omega}\right)\left(2e^{-j\omega} + 2e^{-j2\omega}\right)$$
$$= 2e^{-j2\omega} - 2e^{-j6\omega}$$

d)

$$\frac{Y_1\left(e^{j\omega}\right)}{X_1\left(e^{j\omega}\right)} = 2e^{-j2\omega} - 2e^{-j6\omega}$$
$$Y\left(e^{j\omega}\right) = 2e^{-j2\omega} \times e^{j\omega} - 2e^{-j6\omega} \times e^{j\omega}$$
$$= 2x(n-2) - 2x(n-6)$$

Problem 7.5

a)

$$\left(1 + e^{-j\omega}\right)\left(1 - e^{j\frac{\pi}{3}}e^{-j\omega}\right)\left(1 - e^{-j\frac{\pi}{3}}e^{-j\omega}\right)$$
$$= \left(1 + e^{-j\omega}\right)\left[1 - \left(e^{j\frac{\pi}{3}} + e^{-j\frac{\pi}{3}}\right)e^{-j\omega} + e^{-j2\omega}\right]$$
$$= 1 - 2e^{-j\omega} + 2e^{-j2\omega} - e\text{^} - j3\omega$$

b)

$$h[n] = \delta[n] - 2\delta[n-1] + 2\delta[n-2] - \delta[n-3]$$

c)

$$\left(1 - e^{-j\omega}\right) = 0$$
$$\left(1 - e^{\frac{j\pi}{3}}e^{-j\omega}\right) = 0$$
$$\left(1 - e^{-\frac{j\pi}{3}}e^{-j\omega}\right) = 0$$

$$\left(e^{j\omega} - e^{\frac{j\pi}{3}}\right) = 0$$
$$e^{j\omega} = e^{\frac{j\pi}{3}} \rightarrow \omega = \frac{\pi}{3}$$