ECES-352

Winter 2019

Exam #1

NAME: Solutions

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

- One sheet (8.5x11) of notes (front and back) is permitted and scientific (non-graphing) calculator is permitted.

- Justify your reasoning clearly to receive any partial credit. Explanations are required to receive full credit for any answer.

- You must write your answer in the space provided on the exam paper itself – only these answers will be graded. Circle your answers. If space is needed for scratch work, use the backs of the previous pages.

- Question 1

Define $x(t)$ as

$$x(t) = \cos(1.5\pi t - 5\pi/4) + 1.6\cos(1.5\pi(t-7))$$

Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

$\omega_0 = 1.5\pi$ $\quad A = 1.139$ $\quad \phi = -0.713\pi$ or 1.287π } 3rd Quadrant

$$e^{-j5\pi/4} + 1.6\, e^{-j10.5\pi}$$

$$-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} - j\,1.6$$

$$-\frac{\sqrt{2}}{2} - j\left(1.6 - \frac{\sqrt{2}}{2}\right)$$ } both negative so third quadrant

$$-.707 \quad -j\,.893 = 1.139 \angle -2.24\text{ rad}$$

$$= -0.713\pi$$

$$\text{atan}\left(\frac{0.893}{0.707}\right) = \underbrace{.9011}_{\text{1st quadrant}}$$

so

$$0.9011 - \pi = \boxed{-2.24\text{ rad}}$$

or $$0.9011 + \pi = \boxed{4.04\text{ rad}}$$

$$= 1.287\pi$$

Question 2

The MATLAB command

```
>> y = cos(0.002*pi*(0:250));
```

produces a vector that contains values of a sinusoid of the form $y(t) = A\cos(\omega t + \phi)$ for a number of values of time t.

(a) How long is the vector y, i.e., what would MATLAB give give for a numerical answer to the command `length(y)`?

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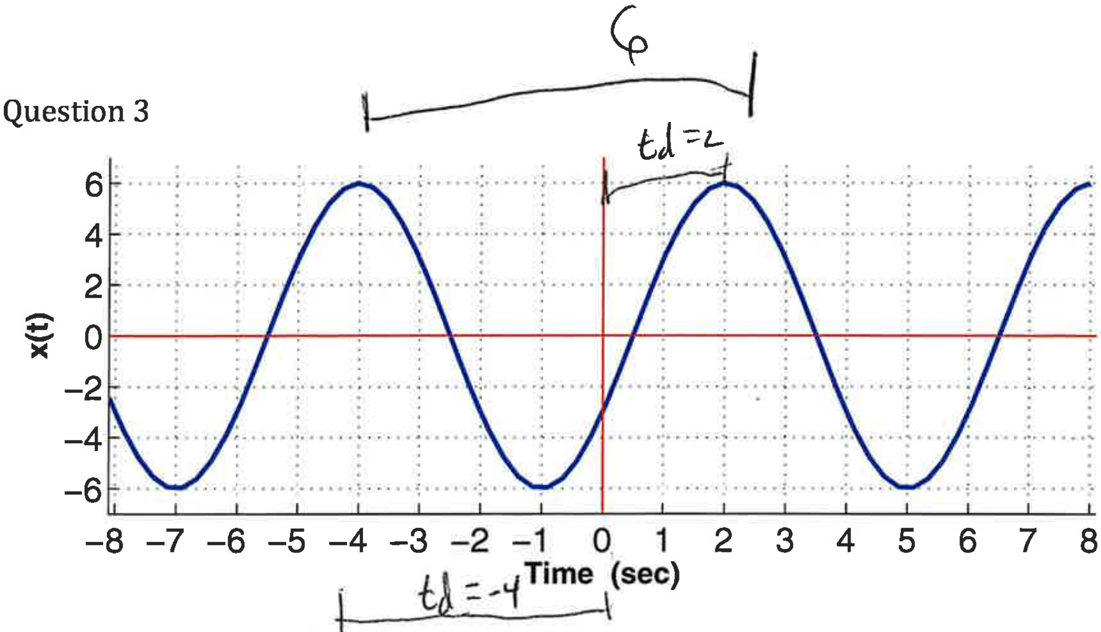
(b) Modify the MATLAB command above to generate 1001 values of the signal

$$y(t) = 3\cos(2\pi(10)(t - 0.01))$$

between $t = 0$ and $t = 0.1$ seconds.

y = 3*cos(2*10*pi*((0:1/1000:0.1)))

Question 3



(a) The graph above is a plot of a sinusoidal signal $x(t) = A\cos(\omega_0 t + \phi)$. Determine numerical values for A, ω_0 and ϕ with $-\pi < \phi \leq \pi$.

$$-2 \cdot \pi/3$$

$$\phi = -t_d\,\omega$$

$\omega_0 = \frac{2\pi}{6} = \pi/3$ $\quad A = 6 \qquad \phi = -2\pi/3$

or

$$\phi = -(-4)\pi/3$$

$$= 4\pi/3$$

(b) By a suitable choice of delay t_d, we can shift $x(t)$ to obtain the new signal

$$y(t) = x(t - t_d) = A\cos(\omega_0 t) \tag{1}$$

There are an infinite number of values of t_d that satisfy Equation (1). Give an equation for these values. *If you cannot write the general expression, give at least* **two** *different values of* t_d.

$y(t) = x(t+2) = x(t-4)$

$t_d = -2, 4 \pm 6$

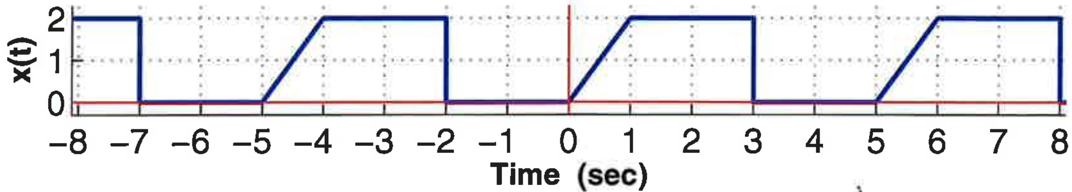
so $t_d = 6K - 2$

or

$6K + 4$

Question 4

Suppose that a periodic signal $x(t)$ is defined by the plot below (only the section $-8 \le t \le 8$ is shown):



$T_0 = 5$

(a) Determine the $\boxed{\text{fundamental frequency}}$ of $x(t)$ in Hz.

$$f_0 = \frac{1}{T_0} = \frac{1}{5}\ \text{Hz}$$

(b) Determine the $\boxed{\text{DC value}}$ of $x(t)$.

$$\frac{1}{T_0}\left(\overset{4}{\boxed{}} + \underbrace{\frac{1}{2}\cdot \underbrace{\text{base}}_{1}\cdot \underbrace{\text{height}}_{2}}_{1} \right) = \frac{5}{5} = \boxed{1}$$

(c) Write the $\boxed{\text{Fourier integral}}$ expression for the coefficient a_3 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral(s) should have numeric values. NOTE: more than one integral might be needed.*

$$a_k = \frac{1}{T_0}\int_{\emptyset}^{T_0} x(t)\, e^{-j\frac{2\pi k}{T_0}t}$$

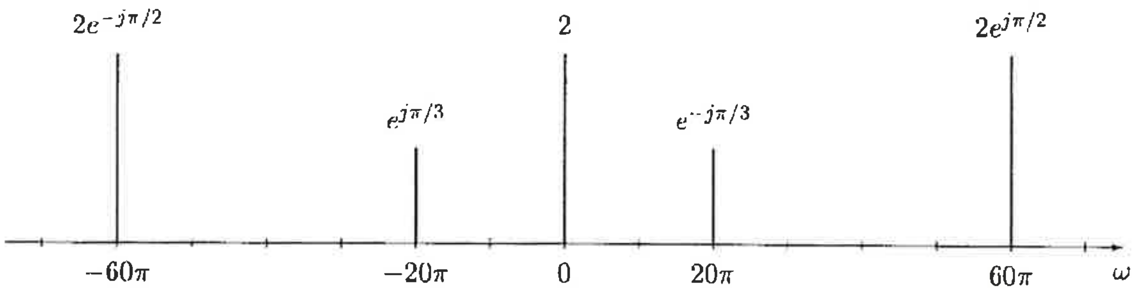
$k = 3 \quad T_0 = 5$

$$a_3 = \frac{1}{5}\int_0^5 x(t)\, e^{-j\frac{2\pi(3)}{5}t}$$

$$a_3 = \frac{1}{5}\int_0^1 2t\, e^{-j\frac{6\pi}{5}t} + \frac{1}{5}\int_1^3 2\, e^{-j\frac{6\pi}{5}t}$$

Question 5

The spectrum of a signal $x(t)$ is shown in the following figure:



Note that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

(a) Write an equation for $x(t)$ in terms of cosine functions.

- $2\left(\frac{e^{-j\pi/3}e^{j20\pi t} + e^{j\pi/3}e^{-j20\pi t}}{2}\right) = 2\cos(20\pi t - \pi/3)$
- Therefore same with $4\cos(60\pi t + \pi/2)$
- 2 DC

$$x(t) = 2\cos(20\pi t - \pi/3) + 4\cos(60\pi t + \pi/2) + 2$$

(b) This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

Greatest common factor of $(20\pi, 60\pi)$ is 20π

$T = \frac{2\pi}{\omega_0}$

$\omega_0 = 20\pi$

$T = \frac{2\pi}{20\pi} = \frac{1}{10}$

(c) Using x(t) above, a new signal is defined as:

$$y(t) = x(t) + \cos(\alpha t + \pi)$$

It is known that $y(t)$ is periodic with period $T_0 = 0.2$ sec.

Determine a value for α that will satisfy this condition.

$$x(t) = 2 + 2\cos(20\pi t - \pi/3) + 4\cos(60\pi t + \pi/2)$$

$$T_x = 1/10$$

$$T_x < 0.2 = 1/2\, T_0$$

$$f_0 = \frac{2\pi}{0.2} = 10\pi$$

therefore, to get T_0 (of y(t)) down to 10π from x(t)'s 20π, α must be 10π

α= 10π