ECES-352
Winter 2019
Homework #3

Reading: In Signal Processing First, Chapter 3 on Spectrum Representation.

Your homework is due at the beginning of class each Tuesday. See syllabus for late policy.

Problem 1

Define $x(t)$ as

$$x(t) = 2\sqrt{2}\cos(25\pi t + \pi/3) + A\cos(25\pi t + \phi) \tag{1}$$

where A is a *positive* number. In addition, assume that $x(t)$ has a phase of zero, so that it may be written as

$$x(t) = B\cos(25\pi t), \tag{2}$$

where B is a *positive* number.

(a) What relationship must exist between A and ϕ in order for $x(t)$ to have zero phase as indicated in Eq. 2?

(b) If $B = 25$, what are the values for A and ϕ?

(c) Now assume that B is unspecified. Find the values for A, B, and ϕ so that the value of A is *minimized*. Draw a plot of the complex amplitudes to prove using a geometrical argument that you have found the minimum for A. *Hint: Recall the geometrical "theorem" that tells you how to find the shortest distance between a line and a point that is not on the line (have you heard the term "projection"?).*

Problem 2

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = [v(t) + A]\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 1 kHz sinusoid, $v(t) = \cos(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a) and assume that $A = 2$. *Hint: Substitute for $v(t)$ and expand $x(t)$ into a sum of cosine terms of three different frequencies. Note that the product of two cosines is equivalent to a sum of sinusoids. If you do not recall this, substitute Euler's relation for each of the cosine terms, and expand the product - tedius, but it works.*

(c) How would the spectrum of the AM radio signal change if the carrier frequency is changed to 680 kHz (WCNN) and $v(t)$ and A are the same as defined in parts (a) and (b).

Problem 4

Consider the signal

$$x(t) = 8[\cos(100\pi t)] * [\sin(200\pi t)].$$

(a) Using the inverse Euler relation for the sine and cosine functions, express $x(t)$ as a sum of complex exponential signals with positive and negative frequencies.

(b) Use your result in part (a) to express $x(t)$ in the form $x(t) = A_1\cos(\omega_0 t+\phi_1)+A_2\cos(\omega_1 t+\phi_2)$.

(c) Determine the period T_0 of $x(t)$ and sketch its waveform over the interval $-T_0 \leq t \leq 2T_0$. Carefully label the graph.

(d) Plot the spectrum of $x(t)$.

Problem 5

A signal composed of sinusoids is given by the equation

$$x(t) = 2\cos(300\pi t) + 5\sin(600\pi t + \pi/4).$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the period? Which harmonics are present?

(c) Now consider a new signal $y(t) = -x(t) + 2\cos(450\pi t + \pi/3)$. How is the spectrum changed? Is $y(t)$ periodic? If so, what is the period of $y(t)$?

Problem 6

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with A-440 and ending with A-880 are:

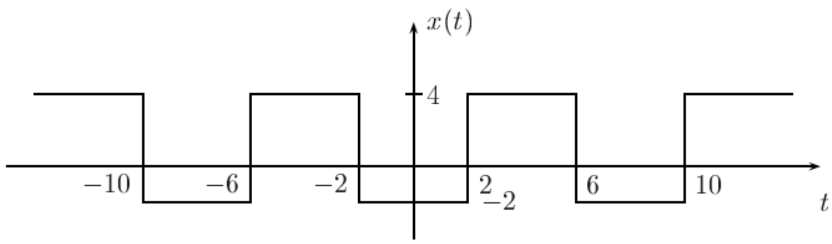
note name	A	$B^{\flat}$	B	C	$C\#$	D	$E^{\flat}$	E	F	$F\#$	G	$G\#$	A
note number	49	50	51	52	53	54	55	56	57	58	59	60	61
frequency													

(a) Make a table of the frequencies of the tones of the octave beginning with A-440 and ending at A-880. Recall that A-440 is the A above middle C (note #49) which is tuned to 440 Hz.

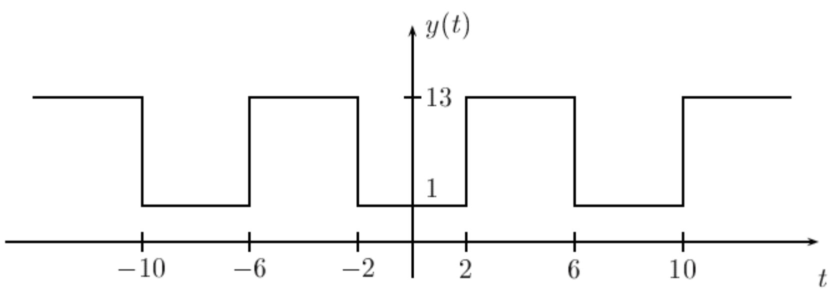
(b) The notes (from part (b)) on a piano keyboard are numbered 49 through 61. If n denotes the note number, and f denotes the frequency of the corresponding tone in Hertz, give a formula for the frequency of the tone as a function of the note number.

Problem 7

Let $x(t)$ be the periodic signal shown in the figure below, with Fourier series coefficients, a_k.



(a) Consider the signal shown below,

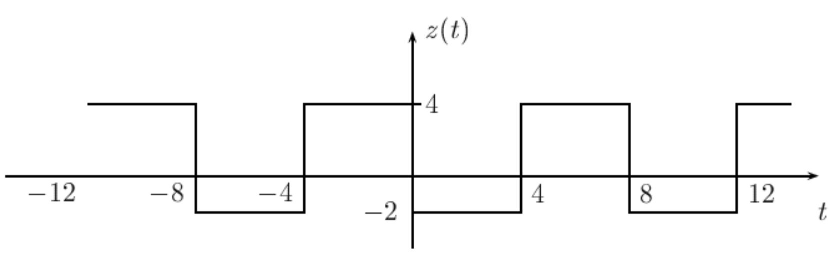


which is related to $x(t)$ by

$$y(t) = 2x(t) + 5$$

Express the Fourier series coefficients for this signal, b_k, in terms of the coefficients a_k for $x(t)$. Hint: This is a simple relationship, and finding it should not require that you compute any coefficients explicitly.

(b) Consider the signal shown below,



which is related to $x(t)$ by

$$z(t) = x(t-2)$$

Express the Fourier series coefficients for this signal, b_k, in terms of the coefficients a_k for $x(t)$. Hint: Again, this is a simple relationship, and finding it should not require that you compute any coefficients explicitly.