Measuring Financial Sector Systemic Risk in Israel : A Non-Parametric Approach

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Abstract

This paper models and estimates the loss distribution of the financial sector in Israel as a joint density through a non-parameteric model. The approach applied solves a constrained cross entropy optimization problem to generate probability multivariate densities. Given the joint density of the financial institutions subsets of sectors and entities are integrated out to measure the level of stress they induce on the system as a whole. A forward looking regression panel consisting of banking sector institutions is modeled to estimate the affect of current institution balance sheet data on future systemic risk estimated by the joint density.

1 Introduction

Measuring and monitoring instability in banking and finance has evolved in conjunction with the growth of the global financial markets. While having major advancements over the past century at the crux of the methodology is the attempt to quantify a latent phenomenon. This characteristic of the quantification of financial instability has allowed for many models to be defined and tested, while none has become the longterm consensus. A large portion of the methodology is retrospective and depends on the historic instability magnitude and characteristics to test for current crises. Regulation of the financial sector is predominately reactionary where constraints on the participants are updated and revised post crisis in an attempt to safeguard against past events from reoccurring.

Basel II attempted to define exposures of the banking system with regards to credit risk and operational risk. The former being the expected loss of capital, i.e. regulatory capital, which buys the bank a license to operate and the latter being the unexpected loss of capital which addresses the loss found in the long tail of the loss distribution as its spikes. The latter definition can be seen as a type of *signaling capital* providing information to the market that the institution has sufficient capital to withstand exceptional shocks. While the operational risk is categorized as the more volatile of the two, stress testing is conducted with relation to the expected loss, which has methodological ties to economic theory. The difficulties found in modeling the unexpected loss is that it is a stochastic process and that is institution specific. This is conditional to the institutions risk appetite found in its operational portfolio. Which in contrast to the credit risk that has a common pattern among banks and their distribution of credit lending by sectors.

This simplification lends itself to more parametric models that often carry heavy assumptions of linearity and process distribution. These distribution assumptions and model specifications are applied in measuring past market downturns which may cause future, out of sample, errors when market participants change strategy as a result of prior losses in an attempt to maximize their future returns on investment. In addition, information constraint has also been found to degrade the quality of risk assessments produced by parametric models that compensate with additional heavy use of assumptions to impute missing information. Caprio and Klingebiel (1996) cite numerous episodes in which the credit losses of banks have compromised the institution's capital obligations. In Israel 1977-1983, virtually the entire banking sector was affected, representing 60% of stock market capitalization and bank share prices fell over 40%. The loss was equivalent to approximately 30% of GDP in 1983. Goodhart et al. (2004) show that in the early 90's in Norway and Mexico unexpected losses from the bank's portfolios increased by 48% and 70% respectively compared to pre-crisis.

Modeling the portfolio profit and loss distribution is commonly approached through structural models, the Moody's KMV framework is a well documented example, Crosby and Bohn (1999). In this framework the firm's latent asset value is a time variant stochastic process and a default state is achieved by a drop in the firm's asset value below a critical value. This critical value is readily modeled as a function of the firm's financial structure. Merton (1974) assumes that the firm's logarithmic asset returns are Gaussian, thus defining the credit risk of a portfolio as a multivariate normal distribution with a time invariant dependence structure. Empirically these assumptions are not valid for modeling financial instruments, thus practitioners have adopted different parametric and dependence assumptions to define structures to measure portfolio credit risk. In asset return modeling it is pertinent that such dependence structures have the ability to take into account asymmetric dependence, where two asset returns exhibit greater correlation during market downturns than market upturns, Ang and Chen (2002). Examples of different model assumptions would be multivariate t-distributions with varying marginal degrees of freedom, Danielsson and de Vries (1997), Hosking et al. (2000), Glasserman et al. (2002) which allow for heavy tails and higher probability of extreme values. This distribution has been found to provide a reasonable fit to the conditional univariate distribution of daily asset returns, as in Andersen and Bollerslev (1998). Other proposed methods include mixture models which allow for more complex data process modeling, McLachlan and Basford (1988), but carry a constraint of a single value of degrees of freedom for each marginal or that the mixture is from the same distribution family which is ideal but empirically not always the case, Zangari (1996). Parametric copula functions have become the industry standard for estimating portfolio loss distributions, Nelsen (2013), Noss (2010), Perez-Gurrola and Murphy (2015). While these approaches are an improvement in many cases to the basic structural model for portfolio risk they still suffer from incomplete information due to the level of information given to the public relating the firms health and asset allocation. The information that is usually available is the market price of publicly held stocks or the value of financial instruments that represent a proxy of the credit risk quality of a company or a sector.

This paper will estimate the multivariate density of the core financial entities currently active in Israel 1998-2015, following methodology derived by Segoviano (2006), and applied by the International Monetary Fund (IMF), regarding the non-parametric estimation of multivariate densities under asymmetric and non-linear interdependency structures. These systemically important entities are the five major banks and the five major insurance companies[1]. The multivariate density, estimated by applying the consistent information multivariate density optimization (CIMDO) methodology, Segoviano (2006), will be defined as a systemic portfolio and corollary banking stability measures (BSM), Segoviano and Goodhart (2009), are derived. These measures are based on time-dependent non-linear systemic distress interdependence structure, which are themselves a function of each entity's probability of distress, thus incorporating distress based

[1]The institutions include, Banks: Bank Leumi Le-Israel B.M, Bank Hapoalim B.M, Mizrahi Tefahot Bank Ltd, Israel Discount Bank Ltd, The First International Bank of Israel Ltd; Insurance: Clal Insurance Company Ltd., Harel Insurance Investments & Financial Services Ltd., Menora Insurance Company Ltd, Migdal Insurance & Financial Holdings Ltd., Phoenix Insurance Ltd.

on the economic cycle. The stability measures are designed to have a hierarchy structure which give complimentary perspective on the distress estimated within the joint density. These different levels are the joint distress of the whole system, distress between subgroups of entities conditioned on group characteristics and systemic risk generated by a single entity.

In previous research by Michelson and Sidi (2014) a non-linear parametric approach was applied through estimating VaR using quantile regressions, thereby drawing inference on covariates throughout the business cycle. This was then applied to a forward looking model, Adrian and Brunnermeier (2011), that attempts to bridge between regulatory banking book data and the market VaR. The main drawback of this previous model was the use of window functions to attempt to capture the dynamic nature of asset returns and their correlations within the portfolio. This paper will attempt to overcome this constraint by estimating the joint density without the use of windows. The same forward looking model will be applied in this paper to measure the validity of previous period book data on the stability of system as measured by the nonparametric model and contrast the results to the competing methodology.

The remainder of the paper is structured as follows. Section 2 provides a short review of copula functions and defines the consistent information multivariate density function, a nonparametric copula, which infers the financial system's multivariate density. Section 3 defines the different stability measures. Section 4 presents the empirical estimates pertaining to the Israeli financial system. Section 5 reviews the forward stability panel model and its estimates with a comparison to the Conditional Value at Risk, CoVaR, systemic distress measure. Section 6 concludes.

2 Copulas

2.1 Parametric Copula

The past decade has seen a rise in the application of copulas to model multivariate dependence structures. They are widely accepted as the industry standards in fields such as finance, insurance and hydrology. Copulas have come to the forefront of modeling portfolio dependence due to its handling of both non-linear and asymmetric dependence structures regardless of multivariate density. Copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval (0,1). This can be understood as that the copula function contains all the information in the joint distribution not captured by the marginal distributions. Copulas have two major purposes, the first is they are a way of studying scale-free measures of dependence which are invariant under strictly increasing transformations and the second is that they are a relatively straight forward starting point for constructing families of bivariate distributions for simulations. We will concentrate on the former in this section and the rest of the paper as it pertains to the estimation of systemic financial stability.

The seminal work by Sklar (1959) showed that an n-dimensional joint distribution function may be decomposed into its n marginal distributions, and a copula, which completely describes the dependence between the n variables. This gives a canonical decomposition of any joint density, allowing for the isolation of the dependence term.

Theorem 2.1 (Sklar's theorem for continuous distributions)**.** *Let F be the distribution of X, G be the distribution of Y , and H be the joint distribution of (X, Y). Assume that F and G are continuous. Then there exists a unique copula C such that*

$$H(x, y) \Leftrightarrow C(F(x), G(y)), \quad \forall (x, y) \in \mathbb{R} \times \mathbb{R} \tag{1}$$

Conversely, if we let F and G be distribution functions and C be a copula, then the function H defined by Equation 1 is a bivariate distribution function with marginal distributions F and G.

The converse has great implications for working with multivariate distributions, it shows that any two univariate distributions, from any family, can be linked together with any copula and it will result in a valid bivariate distribution. While there is a rich collection of univariate parametric distributions, the multivariate case is still comparatively sparse due to its complexity and low main stream popularity. As shown in Patton (2002) by applying Sklar's theorem we can see that if we define for example: M as the number of parametric multivariate distributions, N as the univariate distributions and P as the copulas previously reported in the literature, and note that $N \gg M$; then the set of possible parametric multivariate distributions is increased from M to $N^2 \cdot P \gg M$. Thus relaxing the sparsity problem of generating complex multivariate distributions.

Nelsen (2013) expanded this idea with the following corollary where a copula can be extracted from any given multivariate distribution and use it independently of the marginal distributions of the original distribution.

Corollary 2.1.1. *Let H be any bivariate distribution with continuous marginal distributions F and G. Let F^{-1} and G^{-1} denote the inverses of the marginal distributions. Then there exists a unique copula $C : [0, 1] \times [0, 1] \to [0, 1]$ such that*

$$C(u, v) \Leftrightarrow H\left(F^{-1}(u), G^{-1}(v)\right) \quad \forall (u, v) \in [0, 1] \times [0, 1]. \tag{2}$$

This shows that by applying the probability integral transform on X and Y and transforming them to U and V we filter out the information in the marginal distributions. What is left in H is not the information found in the marginal distributions, but only the dependence information. Thus C contains all of the information on the dependence between X and Y, but no information on the univariate characteristics of X or Y.

The standard use of the copulas under parametric assumptions is to select the model through the calibration of the dependence parameter(s) that connect the marginal densities. For example the calibration of the parameter θ is needed to define the bivariate Gaussian copula:

$$C_\theta(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left\{ \frac{-(s^2 - 2\theta st + t^2)}{2(1-\theta^2)} \right\} dsdt \tag{3}$$

Corollary 2.1.1 allows the extraction of the Gaussian copula from the standard bivariate gaussian distribution. Gaussian copulas are one family that has been extensively described in literature. While another readily used family is the Archimedean copula which are very popular because they model dependence in arbitrarily high dimensions with only one parameter that calibrates the strength of dependence. Table 1 in Appendix B defines the most popular Archimedean copulas. Commonly used copulas in finance from this family are Clayton, Equation 4, which are calibrated using Kendall's tau, τ, and allow for lower tail dependency which capture the nonlinear nature of asset return in during downturns in publicly traded financial markets.

$$C_\theta(u,v) = \left[\max\left\{u^{-\theta} + v^{-\theta} - 1; 0\right\}\right]^{-1/\theta}, \tag{4}$$

where $\hat{\theta} = 2\tau/(1-\tau)$. This being said the parametric copula is still structure dependent making the choice copula analogous to model selection. In the framework of measuring distress from a readily unavailable variables of interest which are needed to correctly specify the functional form and parameters in order to estimate the joint distribution. In the case of the calibration of the copulas the parameters of interest are the dependence parameters (Kendall's τ, Spearman's ρ), which are either assumed fixed through time or updated through rolling windows to synthetically create dynamic processes. In the latter case there is fixed dependence within the windows and they frequently suffer from heuristically chosen lengths of windows. To overcome these issues a non-parametric copula was formulated by Segoviano and Goodhart (2009) which utilize Sklar (1959) and estimate the marginal densities from the multivariate density called the CIMDO copula.

2.2 CIMDO Density Estimation

To overcome the drawbacks of the popular parametric copulas in portfolio density estimation Segoviano (2006) formulated the Consistent Information Multivariate Density Optimization (CIMDO) methodology. The CIMDO estimates the multivariate density, p, by solving the minimum cross-entropy optimization problem defined by Kullback (1959). This approach updates a known initial probability, q, under moment constraints. In our case q is the multivariate t distribution, with varying degrees of freedom, which is consistent with the structural approach of modeling risk. However, under incomplete information the parametric density is inconsistent with the empirically observed measures of distress. Therefore, moment constraints are defined as the empirically estimated probabilities of default (PoD) of each financial institution. The optimal density, $\hat{p}$, is the density that is closest to q which is consistent with the PoD's of the financial system.

The problem is formally defined as a portfolio of institutions, w.l.o.g. two institutions, institution X and institution Y. We define the random variables x,y as the log return of each institution giving the objective function

$$C[p,q] = \int\int p(x,y) ln\left[\frac{p(x,y)}{q(x,y)}\right] dxdy, \quad q(x,y), p(x,y) \in \mathbb{R}^2, \tag{5}$$

with respect to the moment constraints which are imposed on the marginal densities

$$\int\int p(x,y) 1_{[X_d^x,\infty)} dxdy = PoD_t^x \text{ and } \int\int p(x,y) 1_{[X_d^y,\infty)} dydx = PoD_t^y \tag{6}$$

where PoD_t^x and PoD_t^y are the empirically estimated probabilities of distress of each institution in the system, $1_{[X_d^x,\infty)}$ and $1_{[X_d^y,\infty)}$ are indicator functions defining the support of the distress thresholds, X_d^x and X_d^y, for each institution in the portfolio. A final constraint to ensure that $p(x,y)$ is a valid density is defined as

$$\int\int p(x,y) dxdy = 1. \tag{7}$$

The CIMDO-density is recovered by minimizing the functional

$$\begin{aligned} L\left[p,q\right] = & \int\int p(x,y) \ln\left[\frac{p(x,y)}{q(x,y)}\right] dxdy \\ & +\lambda_1 \left[\int\int p(x,y) 1_{[X_d^x,\infty)} dxdy - PoD_t^x\right] \\ & +\lambda_2 \left[\int\int p(x,y) 1_{[X_d^y,\infty)} dydx - PoD_t^y\right] \\ & +\mu \left[\int\int p(x,y) dxdy - 1\right], \end{aligned} \tag{8}$$

where $\lambda_1, \lambda_2, \mu$ represent the Lagrange multipliers for the consistency constraints and the probability additivity constraint. The optimization procedure is carried out using calculus of variations, as seen in the Appendix A, resulting in

$$\hat{p}(x,y) = q(x,y) \exp\left[-\left(1 + \hat{\mu} + \hat{\lambda}_1 1_{[X_d^x,\infty)} + \hat{\lambda}_2 1_{[X_d^y,\infty)}\right)\right]. \tag{9}$$

Through extensive robustness testing Segoviano (2006) found that the CIMDO recovered distributions outperform most commonly used parametric multivariate densities in the modeling of portfolio risk, i.e. standard, conditional, mixture Gaussian, and multivariate t distributions. This is primarily because the CIMDO incorporates the distress information from each institution to adjust the shape of the multivariate density.

2.3 CIMDO Copula

The dependence structure of the distress financial institutions is characterized as linear in the mass and nonlinear in the tail of the loss distribution. This structure is modeled in the CIMDO copula which consistently updates at each point in time conditional on the empirically observed probabilities of distress. Using Sklar (1959) and its corollary by Nelsen (2013) the CIMDO copula is extracted from the CIMDO density explained in Section 2.2. This is done by estimating the marginal densities from the multivariate density and applying the probability integral transform. In addition, the dependence structure is defined along the entire domain of the estimated CIMDO density, with higher resolution at the lower tail that captures the distress.

$$C_c(u,v) = \frac{q\left[F_c^{-1}(u), G_c^{-1}(v)\right] \exp(-(1+\hat{\mu}))]}{\int q\left[F_c^{-1}(u), y\right] \exp(-\hat{\lambda}_2 1_{X_d^y(y)})dy \int q\left[x, G_c^{-1}(v)\right] \exp(-\hat{\lambda}_1 1_{X_d^x(x)})dx}, \tag{10}$$

where $u = F_c(x) \Leftrightarrow x = F^{-1}(u)$ and $v = G_c(y) \Leftrightarrow y = G^{-1}(v)$. From Equation 10 we see that the CIMDO copula is a nonlinear function of the Lagrange multipliers $\lambda_1, \lambda_2, \mu$. These multipliers convey the change of the optimized cross entropy as a function of the marginal change of the empirical probabilities of distress. This enables the copula to change at each time period and thus have the dependence measure of systemic risk of the portfolio (joint density) change at each period of time. This characteristic bypasses the problem most parametric models have of window calibration to synthetically create dynamic systemic dependency.

3 Stability Measures

We interpret the multivariate density, estimated by the CIMDO, as a systemic portfolio and derived from it a number of banking stability measures (BSM), Segoviano and Goodhart (2009). These measures are expanded upon in this paper to include banks and insurance sectors. The BSMs can be seen as a hierarchy of distress measurements; joint distress of the whole system, distress between subgroups of entities conditioned on group characteristics and systemic risk generated by a single entity. In the following section we will define these distress measures and show how they give complementary perspectives on the multifaceted problem of analyzing systemic risk in the financial sector. It is emphasized that while each measure individually may be informative on a given system level, the combination of perspectives is the intended mode of analysis.

We define a portfolio of three institutions and its estimated CIMDO density to illustrate the different measures.

$$\hat{p}(x,y,z) = q(x,y,z) \exp\left[-\left(1+\hat{\mu}+\hat{\lambda}_1 1_{[X_d^x,\infty)} + \hat{\lambda}_2 1_{[X_d^y,\infty)} + +\hat{\lambda}_3 1_{[X_d^z,\infty)}\right)\right], \tag{11}$$

where $p(x,y,z)$ and $q(x,y,z) \in \mathbb{R}^3$.

3.1 Univariate Distress

The base of the measurement of distress is on the individual institution level. With regards to the CIMDO the PoD is an exogenous variable thereby any model one decides to use can be inserted and implemented. There are a number of approaches to estimate the PoD a financial institution is under. The prevalent ones being the estimated default frequency (EDF) produced by Moody's MKV-Merton model which applies the structural approach (SA) as described by Merton (1974), credit default swaps (CDS) spreads based PoDs and out-of-the-money (OOM) option prices. In the Israeli market both the CDS and the OOM are financial instruments that suffer from immaturity, shallow markets and are not tradable for all financial institutions [2]. The structural approach on the other hand has an acceptable historical length, but as in many other markets there is great difficulty in proper parameterization of the model for the Israeli firms. This difficulty has lead to inconsistent results for the EDF of the banking sector which showed higher risk levels in 2009-2010 for all the banks comparable to the Global Financial Crisis (GFC) in 2008.

Due to these shortcoming we have decided to use the empirical value at risk (VaR) as the estimated PoD. This is defined as the stock price implied PoD, in which the probability of the stock price to reach a predetermined loss threshold. In this paper it was chosen to use the 99th percentile of the loss distribution of each institution. The distribution of the stock returns are assumed to be a Gaussian process. The distribution parameters, mean and variance, are estimated in a time-varying technique using a moving window of 125 days [3]. Finally the estimated process is then adjusted to the empirical frequencies. Defining the daily stock price as P_t, and the daily stock price return as $\ln(P_t) - \ln(P_{t-1})$. This return is then normalized using the full sample return mean μ_r and standard deviation σ_r, $r'_t = \frac{r_t - \mu_r}{\sigma_r}$. We then adjust this normalized return to behave more like a Gaussian process by using an adjustment factor $\tilde{r}_t(\alpha) = r'_t e^{-\alpha|r'_t|}$, $\alpha \in (0, 1)$. This factor dampens the tails of the distributions thereby approximating the intended process. The range of $\alpha \in (0, 0.5)$ was tested for Gaussian distribution and robustness. It was found that $\alpha = 0.3$ showed the closet fit to a Gaussian process, which confirms the findings of Segoviano and Goodhart (2009). Using $\tilde{r}$ we find the VaR with regards the 99th percentile and its default threshold for each 125 day window, giving the PoD at each time period for a given asset. With regards to the CIMDO the PoD is an exogenous variable thereby any model one decides to use can be inserted and implemented. While this approach is simple both practically and methodologically, future work could include modeling the PoD using a AR t-GARCH model as implemented in Patton (2002), which has a model based time-varying framework. Furthermore, the exogenous nature of the PoD in the CIMDO density estimation opens the avenue to integrate macroprudential stress testing scenarios into the calculation

[2] For a methodological review of the shortcomings regarding CDS's and OOM's please refer to Segoviano and Goodhart (2009).

[3] A range of 75-150 day windows were tested with no neglible difference in estimated multivaraite density, 125 days were chosen to be consistent with original IMF methodology

of the marginal PoD's. This advantage allows regulatory agencies to integrate the CIMDO into impulse response scenarios and conduct offline tests using both public and proprietary system information.

3.2 Bivariate Conditional Distress

Distress for a pair of institutions is estimated using pairwise conditional probabilities, i.e. the probability of distress a bank comes under distress given the other bank becomes distressed. This is not to be confused with the difficult question of causality, but attempts to give insight to contemporaneous asymmetric interlinkages and contagion levels between the two institutions within the system. Given two institutions X,Y this estimate is defined as

$$P(X \geq X_d^x | Y \geq X_d^y) = \frac{P(X \geq X_d^x, Y \geq X_d^y)}{P(Y \geq X_d^y)} \tag{12}$$

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3.3 Conditional Distress

Conditional distress among groups of institutions is also of interest. This could be among banks of different debt diversification amongst business sectors, the conditional distress the banking and non-banking (insurance) institutions project on each other or the stress a single institution can put on the whole system.

The systemic importance of a given institution is also a significant regulatory question that can be derived from the BSM. That is the probability that at least one (PAO) institution becomes distress given that a specific bank becomes distressed, i.e. a "cascade" effect within a given system.

$$\begin{aligned} PAO =& P(Y|X) + P(Z|X) + P(R|X) \\ &- [P(Y \cap R \setminus X) + P(Y \cap Z \setminus X) + P(Z \cap R \setminus X)] \\ &+ P(Y \cap R \cap Z \setminus X). \end{aligned} \tag{13}$$

Complimentary probabilities to the PAO that shed light on different facets of the interlinkages between institutions in the system are:

- The probability of exactly one institution coming under stress given a specific institution comes under stress
- The probability of a specific institution coming under stress given the system are under stress
- The probability of the system comes under stress given a specific institution coming under stress.

3.4 Multivariate Distress

The estimation of systemic distress is an innate results of the CIMDO density. Such a measure gives the tail risk of the entire portfolio. Due to the use of the CIMDO copula we can isolate changes of distress levels estimated by the joint distribution which are derived from changes in the dependence structure of the system. Under the non-parameteric framework this dependence is not bound by linearity constraints but is constructed to accommodate non-linear changes of dependence found at different points of the tail loss. We define the joint probability of distress (JPOD) that may act differently than distress of an individual institution. Given institutions $\{a_1, a_2, a_3, ..., a_K\} \in A$ where A is a portfolio of K financial institutions the JPOD is defined as

$$\mathrm{JPOD}_A = \int_{X_d^{a_1}}^{\infty} \cdots \int_{X_d^{a_K}}^{\infty} \hat{p}(a_1, \ldots, a_K) da_1 \ldots da_K. \tag{14}$$

The Banking Stability Index (BSI), Equation 15, is an additional systemic stress measure based on conditional expectation of default probability intially derived by Huang (1991). This measure reflects the expected number of banks becoming distressed given at least one bank is in a state of distress. The range of the BSI is [1,K], where one could interpret the measure as relative linkage in the system, ranging from asymptotic independence to asymptotic dependence.

$$\mathrm{BSI}_A = \frac{\sum_{i=1}^{K} P(a_i \geq x_d^{a_i})}{1 - P(\cap_{a_i < x_d^{a_i}})} \tag{15}$$

4 Empirical Stability Estimates

The CIMDO methodology has become a benchmark model implemented by the IMF to model banking distress levels in many countries, such as USA, EU and Mexico. In those instances the PoD was estimated using CDS-PoD's and showed that movement in the JPOD Equation 14 and the BSI Equation 15 coincide with events that were considered relevant by the markets on the given dates and differing levels of risk conditioned on business line of the banks during the Global Financial Crisis 2007-2008, Segoviano and Goodhart (2009).

Due to the immaturity of the financial instruments market in Israel the empirically estimated PoDs are used in the CIMDO optimization. Different levels of distress found in the Israeli financial sector will be analyzed in this section. To capture as many financial and economic cycles as possible historical data of the stock prices of the 5 major domestic banks and 5 major domestic insurance companies currently operating in Israel were compiled from 6/10/1998 to 6/10/2015. This range includes both domestic and global periods of distress that had an affect on the Israeli financial landscape, Djivre and Yakhin (2011). We will highlight in our results the different levels of distress probability per hierarchy.

While being the lowest resolution of systemic distress the Joint Probability of Distress (JPOD) and the Banking Stability Index (BSI) give a straight forward measurement of both the systemic probability and the estimated number of institutions under stress. In Figure 1 the JPOD and BSI are shown for the full portfolio of institutions (red) and each of the sectors Banks (green) and Insurance (blue). Since the second half of 2013 the systemic probability of

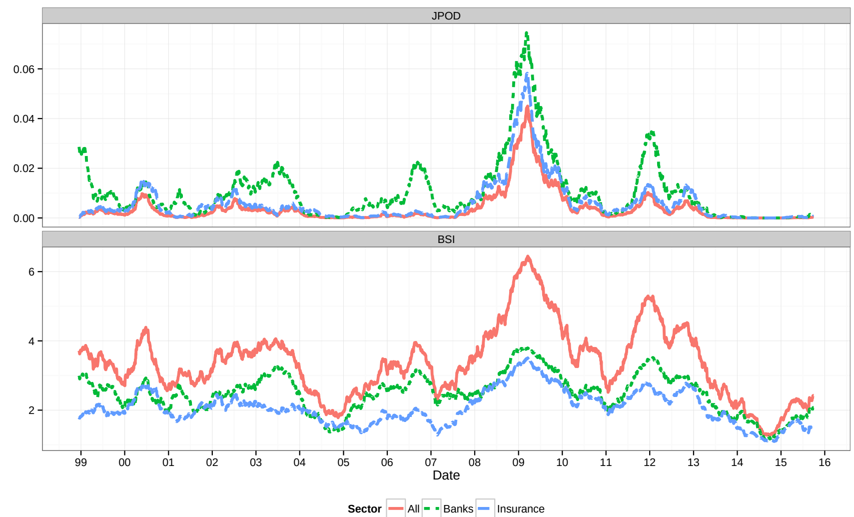

Figure 1: JPOD (upper panel) and BSI (lower panel) of the Israeli Financial System. The full portfolio of institutions (red) and the two major sectors, Banks (green) and Insurance (blue).

distress has decreased to very low levels near zero. This can be seen as the robust confidence of the market in the stability of the financial system since increased banking regulations and the central role the banks have in the current housing price surge. Previous to this we see that the insurance sector has shown high stability throughout the time period except for the GFC where systemic probability of distress levels reached nearly 6%. The banking sector is more susceptible to systemic distress, where during the GFC was the maximum stress estimated thus far at 8%. There were four other episodes of high stress: LTCM 1998-1999, Dot Com and Second Intifada 2003-2004, Second Lebanon War 2006-2007 and the European Sovereign Debt Crisis (ESDC) 2011-2012. The dependency measurement of the CIMDO are apparent in the result of the JPOD where during the GFC the systemic distress of the whole system was lower than a given sector. This could give an indication that the contagion affect between the two sectors was not elevated, thus lowering the whole portfolio risk. Later in this section we will drill down into specifically measuring the evolution of the contagion affect through the GFC and ESDC cycles. The BSI, lower panel of Figure 1, presents a different perspective of intra-sector linkage. The BSI is the expected number of institutions in distress given at least one is in distress. This measure is still low resolution because it does not specifically show which institutions are expected to go into distress. During the GFC over half of the institutions in the distribution of the full portfolio were expected to go into stress. This is intriguing because there are 5 institutions in each sector within the system portfolio thus giving a signal that there are cross sector distress spillover possibilities. Within the banking sector four out of the five banks had contagion affect within the banking portfolio. This sharp increase of dependence is captured by the CIMDO copula function, Equation 10, which contains the time variant dependence within the CIMDO density. In addition although the JPOD from 2013 is near zero the BSI is showing increased linkage from the end of the 2014 in both the full portfolio and more consistently in the banking sector, where at the end of the time range there are two banks that are exhibiting high levels of dependence.

Figure 2 focuses on the period of 2008/01/01-2012/06/01. This period includes peaks of systemic distress in both the banking and insurance portfolios during the GFC and the ESDC. The upper panel shows that the JPOD has larger daily percent changes compared to the marginal PoDs. The lower panel compares the distribution of the difference between the daily percent change of the JPOD and the average PoD's. It shows evidence that the CIMDO copula captures nonlinear dependence which is embedded in the CIMDO density. Distribution location (median) shift was found to be present between the two JPOD and average of PoDs in the upper distress levels of each sector, insurance $(0.05, 0.06]$ and banks $(0.07, 0.08]$. This was tested with the Wilcoxon Rank Sum test, a nonparametric test of difference in the median of two distributions, finding to have a significance level of 10% for the banks sector and 5% for the insurance sector. This confirms the assumption that there is nonlinear dependence during times of high stress in the financial sector.

Enhancing the resolution of the stress measurements we show the probability of each institution within each sector causing their respective sectors to enter distress. This can be interpreted as the probability of contagion within a sector. We focus on two different types of contagions first is of the systemic nature in which is defined as the probability of at least one (PAO) institution going into distress given a different institution is in distress. The other type is restricted to a bivariate contagion which is defined as the probability of exactly one (PEO) institution going into distress given a specific institution is in distress. These two measurements can be seen in Figure 3 in which the upper panel is the probability of systemic contagion and the lower is the restricted contagion. We see that during times of high joint distress (shaded regions) the across all institutions the PAO is near 100% (upper limit) and the PEO is near 0% (lower limit), this reflects the high level of dependence within the sectors during the GFC and the ESDC. Post 2013 we see that the levels of the PAO quickly declined among the institutions and conversely the PEO began to increase, thus allowing for idiosyncratic levels of dependence. From the end of 2014 the co-movement of the institutions in both sectors has increased resulting in elevated levels of probability of systemic contagion, although this is still tempered due to the

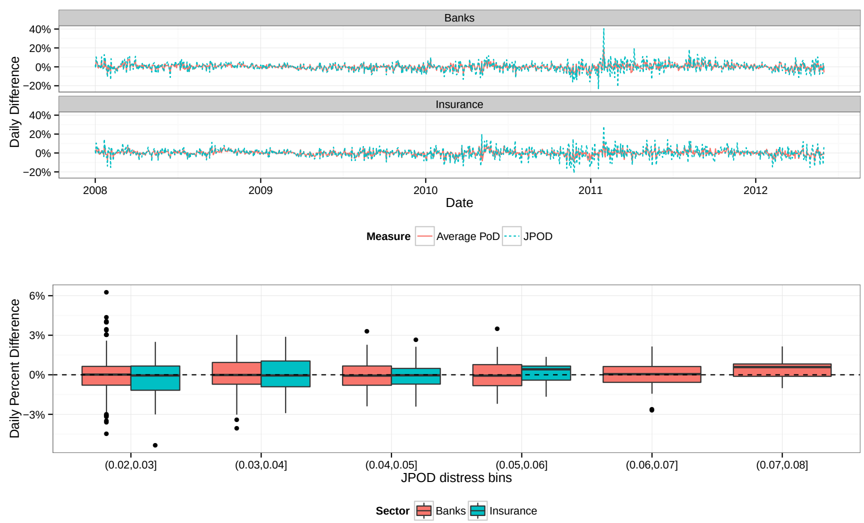

Figure 2: Daily Percentage Increase of the JPOD and the average Probability of Distress (upper panel) within the banking and insurance sectors (2008/01/01-2012/06/01). Distribution of the difference between the daily percent change of JPOD and Average of the PoDs in sectorial portfolios, Banks (red), Insurance (Blue) conditioned on JPOD above 0.02% (lower panel). Distribution location (median) shift between the two JPOD and average of PoDs in the upper distress levels of each sector, insurance $(0.05, 0.06]$ and banks $(0.07, 0.08]$.

relatively moderate levels of the PEO.

The highest distress dependence resolution we show is the distress between specific banks. In Figure 4 the evolution of dependence is shown within the Banking sector during 2008/01/01-2013/12/31. The upper panel of the figure is the sector JPOD with highlights on the GFC and the ESDC. The lower panel shows the probability of the institution in the panel going into distress given the institution of each of the institutions in the legend being in distress, as defined in equation 12. From the different panels it is apparent that the estimated conditional stress is not, Equation symmetric between two banks. This is evident between banks Discount and the other three banks. The probability of distress given Discount enters a state of stress is has increased from the second half of 2014 in those institutions while conversely Discount itself has not suffered from increased conditional risk. The most prominent increase in tail dependency has been between Discount and Mizrahi which both hold portfolios heavily allocated in real-estate related lending. This result can be applied as the decomposition of distress measured in Figure 3, in which the POE and the PAO give a portfolio level measurement of the estimated conditional probability of distress.

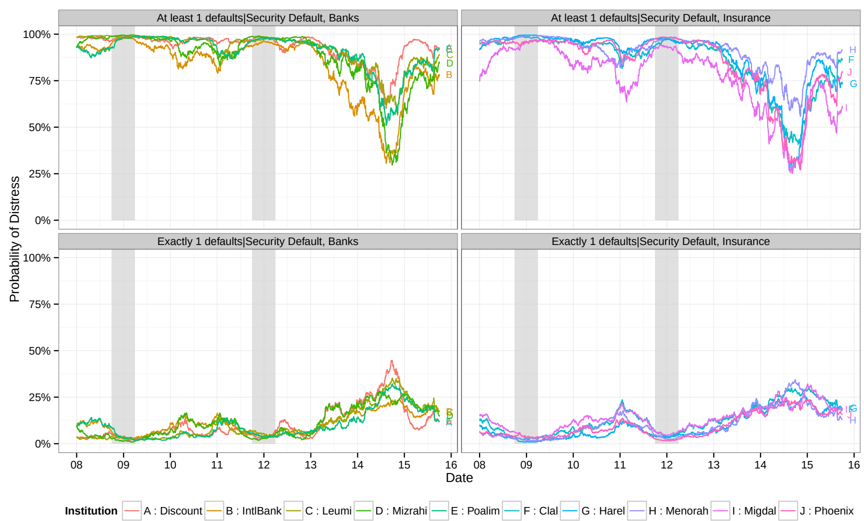

Figure 3: Contagion levels are calculated under two scenarios during 2008/01/01-2015/06/10. The upper panel depicts the probability of at least one (PAO) institution goes into distress given a different institution is in distress (Systemic Contagion). The lower panel depicts the probability of exactly one institution going into distress given a specific institution is in distress (Bivariate Contagion). The institutions in the legend represent the specific institution that is conditioned to be in distress. The right side of the figure contains the banking Sector institutions and the left side contains the Insurance sector institutions.

5 Forward Stability Model

The CIMDO proves to be an appealing method to measure real-time systemic risk, using available high-frequency data. However, lacking the transparent mechanisms of contagion and spillovers, it should be established that there is a connection between this measure and more well-known risk and stability measures of financial institutions. In this section we tie between each institution's contribution to systemic risk, as measured by the CIMDO estimated systemic probability of distress, and other financial conditions indicators. We do this only for banks, since their accounting measures of risk and stability are well-known and intuitive. We construct a forward looking model which predicts current levels of systemic stress conditioned on individual institution lagged book and market variables.

Our purpose in this exercise is twofold. First, as mentioned above, it is somewhat a test for the reasonableness of the CIMDO distress measure that an institution can put on the system. Second, it can give a qualitative, and even more important: a quantitative, framework for assessing the extent regulatory

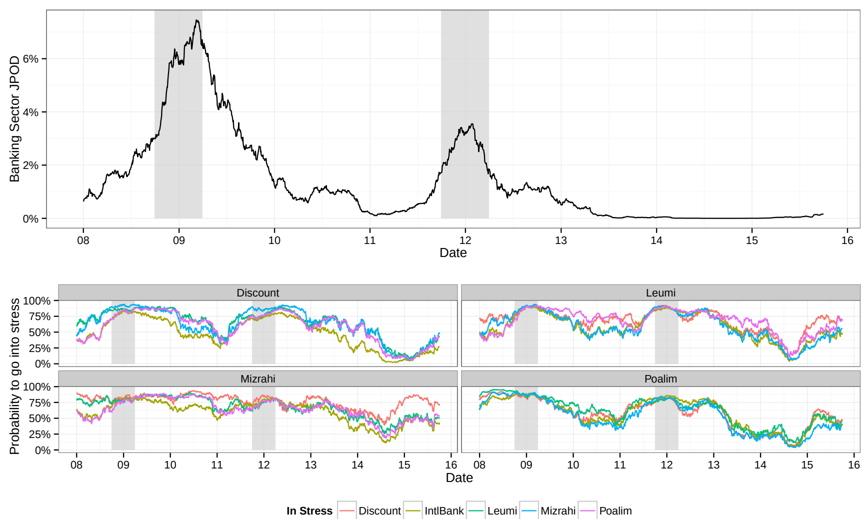

Figure 4: Bivariate Conditional Probability of Stress (2008/01/01-2013/12/31). The upper panel of the figure is the sector JPOD with highlights on the GFC and the ESDC. The lower panel shows the probability of the institution in the panel going into distress given the institution of each of the institutions in the legend being in distress.

measures which focus on CIMDO estimate distress drivers that might reduce institution's systemic risk contribution. Additionally we compare the estimated model coefficients to the results from previous work by Michelson and Sidi (2014) which applied the ΔCoVaR methodology.

Following Adrian and Brunnermeier (2011), we look at the following variables: capital-asset-ratio (LEVERAGE), calculated as the total capital to risk-weighted-assets; maturity mismatch (MM), calculated as the share of the difference between short term liabilities (0-3 months) and cash and deposits in total liabilities; market-to-book value (MTB) of the capital, calculated as the ratio between the market capitalization of each bank and its book value of its capital, therefore it is a non "pure accounting" measure, and size (SIZE), measured by the natural log of total assets. All standard deviations are generalized Newey-West standard deviations Driscoll and Kraay (1998), adjusted for autocorrelation with 3 lags. Fixed effects are used in order to catch unobservable effects of each institution. A descriptive statistics of the variables for each bank is in Table 3.

We start our analysis by running the quarterly average probability of systemic distress given a bank is in distress over the 4 accounting measures. Since financial reports publications are published up to one quarter after the end of the period, we need to use as explanatory variables a lag of these variables. During a given quarter (e.g. 2014Q3), the financial report of last quarter (2014Q2) is available only in the middle of it. Hence, data from these reports might be only partially reflected in market pricing and therefore in the CIMDO estimated distress as well. For this reason we use the second lag of the accounting variables. The comparative results to the model fitted to the ΔCoVaR can be found in Figure 5. The results show that the two methods have similar effects from the balance sheet data, giving motivation of using both models in a checks and balances regulatory framework.

The LEVERAGE and MM has the expected sign: higher LEVERAGE produce lower systemic distress while higher MM adding more to it. Higher levels of SIZE produce increased probability of systemic distress which can be interpreted as predicts that the larger bank's conditional distress given per unit of capital is 350 basis points larger than the small bank?s conditional distress. This has also been found to be in accordance with the findings in Zhou (2010). MTB was found to have no significant effect which is in contradiction to the results found under ΔCoVaR.

In expanded model we add two market variables that also appear in Adrian and Brunnermeier (2011) regressions: the market beta of each bank (MB) and the volatility of the share price return (VOL). Since these measures that are available daily, two lags are not needed. However, in order to avoid too high correlation and in order to give these models a "forward" notion, we choose to insert these variables with one lag. Including the market variables results in lower, but still significant, effect of the CAR and the MM and no significant effect of MTB. Both market variables estimates are 5% significant, where the VOL has a much larger affect on the probability of distress.

6 Conclusions

This paper applies the CIMDO density estimation derived by Segoviano (2006) and calculates estimates of probabilities of distress found on different levels of the Israeli financial system. These measures are tracked over time and in two different financial sectors, the Banking and the Insurance, while estimating the non-linear tail dependence present within the entire system. The different levels of measurements allow to monitor evolving distress from complementary perspectives and specify the resolution needed to properly calibrate regulatory policy. The estimated joint probability of distress and the corollary stress measurements correctly displayed signals of increased probability, and their proportional amplitude, during the Global Financial Crisis and the European Sovereign Debt Crisis. Within the banking sector there were additional episodes of distress signaled that coincided with both global financial events (LTCM and Dot Com) and domestic conflicts (second intifada and the second Lebanon War) which were expressed as financial market uncertainty.

The non-linear characteristics of the default interdependence within the financial system, estimated by the CIMDO copula function, was tested and ver-

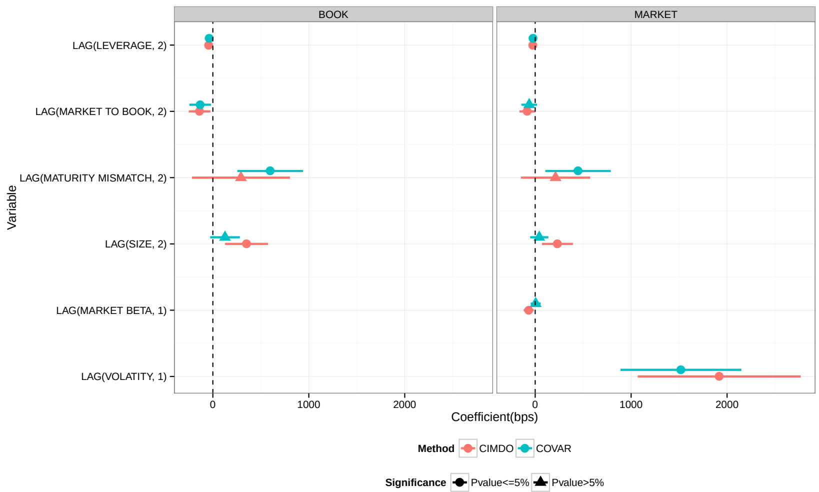

Figure 5: Coefficients of the Forward Panel Regression by Model Type and Method. The right figure depicts the model with only the 2 quarter lagged book variables and the left figure includes the 1 quarter lagged market variables. A comparison of the CIMDO (Red) and ΔCoVaR (Blue) is done for each variable with a distinction between coefficients that meet the 5% pvalue significance criterion. All standard deviations are generalized Newey-West standard deviations adjusted for autocorrelation with 3 lags.

ified to exist at the extreme levels of the tail both in the Banking and the Insurance sector. This gives additional credence to the use of the JPOD estimated by the CIMDO density and its ability to correctly capture the evolution of stress generated in concert by a variety of financial institutions. This characteristic can be monitored using surveillance techniques which track in real time the evolution of distress probabilities and signal when approaching the non-linear threshold. In the banking sector this threshold was found to be at 7% JPOD and in the insurance sector at 5% JPOD. The important distinction to be made is not reaching the threshold but the convergence to it. In addition, when enhancing the resolution of the distress measurements, specific subsets of institutions can be identified as having the largest marginal contribution.

Finally, we modeled a forward looking regression which predicted current levels of systemic risk conditional on lagged book and market values of individual banks. This analysis points out that there are strong ties between systemic risk and regulatory measures which were found comparable to past research conducted both in Israel and globally.

The construct of the CIMDO estimation allows for exogenous estimation of the Probability of Default for each financial institution. In this paper an empirical VaR was used to calculate the stock price implied PoD due to the limited financial instruments found in the Israeli market, future work could include modeling the PoD using an AR t-GARCH model as implemented in Patton (2002), which has a model based time-varying framework. Furthermore, the exogenous nature of the PoD in the CIMDO density estimation opens the avenue to integrate macroprudential stress testing scenarios into the calculation of the marginal PoD's. This advantage allows regulatory agencies to integrate the CIMDO into impulse response scenarios and conduct offline tests using both public and proprietary system information

Future research will focus on integrating portfolio characteristics of institutions to further understand the distress dependence signals. Three potential characteristics include cross bank lending which indirectly can cause increased linkage and higher propensity of contagion effects; the allocation of mortgage lending of banks by household income distribution to identify subsets of banks with similar household lending portfolios; and defining subsets of insurance companies that share similar investment portfolios or banks with significantly high levels of correlations in their credit portfolios.

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A CIMDO Functional Solution

Defining the constrained cross entropy problem as $L[p,q]$ where $p(x,y)$ is the unknown distribution and $q(x,y)$ is the assumed distribution, we want to approximate $p(x,y)$ to $q(x,y)$ as close as possible while keeping lower tail characteristics of the marginal distributions of x and y, which are denoted as PoD_t^x and PoD_t^y respectively.

$$
\begin{aligned}
L[p,q] &= \int\int p(x,y)\left[\ln p(x,y) - \ln q(x,y)\right] dxdy \\
&+ \int\int p(x,y)\left[\lambda_1 1_{[X_d^x,\infty)} + \lambda_2 1_{[X_d^y,\infty)} + \mu\right] dxdy \\
&-\lambda_1 \text{PoD}_t^x - \lambda_2 \text{PoD}_t^y - \mu \\
\text{PoD}_t^x &= \int\int p(x,y) 1_{[x^d,\infty)} dxdy \\
\text{PoD}_t^y &= \int\int p(x,y) 1_{[y^d,\infty)} dydx
\end{aligned}
$$

We solve for the stationary solution of the functional:

$$
\frac{\partial L\left[p(x,y)+\epsilon\gamma(x,y), q(x,y)\right]}{\partial\epsilon}|_{\epsilon=0} = 0,
$$

where $\epsilon\gamma(x,y)$ is a variation of $p(x,y)$.

$$
\begin{aligned}
L\left[p(x,y)+\epsilon\gamma(x,y), q(x,y)\right] = \\
&\int\int \left[p(x,y)+\epsilon\gamma(x,y)\right] \ln\left[p(x,y)+\epsilon\gamma(x,y)\right] dxdy \quad \text{(A)} \\
&-\int\int \left[p(x,y)+\epsilon\gamma(x,y)\right] \ln q(x,y) dxdy \quad \text{(B)} \\
&+\int\int \left[p(x,y)+\epsilon\gamma(x,y)\right]\left[\lambda_1 1_{[X_d^x,\infty)} + \lambda_2 1_{[X_d^y,\infty)} + \mu\right] dxdy \quad \text{(C)}
\end{aligned}
$$

$$
\frac{\partial L\left[p(x,y)+\epsilon\gamma(x,y), q(x,y)\right]}{\partial\epsilon} =
$$

$$
\text{(A)} \quad \int\int \gamma(x,y)\left(\ln\left[p(x,y)+\epsilon\gamma(x,y)\right]\right) + \left(p(x,y)+\epsilon\gamma(x,y)\right)\left(\frac{\gamma(x,y)}{p(x,y)+\epsilon\gamma(x,y)}\right) dxdy =
$$

$$
\int\int \gamma(x,y)\left(\ln\left[p(x,y)+\epsilon\gamma(x,y)\right]\right) + \frac{p(x,y)+\epsilon\gamma(x,y)}{p(x,y)+\epsilon\gamma(x,y)}|_{\epsilon=0} = \int\int \gamma(x,y)\left[\ln p(x,y) + 1\right] dxdy
$$

$$
\text{(B+C)} \quad -\int\int \gamma(x,y) \ln q(x,y) dxdy + \int\int \gamma(x,y)\left[\lambda_1 1_{[X_d^x,\infty)} + \lambda_2 1_{[X_d^y,\infty)} + \mu\right] dxdy
$$

$$
= \int\int \gamma(x,y)\left[\ln p(x,y) + 1 - \ln q(x,y) + \lambda_1 1_{[X_d^x,\infty)} + \lambda_2 1_{[X_d^y,\infty)} + \mu\right] dxdy
$$

Solving the term in $[\cdot] = 0$ will give the stationary solution $\hat{p}(x, y)$

$$\begin{aligned}\ln p(x,y) =& \ln q(x,y) + 1 - \ln q(x,y) - \lambda_1 1_{[X_d^x, \infty)} - \lambda_2 1_{[X_d^y, \infty)} - \mu \\ p(x,y) =& q(x,y) \exp -(1 + \lambda_1 1_{[X_d^x, \infty)} + \lambda_2 1_{[X_d^y, \infty)} + \mu),\end{aligned}$$

solving for $\lambda_1, \lambda_2, \mu$ we have the solution as in 9

$$\hat{p}(x,y) = q(x,y) \exp\left[-(1 + \hat{\mu} + \hat{\lambda}_1 1_{[X_d^x, \infty)} + \hat{\lambda}_2 1_{[X_d^y, \infty)})\right]$$

B Archimedian Copulas

Table 1: Common Archimedean Copulas

Name of Copula	$C_\theta(u, v)$	θ
Ali-Mikhail-Haq	$\frac{uv}{1-\theta(1-u)(1-v)}$	$[-1, 1)$
Clayton	$\left[\max\left\{u^{-\theta} + v^{-\theta} - 1; 0\right\}\right]^{-1/\theta}$	$[-1, \infty) \setminus \{0\}$
Frank	$\frac{1}{\theta} \log\left[1 + \frac{(\exp(\theta u)-1)(\exp(-\theta v)-1)}{\exp(\theta)-1}\right]$	$\mathbb{R} \setminus \{0\}$
Gumbel	$\exp\left(-\left[(-\log(u))^\theta (-\log(v))^\theta\right]^{1/\theta}\right)$	$[-1, \infty)$
Independence	uv	
Joe	$1 - \left[(1-u)^\theta + (1-v)^\theta - (1-u)^\theta (1-v)^\theta\right]^{1/\theta}$	$[-1, \infty)$

Table 2: Common Archimedean Copulas: Generators

Name of Copula	$\psi_\theta(t), \quad t \in [0, \infty]$	Kendall's τ	Tail Dependence
Ali-Mikhail-Haq	$(1-\theta)/(\exp(t) - \theta)$	$[0, 1/3)$	None
Clayton	$(1+\tau)^{-1/\theta}$	$(0, 1)$	Lower
Frank	$-\log(1 - (1 - \exp(-\theta)\exp(-t)))/\theta$	$(0, 1)$	None
Gumbel	$\exp(-t^{1/\theta})$	$[0, 1)$	Upper
Independence	$\exp(-t)$	$[-1, 1]$	None
Joe	$1 - (1 - \exp(-t))^{1/\theta}$	$[0, 1)$	Upper

C Additional Results

Figure 6: Adjusted Stock Returns of the 10 major Financial Institutions in Israel (1998/06/10-2015/06/10)

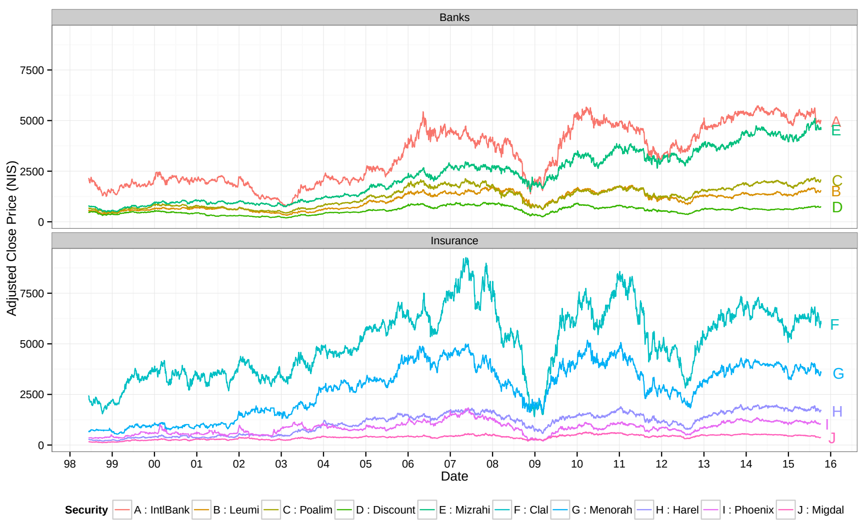

Table 3: Summary Statistics of Covariates for Forward Panel Regession

Variable	Bank	Min	Max	Mean	St.Dev
Market Beta (MB)	Benleumi	-0.04	1.68	0.76	0.38
	Discount	-0.30	1.58	0.58	0.39
	Leumi	0.05	1.71	0.63	0.36
	Mizrahi	0.01	1.22	0.49	0.28
	Poalim	0.20	1.25	0.63	0.27
Volatility (VOL)	Benleumi	0.02	0.17	0.06	0.03
	Discount	0.02	0.13	0.05	0.02
	Leumi	0.02	0.13	0.05	0.02
	Mizrahi	0.02	0.13	0.04	0.02
	Poalim	0.02	0.11	0.05	0.02
Capital-Assets Ratio (CAR)	Benleumi	10.00	15.27	12.26	1.48
	Discount	9.29	14.42	11.41	1.81
	Leumi	10.80	15.07	12.84	1.53
	Mizrahi	9.38	14.25	11.63	1.65
	Poalim	9.54	15.70	12.19	1.94
Market to Book (MTB)	Benleumi	0.20	0.96	0.59	0.18
	Discount	0.34	1.27	0.74	0.24
	Leumi	0.48	1.56	0.96	0.27
	Mizrahi	0.66	1.36	1.01	0.18
	Poalim	0.53	2.03	1.02	0.32
Maturity Mismatch (MM)	Benleumi	0.49	0.69	0.58	0.05
	Discount	0.39	0.58	0.48	0.06
	Leumi	0.37	0.57	0.46	0.06
	Mizrahi	0.34	0.56	0.44	0.06
	Poalim	0.34	0.58	0.50	0.05
Size	Benleumi	15.01	15.73	15.43	0.22
	Discount	15.46	16.32	15.96	0.25
	Leumi	16.41	17.08	16.78	0.20
	Mizrahi	15.12	16.07	15.57	0.28
	Poalim	16.41	17.16	16.77	0.21