# Non Parametric Detection Schemes and Their Application to the Surveillance of Exponentially Tilted Joint Distributions

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**Abstract**

Estimation of patient pero-operative instability requires the modeling of the patient's hemodynamic measurements as a multivariate distribution. This would describe the joint likelihood of changes in the physiological measurements that characterize the continuous well-being of a patient during a clinical procedure. Such data is received in a continuous stream at irregular time periods throughout the procedure.

Prospective monitoring on simultaneous data streams has been a growing field within statistical process control. Monitoring in this setting is characterized by interdependent and autocorrelated continuous processes. A formal definition to objectively measure such a framework can be achieved through stochastic differential equations (SDE). Parameters that are defined within the SDE change throughout the continuous process. These changes can be monitored and decision rules can be formulated to alert when the state of the process has changed. When several of these process are monitored simultaneously both the system and marginal processes must be monitored for changes.

This paper proposes a cross-entropy approach to estimate the joint density from the partial information received at real-time, thus removing the constraint of parametric assumptions. Given the estimated multivariate density, detection schemes are applied to monitor and alert when patient stability thresholds are either crossed or estimate the probability of being crossed. In addition, this framework allows conditional probabilities of distress to be estimated to specify the origin of the instability.

# 1. Introduction

Several data streams containing information regarding the stability of a system is a challenging problem to model and monitor. The most common approach to handle such a problem is through dimension reduction to a single statistic and monitor it using standard surveillance methodology. This reduction has been found sufficient when the changes occur simultaneously across all information streams. Another approach is to monitor the data streams, or a statistic of them, in parallel and raise an alarm when at least one of the streams diverges from its baseline process. More sophisticated approaches allow for the possibility of unsynchronized change times for multiple streams. Statistics such as a generalized average run length to false alarm (ARL2FA) and the control of the false discovery rate are of great interest, while the question of optimality becomes a difficult one under a multivariate setting. Multivariate surveillance problems are prevalent in financial portfolio allocation problems, in which trading strategies are triggered based on process monitoring algorithms. Sequential monitoring of optimal portfolio weights can be calculated through a multivariate exponentially weighted moving average and the mahalanobis control charts. These solutions still reduce the dimension of the multivariate to a single statistic, thus keeping the same problematic characteristics.

This research applies a different tool to the multivariate monitoring problem. We suggest that the process by estimated using a tilted-likelihood via a non-parameteric copula function. Thereby allowing for the joint density to be decomposed into the three parts as defined by Sklar (1959): the copula and the marginal densities.

$$H(x, y) \Leftrightarrow C(F(x), G(y)) \quad \forall x, y \in \bar{\Re}, \tag{1.1}$$

where $H(x, y)$ is the joint distribution of $x, y$, $C(\cdot, \cdot)$ is the copula function and $F(x), G(y)$ are the marginal distribution of x and y respectively. This decomposition allows for the surveillance of joint distribution or their decomposed parts, i.e. the marginals and the copula function that captures the linear and non-linear interdependencies between the random variables.

Copulas have become common place in the estimation of interdependencies of assets portfolios in finance. In this setting it has been applied to the pricing options with more than one underlying asset, calculating Value-at-Risk of a portfolio of assets, and multivariate density forecasting. In the calculation of Value-at-Risk, the tail return loss dependencies differ from the expected loss distribution. These complex multivariate distributions are both simulated and estimated using copula functions, which can accommodate asymmetric equity correlations. Copula's are normally generated using parametric approaches through the definition of marginal distributions as gaussian or student's t distribution, or a multivariate t. In these cases the copula is a function of a correlation parameter.

In this paper we are going to take the idea of portfolio distribution estimation and apply it to continuous pero-operative physiological measurements that convey hemodynamic stability at a patient level. This would allow for a framework to define a joint distribution that would be hierarchical by nature, the joint distribution would be the patient and the marginal distributions would be each vital measured separately. The multivariate distribution is estimated using a tilted-likelihood ratio, which is a solution to the cross entropy problem with moment constraints. The tilted-likelihood is then used in a surveillance scheme which is based on the Shiryayev-Roberts approach, under the constraint that the baseline in unknown. In such a case any part of the decomposition can be monitored throughout a medical procedure.

## References

M. Sklar. *Fonctions de répartition à n dimensions et leurs marges.* Université Paris 8, 1959.