Measuring Portfolio Systemic Risk: A Non-Parametric Approach

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2 Minutes about me...

Academic: Hebrew University of Jerusalem

- BA: Statistics and Economics
- MA: Statistics
  - Dissertation: Nowcasting Israel GDP Using High Frequency Macroeconomic Disaggregates
- PhD: Statistics (Expected Submission in October)
  - Thesis Subject: Signal Process Classification and Monitoring in a High Dimension Setting

Non Academic

- Bank of Israel: Research Statistician and Economist
- Israel useR
- The Public Knowledge Workshop
- R package/Shiny development

A number of methods were applied in the Bank of Israel with the goal of building a set of tools to measure systemic risk in the Israeli Financial Sector.

Each method built upon each other in order to create a system of checks and balances for each successive method.

These methods originally derived to measure asset portfolio risk were augmented to measure patient level pero-operative hemodynamic risk.

Methods for Measuring Agent and Systemic Risk

Parametric	Gaussian, Multivariate t, Quantile Regressions GARCH, Factor based VARs Option Based (Finance)
Non-Parametric	Copulas,Historical analysis, Extreme Value
Networks	Modeling the Clearance of Inter-agent Networks Eisenberg and Noe (2001)

We focus on methods derived by Segoviano (2006) which use non-parameteric copula functions to estimate probability multivariate densities. These densities have each agent time variant probability of distress embedded within them.

What we intend to show

- A non-parametric method to estimate multi-agent portfolio density
- A function that measures non-linear, asymmetric and time variant tail dependence in multivariate distributions
- A set of probabilistic measures to quantify contemporaneous lower tail distress

- Case studies
  - Israeli Financial System
  - Significant thresholds of moving from linear to non-linear systemic distress in the banking and insurance sectors in Israel

  - Patient Hemodynamic Stability
  - Motivation to change current model to better suite patient risk characteristics

What we Don't intend to show

- Structural Model
- Causality

Background

- Measuring and monitoring instability in banking and finance has evolved in conjunction with the growth of the global financial markets.
- The core challenge of measuring financial instability is the attempt to quantify a latent phenomenon.
- This characteristic has allowed for many models to be defined and tested, while none has become the longterm consensus

Background

- The credit risk of a institution's portfolio of loans is reflected in its profit and loss distribution (PLD)
- These losses or gains are prime importance for institutions' economic capital decisions and their risk management strategies
- The proper measurement of the PLD has become a key objective in financial risk management firms

Background

- Modeling the PLD requires modeling the marginal and portfolio multivariate distributions that describe the individual and joint likelihood of changes in the value of the assets of the borrowing firms
- A significant problem for portfolio credit risk measurement is the greatly restricted data that are available for its modeling
- Convenient parametric assumptions are frequently made in order to represent the nonexistent information

Structural Approach

- The structural approach, Merton (1974), is one of the most common approaches to the modeling the PLD
- The change in the credit risk quality of a firm is a function of changes in the value of its assets
- The firm's underlying asset value evolves stochastically over time and default is triggered by a drop in the firm's asset value below a threshold value
- The probability of the firm to exceed this threshold is the Probability of Default (PoD).

Capital Loss Types

- Credit risk is the expected loss of capital, i.e. regulatory capital, which buys the bank a license to operate
- Operational risk is the unexpected loss of capital which addresses the loss found in the long tail of the loss distribution as its spikes. This can be seen as a type of *signaling capital* providing information to the market that the institution has sufficient capital to withstand exceptional shocks.

Estimating Financial Institution Level PoD

- Banking and Insurance , so-called arm's length companies, publicly available data is aggregated, and thus does not include the institution's perceived risk to the loan portfolio
- Publicly available data on the risk level of such institutions:
    - Asset market price
    - Publicly traded institution bonds
    - Credit Default Swap spreads (CDS-PoD)
    - Contingent Convertible bonds spreads (CoCo-PoD)

Betrayed by Asymmetric and Non-linear Dependency

- Asymmetric equity correlations: stock returns seem more dependent on downturns than on upturns, Erb et al. (1994)
- Markets crashes induce cascading losses that translate to nonlinear systemic loss compared to single institution loss.
- Asset pricing when you do not believe that structural model assumptions hold such as the multivariate normally distribution of the asset returns

More flexible methodology is needed to create multivariate densities that can capture the non-linear and asymmetric dependence characteristics that are within the asset portfolios in financial institutions

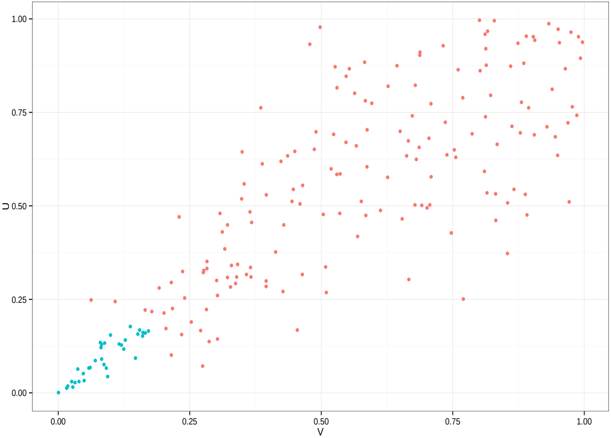
Copula

Copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval [0,1]

- Contain all the information in the joint distribution not captured by the marginal distributions
- A way of studying scale-free measures of dependence which are invariant under strictly increasing transformations (i.e. CDF's)
- Starting point for constructing families of bivariate distributions for simulations

Tail Dependence

The object is to estimate this type of multivariate distribution under the constraints that each marginal institution PoD is embedded in the joint density



Sklar's Theorem

Sklar (1959) showed that an n-dimensional joint distribution function may be decomposed into its n marginal distributions, and a copula, which completely describes the dependence between the n variables.

Let F be the distribution of X, G be the distribution of Y , and H be the joint distribution of (X, Y). Assume that F and G are continuous. Then there exists a unique copula C such that

$$H(x, y) \Leftrightarrow C(F(x), G(y)), \quad \forall (x, y) \in \mathbb{R} \times \mathbb{R} \tag{1}$$

Conversely, if we let F and G be distribution functions and C be a copula, then the function H defined by Equation 1 is a bivariate distribution function with marginal distributions F and G.

Implications of Skalr's Theorem

$$H(x, y) \Leftarrow C(F(x), G(y))$$

- The converse has great implications for working with multivariate distributions, it shows that any two univariate distributions, from any family, can be linked together with any copula and it will result in a valid bivariate distribution.
- While there is a rich collection of univariate parametric distributions, the multivariate case is still comparatively sparse due to its complexity and low main stream popularity.
- This theorem allows for the construction of complex multivariate distribution through a simple function.

Parameteric Copula Problems

- The parametric copula is still **structure dependent** making the choice copula analogous to **model selection**.
- In the framework of measuring distress from a readily **unavailable variables** of interest. They are needed to correctly specify the functional form and parameters in order to estimate the joint distribution.
- In the case of the calibration of the copulas the parameters of interest are the dependence parameters (Kendall's τ, Spearman's ρ), which are either assumed fixed through time or updated through rolling windows to synthetically create dynamic processes.

CIMDO

- The International Monetary Fund (IMF) introduced the Consistent Information Multivariate Density Optimization (CIMDO) methodology, Segoviano (2006)
- This methodology has become a benchmark of the IMF for measuring and testing financial stress within and cross country banking systems, with implementation in Mexico, USA, Europe and currently in Columbia
- This method estimates multivariate densities using the minimum cross entropy approach derived by Kullback (1959)
- In this method the PoD's are exogenous to the estimation of the multivariate density, which creates an extremely flexible framework depending on the level of information available

CIMDO Formally

We formally defined a portfolio of agents, w.l.o.g. two agents, institution X and institution Y. Denoting random variables x,y as the log return of each institution giving the objective function

$$C[p,q] = \int\int p(x,y) ln\left[\frac{p(x,y)}{q(x,y)}\right] dxdy, \quad q(x,y), p(x,y) \in \mathbb{R}^2, \tag{2}$$

with respect to the moment constraints which are imposed on the marginal densities

$$\begin{aligned} \int\int p(x,y) 1_{[X_d^x,\infty)} dxdy &= PoD_t^x \\ \int\int p(x,y) 1_{[X_d^y,\infty)} dydx &= PoD_t^y \end{aligned} \tag{3}$$

CIMDO Optimization Problem

The CIMDO-density is recovered by minimizing the functional

$$
\begin{aligned}
L[p,q] = & \int\int p(x,y)\ln\left[\frac{p(x,y)}{q(x,y)}\right] dxdy \\
& + \lambda_1 \left[\int\int p(x,y) 1_{[X_d^x,\infty)} dxdy - PoD_t^x\right] \\
& + \lambda_2 \left[\int\int p(x,y) 1_{[X_d^y,\infty)} dydx - PoD_t^y\right] \\
& + \mu \left[\int\int p(x,y) dxdy - 1\right],
\end{aligned}
\tag{4}
$$

where $\lambda_1, \lambda_2, \mu$ represent the Lagrange multipliers for the consistency constraints and the probability additivity constraint.

CIMDO Solution

The optimization procedure is carried out using calculus of variations resulting in

$$\hat{p}(x,y) = q(x,y)\exp\left[-\left(1+\hat{\mu}+\hat{\lambda}_1 1_{[X_d^x,\infty)}+\hat{\lambda}_2 1_{[X_d^y,\infty)}\right)\right] \quad (5)$$

- Through extensive robustness testing Segoviano (2006) found that the CIMDO recovered distributions outperform most commonly used parametric multivariate densities in the modeling of portfolio risk
  - standard, conditional, mixture Gaussian, and multivariate t distributions
- This is primarily because the CIMDO incorporates the distress information from each institution at each point in time to adjust the shape of the multivariate density

Capturing Evolving Tail Dependence with CIMDO-Copula

- The CIMDO copula is a nonlinear function of the Lagrange multipliers $\lambda_1, \lambda_2, \mu$
- These multipliers convey the change of the optimized cross entropy as a function of the marginal change of the empirical probabilities of distress
- This enables the copula to change at each time period and thus have the dependence measure of systemic risk of the portfolio (joint density) change at each period of time
- This characteristic bypasses the problem most parametric models have of window calibration to synthetically create dynamic systemic dependency

Portfolio Stability Measures

The multivariate density, estimated by the CIMDO, as a systemic portfolio and derived from it a number of portfolio stability measures (PSM), Segoviano and Goodhart (2009)

- The PSMs can be seen as a hierarchy of distress measurements that give complementary perspectives on the problem of analyzing systemic risk
  - Joint distress of the whole system
  - Conditional distress between subgroups of agents
  - Systemic risk generated by a single agent
- While each measure individually may be informative on a given system level, the **combination of perspectives is the intended mode of analysis**

Joint Probability of Distress

- The estimation of systemic distress is an innate results of the CIMDO density
- This measure gives the tail risk of the entire portfolio, i.e. the probability of all agents to go into distress at once.

$$\mathrm{JPOD}_A = \int_{X_d^{a_1}}^{\infty} \cdots \int_{X_d^{a_K}}^{\infty} \hat{p}(a_1, \ldots, a_K) da_1 \ldots da_K \tag{6}$$

- With the CIMDO copula we can isolate changes of distress levels estimated by the joint distribution which are derived from changes in the dependence structure of the system

Portfolio Stability Index

- The Portfolio Stability Index (PSI) the conditional expectation of default probability intially derived by Huang (1991)
- Reflects the expected number of agents becoming distressed given at least one agent is in a state of distress

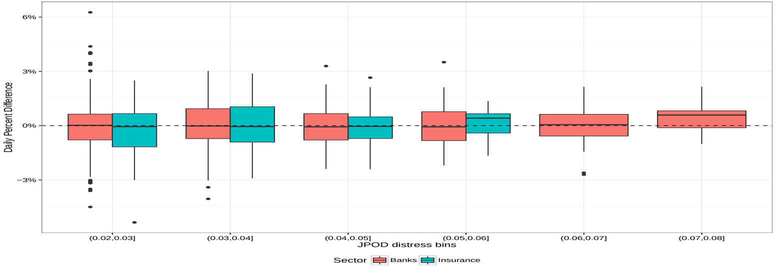
$$\mathrm{PSI}_A = \frac{\sum_{i=1}^{K} P(a_i \geq x_d^{a_i})}{1 - P(\cap_{a_i < x_d^{a_i}})} \tag{7}$$

- The range of the PSI is [1,K], where one could interpret the measure as relative linkage in the system, ranging from asymptotic independence to asymptotic dependence

Testing for Non-linear Dependence

To test for the existence of systemic non-linear dependence we calculate the difference between the daily percent change of the JPOD and the average PoD daily percent change:

$$\Delta D_t = \ln\left(\frac{JPOD_t}{JPOD_{t-1}}\right) - \ln\left(\frac{1/K\sum_{k=1}^{K} PoD_t^k}{1/K\sum_{k=1}^{K} PoD_{t-1}^k}\right)$$



We then split ΔD into different levels of JPOD distress ΔD_i, $i \in (1,\ldots,m)$ and for each ΔD_i we apply the Wilcoxon Rank Sum Test H0:median(ΔD_i)>0

Cimdo Dashboard

Finance

- 5 Banks and 5 Insurance
- Date Range 1999-2016
- 3 major economic and financial downturns

Hemodynamics

- 3 Hemodynamic Measurements (HR,Systolic,Diastolic)
- 10 cases of at least 4 hours
- Retrospective labeling of stress by Physicians

CIMDO Dashboard

What we showed

- CIMDO was presented as a non-parametric method to estimate multi-asset portfolio density
- The CIMDO captures non-linear, asymmetric and time variant tail dependence in multivariate distributions
- A set of probabilistic measures were shown to quantify contemporaneous financial distress
    - Non-parameteric tests for the existence non-linear distress as a function of different levels of systemic distress
- A case study for the Israeli Financial System
- Motivation and application of the CIMDO to real time patient hemodynamic surveillance

Thank you

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