Construction of an Asymmetrical Prediction Interval for CBS GDP Publications via the Distribution Analysis of Vintage GDP

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- A prediction interval for the next revision of seasonally adjusted GDP annualized growth was created around the last available estimate of GDP growth. This prediction interval defines the upper and the lower bound of the GDP growth rate in the subsequent revision and constitutes as a result an important piece of information for policy makers.

- Revision distributions were examined in three different stages of the economic growth rate cycle : Negatively accelerating Growth Rate, Stable Growth Rate and Positively accelerating Growth Rate. It was found that they are differently distributed depending on the state of the rate of GDP growth.

- Evidence was found that if the rate of growth is negatively accelerating, then the initial publication will be overestimated and subsequent revisions will lower the GDP, and conversely when the rate of growth is positively accelerating the initial publication is underestimated and the subsequent revisions will be higher.

- Out of sample testing shows that the prediction interval produced at the time of initial release correctly bounds the initial revision of the GDP at a rate of 80% and the future third revision at a rate of 78%.

- The current GDP publication of 2.13% growth in 2014Q1 was estimated to have a prediction interval of (1.69%,2.61%) in the forthcoming release of the GDP. The fact that the last publication showed a 2.92% rate of GDP growth for this quarter indicates that the initial growth estimate has deviated substantially from its prediction interval, thus implying that the initial estimate was an outlier with respect to the publication distribution.

1 Objective and Results

Quarterly national accounts data are published once a month by the Israeli Central Bureau of Statistics (CBS). Each quarterly data report is revised after the initial publication, because of new information and seasonal adjustments. Within the 40 months following the initial publication new information is received by the CBS through surveys and financial reports. Once this information has been taken into account in the revised publication, the full series is once again seasonally adjusted.

Once a month the Bank of Israel (BoI) reassesses its monetary policy stance and a lot of weight is put on the marginal series of the GDP disregarding the divergence between the current GDP publication and the ”full information” GDP. This exercise attempts to characterize this divergence on the basis of vintage series of GDP data and construct a prediction interval to the current publication of the seasonally adjusted GDP. The objective of the prediction interval is to broaden the discussion of economic growth from a point estimate constructed from ”partial information” found in the initial publications to an estimation of the ”full information” GDP, which is found in the revised GDP.

Previous work in the BoI, Recent Economic Developments 135 (pg 21), has tested the revision convergence rate of the GDP and the possibility of a revision bias with respect to the long term growth trend. The latter test was conditioned on if the GDP growth being above or below the long run growth trend measured by the HP filter. The findings of this work were that the GDP is updated within 12 periods from initial publication and there was no significant bias in the revisions with relation to the long term growth trend.

The present exercise constructs a prediction interval for the next revision of GDP growth around the last available estimate of GDP growth. This prediction interval is formulated as a function of the change in GDP growth rate. We found that the revisions are distributed differently depending on the state of the rate of growth. To depict different rates of the growth the revision distributions were examined in three different stages of the economic growth rate cycle: Negatively accelerating Growth Rate, Stable Growth Rate and Positively accelerating Growth Rate. We found evidence that if the rate of growth is negatively accelerating, then the initial publication will be overestimated and subsequent revisions will lower the GDP, and conversely when the rate of growth is positively accelerating the initial publication is underestimated and the subsequent revisions will be higher. Out of sample testing shows that the prediction interval produced at the time of initial release correctly bounds the initial revision of the GDP at a rate of 80% and the future third revision at a rate of 78%. Finally, it was estimated that the current GDP publication of 2.13% growth in 2014Q1 will have a prediction interval of (1.69%,2.61%) in the forthcoming release of the GDP. The following sections of the report will describe the stages to construct the prediction interval and show its implementation for the current GDP publication.

2 GDP Publication Prediction Interval

The main objective of this exercise is to generate a prediction interval around the current release of the seasonally adjusted GDP annualized growth rate. In practice, while the frequency of the GDP is quarterly it is updated and released every month by the CBS. More specifically, every three months a new quarter is added to the GDP series, after-which this new data point is updated monthly. As a result the prediction interval is generated monthly in accordance to the revision number of each quarter. In addition the data revision has been categorized into three groups with respect to the change in growth rate of the GDP, where it was found that there is a significant difference in data revision between negatively accelerating GDP growth rates and positively accelerating GDP growth rates.

Four statistics are estimated from vintage GDP data to create two prediction intervals around the upcoming series revision: $\{10^{th}, 25^{th}, 75^{th}, 90^{th}\}$ percentiles. Using these four statistics we can construct two bands that give 50% and 80% coverage probability of the next revision location. A depiction of such prediction intervals can be seen in Figure 1, where the current publication of the GDP includes the initial release of 2014Q1. We can see that the initial release of 2014Q1 is estimated to have a prediction intervals of (1.99%,2.32%) and (1.69%,2.61%) with 50% and 80% coverage probability respectively in the upcoming publication. Table 1 presents all the estimates and prediction intervals from 2013Q1-2014Q1 and how many revisions each quarter has had up to this date.

Figure 1: Current GDP Publication and Upcoming Release Revision Range

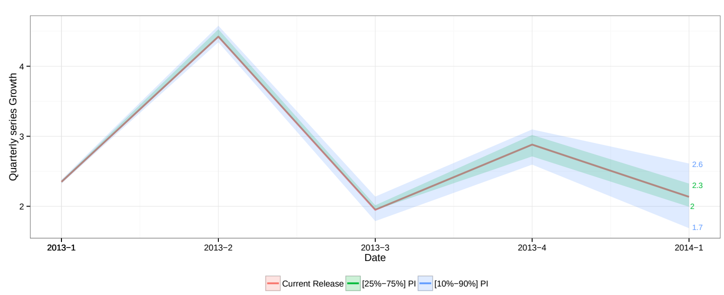


Table 1: Current GDP Publication and Upcoming Release Revision Range

Publication Number	Quarter	Current GDP	Upcoming Revision 10%	25%	75%	90%
13	2013Q1	2.35%	2.32%	2.33%	2.37%	2.38%
10	2013Q2	4.42%	4.34%	4.41%	4.53%	4.58%
7	2013Q3	1.95%	1.79%	1.95%	2.01%	2.14%
4	2013Q4	2.88%	2.6%	2.71%	3.02%	3.1%
1	2014Q1	2.13%	1.69%	1.99%	2.32%	2.61%

3 Construction of Prediction Interval

3.1 Analyzing the Vintage GDP

The construction of the prediction interval utilizes information found in the vintage series of GDP publications. We define the GDP vintage as a two dimensional time series $Y^{(k)} = \{Y_1^{(k)}, Y_2^{(k)}, \dots Y_q^{(k)}, \dots\}$, where q is a time period of a given publication k, $k \in \{1 \dots K\}$. We will define formally the GDP growth rate, the change in GDP growth rate and the revision rate of a given quarter.

Growth Rate (Across Quarters) For a given publication k^* the GDP can be deconstructed to show growth rate and the change in growth rate. To define these two measures we apply a percent change operator Δ, where $\Delta X(t) = \frac{X_t - X_{t-1}}{X_{t-1}}$ for all time periods $t > 1$. Using this operator we can define $\Delta GDP_{k^*}(q)$ as the GDP growth rate and $\Delta(1 + \Delta GDP_{k^*}(q)) = \Delta^2 GDP_{k^*}(q)$ as the change in growth rate.

Revision Rate (Within Quarters) For a given time period q^*, one can define $GDP^k_{(q^*)} = \{GDP^{k_1}_{(q^*)}, GDP^{k_2}_{(q^*)}, \dots GDP^{K_{q^*}}_{(q^*)}\}$, with which $\Delta GDP_{q*}(k)$ is defined as the the revision rate.

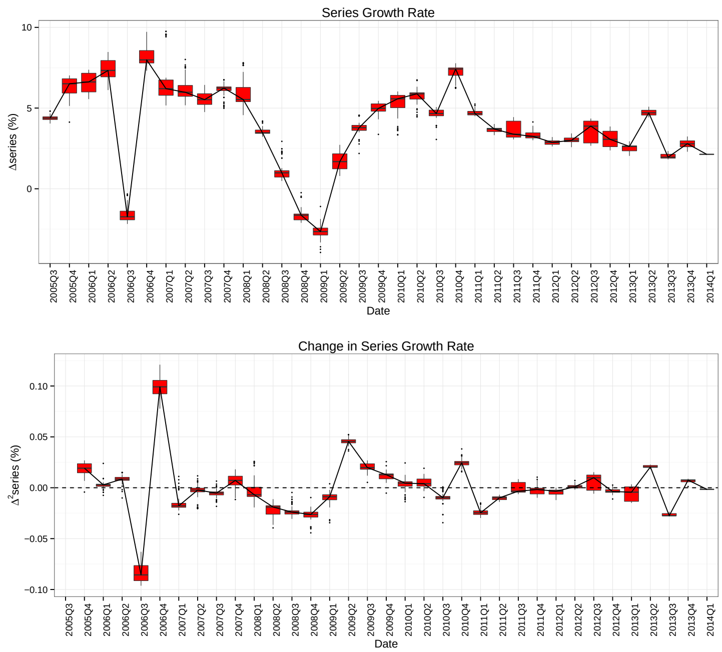
Figure 2 depicts the GDP growth rate and the change in growth rate through boxplots connected at the median value of each quarter in order to visualize the publication volatility. It was found that the revision volatility increases as the absolute value of $\Delta^2 GDP$ increases, and that the horizon for volatility convergence is longer with the increase of $\Delta^2 GDP$. When testing the convergence rate of the GDP we found that, at a significance of 95%, after 40 revisions following the initial publication of a quarter the volatility of the publications stabilize [1].

3.2 Revision Distribution by GDP Growth Rate Regime

An analysis of the categorization of the GDP data revision rate was conducted to test if there are different types of revisions depending on the change in the GDP growth rate. Under the assumption of varying revision rates between different types of GDP growth rates, the one-sided Wilcoxon location test statistic (non-parametric location test) was used as a target function to locate the maximum revision rate difference between low GDP growth rate and high GDP growth rates.

[1] Analysis of the revision volatility horizon can be found in the Appendix 5.1

Figure 2: Distribution of Vintage GDP Growth Rate and Change in Growth Rate
Sample Period 2005m9-2014m3, Median Connected



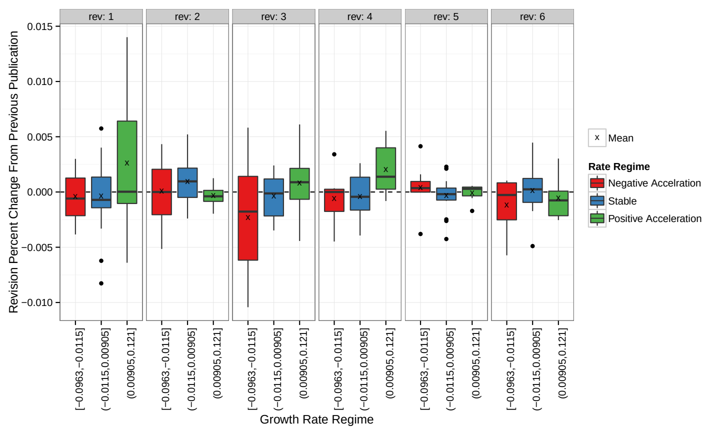
More specifically, the change in growth rate between two consecutive quarters, $\Delta^2 GDP_k(q)$, was calculated for each revision. We then tested if the median revision of the High Growth regime is significantly larger than revision of the Low Growth regime in the first 12 revisions after publication. This was done by applying the one-sided Wilcoxon Test for the median value with a confidence level of 95%. As seen in Table 2 the median in the first revision of the High Regime is larger than the Low Regime with a P-Value of 20.4% and in the fourth revision it the P-Value has a level of 6.9% for the median difference. This result gives validation for the construction of a prediction interval on the current publication GDP while taking into account the growth rate regime.

Table 2: Wilcoxon Test for Location Difference Between High Growth and Low Growth Regimes First 12 Revisions, Sample Period 2005m9-2014m3

Test	Revision (P-Value)											
	1	2	3	4	5	6	7	8	9	10	11	12
Wilcoxon	0.20	0.70	0.14	0.07	0.75	0.47	0.01	0.71	0.29	0.08	0.63	0.27

The optimal grouping of the GDP growth rate was found to be (percentiles) [p0-p27.5],(p27.5-p72.5],(p72.5-p100], where there are (27.5%,45.0%,27.5%) data points in each group respectively [2]. The distribution of the GDP revision across quarters is then calculated by growth rate regime, e.g. the first revision (k=1) of all quarters (q) is the upper left panel of Figure 3. We will denote the regime for each quarter by q_r where $r = 1, 2, 3$ corresponding to the three groups defined above, thus defining the revision rate of a given quarter under growth rate regime q_r as $\Delta GDP_{(q^*, q_r^*)}(k)$.

Figure 3: Revision Distribution by Regime
Sample Period 2005m9-2014m3



3.2.1 Initial Release Grouping Uncertainty

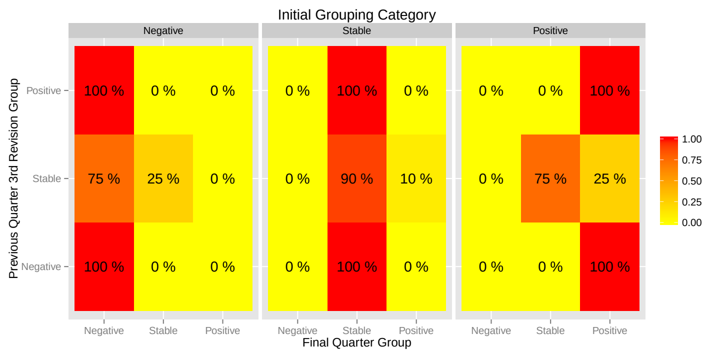
The prediction intervals with respect to the initial release of a new quarter was treated differently than subsequent revisions due to the fact that there is uncertainty regarding the true level of the growth rate. This uncertainty would create larger prediction intervals for the initial revision. To mitigate this uncertainty, and underestimation, weights were constructed on the propensity of a new quarter to be in a given growth regime on the third revision conditioned on the growth rate of the new quarter and the growth rate of the previous quarter under the third revision. This is depicted in equation 1:

$$\omega(r) = \Pr(\Delta GDP_{k=3}(q) = i | \Delta GDP_{k=0}(q) = j, \Delta GDP_{k=3}(q-1) = k), \quad j,k \in \{1,2,3\} \qquad (1)$$

if we are estimating the prediction interval for a non-initial release $\omega = 1$. Using this distribution we can produce a weighted prediction interval.

[2] An outline of the grouping optimization can be found in the Appendix 5.2

Figure 4: Probability Distribution of First Revision Categorization



The probability distribution for this procedure is shown in Figure 4. In this figure we can see that for an initial growth rate categorization $\Delta GDP_{k=0}(q) = j$, depicted by the panels, and the previous quarter's third revision $\Delta GDP_{k=3}(q-1) = k$ a probability of the third revision being in a given growth regime, $\Delta GDP_{k=3}(q) = r$. Where j is a panel and r and k are rows and columns within a panel. Thus for example, if the initial growth regime is "positively accelerating" and the previous quarter growth regime is "stable", then the probability of the new quarter to be in the negatively accelerating,stable and positively accelerating growth regimes in the third revision is (0%,75%,25%).

3.3 Updating Current GDP Publication to Next Revision

The current CBS publication for the GDP can be defined as $GDP_q^{(K_q)}$. In this one dimensional time series each quarter is at a different publication length K_q. To update $GDP_q^{(K_q)}$ to $GDP_q^{(K_q+1)}$ we use the characteristics of the revision distribution described in the previous section, i.e. updating the current GDP publication through identifying the growth rate regime for each quarter in GDP and applying the corresponding revision percentile bounds. This is defined in the following formulas:

$$\begin{aligned} P_{10}(GDP_q^{(K_q+1)}) &= \left(1+\Delta GDP_{K_q}(q)\right) \times \left(1+\mathrm{p10}(\Delta GDP_{(q,q_r)}(K_q+1))\right) \\ P_{25}(GDP_q^{(K_q+1)}) &= \left(1+\Delta GDP_{K_q}(q)\right) \times \left(1+\mathrm{p25}(\Delta GDP_{(q,q_r)}(K_q+1))\right) \\ P_{75}(GDP_q^{(K_q+1)}) &= \left(1+\Delta GDP_{K_q}(q)\right) \times \left(1+\mathrm{p75}(\Delta GDP_{(q,q_r)}(K_q+1))\right) \\ P_{90}(GDP_q^{(K_q+1)}) &= \left(1+\Delta GDP_{K_q}(q)\right) \times \left(1+\mathrm{p90}(\Delta GDP_{(q,q_r)}(K_q+1))\right) \end{aligned} \tag{2}$$

During initial release publications the weights are implemented as below to produce percentile p^*:

$$p^*(GDP_q^{(K_q+1)}) = \left(1+\Delta GDP_{K_q}(q)) \times (1+\sum_{r=1}^{3}\mathrm{p}^*(\Delta GDP_{(q,q_r=r)}(K_q+1)\omega(r))\right) \tag{3}$$

3.4 Out of Sample Testing

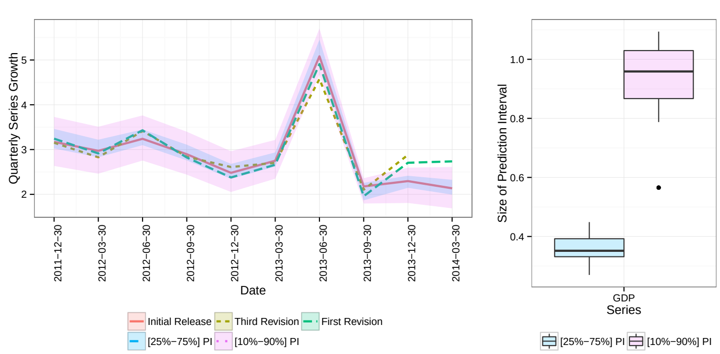
An out of sample test was conducted to measure the ability of the algorithm to correctly identify the bounds of an upcoming revision. The algorithm was applied to an expanding window from 2011Q4 to 2014Q1 in which the input was the initial publication of the CBS GDP for each quarter. The algorithm was set to create the bounds for the initial revision of the relevant quarter. The measure used for procedure performance is the percent of actual data points that fall within the estimated prediction intervals. In addition a similar measure was calculated for the third future revision, this was done to see if the algorithm could cover up to three months of future GDP publication which are closer to "full information". As can be seen from Table 3 in the first revision estimation the 50% coverage correctly identified the upcoming revision 70% of the time and the 80% coverage correctly identified 80% of the future first revisions. Regarding the third revision the 50% and 80% coverage correctly identified 66% and 78% respectively.

Table 3: Percent of Future GDP Publications in the Prediction Intervals

	Prediction Interval	
	[25%-75%]	[10%-90%]
First Revision	70%	80%
Third Revision	66.67%	77.78%

Figure 5 shows the out of sample estimation over the expanding window in the left panel, and the distribution of the prediction interval size in the right panel. We can see that the 50% prediction interval covers the future GDP revisions and that the prediction intervals are tight, i.e. 0.35% for the 50% coverage and 0.95% for the 80% coverage (annualized median growth rate).

Figure 5: Out of Sample One Step Ahead GDP Revision
2011Q4-2014Q1 Seasonally Adjusted Annualized Quarterly Growth



4 Conclusion

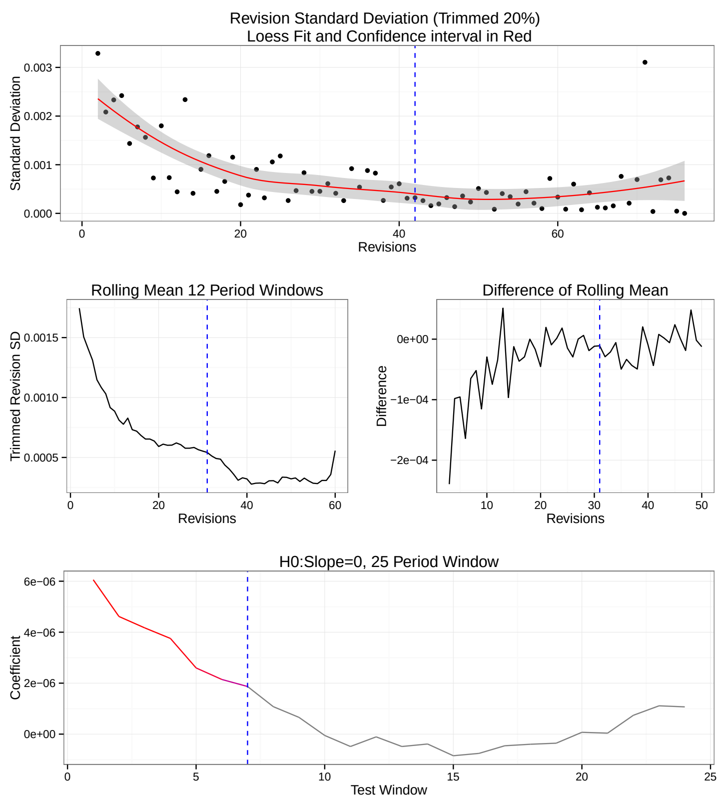
This paper describes an exercise which attempts to generate a prediction interval of the upcoming revision to the current CBS publication of the GDP. This interval characterizes the upper and lower percentiles based on revision distributions from the vintage GDP. It was found that there is a significant difference in the size and sign of revisions dependent on the state of the GDP growth rate. More specifically, when the initial GDP publication is in a low growth quarter, the quarter is overestimated and subsequent revisions lower the GDP, and conversely when the initial GDP publication is in a high growth quarter the quarter is underestimated and the subsequent revisions increase the GDP. This finding was implemented into the calculation of the estimated revision and its prediction interval.

We conclude that this technique constitutes an improvement upon the current point estimates of GDP and GDP growth serving as an input in the assessment of economic activity affecting the monetary policy stance because it provides in addition to this point estimate a prediction interval for the range of fluctuation of the growth rate allowing a more reliable assessment of the strength of economic activity. Moreover, it could improve the results of the Nowcasting methodology in the bank, by adjusting the target GDP series from current publication to the estimated revision of the GDP.

5 Appendix

5.1 Revision Decay

Figure 6: Revision Decay Analysis

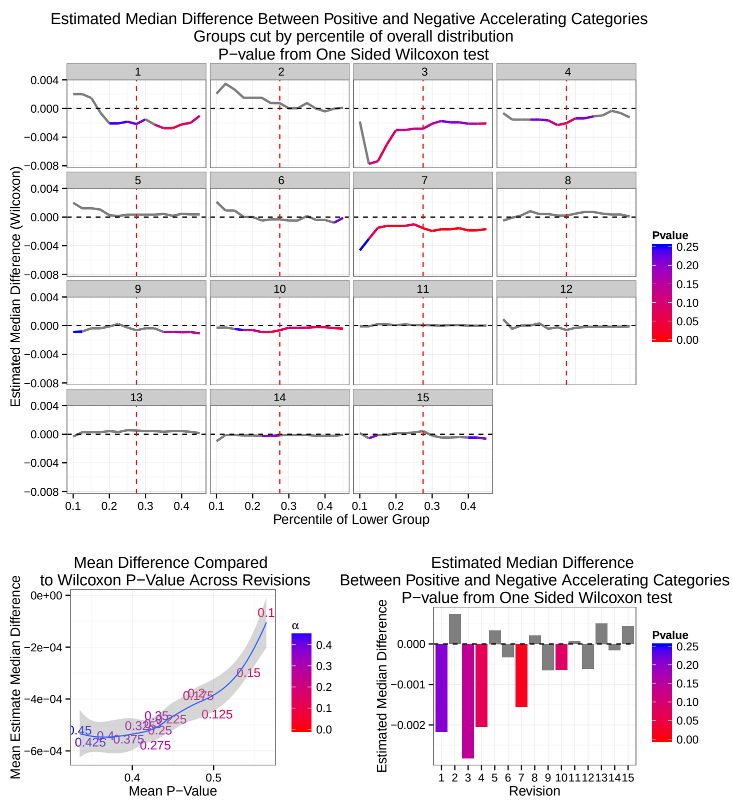


The revision decay rate was estimated using the seasonally adjusted GDP vintage series 2005m9-2013m8, encompassing the historical monthly series until the change of seasonal adjustment of the GDP known as SNA2013. The trimmed (+/-10%) standard deviation of each GDP revision was calculated. This can be seen in the first row of Figure 6 along with a loess fit for to visualize a linear decay. When looking at the long term horizon of the revisions one can see that the standard deviation increases once again after 60 revisions (approximately five years after initial release), this is taken as a technical noise due to seasonal adjustments and not new data to the system. The following analysis is done using the first 60 revisions in the vintage GDP data.
To find the horizon length when the standard deviation becomes stable a rolling mean with a 12 period window was calculated smooth the trimmed standard deviation. Where after the difference between two consecutive rolling windows

was calculated, $\Delta sd(GDP_q(k))$. Finally a rolling regression with a window of 25 periods was applied to test when the slope is not significantly different from zero, we identify the stabilization of the standard deviation when this occurs. We found that the slope coefficient is not significantly different from zero at window 7 which corresponds to rolling mean windows 7-31. Taking into account the 12 period rolling mean, this gives the time period of 18-42 revisions on the original time scale. The vertical dashed blue line is used as a visual aid to locate these time intervals in Figure 6. One can conclude that up to 42 revisions after then initial publication are needed for the GDP revision to stabilize.

5.2 Growth Regime Calibration

Figure 7: Growth Regime Calibration



Calibration of the growth rate regime was conducted through the application of the one-sided wilcoxon test. This test was used to measure the location (median) difference between the negatively accelerating growth regime and the positively accelerating growth regime.

The parameter α was used to split the revision distribution into three sections by percentile:

$$\{[p_0 - p_\alpha], (p_\alpha - p_{(1-\alpha)}], (p_{(1-\alpha)} - p_{100}]|\alpha\}.$$

For each level of alpha in (0.05-0.45) the wilcoxon test was performed and the estimated difference of the medians and the pvalue was measured. We can see the result of this procedure in the top panel Figure 7, where for each revision the y-axis is the estimated difference and the color scale is the p-value. After which, the measures were averaged across revisions bottom left panel and the maximum negative difference was chosen as the optimal α level, α^*. In the top panel the vertical dashed line is for reference on the location of α^* and the lower right panel is the specific location differences and pvalues with respect to α^*.