Solutions of Mathematica based Questions for Assignment 2, MATH7501, Sem 1, 2021

Question 1iii

In[1]:=
```
Sum[1/k^2, {k, 1, ∞}]
```

Out[1]= $\frac{\pi^2}{6}$

Question 1iv

In[21]:=
```
P[n_] := Total[Table[N[1/k], {k, 1, n}]]
```

In[22]:=
```
Log[E]
```

Out[22]= 1

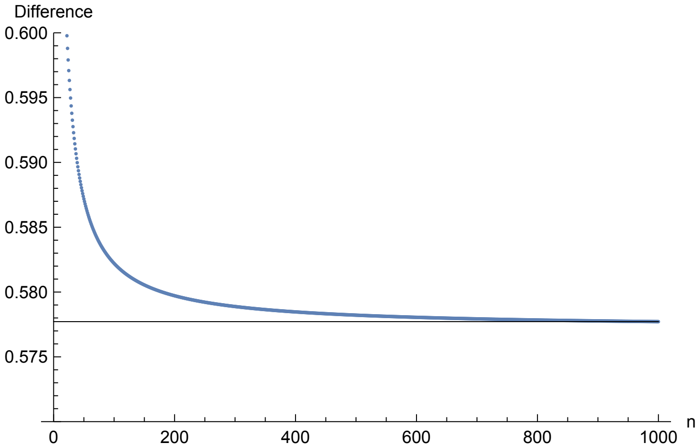
In[36]:=
```
diffSequence = Table[P[n] - Log[n], {n, 1, 1000}];
```

In[37]:=
```
γApprox = Last[diffSequence]
```

Out[37]= 0.577716

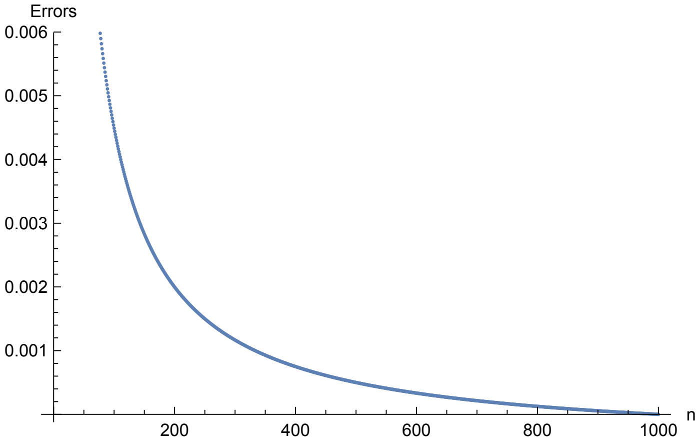
In[48]:=
```
ListPlot[diffSequence, AxesOrigin → {0, 0.57}, PlotRange → {0.57, 0.6},
 AxesLabel → {"n", "Difference"}, Epilog → {Line[{{0, γApprox}, {1000, γApprox}}]}]
```

Out[48]=



In[52]:=
```
errors = diffSequence - γApprox;
ListPlot[errors, AxesLabel → {"n", "Errors"}]
```

Out[53]=



Question 1vi

```
Module[{},
 n = 10;
 data = Table[RandomReal[], {n}]


]
```

Out[55]=
```
{0.614405, 0.121693, 0.0164655, 0.866571,
 0.980211, 0.369698, 0.422367, 0.308614, 0.893082, 0.944189}
```

In[56]:=
```
? For
```

Out[56]=

Symbol

For[*start*, *test*, *incr*, *body*] executes *start*, then repeatedly evaluates *body* and *incr* until *test* fails to give True.

In[126]:=
```
doIt[n_] := (
  For[
   (*start*)
   data = Table[RandomReal[], {n}];
   m = -∞;
   L = 0;
   i = 1;
   ,
   (*test*)
   i ≤ n
   ,
   (*incr*)
   i = i + 1;
   ,
   (*body*)
   If[data[[i]] > m,
      m = data[[i]];
      L = L + 1;
     ];
    ;
  ];
  N[L])
```

In[133]:=
```
statEst[n_] := Mean[Table[doIt[n], {10 000}]]
```

In[134]:=
```
est[n] := Log[n] + γApprox
```

In[143]:=
```
Print["n \t Difference"]
Table[{n, est[n] - statEst[n]}, {n, 1, 10}] // TableForm
```

n Difference

Out[144]//TableForm=

1	0.
2	-0.0025
3	0.
4	-0.0002
5	-0.0009
6	0.0101
7	-0.0007
8	-0.0022
9	0.017
10	-0.0042

Question 5iii

In[178]:=
```
f[x_] := E^(-x^2/2)
taylorF[x_, K_] := Sum[(D[f[x], {x, k}] /. x → 0) x^k/k!, {k, 0, K}]
```

$$f[x\_] := E^{-\frac{x^2}{2}}$$

$$\text{taylorF}[x\_, K\_] := \text{Sum}\left[\left(D[f[x], \{x, k\}] \;/.\; x \to 0\right) \frac{x^k}{k!}, \{k, 0, K\}\right]$$

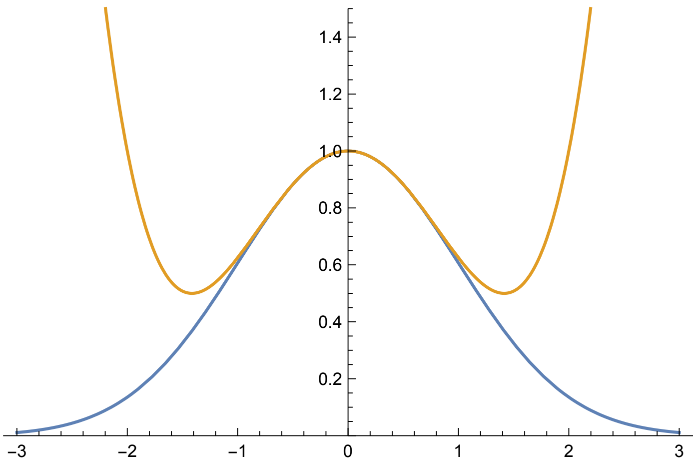
In[180]:=
```
taylorFs = Table[taylorF[x, K], {K, 1, 20}];
taylorFs // TableForm
```

Out[181]//TableForm=

$$1$$
$$1-\frac{x^2}{2}$$
$$1-\frac{x^2}{2}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}+\frac{x^{16}}{10\,321\,920}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}+\frac{x^{16}}{10\,321\,920}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}+\frac{x^{16}}{10\,321\,920}-\frac{x^{18}}{185\,794\,560}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}+\frac{x^{16}}{10\,321\,920}-\frac{x^{18}}{185\,794\,560}$$
$$1-\frac{x^2}{2}+\frac{x^4}{8}-\frac{x^6}{48}+\frac{x^8}{384}-\frac{x^{10}}{3840}+\frac{x^{12}}{46\,080}-\frac{x^{14}}{645\,120}+\frac{x^{16}}{10\,321\,920}-\frac{x^{18}}{185\,794\,560}+\frac{x^{20}}{3\,715\,891\,200}$$

In[185]:=
```
Plot[{f[x], taylorFs[[5]]}, {x, -3, 3}, PlotRange → {0, 1.5}]
```

Out[185]=



In[201]:=
```
maxError[a_, K_] := Max[Table[Abs[f[x] - taylorFs[[K]]], {x, -a, a, 0.1}]]
```

In[205]:=
```
Table[maxError[0.5, K], {K, 1, 6}] // TableForm
```
Out[205]//TableForm=
```
0.117503
0.0074969
0.0074969
0.000315597
0.000315597
9.92342 × 10^-6
```
```
(*So we see that for a=0.5, K=6*)
```

In[211]:=
```
Table[maxError[1.0, K], {K, 1, 10}] // TableForm
```
Out[211]//TableForm=
```
0.393469
0.106531
0.106531
0.0184693
0.0184693
0.00236399
0.00236399
0.000240174
0.000240174
0.000020243
```

In[212]:=
```
(*So we see that for a=1.0, K=10*)
```

In[216]:=
```
Table[maxError[1.5, K], {K, 1, 14}] // TableForm
```
Out[216]//TableForm=
```
0.675348
0.449652
0.449652
0.18316
0.18316
0.0541447
0.0541447
0.0125973
0.0125973
0.00241965
0.00241965
0.000396027
0.000396027
0.0000564923
```
```
(*So we see that for a=1.5, K=14*)
```

Question 5iv

In[229]:=
```
H[n_] := Expand[Simplify[(-1)^n E^(x^2/2) D[E^(-x^2/2), {x, n}]]]
```

In[230]:=
```
Table[H[n], {n, 0, 12}] // TableForm
```

Out[230]//TableForm=

1

x

$-1 + x^2$

$-3x + x^3$

$3 - 6x^2 + x^4$

$15x - 10x^3 + x^5$

$-15 + 45x^2 - 15x^4 + x^6$

$-105x + 105x^3 - 21x^5 + x^7$

$105 - 420x^2 + 210x^4 - 28x^6 + x^8$

$945x - 1260x^3 + 378x^5 - 36x^7 + x^9$

$-945 + 4725x^2 - 3150x^4 + 630x^6 - 45x^8 + x^{10}$

$-10\,395x + 17\,325x^3 - 6930x^5 + 990x^7 - 55x^9 + x^{11}$

$10\,395 - 62\,370x^2 + 51\,975x^4 - 13\,860x^6 + 1485x^8 - 66x^{10} + x^{12}$

Question 6iii

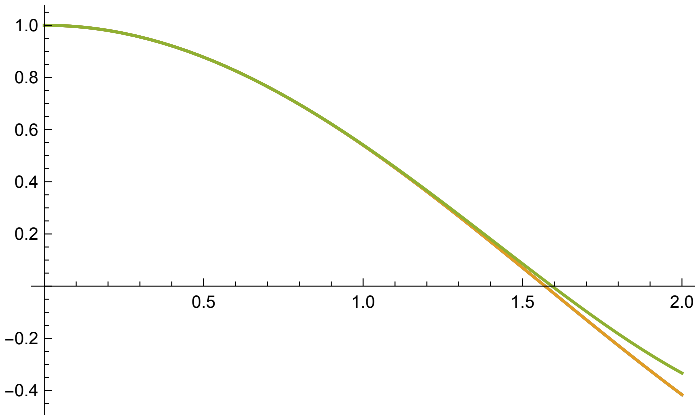
In[246]:=

$$\text{CosTaylor4[x\_] := } 1 - \frac{x^2}{2} + \frac{x^4}{4!}$$

$$\text{CosTaylor10[x\_] := } 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$$

```
Plot[{Cos[x], CosTaylor10[x], CosTaylor4[x]}, {x, 0, 2}]
```

Out[248]=



In[250]:=
```
(*We see the 10'th order Taylro approximation is very close to the actual
  function. For illustration - also plotting the 4'th order approximation*)
Cos[1.5] - CosTaylor10[1.5]
```

Out[250]= 2.67551×10^{-7}

In[285]:=
```
numBisections = 20;
a = 0.0;
b = 2.0;
bisectionTrace = Table[
  mid = (a + b)/2;
  If[CosTaylor10[mid] > 0, a = mid, b = mid];
  {a, b},
  {numBisections}]
```

Out[288]=
```
{{1., 2.}, {1.5, 2.}, {1.5, 1.75}, {1.5, 1.625}, {1.5625, 1.625},
 {1.5625, 1.59375}, {1.5625, 1.57813}, {1.57031, 1.57813}, {1.57031, 1.57422},
 {1.57031, 1.57227}, {1.57031, 1.57129}, {1.57031, 1.5708},
 {1.57056, 1.5708}, {1.57068, 1.5708}, {1.57074, 1.5708}, {1.57077, 1.5708},
 {1.57079, 1.5708}, {1.57079, 1.5708}, {1.57079, 1.5708}, {1.5708, 1.5708}}
```

In[291]:=
```
(*Final error*)
Last[bisectionTrace][[1]] - Last[bisectionTrace][[2]]
```

Out[291]= -1.90735×10^{-6}

In[292]:= $p = \frac{\text{Last[bisectionTrace][[1]]} + \text{Last[bisectionTrace][[2]]}}{2}$

Out[292]= 1.5708

In[298]:= $N\left[\frac{\pi}{2}\right]$

Out[298]= 1.5708