MATH7501 Practical 9 (Week 10), Semester 1-2021

Topic: Sequences & Series, Limits and Derivatives

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Pre-Tutorial Activity

- Students must have familiarised themselves with units 5 to 8 contents of the reading materials for MATH7501

Resources

- Chapters 5 to 8 of course reader

Q1 Limit

Calculate $\text{limit}_{x \to \text{Infinity}} \sin(e^{-x})$ and explain why this is true

In[33]:=
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Plot[Sin[Exp[-x]], {x, -5, 10}]
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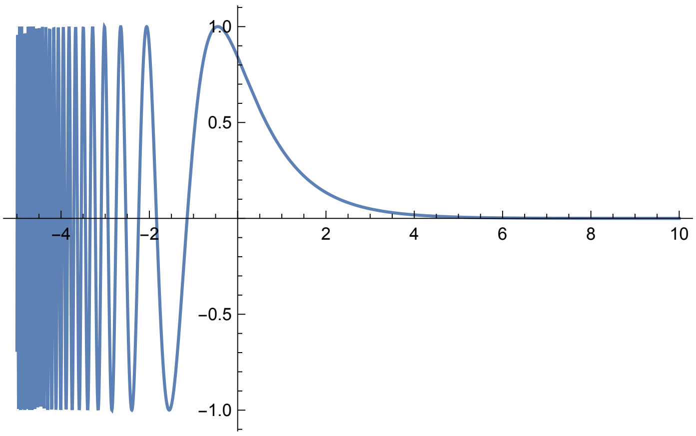
Out[33]=



limit e ^ {-x} is 0 as x approaches Infinity. sin (0) is continuous. Then sin (e^{-x}) = 0 as x approaches Infinity.

Q2 Limit

Calculate $\text{limit}_{x \to \text{Infinity}} \dfrac{1}{\cos(x) + x}$

$-1 + x \leq \cos(x) + x \leq 1 + x$

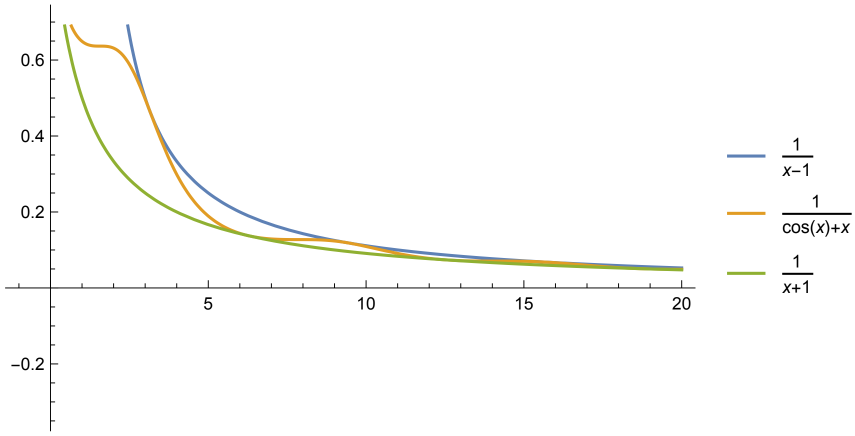
For $x > 1$, $\frac{1}{1+x} \leq \frac{1}{\cos(x)+x} \leq \frac{1}{x-1}$

As $\text{limit}_{x\to\text{Infinity}} \frac{1}{-1+x} = 0$ and $\text{limit}_{x\to\text{Infinity}} \frac{1}{1+x} = 0$,

then by the sqeez theorem $\text{limit}_{x\to\text{Infinity}} \frac{1}{\cos(x)+x} = 0$

In[35]:= $\text{Plot}\left[\left\{\frac{1}{x-1}, \frac{1}{\text{Cos}[x]+x}, \frac{1}{x+1}\right\}, \{x, -1, 20\}, \text{PlotLegends} \to \text{"Expressions"}\right]$

Out[35]=



Q3 Continuous function

Consider the function

$$f(x) = \begin{cases} \frac{\sin(x)}{x} & , x > 0 \\ 1 & , x \leq 0 \end{cases}$$

Show that this function is continuous for all x.

Hint : Continuous at a point (see Section 6.4, Definition 12)

Continuous on interval (see pg 104)

$f(x)$ is continuous everywhere except possibly $x = 0$. So check continuity at $x = 0$.

Check : (1) Is $f(0)$ defined? yes $f(0) = 1$

(2) Does $\text{limit}_{x\to 0} f(0)$ exist?

(3) Is $\text{limit}_{x\to 0} f(x) = f(0)$

$\text{limit}_{x\to 0^-} f(x) = f(0) = 1$

$\text{limit}_{x\to 0^+} f(x) = \sin(x)/x = 1$

Thus condition (3) is satisfied. Thus $f(x)$ is continuous for all x.

In[36]:=
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Plot[Piecewise[{{1, x ≤ 0}, {Sin[x]/x, x > 0}}], {x, -3, 10}]
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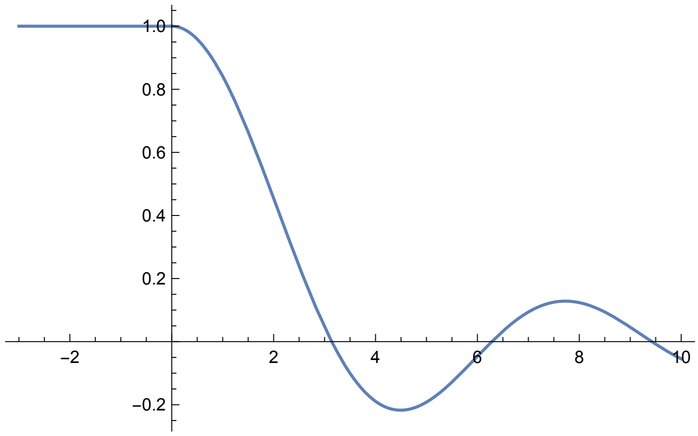
Out[36]=



Q4 IVT and MVT

a) Consider the function $f(x) = x^4 + x + 1$. Show that this function does not cross the x – axis.

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let y = f (x)
Find x value at f' (x) = 0 (this gives any critical points)
Substitute the value of x in f (x) to get the value of y

step1 : solve for x in f' (x) = 0 (you should get just one critical point)
step 2 : find f'' (x) and show that the critical point is global minimum (f'' (x) > 0)
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b) Consider the new function $f(x) =$
$x^4 + x - 1$. Show that there are exactly two solutions to $f(x) = 0$,
using Intermediate Value Theorem (IVT) and Mean Value Theorem (MVT).

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IVT (look at pg 105)
MVT (look at pg 115)
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Q5 Series

Determine if the following sums converge or not. If they do, work out their limit

a) $\sum_{n=1}^{\infty} n^{2/e}$

(b) $\sum_{n=0}^{\infty} (\pi - e)^n$