MATH7501 Practical 8 (Week 9), Semester 1-2021

Topic: Sequences, Limits and Series

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Pre-Tutorial Activity

- Students must have familiarised themselves with units 5 to 7 contents of the reading materials for MATH7501

Resources

- Chapters 5 to 7 of course reader
- https://reference.wolfram.com/language/tutorial/SeriesLimitsAndResidues.html#3309
- http://web.mit.edu/kayla/www/calc/06-summary-discontinuities-derivatives.pdf

Q1 Construction of a function

Construct an example of a function f (x) so that :

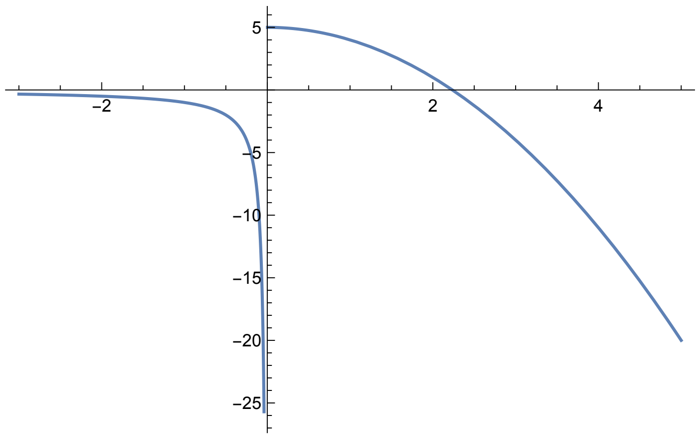
• f (x) is defined for all real x;

• f' (x) exists and is negative for all x not equal to 0

• The point x = 0 is a local maximum of f.

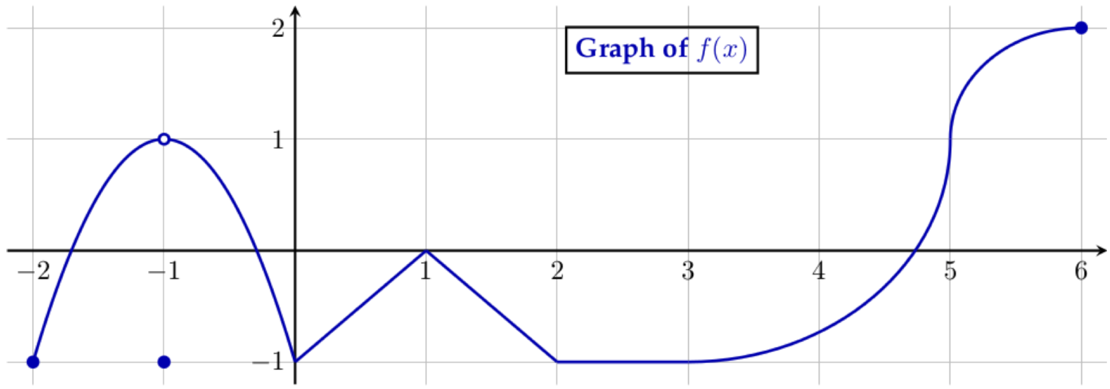
```
(* an exaple could be the following piecewise function*)
Plot[Piecewise[{{1 / x, x < 0}, {-x^2 + 5, 0 ≤ x}}], {x, -3, 5}]
```

Out[204]=



Q2 Turning points of f(x)

The following graph describes a function f (x)
on [−2, 6]. List all critical points in the interval (−2, 6).



Note that critical points occur when the first derivative is zero or undefined.
Critical Points and classifications.

```
(-1, -1) - discontinuity
  (0, -1) - corner
  (1, 0) - corner
  (2, -1) - corner
  (5, 1) - inflection point
  values in the interval (2, 3] - derivative is 0
```

Q3 Derivatives

Show why P (t) = C e^{kt} is the ONLY solution to the growth equation P' (t) = k P (t),
with P (0) = C.

Let P (t) = A (t) e^{kt}. Compute P' (t) and substitute into P' (t) = k P (t).
P' (t) = A' (t) e^{kt} + kA (t) e^{kt}

A' (t) e^{kt} + kA (t) e^{kt} = kA (t) e^{kt}

A' (t) + kA (t) = kA (t)
This means A' (t) = 0, and thus A (t) is a constant (say C). Therefore P (t) = Ce^{kt}

Q4 Maclaurin series

Use Mathematica to plot the Maclaurin series approximations for e^x.
Use these approximations to get an approximate value for e itself.

In[135]:=
```
(* this computes Taylor series expansion of f(x) at x=a upto order 4*)
Clear[f]
Series[f[x], {x, a, 4}]
```

Out[136]= $f[a] + f'[a]\,(x-a) + \frac{1}{2} f''[a]\,(x-a)^2 + \frac{1}{6} f^{(3)}[a]\,(x-a)^3 + \frac{1}{24} f^{(4)}[a]\,(x-a)^4 + O[x-a]^5$

In[205]:=
```
Series[f[x], {x, 0, 4}]
```

Out[205]= $f[0] + f'[0]\, x + \frac{1}{2} f''[0]\, x^2 + \frac{1}{6} f^{(3)}[0]\, x^3 + \frac{1}{24} f^{(4)}[0]\, x^4 + O[x]^5$

In[208]:=
```
Series[Exp[x], {x, 0, 5}]
```

Out[208]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + O[x]^6$

In[209]:=
```
Sum[x^n/n!, {n, 0, Infinity}]
```

Out[209]= e^x

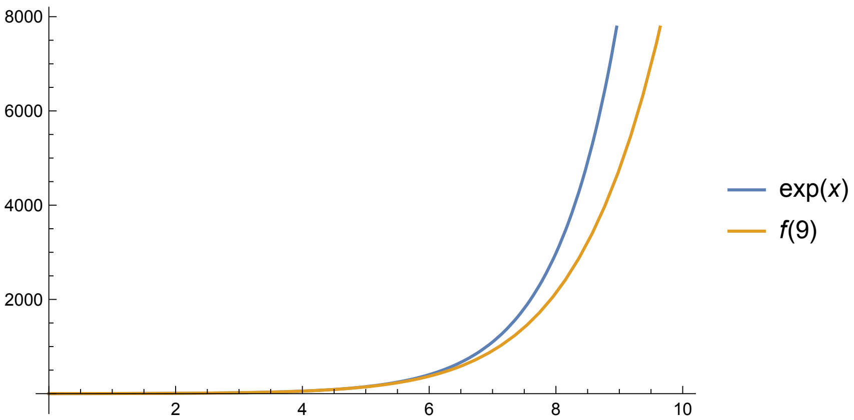
```
(* first create a function to compute the
  Maclaurin series expansion of eˣ at x=0 upto order n*)
Clear[f]
f[k_] := Sum[x^n/n!, {n, 0, k}]
```

In[213]:=
```
f[5]
```

Out[213]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$

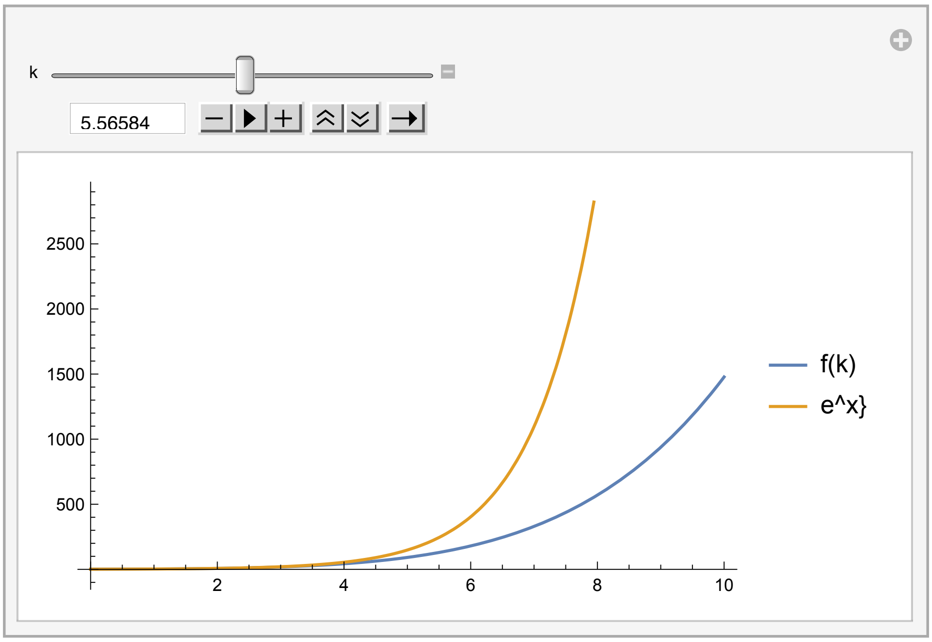
```
(* this plots eˣ and f[k] for a specific value of k *)
Plot[{Exp[x], f[9]}, {x, 0, 10}, PlotLegends → "Expressions"]
```

Out[220]=



```
(* this plot shows how f[k] is
 approximated to e^x as k is increased from 1 to 10*)
Manipulate[Plot[{f[k], Exp[x]}, {x, 0, 10},
  PlotLegends → {"f(k)", "e^x}"}], {k, 1, 10}]
```

Out[221]=




```
(* To get an approximate value for e,
you need the Maclaurin series for e^x for x=1,
which is given by the function g[]*)
```

In[222]:=
```
Clear[g]
g[k_] := Sum[1 / n!, {n, 0, k}]
```

In[224]:=
```
With[{nmax = 30},
 (*nmax is the upper limit of the sum*)
  Show[ DiscretePlot[g[k], {k, 1, nmax},
   Epilog → {Red, Line[{{0, Exp[1]}, {nmax, Exp[1]}}]},
   (*This plots a hrizontal red line showing the actual value of e*)
   PlotRange → All,
   AxesLabel → {k, Approximate value}
  ]]
]
```

Out[224]=