Statistics with Julia from the ground up

A JuliaCon 2021 (https://juliacon.org/2021/) workshop by Yoni Nazarathy (https://yoninazarathy.com/)

Many of the code examples for this workshop are adapted from Statistics with Julia: Fundamentals for Data Science, Machine Learning and Artificial Intelligence by Yoni Nazarathy and Hayden Klok (https://statisticswithjulia.org/).

Also related (and recommended in this JuliaCon):

- Dataframes tutorial (https://github.com/bkamins/JuliaCon2021-DataFrames-Tutorial) or here (https://pretalx.com/juliacon2021/talk/FXZXMB/) by Bogumił Kamiński.
- Introduction to Bayesian Data Analysis (https://pretalx.com/juliacon2021/talk/J7BFBM/) by Kusti Skytén.
- Dozens of other very exciting talks...

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10. Basic Machine learning.

Before we start

The tutorial was developed and tested under Julia 1.6.0. It is best to run it with the `Project.toml` and `Manifest.toml` files present in the working directory of the notebook. It also uses the following data files:

In [1]:
```
readdir("./data")
```

Out[1]:
```
8-element Vector{String}:
 "L1L2data.csv"
 "fertilizer.csv"
 "machine1.csv"
 "machine2.csv"
 "machine3.csv"
 "purchaseData.csv"
 "temperatures.csv"
 "weightHeight.csv"
```

You will find all these files and this notebook in the Github repo for this workshop (https://github.com/yoninazarathy/JuliaCon2021-StatisticsWithJuliaFromTheGroundUp). You can either "clone" the repo or download a zip file.

We use a Jupyter Notebook (https://jupyter.org/). Here is a qjuuick reference sheet (edureka.co/blog/wp-content/uploads/2018/10/Jupyter_Notebook_CheatSheet_Edureka.pdf). Many other reserouces on the web as well for Jupyter - many of which use Python and not Julia, but Jupyter is the same. BTW the "J" in "Jupyter" is for "Julia".

Load the packages we will use:

In [2]:
```
using Pkg
Pkg.activate(".")
Pkg.instantiate()
```

```
  Activating environment at `~/git/mine/JuliaCon2021-StatisticsWithJuli
```

In [3]:
```
Pkg.status()
```

```
      Status `~/git/mine/JuliaCon2021-StatisticsWithJuliaFromTheGroundU
  [336ed68f] CSV v0.8.5
  [aaaa29a8] Clustering v0.14.2
  [861a8166] Combinatorics v1.0.2
  [a93c6f00] DataFrames v1.2.0
  [31c24e10] Distributions v0.25.11
  [587475ba] Flux v0.12.5
  [38e38edf] GLM v1.5.1
  [09f84164] HypothesisTests v0.10.4
  [5ab0869b] KernelDensity v0.6.3
  [b964fa9f] LaTeXStrings v1.2.1
  [b4fcebef] Lasso v0.6.2
  [eb30cadb] MLDatasets v0.5.7
  [442fdcdd] Measures v0.3.1
  [dbeba491] Metalhead v0.5.3
  [6f286f6a] MultivariateStats v0.8.0
  [91a5bcdd] Plots v1.19.2
  [ce6b1742] RDatasets v0.7.5
  [f2b01f46] Roots v1.0.10
  [276daf66] SpecialFunctions v1.5.1
  [2913bbd2] StatsBase v0.33.8
  [f3b207a7] StatsPlots v0.14.25
```

In [4]:
```
using Random, Statistics, LinearAlgebra, Dates #Shipped with Julia
using Distributions, StatsBase #Core statistics
using CSV, DataFrames #Basic Data
using Plots, StatsPlots, LaTeXStrings, Measures #Plotting and Output
using HypothesisTests, KernelDensity, GLM, Lasso, Clustering, Multivaria
using Flux, Metalhead #Deep learning
using Combinatorics, SpecialFunctions, Roots #Mathematical misc.
using RDatasets, MLDatasets #Example datasets
#uncomment if using R:  using RCall #Interface with R
```

In [5]:
```
# We run this before many examples for reproducibility
fix_seed!() = Random.seed!(0)
```

Out[5]:
```
fix_seed! (generic function with 1 method)
```

Why Julia?

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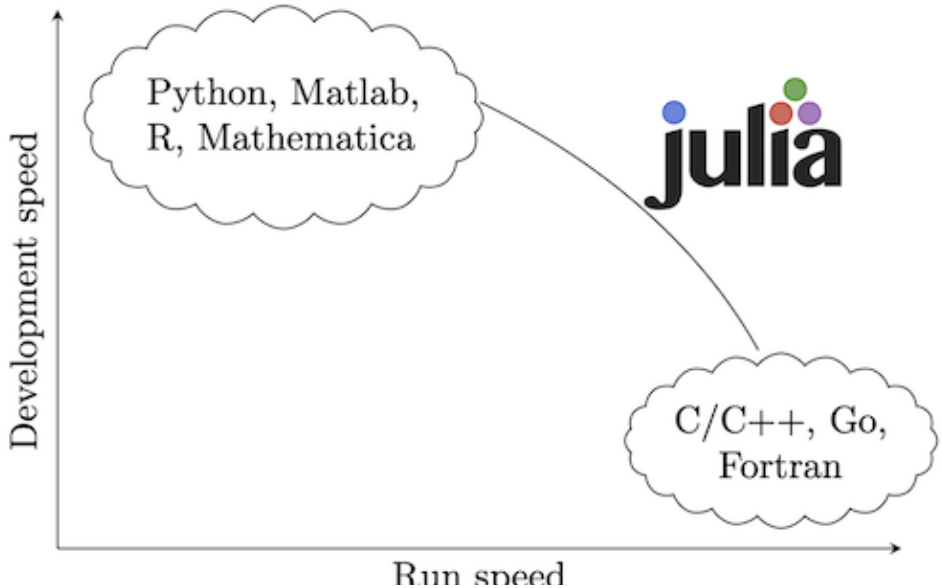


Some ways to run Julia

- REPL
  - As an application
  - Out of your shell
  - As part of an IDE
- Jupyter (IJulia)
  - In your web browser
  - Jupyter Lab
- Google collab
- Pluto
- Visual Studio Code
- Legacy: Atom (Juno)
- JuliaHub
- In RMarkdown with IJulia (e.g. in R studio)
- ...

Key resources (My favorites)

- Main Julia Page: https://julialang.org/ (https://julialang.org/)
- Docs: https://docs.julialang.org/ (https://docs.julialang.org/)
- Julia Express: https://github.com/bkamins/The-Julia-Express (https://github.com/bkamins/The-Julia-Express)
- Think Julia: https://www.oreilly.com/library/view/think-julia/9781492045021/ (https://www.oreilly.com/library/view/think-julia/9781492045021/)
- MIT Course, computational thinking: https://computationalthinking.mit.edu/Spring21/ (https://computationalthinking.mit.edu/Spring21/)
- A University of Queensland Course: https://courses.smp.uq.edu.au/MATH2504/ (https://courses.smp.uq.edu.au/MATH2504/)
- Statistics with Julia: https://statisticswithjulia.org/ (https://statisticswithjulia.org/) (use image gallary)
- Package documentation: Searching for the package, e.g. `Plots.jl` , typically gets you to GitHub. From there find the docs.
- Julia Discourse: https://discourse.julialang.org/ (https://discourse.julialang.org/)
- Julia Slack: https://julialang.org/slack/ (https://julialang.org/slack/)
- Your local Julia "club": E.g. in my area: https://www.meetup.com/en-AU/brisbane-julia-language-meetup/ (https://www.meetup.com/en-AU/brisbane-julia-language-meetup/)
- YouTube...

What do you `mean`?

home

In [6]:
```
fix_seed!()
data = rand(Normal(),5)
```

Out[6]:
```
5-element Vector{Float64}:
  0.6791074260357777
  0.8284134829000359
 -0.3530074003005963
 -0.13485387193052173
  0.5866170746331097
```

```
In [7]: n = length(data)
Out[7]: 5

In [8]: sum(data)/n
Out[8]: 0.3212553422675611

In [9]: +(data...)/n
Out[9]: 0.3212553422675611
```

In [10]:
```
? +
```

```
search: +
```

Out[10]:
```
+(x, y...)
```

Addition operator. `x+y+z+...` calls this function with all arguments, i.e. `+(x, y, z, ...)`.

Examples

```
jldoctest
julia> 1 + 20 + 4
25

julia> +(1, 20, 4)
25
```

```
dt::Date + t::Time -> DateTime
```

The addition of a `Date` with a `Time` produces a `DateTime`. The hour, minute, second, and millisecond parts of the `Time` are used along with the year, month, and day of the `Date` to create the new `DateTime`. Non-zero microseconds or nanoseconds in the `Time` type will result in an `InexactError` being thrown.

```
bb1 + bb2 -> bb
```

Compute the BoundingBox `bb` that minimally contains `bb1` and `bb2`

In [11]:
```
"""
This is a function that takes in data and returns its sum.
"""
function my_sum(data)
    s = 0.0
    for d in data
        s += d
    end
    return s
end
```

Out[11]:
```
my_sum
```

In [12]:
```
? my_sum
```

```
search: my_sum
```

Out[12]: This is a function that takes in data and returns its sum.

In [13]:
```
my_sum(data)/length(data)
```

Out[13]:
```
0.3212553422675611
```

In [14]:
```
mean(data)
```

Out[14]:
```
0.3212553422675611
```

In [15]:
```
data'*ones(n)/n
```

Out[15]:
```
0.3212553422675611
```

In [16]:
```
dot(data,ones(n))/n
```

Out[16]:
```
0.3212553422675611
```

In [17]:
```
? dot
```

```
search: dot dotplot dotplot! @__dot__ stdout dropterm DEPOT_PATH groupe
```

Out[17]:
```
dot(x, y)
x · y
```

Compute the dot product between two vectors. For complex vectors, the first vector is conjugated.

`dot` also works on arbitrary iterable objects, including arrays of any dimension, as long as `dot` is defined on the elements.

`dot` is semantically equivalent to `sum(dot(vx,vy) for (vx,vy) in zip(x, y))`, with the added restriction that the arguments must have equal lengths.

`x · y` (where `·` can be typed by tab-completing `\cdot` in the REPL) is a synonym for `dot(x, y)`.

Examples

```
jldoctest
julia> dot([1; 1], [2; 3])
5

julia> dot([im; im], [1; 1])
0 - 2im

julia> dot(1:5, 2:6)
70

julia> x = fill(2., (5,5));

julia> y = fill(3., (5,5));

julia> dot(x, y)
150.0
```

```
dot(x, A, y)
```

Compute the generalized dot product `dot(x, A*y)` between two vectors `x` and `y`, without storing the intermediate result of `A*y`. As for the two-argument `dot(_,_)` (@ref), this acts recursively. Moreover, for complex vectors, the first vector is conjugated.

!!! compat "Julia 1.4" Three-argument `dot` requires at least Julia 1.4.

Examples

```
jldoctest
julia> dot([1; 1], [1 2; 3 4], [2; 3])
26

julia> dot(1:5, reshape(1:25, 5, 5), 2:6)
4850

julia> ⋅(1:5, reshape(1:25, 5, 5), 2:6) == dot(1:5, reshape(1:25, 5,
5), 2:6)
true
```

Doing it a little differently with the "running mean" formula

[Math Processing Error]

In [18]:
```
mn = 0
for i in 1:length(data)
    global mn = (1/i)*data[i] + (i-1)/i*mn
end
mn
```

Out[18]:
```
0.32125534226756114
```

In [19]:
```
mn = 0
for (i,d) in enumerate(data)
    global mn = (1/i)*d + (i-1)/i*mn #Note that in Jupyter `global` isn
end
mn
```

Out[19]:
```
0.32125534226756114
```

In [20]:
```
function my_mean(data)
    mn = 0
    for (i,d) in enumerate(data)
        mn = (1/i)*d + (i-1)/i*mn
    end
    return mn
end
```

Out[20]:
```
my_mean (generic function with 1 method)
```

In [21]:
```
my_mean(data)
```

Out[21]:
```
0.32125534226756114
```

In [22]:
```
fix_seed!()
data = rand(Normal(),n) + im*rand(Normal(),n)
```

Out[22]:
```
5-element Vector{ComplexF64}:
   0.6791074260357777 + 0.29733585084941616im
   0.8284134829000359 + 0.06494754854834232im
  -0.3530074003005963 - 0.10901738508171745im
 -0.13485387193052173 - 0.514210390833322im
   0.5866170746331097 + 1.5743302021369892im
```

In [23]:
```
mean(data)
```

Out[23]:
```
0.3212553422675611 + 0.2626771651239416im
```

In [24]:
```
methods(mean)
```

Out[24]: # 105 methods for generic function **mean**:

- mean(M::**PPCA**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/ppca.jl:15 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/ppca.jl)
- mean(d::**Geometric**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/geometric.jl:55 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/geometric.j
- mean(M::**MulticlassLDA**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/lda.jl:131 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/lda.jl)
- mean(d::**InverseGamma{T}**) *where T* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/inversegamm (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/inverseg
- mean(d::**Normal**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/normal.jl:67 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/normal.j
- mean(d::**PoissonBinomial**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/poissonbinomial (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/poissonbin
- mean(d::**FisherNoncentralHypergeometric**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/noncentralhyper (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/noncentrall
- mean(M::**PCA**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/pca.jl:31 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/pca.jl)
- mean(d::**Chi**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/chi.jl:51 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/chi.jl)
- mean(d::**EdgeworthSum**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/edgeworth.jl:87 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/edgeworth.jl)
- mean(d::**Triweight**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/triweight.jl:36 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/triweigh

- mean(d::**Product**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/product.jl:38 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/product.jl)
- mean(d::**NormalCanon**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/normalcanon. (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/normalc
- mean(d::**Laplace**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/laplace.jl:63 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/laplace.
- mean(d::**Biweight**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/biweight.jl:27 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/biweigh
- mean(r::**AbstractRange{var"#s832"} where var"#s832"<:Real**) in Statistics at /Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/stdlib/v1.6/Statistics/src/Statistics.jl:185 (file:///Users/julia/buildbot/worker/package_macos64/build/usr/share/julia/stdlib/v1.6/Statisti
- mean(d::**TDist{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/tdist.jl:50 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/tdist.jl)
- mean(S::**MulticlassLDAStats**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/lda.jl:74 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/lda.jl)
- mean(d::**Epanechnikov**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/epanechniko (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/epanecl
- mean(d::**Beta**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/beta.jl:63 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/beta.jl)
- mean(d::**Poisson**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/poisson.jl:52 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/poisson.jl)
- mean(M::**SubspaceLDA**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/lda.jl:222 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/lda.jl)
- mean(f::**Function**, a::**StaticArrays.StaticArray**; *dims*) in StaticArrays at /Users/uqjnazar/.julia/packages/StaticArrays/AHT47/src/mapreduce.jl:275 (file:///Users/uqjnazar/.julia/packages/StaticArrays/AHT47/src/mapreduce.jl)

- mean(d::**Cauchy{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/cauchy.jl:62 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/cauchy.
- mean(d::**PGeneralizedGaussian**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/pgeneralized((file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/pgenera
- mean(d::**MvNormalCanon**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvnormalcanon.jl:151 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvnormalcanon.jl
- mean(d::**Skellam**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/skellam.jl:60 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/skellam.jl)
- mean(d::**UnivariateGMM**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/mixtures/unigmm.jl:26 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/mixtures/unigmm.jl)
- mean(d::**MultivariateMixture{S, C} where {S<:ValueSupport, C<:Distribution}**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/mixtures/mixturemodel.jl:179 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/mixtures/mixturemodel.jl)
- mean(d::**UnivariateMixture{S, C} where {S<:ValueSupport, C<:Distribution}**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/mixtures/mixturemodel.jl:173 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/mixtures/mixturemodel.jl)
- mean(f::**Whitening**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/whiten.jl:36 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/whiten.jl)
- mean(d::**MatrixReshaped**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixreshaped.jl:54 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixreshaped.jl)
- mean(d::**Binomial**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/binomial.jl:68 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/binomial.jl)
- mean(d::**MatrixTDist**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixtdist.jl:106 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixtdist.jl)
- mean(d::**InverseGaussian**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/inversegauss

(file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/inverseg

- mean(d::**Distributions.GenericMvTDist**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvtdist.jl:89 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvtdist.jl)
- mean(d::**SymTriangularDist**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/symtriangular (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/symtriar
- mean(d::**LogNormal**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/lognormal.jl:6 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/lognorm
- mean(d::**Levy{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/levy.jl:55 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/levy.jl)
- mean(d::**Erlang**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/erlang.jl:61 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/erlang.jl
- mean(d::**VonMises**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/vonmises.jl:5 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/vonmise
- mean(d::**LocationScale**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/locationscale.jl:85 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/locationscale.jl)
- mean(a::**StaticArrays.StaticArray**; *dims*) in StaticArrays at /Users/uqjnazar/.julia/packages/StaticArrays/AHT47/src/mapreduce.jl:274 (file:///Users/uqjnazar/.julia/packages/StaticArrays/AHT47/src/mapreduce.jl)
- mean(d::**DirichletMultinomial**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/dirichletmultinomial.jl:2 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/dirichletmultinomi
- mean(d::**EdgeworthZ**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/edgeworth.jl:23 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/edgeworth.jl)
- mean(d::**Rayleigh**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/rayleigh.jl:55 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/rayleigh
- mean(d::**Bernoulli**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/bernoulli.jl:60 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/bernoulli.jl)

- mean(d::**Exponential**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/exponential.jl (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/exponer
- mean(d::**MatrixFDist**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixfdist.jl:90 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixfdist.jl)
- mean(d::**GeneralizedPareto{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/generalizedpa (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/generali
- mean(d::**Wishart**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/wishart.jl:108 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/wishart.jl)
- mean(d::**MvNormal**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvnormal.jl:237 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvnormal.jl)
- mean(d::**Semicircle**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/semicircle.jl:3 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/semicirc
- mean(d::**MatrixBeta**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixbeta.jl:84 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixbeta.jl)
- mean(d::**Truncated{var"#s464", Continuous, T} where {var"#s464"<:Exponential, T<:Real}**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/truncated/exponential.jl:5 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/truncated/exponential.jl)
- mean(d::**Truncated{Normal{T}, Continuous, T1} where T1<:Real**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/truncated/normal.jl:96 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/truncated/normal.jl)
- mean(d::**NoncentralT{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/noncentralt.jl: (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/noncent
- mean(d::**Chisq**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/chisq.jl:50 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/chisq.jl)
- mean(d::**Dirac**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/dirac.jl:33

(file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/dirac.jl)

- mean(d::**Cosine**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/cosine.jl:47 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/cosine.j
- mean(d::**Uniform**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/uniform.jl:58 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/uniform
- mean(d::**LKJ**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/lkj.jl:71 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/lkj.jl)
- mean(d::**SkewNormal**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/skewnormal.j (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/skewno
- mean(d::**NoncentralChisq**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/noncentralchi (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/noncent
- mean(d::**Gamma**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/gamma.jl:60 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/gamma
- mean(d::**NoncentralF{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/noncentralf.jl: (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/noncent
- mean(M::**FactorAnalysis**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/fa.jl:13 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/fa.jl)
- mean(d::**WalleniusNoncentralHypergeometric**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/noncentralhyper (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/noncentrall
- mean(d::**NegativeBinomial{T}**) *where T* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/negativebinomia (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/negativebir
- mean(d::**IntervalSets.AbstractInterval**) in IntervalSets at /Users/uqjnazar/.julia/packages/IntervalSets/VeVgo/src/IntervalSets.jl:100 (file:///Users/uqjnazar/.julia/packages/IntervalSets/VeVgo/src/IntervalSets.jl)
- mean(d::**MvLogNormal**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvlognormal.jl:207 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/mvlognormal.jl)

- mean(d::**Multinomial**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/multinomial.jl:59 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/multinomial.jl)
- mean(d::**Gumbel**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/gumbel.jl:70 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/gumbel.
- mean(d::**BetaPrime{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/betaprime.jl:6 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/betapri
- mean(d::**TriangularDist**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/triangular.jl:65 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/triangul
- mean(d::**Frechet{T}**) *where T* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/frechet.jl:62 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/frechet.
- mean(d::**Logistic**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/logistic.jl:64 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/logistic.
- mean(d::**GeneralizedExtremeValue{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/generalizedex (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/generali
- mean(d::**InverseWishart**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/inversewishart.jl:81 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/inversewishart.jl)
- mean(d::**Dirichlet**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/dirichlet.jl:69 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/multivariate/dirichlet.jl)
- mean(d::**NormalInverseGaussian**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/normalinverse (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/normali
- mean(M::**ICA**) in MultivariateStats at /Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/ica.jl:12 (file:///Users/uqjnazar/.julia/packages/MultivariateStats/HTpHt/src/ica.jl)
- mean(d::**MatrixNormal**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixnormal.jl:88 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/matrix/matrixnormal.jl)

- mean(d::**FDist{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/fdist.jl:57 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/fdist.jl)
- mean(qvec::**AbstractArray{Rotations.UnitQuaternion{T}, 1}**) *where T* in Rotations at /Users/uqjnazar/.julia/packages/Rotations/QaTBi/src/mean.jl:32 (file:///Users/uqjnazar/.julia/packages/Rotations/QaTBi/src/mean.jl)
- mean(qvec::**AbstractArray{Rotations.UnitQuaternion{T}, 1}**, method::**Integer**) *where T* in Rotations at /Users/uqjnazar/.julia/packages/Rotations/QaTBi/src/mean.jl:32 (file:///Users/uqjnazar/.julia/packages/Rotations/QaTBi/src/mean.jl)
- mean(vec::**AbstractVector{R}**) *where R<:Rotations.Rotation* in Rotations at /Users/uqjnazar/.julia/packages/Rotations/QaTBi/src/mean.jl:49 (file:///Users/uqjnazar/.julia/packages/Rotations/QaTBi/src/mean.jl)
- mean(A::**AbstractArray{T, N} where N**, w::**AbstractWeights{W, T, V} where {T<:Real, V<:AbstractVector{T}}**, dims::**Int64**) *where {T<:Number, W<:Real}* in StatsBase at deprecated.jl:70 (file:///Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/base/deprecated.jl)
- mean(A::**AbstractArray**; *dims*) in Statistics at /Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/stdlib/v1.6/Statistics/src/Statistics.jl:164 (file:///Users/julia/buildbot/worker/package_macos64/build/usr/share/julia/stdlib/v1.6/Statisti
- mean(A::**AbstractArray**, w::**UnitWeights**; *dims*) in StatsBase at /Users/uqjnazar/.julia/packages/StatsBase/DU1bT/src/weights.jl:625 (file:///Users/uqjnazar/.julia/packages/StatsBase/DU1bT/src/weights.jl)
- mean(A::**AbstractArray**, w::**AbstractWeights**; *dims*) in StatsBase at /Users/uqjnazar/.julia/packages/StatsBase/DU1bT/src/weights.jl:618 (file:///Users/uqjnazar/.julia/packages/StatsBase/DU1bT/src/weights.jl)
- mean(d::**DiscreteNonParametric**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/discretenonpara (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/discretenor
- mean(d::**EdgeworthMean**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/edgeworth.jl:103 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/edgeworth.jl)
- mean(d::**BetaBinomial**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/betabinomial.jl:5 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/betabinom
- mean(d::**Pareto{T}**) *where T<:Real* in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/pareto.jl:57 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/pareto.j

- mean(d::**Arcsine**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/arcsine.jl:64 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/arcsine.
- mean(d::**Weibull**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/weibull.jl:63 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/weibull.
- mean(d::**Chernoff**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/chernoff.jl:22 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/chernof
- mean(d::**Hypergeometric**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/hypergeometric. (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/hypergeom
- mean(d::**Kolmogorov**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/kolmogorov.jl (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/continuous/kolmog
- mean(Ω::**Soliton**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/soliton.jl:108 (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/soliton.jl)
- mean(d::**DiscreteUniform**) in Distributions at /Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/discreteuniform. (file:///Users/uqjnazar/.julia/packages/Distributions/fXTVC/src/univariate/discrete/discreteuni
- mean(itr) in Statistics at /Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/stdlib/v1.6/Statistics/src/Statistics.jl:44 (file:///Users/julia/buildbot/worker/package_macos64/build/usr/share/julia/stdlib/v1.6/Statisti
- mean(f, A::**AbstractArray**; *dims*) in Statistics at /Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/stdlib/v1.6/Statistics/src/Statistics.jl:104 (file:///Users/julia/buildbot/worker/package_macos64/build/usr/share/julia/stdlib/v1.6/Statisti
- mean(f, itr) in Statistics at /Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/stdlib/v1.6/Statistics/src/Statistics.jl:61 (file:///Users/julia/buildbot/worker/package_macos64/build/usr/share/julia/stdlib/v1.6/Statisti

In [25]:
```
@which mean(data)
```

Out[25]: mean(A::**AbstractArray**; *dims*) in Statistics at /Applications/Julia-1.6.app/Contents/Resources/julia/share/julia/stdlib/v1.6/Statistics/src/Statistics.jl:164 (file:///Users/julia/buildbot/worker/package_macos64/build/usr/share/julia/stdlib/v1.6/Statistics/s

In [26]:
```
methods(my_mean)
```

Out[26]: # 1 method for generic function **my_mean**:

- my_mean(data) in Main at In[20]:1

In [27]:
```
function my_mean(data::Vector{Float64})::Float64
    mn = 0.0
    for (i,d) in enumerate(data)
        mn = (1/i)*d + (i-1)/i*mn
    end
    return mn
end
```

Out[27]: `my_mean (generic function with 2 methods)`

In [28]:
```
methods(my_mean)
```

Out[28]: # 2 methods for generic function **my_mean**:

- my_mean(data::**Vector{Float64}**) in Main at In[27]:1
- my_mean(data) in Main at In[20]:1

In [29]:
```
@which my_mean(data)
```

Out[29]: my_mean(data) in Main at In[20]:1

In [30]:
```
@which my_mean([1,2])
```

Out[30]: my_mean(data) in Main at In[20]:1

In [31]:
```
@which my_mean([1.0,2.0])
```

Out[31]: my_mean(data::**Vector{Float64}**) in Main at In[27]:1

In [32]:
```
@which my_mean([1.0,2])
```

Out[32]: my_mean(data::**Vector{Float64}**) in Main at In[27]:1

In [33]:
```
function my_mean(data::Vector{T})::T where T
    mn = zero(data[begin])
    for (i,d) in enumerate(data)
        mn = (1/i)*d + (i-1)/i*mn
    end
    return mn
end
```

Out[33]:
```
my_mean (generic function with 3 methods)
```

In [34]:
```
methods(my_mean)
```

Out[34]:
3 methods for generic function **my_mean**:

- my_mean(data::**Vector{Float64}**) in Main at In[27]:1
- my_mean(data::**Vector{T}**) *where T* in Main at In[33]:1
- my_mean(data) in Main at In[20]:1

The mean of other types

In [35]:
```
fix_seed!()
data = [rand(Normal(),2,2) for _ in 1:5]
```

Out[35]:
```
5-element Vector{Matrix{Float64}}:
 [0.6791074260357777 -0.3530074003005963; 0.8284134829000359 -0.1348538
 [0.5866170746331097 0.06494754854834232; 0.29733585084941616 -0.109017
 [-0.514210390833322 -0.6889071278256981; 1.5743302021369892 -0.7628038
 [0.39748240921816347 -0.34635460427879816; 0.8116296225068749 -0.18757
 [-1.6072563241277753 2.2762328327845243; -2.48079273065994 0.219693467
```

In [36]:
```
mean(data)
```

Out[36]:
```
2×2 Matrix{Float64}:
 -0.091652   0.190582
  0.206183  -0.194911
```

In [37]:
```
my_mean(data)
```

Out[37]:
```
2×2 Matrix{Float64}:
 -0.091652   0.190582
  0.206183  -0.194911
```

In [38]:
```
@which my_mean(data)
```

Out[38]: my_mean(data::**Vector{T}**) *where T* in Main at In[33]:1

A glimpse under the hood

In [39]:
```
@code_lowered my_mean(data)
```

Out[39]:
```
CodeInfo(
1 ─ %1  = $(Expr(:static_parameter, 1))
│   %2  = Base.firstindex(data)
│   %3  = Base.getindex(data, %2)
│         mn = Main.zero(%3)
│   %5  = Main.enumerate(data)
│         @_3 = Base.iterate(%5)
│   %7  = @_3 === nothing
│   %8  = Base.not_int(%7)
└──       goto #4 if not %8
2 ┄ %10 = @_3
│   %11 = Core.getfield(%10, 1)
│   %12 = Base.indexed_iterate(%11, 1)
│         i = Core.getfield(%12, 1)
│         @_5 = Core.getfield(%12, 2)
│   %15 = Base.indexed_iterate(%11, 2, @_5)
│         d = Core.getfield(%15, 1)
│   %17 = Core.getfield(%10, 2)
│   %18 = 1 / i
│   %19 = %18 * d
│   %20 = i - 1
│   %21 = %20 / i
│   %22 = %21 * mn
│         mn = %19 + %22
│         @_3 = Base.iterate(%5, %17)
│   %25 = @_3 === nothing
│   %26 = Base.not_int(%25)
└──       goto #4 if not %26
3 ─       goto #2
4 ┄ %29 = Base.convert(%1, mn)
│   %30 = Core.typeassert(%29, %1)
└──       return %30
)
```

In [40]:
```
@code_llvm my_mean(data)
```

```
;  @ In[33]:1 within `my_mean'
define nonnull {}* @japi1_my_mean_4985({}* %0, {}** %1, i32 %2) #0 {
top:
  %3 = alloca [2 x {}*], align 8
  %gcframe70 = alloca [4 x {}*], align 16
  %gcframe70.sub = getelementptr inbounds [4 x {}*], [4 x {}*]* %gcfram
  %.sub = getelementptr inbounds [2 x {}*], [2 x {}*]* %3, i64 0, i64 0
  %4 = bitcast [4 x {}*]* %gcframe70 to i8*
  call void @llvm.memset.p0i8.i32(i8* nonnull align 16 dereferenceable(
  %5 = alloca {}**, align 8
  store volatile {}** %1, {}*** %5, align 8
  %6 = call {}*** inttoptr (i64 4480861040 to {}*** ()*)() #5
  %7 = bitcast [4 x {}*]* %gcframe70 to i64*
  store i64 8, i64* %7, align 16
  %8 = bitcast {}*** %6 to i64*
  %9 = load i64, i64* %8, align 8
  %10 = getelementptr inbounds [4 x {}*], [4 x {}*]* %gcframe70, i64 0,
  %11 = bitcast {}** %10 to i64*
  store i64 %9, i64* %11, align 8
  %12 = bitcast {}*** %6 to {}***
  store {}** %gcframe70.sub, {}*** %12, align 8
  %13 = load {}*, {}** %1, align 8
;  @ In[33]:2 within `my_mean'
; ┌ @ array.jl:801 within `getindex'
   %14 = bitcast {}* %13 to { i8*, i64, i16, i16, i32 }*
   %15 = getelementptr inbounds { i8*, i64, i16, i16, i32 }, { i8*, i64
   %16 = load i64, i64* %15, align 8
   %.not = icmp eq i64 %16, 0
   br i1 %.not, label %oob, label %idxend

L18:                                              ; preds = %scalar.ph,
   %value_phi3 = phi i64 [ %19, %L18 ], [ %bc.resume.val, %scalar.ph ]
; └
; ┌ @ abstractarray.jl:1085 within `zero'
; │┌ @ array.jl:335 within `fill!'
; ││┌ @ array.jl:839 within `setindex!'
     %17 = add nsw i64 %value_phi3, -1
     %18 = getelementptr inbounds double, double* %58, i64 %17
     store double 0.000000e+00, double* %18, align 8
; ││└
; ││┌ @ range.jl:674 within `iterate'
```

```
; │││┌ @ promotion.jl:410 within `=='
      %.not43.not = icmp eq i64 %value_phi3, %56
; │││└
     %19 = add nuw nsw i64 %value_phi3, 1
; ││└
    br i1 %.not43.not, label %L34, label %L18

L34:                                              ; preds = %middle.blo
; └└
;  @ In[33]:3 within `my_mean'
; ┌ @ iterators.jl:158 within `iterate' @ iterators.jl:159 @ array.jl:7
; │┌ @ array.jl:197 within `length'
    %20 = load i64, i64* %15, align 8
; │└
; │┌ @ int.jl:448 within `<' @ int.jl:441
    %.not44 = icmp eq i64 %20, 0
; │└
   br i1 %.not44, label %L106, label %L43

L43:                                              ; preds = %L34
; │┌ @ array.jl:801 within `getindex'
    %21 = load {}**, {}*** %43, align 8
    %22 = load atomic {}*, {}** %21 unordered, align 8
    %.not45 = icmp eq {}* %22, null
    br i1 %.not45, label %fail8, label %L63

L63:                                              ; preds = %39, %L43
    %value_phi19 = phi i64 [ %value_phi21, %39 ], [ 1, %L43 ]
    %value_phi20 = phi {}* [ %38, %39 ], [ %22, %L43 ]
    %value_phi21 = phi i64 [ %40, %39 ], [ 2, %L43 ]
    %value_phi23 = phi {}* [ %33, %39 ], [ %53, %L43 ]
; └└
;  @ In[33]:4 within `my_mean'
; ┌ @ int.jl:93 within `/'
; │┌ @ float.jl:206 within `float'
; ││┌ @ float.jl:191 within `AbstractFloat'
; │││┌ @ float.jl:94 within `Float64'
      %23 = sitofp i64 %value_phi19 to double
; │└└└
; │ @ int.jl:93 within `/' @ float.jl:335
   %24 = fdiv double 1.000000e+00, %23
   %25 = getelementptr inbounds [4 x {}*], [4 x {}*]* %gcframe70, i64 0
   store {}* %value_phi23, {}** %25, align 8
   %26 = getelementptr inbounds [4 x {}*], [4 x {}*]* %gcframe70, i64 0
```

```
   store {}* %value_phi20, {}** %26, align 16
; └
  %27 = call nonnull {}* @"j_*_4986"(double %24, {}* %value_phi20)
; ┌ @ int.jl:86 within `-'
   %28 = add nsw i64 %value_phi19, -1
; └
; ┌ @ int.jl:93 within `/'
; │┌ @ float.jl:206 within `float'
; ││┌ @ float.jl:191 within `AbstractFloat'
; │││┌ @ float.jl:94 within `Float64'
      %29 = sitofp i64 %28 to double
; │└└└
; │ @ int.jl:93 within `/' @ float.jl:335
   %30 = fdiv double %29, %23
   store {}* %27, {}** %26, align 16
; └
  %31 = call nonnull {}* @"j_*_4987"(double %30, {}* %value_phi23)
  store {}* %31, {}** %25, align 8
  store {}* %27, {}** %.sub, align 8
  %32 = getelementptr inbounds [2 x {}*], [2 x {}*]* %3, i64 0, i64 1
  store {}* %31, {}** %32, align 8
  %33 = call nonnull {}* @"j1_+_4988"({}* inttoptr (i64 4598481024 to {
; ┌ @ iterators.jl:159 within `iterate' @ array.jl:777
; │┌ @ int.jl:923 within `-' @ int.jl:86
    %34 = add nsw i64 %value_phi21, -1
; │└
; │┌ @ array.jl:197 within `length'
    %35 = load i64, i64* %15, align 8
; │└
; │┌ @ int.jl:448 within `<' @ int.jl:441
    %.not46 = icmp ult i64 %34, %35
; │└
   br i1 %.not46, label %L86, label %L106

L86:                                              ; preds = %L63
; │┌ @ array.jl:801 within `getindex'
    %36 = load {}**, {}*** %43, align 8
    %37 = getelementptr inbounds {}*, {}** %36, i64 %34
    %38 = load atomic {}*, {}** %37 unordered, align 8
    %.not47 = icmp eq {}* %38, null
    br i1 %.not47, label %fail24, label %39

39:                                               ; preds = %L86
    %40 = add nuw i64 %value_phi21, 1
```

```
; └┘
  br label %L63

L106:                                             ; preds = %L63, %L34
  %value_phi35 = phi {}* [ %53, %L34 ], [ %33, %L63 ]
  %41 = load i64, i64* %11, align 8
  store i64 %41, i64* %8, align 8
;  @ In[33]:6 within `my_mean'
  ret {}* %value_phi35

oob:                                              ; preds = %top
;  @ In[33]:2 within `my_mean'
; ┌ @ array.jl:801 within `getindex'
   %42 = alloca i64, align 8
   store i64 1, i64* %42, align 8
   call void @jl_bounds_error_ints({}* %13, i64* nonnull %42, i64 1)
   unreachable

idxend:                                           ; preds = %top
   %43 = bitcast {}* %13 to {}***
   %44 = load {}**, {}*** %43, align 8
   %45 = load atomic {}*, {}** %44 unordered, align 8
   %.not41 = icmp eq {}* %45, null
   br i1 %.not41, label %fail, label %pass

fail:                                             ; preds = %idxend
   call void @jl_throw({}* inttoptr (i64 4636188672 to {}*))
   unreachable

pass:                                             ; preds = %idxend
; └
; ┌ @ abstractarray.jl:1085 within `zero'
; │┌ @ array.jl:354 within `similar'
; ││┌ @ array.jl:132 within `size'
     %46 = bitcast {}* %45 to {}**
     %47 = getelementptr inbounds {}*, {}** %46, i64 3
     %48 = bitcast {}** %47 to i64*
     %49 = load i64, i64* %48, align 8
     %50 = getelementptr inbounds {}*, {}** %46, i64 4
     %51 = bitcast {}** %50 to i64*
     %52 = load i64, i64* %51, align 8
; ││└
; ││┌ @ boot.jl:450 within `Array'
     %53 = call nonnull {}* inttoptr (i64 4479306021 to {}* ({}*, i64, 
```

```
; │└└
; │ ┌ @ array.jl:334 within `fill!'
; │ │┌ @ abstractarray.jl:301 within `eachindex' @ abstractarray.jl:311
; │ ││┌ @ array.jl:197 within `length'
      %54 = bitcast {}* %53 to { i8*, i64, i16, i16, i32 }*
      %55 = getelementptr inbounds { i8*, i64, i16, i16, i32 }, { i8*, i
      %56 = load i64, i64* %55, align 8
; │ ││└└
; │ ││ ┌ @ range.jl:670 within `iterate'
; │ ││ │┌ @ range.jl:519 within `isempty'
; │ ││ ││┌ @ operators.jl:305 within `>'
; │ ││ │││┌ @ int.jl:83 within `<'
        %.not42.not = icmp eq i64 %56, 0
; │ ││ ││└└└└
    br i1 %.not42.not, label %L34, label %L18.preheader

L18.preheader:                                    ; preds = %pass
; │ ┌ @ array.jl:335 within `fill!'
; │ │┌ @ array.jl within `setindex!'
     %57 = bitcast {}* %53 to double**
     %58 = load double*, double** %57, align 8
; │ │└
    %min.iters.check = icmp ult i64 %56, 16
    br i1 %min.iters.check, label %scalar.ph, label %vector.ph

vector.ph:                                        ; preds = %L18.prehea
    %n.vec = and i64 %56, 9223372036854775792
    %ind.end = or i64 %n.vec, 1
    br label %vector.body

vector.body:                                      ; preds = %vector.bod
    %index = phi i64 [ 0, %vector.ph ], [ %index.next, %vector.body ]
; │ ┌ @ array.jl:839 within `setindex!'
     %59 = getelementptr inbounds double, double* %58, i64 %index
     %60 = bitcast double* %59 to <4 x double>*
     store <4 x double> zeroinitializer, <4 x double>* %60, align 8
     %61 = getelementptr inbounds double, double* %59, i64 4
     %62 = bitcast double* %61 to <4 x double>*
     store <4 x double> zeroinitializer, <4 x double>* %62, align 8
     %63 = getelementptr inbounds double, double* %59, i64 8
     %64 = bitcast double* %63 to <4 x double>*
     store <4 x double> zeroinitializer, <4 x double>* %64, align 8
     %65 = getelementptr inbounds double, double* %59, i64 12
```

```
     %66 = bitcast double* %65 to <4 x double>*
     store <4 x double> zeroinitializer, <4 x double>* %66, align 8
     %index.next = add i64 %index, 16
     %67 = icmp eq i64 %index.next, %n.vec
     br i1 %67, label %middle.block, label %vector.body

middle.block:                                     ; preds = %vector.body
; ||└
    %cmp.n = icmp eq i64 %56, %n.vec
    br i1 %cmp.n, label %L34, label %scalar.ph

scalar.ph:                                        ; preds = %middle.block
    %bc.resume.val = phi i64 [ %ind.end, %middle.block ], [ 1, %L18.preh
    br label %L18

fail8:                                            ; preds = %L43
; └└
;  @ In[33]:3 within `my_mean'
; ┌ @ iterators.jl:158 within `iterate' @ iterators.jl:159 @ array.jl:7
; │┌ @ array.jl:801 within `getindex'
    call void @jl_throw({}* inttoptr (i64 4636188672 to {}*))
    unreachable

fail24:                                           ; preds = %L86
; └└
;  @ In[33]:4 within `my_mean'
; ┌ @ iterators.jl:159 within `iterate' @ array.jl:777
; │┌ @ array.jl:801 within `getindex'
    call void @jl_throw({}* inttoptr (i64 4636188672 to {}*))
    unreachable
; └└
}
```

In [41]:
```
@code_native my_mean(data)
```

```
        .section        __TEXT,__text,regular,pure_instructions
; ┌ @ In[33]:1 within `my_mean'
        pushq   %rbp
        movq    %rsp, %rbp
        pushq   %r15
        pushq   %r14
        pushq   %r13
        pushq   %r12
        pushq   %rbx
        andq    $-32, %rsp
        subq    $128, %rsp
        movq    %rsp, %rbx
        movq    %rsi, %r14
        movabsq $jl_alloc_array_2d, %r12
        vxorpd  %xmm0, %xmm0, %xmm0
        vmovapd %ymm0, (%rbx)
        movq    %rsi, 96(%rbx)
        leaq    1555019(%r12), %rax
        vzeroupper
        callq   *%rax
        movq    %rax, %r13
        movq    $8, (%rbx)
        movq    (%rax), %rax
        movq    %rax, 8(%rbx)
        movq    %rbx, %rax
        movq    %rax, (%r13)
        movq    (%r14), %r14
; │ @ In[33]:2 within `my_mean'
; │┌ @ array.jl:801 within `getindex'
        cmpq    $0, 8(%r14)
        je      L527
        movq    (%r14), %rax
        movq    (%rax), %rax
        testq   %rax, %rax
        je      L565
; │└
; │┌ @ abstractarray.jl:1085 within `zero'
; ││┌ @ array.jl:354 within `similar'
; │││┌ @ array.jl:132 within `size'
        movq    24(%rax), %rsi
        movq    32(%rax), %rdx
```

```
        movabsq $jl_system_image_data, %rdi
; ┌┌┌└
; │└└└ @ boot.jl:450 within `Array'
        callq   *%r12
        movq    %rax, %r12
; ┌└└└
; │┌┌ @ array.jl:334 within `fill!'
; ┌└└└ @ abstractarray.jl:301 within `eachindex' @ abstractarray.jl:311
; ┌└└└ @ array.jl:197 within `length'
        movq    8(%rax), %rax
; ┌└└└
; │┌┌ @ range.jl:670 within `iterate'
; ┌└└└ @ range.jl:519 within `isempty'
; ┌└└└ @ operators.jl:305 within `>'
; ┌└└└ @ int.jl:83 within `<'
        testq   %rax, %rax
; ┌└└└
        je      L256
; ┌└└
; │┌┌ @ array.jl:335 within `fill!'
; │└└ @ array.jl within `setindex!'
        movq    (%r12), %rcx
        movl    $1, %edx
; ┌└└
        cmpq    $16, %rax
        jb      L229
        movq    %rax, %rsi
        andq    $-16, %rsi
        leaq    1(%rsi), %rdx
        xorl    %edi, %edi
        vxorpd  %xmm0, %xmm0, %xmm0
        nopl    (%rax,%rax)
; ┌└└ @ array.jl:839 within `setindex!'
L192:
        vmovupd %ymm0, (%rcx,%rdi,8)
        vmovupd %ymm0, 32(%rcx,%rdi,8)
        vmovupd %ymm0, 64(%rcx,%rdi,8)
        vmovupd %ymm0, 96(%rcx,%rdi,8)
        addq    $16, %rdi
        cmpq    %rdi, %rsi
        jne     L192
; ┌└└
        cmpq    %rsi, %rax
        je      L256
```

```
L229:
        decq    %rdx
        nopl    (%rax,%rax)
; ┌┌┌ @ array.jl:839 within `setindex!'
L240:
        movq    $0, (%rcx,%rdx,8)
; ┌┌└
; ┌┌┌ @ range.jl:674 within `iterate'
; ┌┌┌┌ @ promotion.jl:410 within `=='
        incq    %rdx
        cmpq    %rdx, %rax
; └└└└
        jne     L240
; ┌└└
;  @ In[33]:3 within `my_mean'
; ┌┌ @ iterators.jl:158 within `iterate' @ iterators.jl:159 @ array.jl:
; ┌┌┌ @ int.jl:448 within `<' @ int.jl:441
L256:
        cmpq    $0, 8(%r14)
; ┌└
        je      L476
; ┌┌ @ iterators.jl:158 within `iterate' @ iterators.jl:159 @ array.jl:
        movq    %r13, 48(%rbx)
        movq    %r14, 56(%rbx)
; ┌┌ @ iterators.jl:158 within `iterate' @ iterators.jl:159 @ array.jl:
; ┌┌┌ @ array.jl:801 within `getindex'
        movq    (%r14), %rax
        movq    (%rax), %rdi
        testq   %rdi, %rdi
        je      L587
; ┌└└
;  @ array.jl within `my_mean'
        movl    $1, %r14d
        movabsq $5909782808, %rax               ## imm = 0x160402118
        vmovsd  (%rax), %xmm0                   ## xmm0 = mem[0],zero
        vmovsd  %xmm0, 64(%rbx)
        movabsq $"*", %r15
        nopw    %cs:(%rax,%rax)
;  @ In[33]:4 within `my_mean'
; ┌ @ int.jl:93 within `/'
; ┌┌ @ float.jl:206 within `float'
; ┌┌┌ @ float.jl:191 within `AbstractFloat'
; ┌┌┌┌ @ float.jl:94 within `Float64'
L336:
```

```
        vcvtsi2sd       %r14, %xmm2, %xmm1
        vmovsd  %xmm1, 72(%rbx)
        vmovsd  64(%rbx), %xmm0                 ## xmm0 = mem[0],zero
; ||└└└
; ||  @ int.jl:93 within `/' @ float.jl:335
        vdivsd  %xmm1, %xmm0, %xmm0
        movq    %r12, 24(%rbx)
        movq    %rdi, 16(%rbx)
; |└
        vzeroupper
        callq   *%r15
        movq    %rax, %r13
; |┌ @ int.jl:93 within `/'
; ||┌ @ float.jl:206 within `float'
; |||┌ @ float.jl:191 within `AbstractFloat'
; ||||┌ @ float.jl:94 within `Float64'
        leaq    -1(%r14), %rax
        vcvtsi2sd       %rax, %xmm2, %xmm0
; ||└└└
; ||  @ int.jl:93 within `/' @ float.jl:335
        vdivsd  72(%rbx), %xmm0, %xmm0
        movq    %r13, 16(%rbx)
; |└
        movq    %r12, %rdi
        callq   *%r15
        movq    %rax, 24(%rbx)
        movq    %r13, 80(%rbx)
        movq    %rax, 88(%rbx)
        movabsq $jl_system_image_data, %rdi
        leaq    80(%rbx), %rsi
        movl    $2, %edx
        movabsq $"+", %rax
        callq   *%rax
        movq    %rax, %r12
        movq    56(%rbx), %rax
; |┌ @ iterators.jl:159 within `iterate' @ array.jl:777
; ||┌ @ int.jl:448 within `<' @ int.jl:441
        cmpq    8(%rax), %r14
; ||└
        jae     L472
; ||┌ @ array.jl:801 within `getindex'
        movq    (%rax), %rax
        movq    (%rax,%r14,8), %rdi
        testq   %rdi, %rdi
```

```
        je      L505
; ┌└
        incq    %r14
        jmp     L336
; │ @ In[33] within `my_mean'
L472:
        movq    48(%rbx), %r13
L476:
        movq    8(%rbx), %rax
        movq    %rax, (%r13)
; │ @ In[33]:6 within `my_mean'
        movq    %r12, %rax
        leaq    -40(%rbp), %rsp
        popq    %rbx
        popq    %r12
        popq    %r13
        popq    %r14
        popq    %r15
        popq    %rbp
        vzeroupper
        retq
; │ @ In[33]:4 within `my_mean'
; │┌ @ iterators.jl:159 within `iterate' @ array.jl:777
; ││┌ @ array.jl:801 within `getindex'
L505:
        movabsq $jl_throw, %rax
        movabsq $jl_system_image_data, %rdi
        callq   *%rax
; │└└
; │ @ In[33]:2 within `my_mean'
; │┌ @ array.jl:801 within `getindex'
L527:
        movq    %rsp, %rax
        leaq    -16(%rax), %rsi
        movq    %rsi, %rsp
        movq    $1, -16(%rax)
        movabsq $jl_bounds_error_ints, %rax
        movl    $1, %edx
        movq    %r14, %rdi
        callq   *%rax
L565:
        movabsq $jl_throw, %rax
        movabsq $jl_system_image_data, %rdi
        callq   *%rax
```

```
; │└
; │ @ In[33]:3 within `my_mean'
; │┌ @ iterators.jl:158 within `iterate' @ iterators.jl:159 @ array.jl:
; ││┌ @ array.jl:801 within `getindex'
L587:
        movabsq $jl_throw, %rax
        movabsq $jl_system_image_data, %rdi
        vzeroupper
        callq   *%rax
; └└└
```

An array of arrays

In [42]:
```
fix_seed!()
data = [rand(Normal(μ,1),5) for μ in 1:5] #\mu + [TAB]
```

Out[42]:
```
5-element Vector{Vector{Float64}}:
 [1.6791074260357777, 1.8284134829000358, 0.6469925996994037, 0.865146
 [2.297335850849416, 2.0649475485483424, 1.8909826149182825, 1.485789
 [2.311092872174302, 2.2371961835895418, 3.3974824092181635, 3.811629
 [3.81242731805484, 2.3927436758722247, 1.5192072693400598, 6.276232
 [4.882862469267065, 4.398746405814617, 6.14227643894132, 4.911383681
```

In [43]:
```
mean.(data)
```

Out[43]:
```
5-element Vector{Float64}:
 1.321255342267561
 2.262677165123942
 2.8822092966420163
 3.644060912718838
 5.122947049582486
```

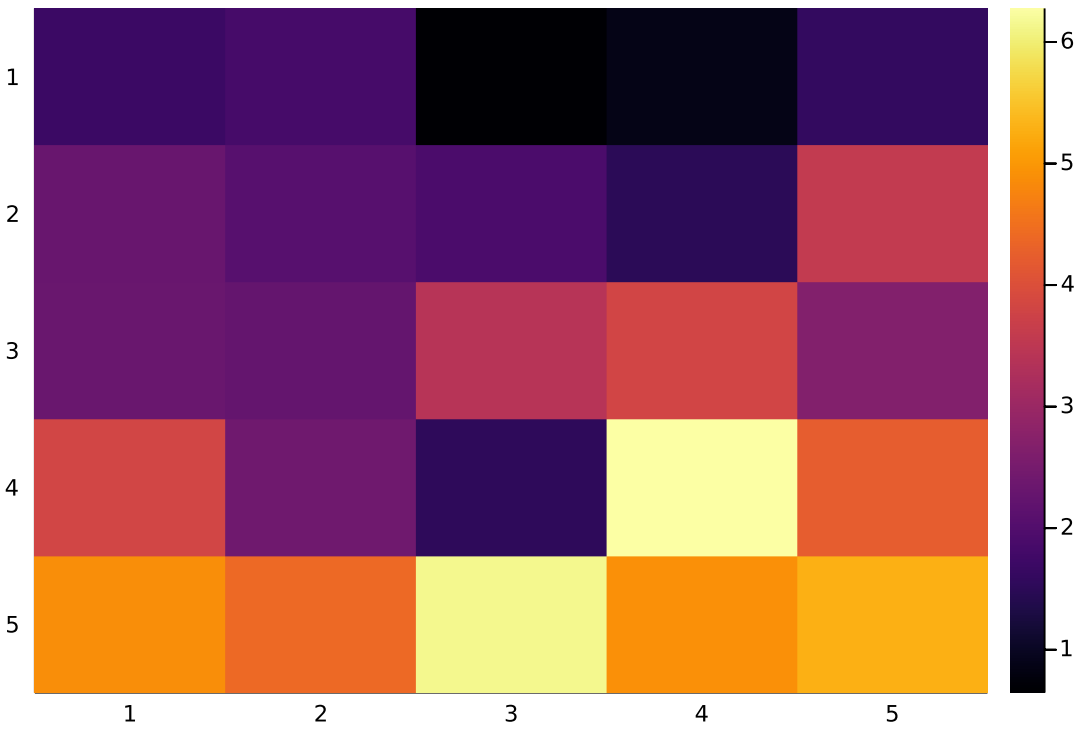
A matrix of data

In [44]:
```
data = hcat(data...)
```

Out[44]:
```
5×5 Matrix{Float64}:
 1.67911   2.29734  2.31109  3.81243  4.88286
 1.82841   2.06495  2.2372   2.39274  4.39875
 0.646993  1.89098  3.39748  1.51921  6.14228
 0.865146  1.48579  3.81163  6.27623  4.91138
 1.58662   3.57433  2.65365  4.21969  5.27947
```

In [45]:
```
heatmap(data',yflip=true)
```

Out[45]:



In [46]:
```
mean.(data) #does nothing useful
```

Out[46]:
```
5×5 Matrix{Float64}:
 1.67911   2.29734  2.31109  3.81243  4.88286
 1.82841   2.06495  2.2372   2.39274  4.39875
 0.646993  1.89098  3.39748  1.51921  6.14228
 0.865146  1.48579  3.81163  6.27623  4.91138
 1.58662   3.57433  2.65365  4.21969  5.27947
```

In [47]:
```
mean(data,dims=1) #better!
```

Out[47]:
```
1×5 Matrix{Float64}:
 1.32126  2.26268  2.88221  3.64406  5.12295
```

Dictionaries

In [48]:
```
data = Dict()
data[:cats] = rand(Normal(2.3,1),5)
data["dogs"] = rand(Normal(5.2,1),5)
data[25] = rand(Normal(7.8,1),5)
data
```

Out[48]:
```
Dict{Any, Any} with 3 entries:
  "dogs" => [6.62305, 5.60839, 5.78862, 4.90372, 5.89111]
  25     => [8.30687, 7.74307, 6.02898, 9.39062, 9.19706]
  :cats  => [2.41142, 1.94212, 2.77371, 2.60023, 1.53732]
```

In [49]:
```
keys(data)
```

Out[49]:
```
KeySet for a Dict{Any, Any} with 3 entries. Keys:
  "dogs"
  25
  :cats
```

In [50]:
```
mean(data[:cats])
```

Out[50]:
```
2.2529618709854136
```

In [51]:
```
mean(data["dogs"])
```

Out[51]:
```
5.762977963045932
```

In [52]:
```
mean(data["cats"])
```

```
KeyError: key "cats" not found

Stacktrace:
 [1] getindex(h::Dict{Any, Any}, key::String)
   @ Base ./dict.jl:482
 [2] top-level scope
   @ In[52]:1
 [3] eval
   @ ./boot.jl:360 [inlined]
 [4] include_string(mapexpr::typeof(REPL.softscope), mod::Module, code:
   @ Base ./loading.jl:1094
```

In [53]:
```
mean(data[Symbol("cats")])
```

Out[53]:
```
2.2529618709854136
```

In [54]:
```
data[Symbol("cats")] |> mean
```

Out[54]:
```
2.2529618709854136
```

Tuples

In [55]:
```
fix_seed!()
rn() = rand(Normal())
data = (rn(), rn(),rn())
```

Out[55]:
```
(0.6791074260357777, 0.8284134829000359, -0.3530074003005963)
```

In [56]:
```
typeof(data)
```

Out[56]:
```
Tuple{Float64, Float64, Float64}
```

In [57]:
```
mean(data)
```

Out[57]:
```
0.3848378362117391
```

In [58]:
```
data[2]
```

Out[58]:
```
0.8284134829000359
```

In [59]:
```
data[2] = 7.8 #Tuples are immutable!
```

```
MethodError: no method matching setindex!(::Tuple{Float64, Float64, Flo

Stacktrace:
 [1] top-level scope
   @ In[59]:1
 [2] eval
   @ ./boot.jl:360 [inlined]
 [3] include_string(mapexpr::typeof(REPL.softscope), mod::Module, code:
   @ Base ./loading.jl:1094
```

Structs

In [60]:
```
struct Person
    weight::Float64
    height::Float64
end
```

In [61]:
```
someone = Person(102, 180)
```

Out[61]:
```
Person(102.0, 180.0)
```

In [62]:
```
someone.weight = 103 #Structs are immutable unless you state immutable
```

```
setfield! immutable struct of type Person cannot be changed

Stacktrace:
 [1] setproperty!(x::Person, f::Symbol, v::Int64)
   @ Base ./Base.jl:34
 [2] top-level scope
   @ In[62]:1
 [3] eval
   @ ./boot.jl:360 [inlined]
 [4] include_string(mapexpr::typeof(REPL.softscope), mod::Module, code:
   @ Base ./loading.jl:1094
```

In [63]:
```
me = Person(102, 180)
you = Person(83, 170)
me + you #can't add persons just like that
```

```
MethodError: no method matching +(::Person, ::Person)
Closest candidates are:
  +(::Any, ::Any, ::Any, ::Any...) at operators.jl:560
  +(::T, ::Any) where T<:Intervals.Interval at /Users/uqjnazar/.julia/p
  +(::ChainRulesCore.Tangent{P, T} where T, ::P) where P at /Users/uqjn
  ...

Stacktrace:
 [1] top-level scope
   @ In[63]:3
 [2] eval
   @ ./boot.jl:360 [inlined]
 [3] include_string(mapexpr::typeof(REPL.softscope), mod::Module, code:
   @ Base ./loading.jl:1094
```

In [64]:
```
import Base: +

function +(x::Person, y::Person)
    Person(x.weight + y.weight, x.height + y.height)
end
```

Out[64]:
```
+ (generic function with 472 methods)
```

In [65]:
```
me + you #Now you can
```

Out[65]:
```
Person(185.0, 350.0)
```

In [66]:
```
import Base: /
function /(x::Person, n::Real)
    Person(x.weight/n, x.height/n)
end
```

Out[66]:
```
/ (generic function with 209 methods)
```

In [67]:
```
mean([me,you]) #With + and / (by number) we can compute the mean of pers
```

Out[67]:
```
Person(92.5, 175.0)
```

Dataframes

In [68]:
```
df = CSV.File("./data/temperatures.csv") |> DataFrame
```

Out[68]: 777 rows × 5 columns

	Year	Month	Day	Brisbane	GoldCoast
	Int64	Int64	Int64	Float64	Float64
1	2015	1	1	31.3	30.9
2	2015	1	2	30.5	30.1
3	2015	1	3	28.9	30.1
4	2015	1	4	30.2	30.1
5	2015	1	5	28.1	28.0
6	2015	1	6	29.5	29.3
7	2015	1	7	27.1	26.4
8	2015	1	8	28.4	27.7
9	2015	1	9	29.5	29.6
10	2015	1	10	30.2	29.5
11	2015	1	11	28.2	28.0
12	2015	1	12	30.4	29.1
13	2015	1	13	28.4	27.2
14	2015	1	14	30.3	29.4
15	2015	1	15	36.0	36.5
16	2015	1	16	32.5	30.9
17	2015	1	17	35.3	31.6
18	2015	1	18	36.7	32.2
19	2015	1	19	31.8	30.8
20	2015	1	20	29.2	27.9
21	2015	1	21	30.0	30.2
22	2015	1	22	28.6	28.9
23	2015	1	23	24.8	25.7

	Year	Month	Day	Brisbane	GoldCoast
	Int64	Int64	Int64	Float64	Float64
24	2015	1	24	30.8	29.4
25	2015	1	25	36.7	33.4
26	2015	1	26	33.1	30.8
27	2015	1	27	30.2	30.3
28	2015	1	28	28.1	27.9
29	2015	1	29	28.9	29.4
30	2015	1	30	29.0	28.8
⋮	⋮	⋮	⋮	⋮	⋮

In [69]:
```
data = df.GoldCoast
mean(data)
```

Out[69]:
```
26.163835263835264
```

All kind of other descriptive statistics

In [70]:
```
println("Sample Mean: ", mean(data))
println("Harmonic ≤ Geometric ≤ Arithmetic ",      #\le + [TAB]
            (harmmean(data), geomean(data), mean(data)))

println("Sample Variance: ",var(data))
println("Sample Standard Deviation: ",std(data))
println("Minimum: ", minimum(data))
println("Maximum: ", maximum(data))
println("Median: ", median(data))
println("95th percentile: ", percentile(data, 95))
println("0.95 quantile: ", quantile(data, 0.95))
println("Interquartile range: ", iqr(data),"\n")

summarystats(data)
```

```
Sample Mean: 26.163835263835264
Harmonic ≤ Geometric ≤ Arithmetic (25.65423319190783, 25.91556340978906
Sample Variance: 12.367253570765161
Sample Standard Deviation: 3.516710618001595
Minimum: 15.4
Maximum: 36.5
Median: 26.8
95th percentile: 30.9
0.95 quantile: 30.9
Interquartile range: 5.5
```

Out[70]:
```
Summary Stats:
Length:         777
Missing Count:  0
Mean:           26.163835
Minimum:        15.400000
1st Quartile:   23.400000
Median:         26.800000
3rd Quartile:   28.900000
Maximum:        36.500000
```

In [71]:
```
min(5,2,-3) #Minimum of individual function arguments
```

Out[71]:
```
-3
```

In [72]:
```
minimum([5,2,-3]) #Minimum of an array (AbstractArray)
```

Out[72]:
```
-3
```

In [73]:
```
min([5,2,-3]) #doesn't work
```

```
MethodError: no method matching min(::Vector{Int64})
Closest candidates are:
  min(::Any, ::Missing) at missing.jl:127
  min(::T1, ::DataValues.DataValue{T2}) where {T1, T2} at /Users/uqjnaza
  min(::Any, ::Any) at operators.jl:433
  ...

Stacktrace:
 [1] top-level scope
   @ In[73]:1
 [2] eval
   @ ./boot.jl:360 [inlined]
 [3] include_string(mapexpr::typeof(REPL.softscope), mod::Module, code:
   @ Base ./loading.jl:1094
```

In [74]:
```
min([5,2,-3]...) #The splat operator passes the arguments
```

Out[74]:
```
-3
```

In [75]:
```
fix_seed!()
data = rand(Normal(),10^6);
```

In [76]:
```
@time begin
    minimum(data)
end
```

```
  0.000859 seconds (1 allocation: 16 bytes)
```

Out[76]:
```
-4.846415885152558
```

In [77]:
```
@time begin
    min(data...)
end
```

```
  0.491609 seconds (2.10 M allocations: 82.025 MiB, 11.12% gc time, 8.9
```

Out[77]:
```
-4.846415885152558
```

Something `rand`.

home

The Monty Hall Problem

In [78]:
```
fix_seed!()


function montyHall(switch_policy)
    prize, choice = rand(1:3), rand(1:3)
    if prize == choice
        revealed = rand(setdiff(1:3,choice))
    else
        revealed = rand(setdiff(1:3,[prize,choice]))
    end

    if switch_policy
        choice = setdiff(1:3,[revealed,choice])[1]
    end
    return choice == prize
end

N = 10^6
println("Success probability with policy I (stay): ",  sum([montyHall(fa
println("Success probability with policy II (switch): ", sum([montyHall(
```

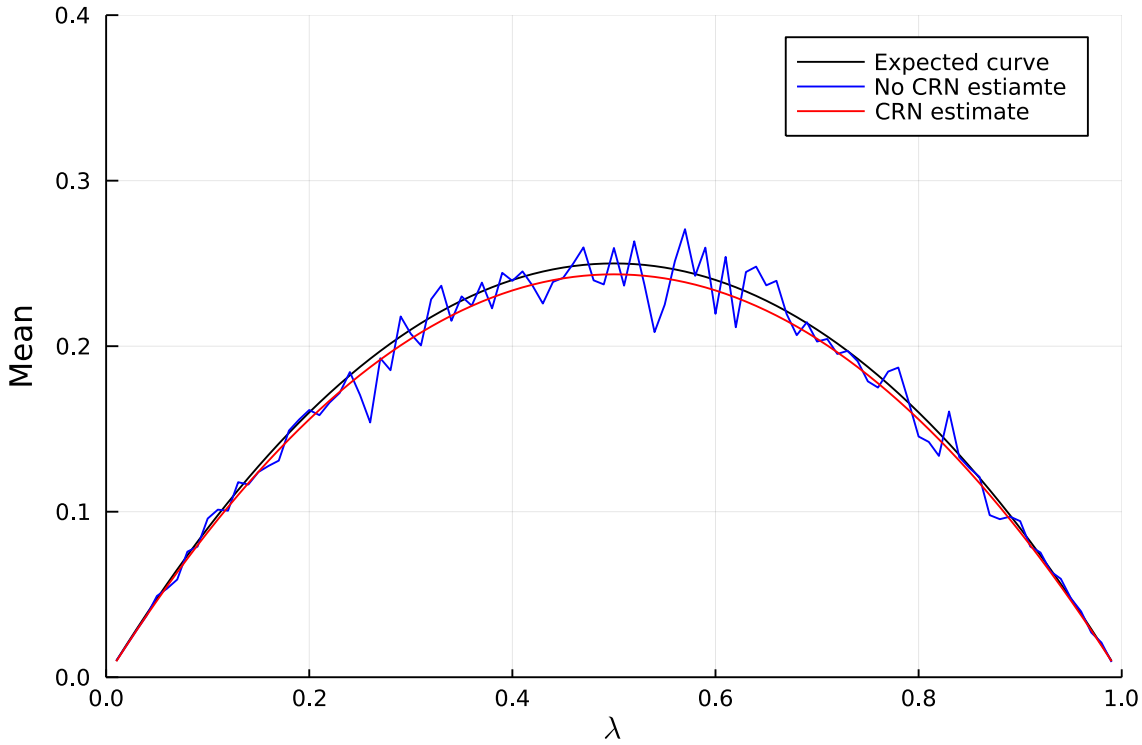
```
Success probability with policy I (stay): 0.334146
Success probability with policy II (switch): 0.666056
```

Common random numbers

[Math Processing Error][Math Processing Error]

In [79]:
```
seed = 1
N = 100
λ_grid = 0.01:0.01:0.99

theorM(λ) = mean(Uniform(0,2λ*(1-λ)))
estM(λ) = mean(rand(Uniform(0,2λ*(1-λ)),N))

function estM(λ, seed)
    Random.seed!(seed)
    estM(λ)
end

trueM = theorM.(λ_grid)
estM0 = estM.(λ_grid)
estMCRN = estM.(λ_grid,seed)

plot(λ_grid, trueM, c=:black, label="Expected curve")
plot!(λ_grid, estM0, c=:blue, label="No CRN estiamte")
plot!(λ_grid, estMCRN, c=:red, label="CRN estimate",
    xlims=(0,1), ylims=(0,0.4), xlabel=L"\lambda", ylabel = "Mean")
```

Out[79]:



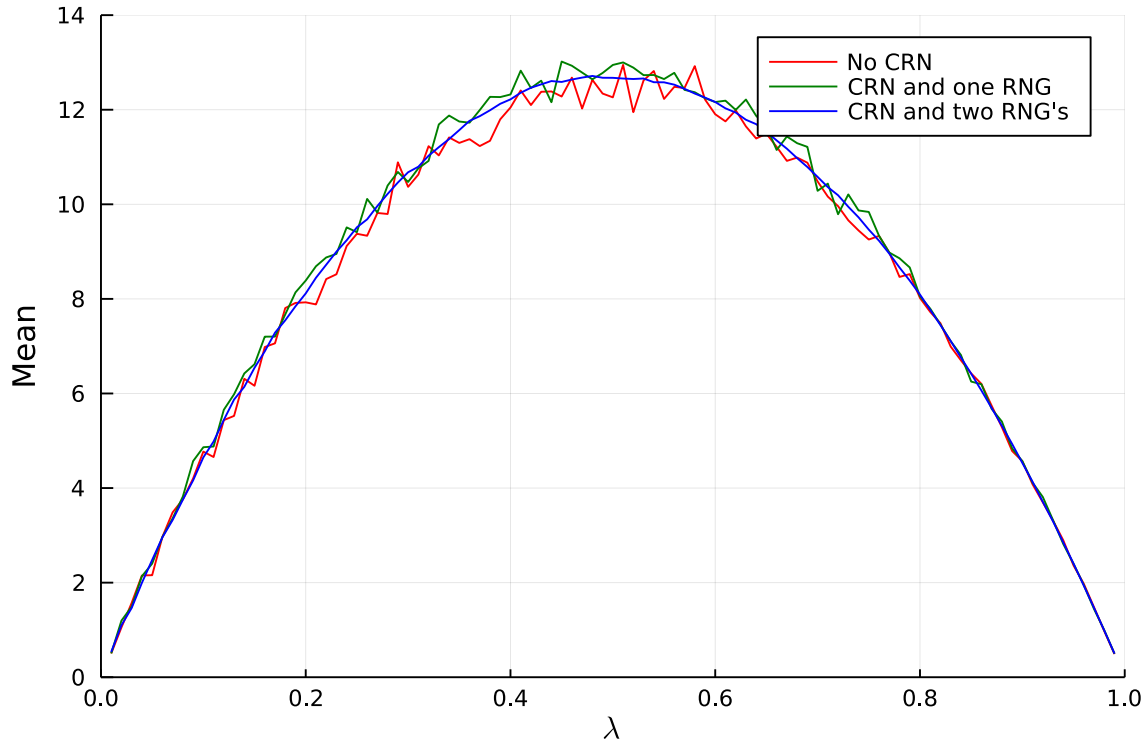
Multiple RNGs

Say we want to estimate the mean of *[Math Processing Error]* with *[Math Processing Error]* and *[Math Processing Error]* with *[Math Processing Error]* and *[Math Processing Error]*.

[Math Processing Error]

In [80]:
```
K = 50

prn(λ,rng) = quantile(Poisson(λ),rand(rng))
zDist(λ) = Uniform(0,2*(1-λ))

rv(λ,rng) = sum([rand(rng,zDist(λ)) for _ in 1:prn(K*λ, rng)])
rv2(λ,rng1,rng2) = sum([rand(rng1,zDist(λ)) for _ in 1:prn(K*λ,rng2)])

mEst(λ,rng) = mean([rv(λ,rng) for _ in 1:N])
mEst2(λ,rng1,rng2) = mean([rv2(λ,rng1,rng2) for _ in 1:N])

#No Common Random Numbers (CRN)
function mGraph0(seed)
    singleRng = MersenneTwister(seed)
    [mEst(λ,singleRng) for λ in λ_grid]
end

#CRN but single random number generator (RNG)
mGraph1(seed) = [mEst(λ,MersenneTwister(seed)) for λ in λ_grid]

#CRN with two random number generators
mGraph2(seed1,seed2) = [mEst2(λ,MersenneTwister(seed1), MersenneTwister(

plot(λ_grid,mGraph0(1987), c=:red, label="No CRN")
plot!(λ_grid,mGraph1(1987), c=:green, label="CRN and one RNG")
plot!(λ_grid,mGraph2(1987,1988), c=:blue, label="CRN and two RNG's", xli
    xlabel=L"\lambda", ylabel = "Mean")
```

Out[80]:



In [81]:
```
argMaxλ(graph) = λ_grid[findmax(graph)[2]]

M = 10^3
std0 = std([argMaxλ(mGraph0(seed)) for seed in 1:M])
std1 = std([argMaxλ(mGraph1(seed)) for seed in 1:M])
std2 = std([argMaxλ(mGraph2(seed,seed+M)) for seed in 1:M])

println("Standard deviation with no CRN: ", std0)
println("Standard deviation with CRN and single RNG: ", std1)
println("Standard deviation with CRN and two RNGs: ", std2)
```

```
Standard deviation with no CRN: 0.03708052002015299
Standard deviation with CRN and single RNG: 0.03411444555309959
Standard deviation with CRN and two RNGs: 0.014645353747396703
```

Do you still miss R? So Just `RCall`.

home

See the docs for RCall.jl (https://juliainterop.github.io/RCall.jl/stable/)

In [82]:
```
using RCall

data1 = DataFrame(CSV.File("./data/machine1.csv", header=false))[:,1]
data2 = DataFrame(CSV.File("./data/machine2.csv", header=false))[:,1]
data3 = DataFrame(CSV.File("./data/machine3.csv", header=false))[:,1]

function R_ANOVA(allData)
    data = vcat([ [x fill(i, length(x))] for (i, x) in enumerate(allDat
    df = DataFrame(data, [:Diameter, :MachNo])
    @rput df

    R"""
    df$MachNo <- as.factor(df$MachNo)
    anova <- summary(aov( Diameter ~ MachNo, data=df))
    fVal <- anova[[1]]["F value"][[1]][1]
    pVal <- anova[[1]]["Pr(>F)"][[1]][1]
    """
    println("R ANOVA f-value: ", @rget fVal)
    println("R ANOVA p-value: ", @rget pVal)
end

R_ANOVA([data1, data2, data3])
```

```
R ANOVA f-value: 10.516968568709089
R ANOVA p-value: 0.00014236168817139574
```

To use Julia from R, use JuliaCall (https://cran.r-project.org/web/packages/JuliaCall/index.html)

Some `Plots.`

home

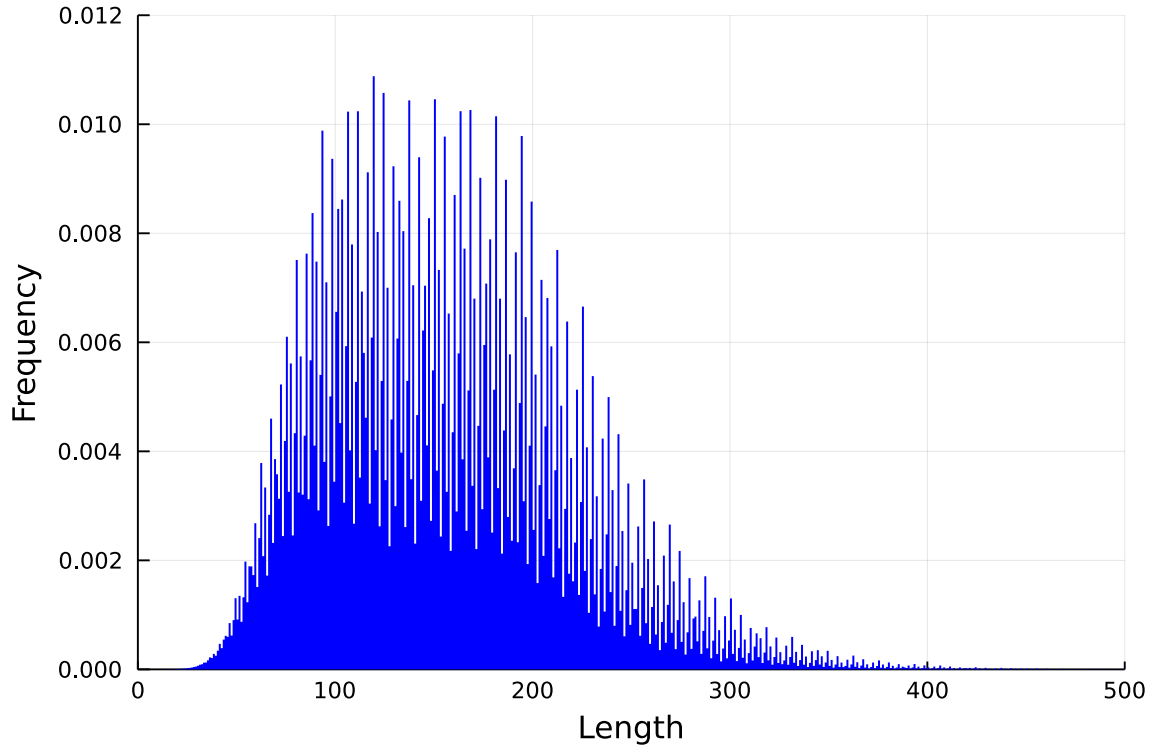
See example image gallery (https://statisticswithjulia.org/gallery.html) with code - part of "Statistics with Julia" book.

See docs for Plots.jl (http://docs.juliaplots.org/latest//) and StatsPlots.jl (https://github.com/JuliaPlots/StatsPlots.jl)

In [83]:

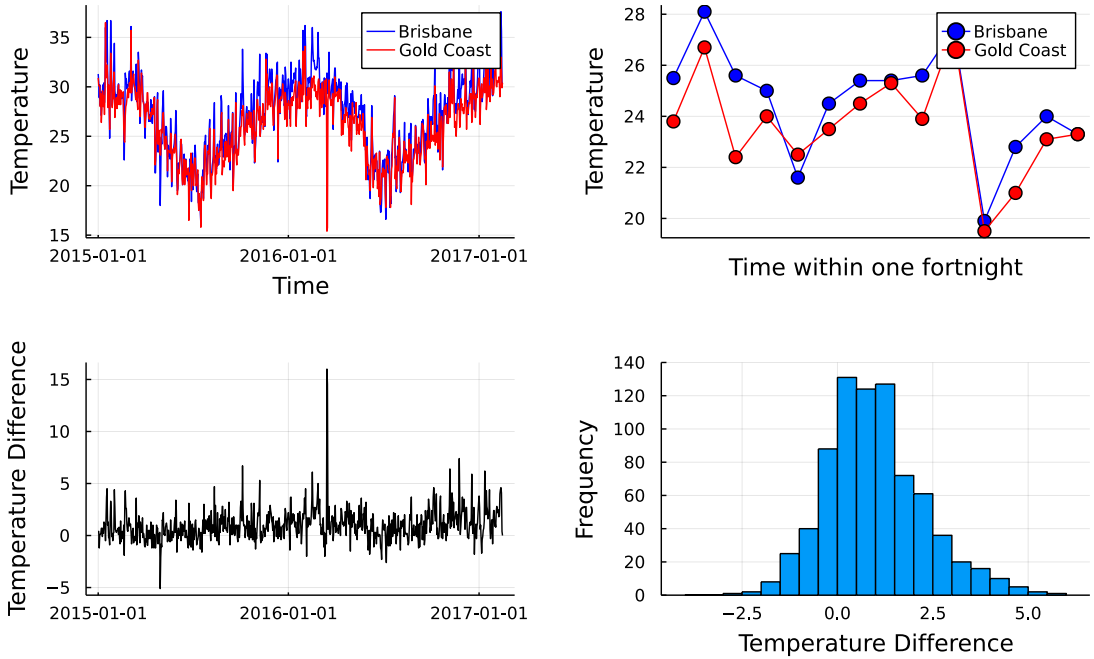
```
function hailLength(x::Int)
    n = 0
    while x != 1
        if x % 2 == 0
            x = x ÷ 2 #\div + [TAB]
        else
            x = 3x +1
        end
        n += 1
    end
    return n
end

lengths = [hailLength(x₀) for x₀ in 2:10^7] #\_0 + [TAB]

histogram(lengths, bins=1000, normed=:true,
    fill=(:blue, true), la=0, legend=:none,
    xlims=(0, 500), ylims=(0, 0.012),
    xlabel="Length", ylabel="Frequency")
```

Out[83]:



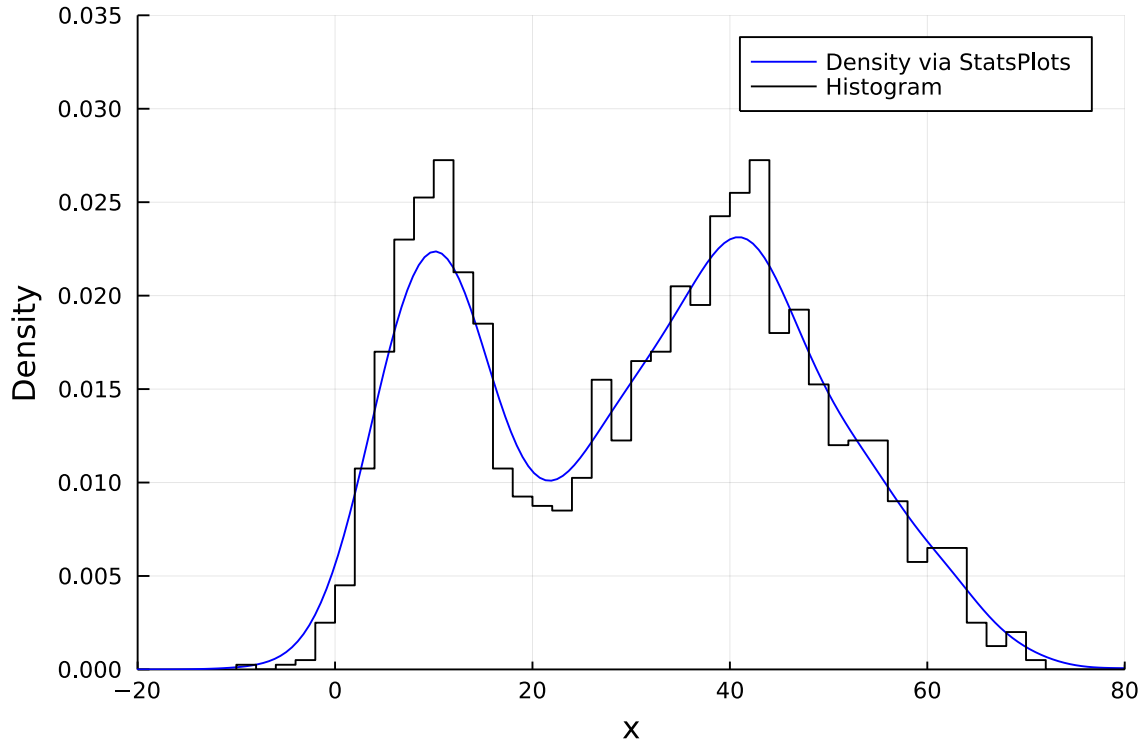
In [84]:
```
data = CSV.File("./data/temperatures.csv") |> DataFrame
brisbane = data.Brisbane
goldcoast = data.GoldCoast

diff = brisbane - goldcoast
dates = [Date(
            Year(data.Year[i]),
            Month(data.Month[i]),
            Day(data.Day[i])
        ) for i in 1:nrow(data)]

fortnightRange = 250:263
brisFortnight = brisbane[fortnightRange]
goldFortnight = goldcoast[fortnightRange]

p1 = plot(dates, [brisbane goldcoast],
        c=[:blue :red], xlabel="Time", ylabel="Temperature", label=["Bri
p2 = plot(dates[fortnightRange], [brisFortnight goldFortnight], xticks=:
        c=[:blue :red], m=(:dot, 5, Plots.stroke(1)), ylabel="Temperatur
        label=["Brisbane" "Gold Coast"], xlabel="Time within one fortnig
p3 = plot(dates, diff,
        c=:black, ylabel="Temperature Difference",legend=false)
p4 = histogram(diff, bins=-4:0.5:6,
        ylims=(0,140), legend = false,
        xlabel="Temperature Difference", ylabel="Frequency")
plot(p1,p2,p3,p4, size = (800,500), margin = 5mm)
```

Out[84]:


Temperature
Time
Brisbane
Gold Coast
2015-01-01
2016-01-01
2017-01-01
Temperature
Time within one fortnight
Brisbane
Gold Coast
Temperature Difference
Frequency
Temperature Difference

In [85]:
```
fix_seed!()

μ₁, σ₁ = 10, 5 #\mu + [TAB] \_1 + [TAB] etc...
μ₂, σ₂ = 40, 12
dist1, dist2 = Normal(μ₁,σ₁), Normal(μ₂,σ₂)
p = 0.3
mixRv() = (rand() <= p) ? rand(dist1) : rand(dist2)

n = 2000
data = [mixRv() for _ in 1:n]

density(data, c=:blue, label="Density via StatsPlots", xlims=(-20,80), y
stephist!(data, bins=50, c=:black, norm=true, label="Histogram", xlabel=
```

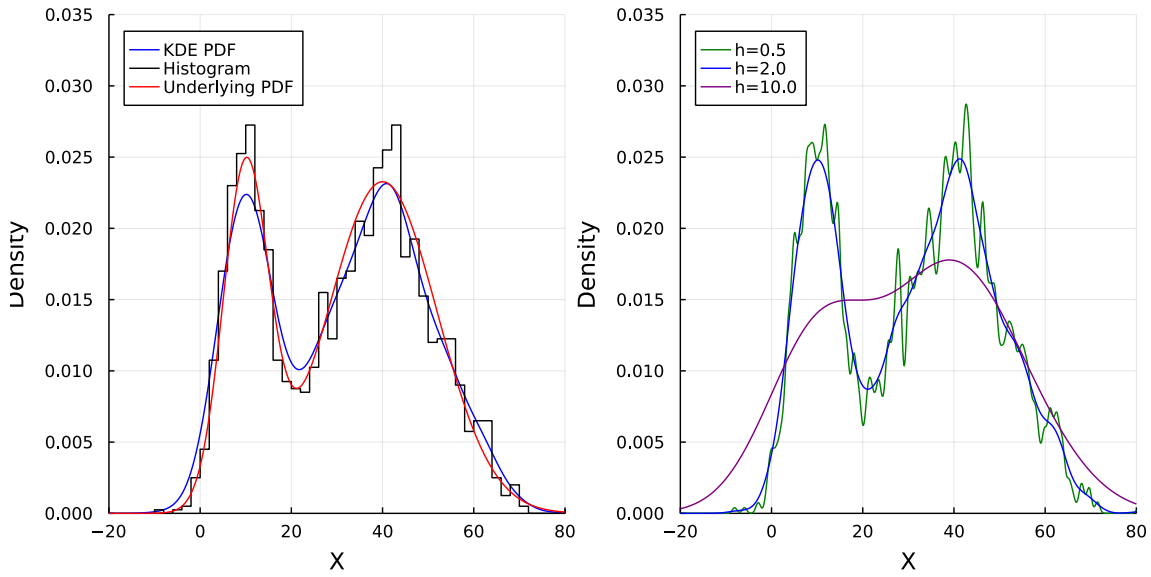
Out[85]:



In [86]:

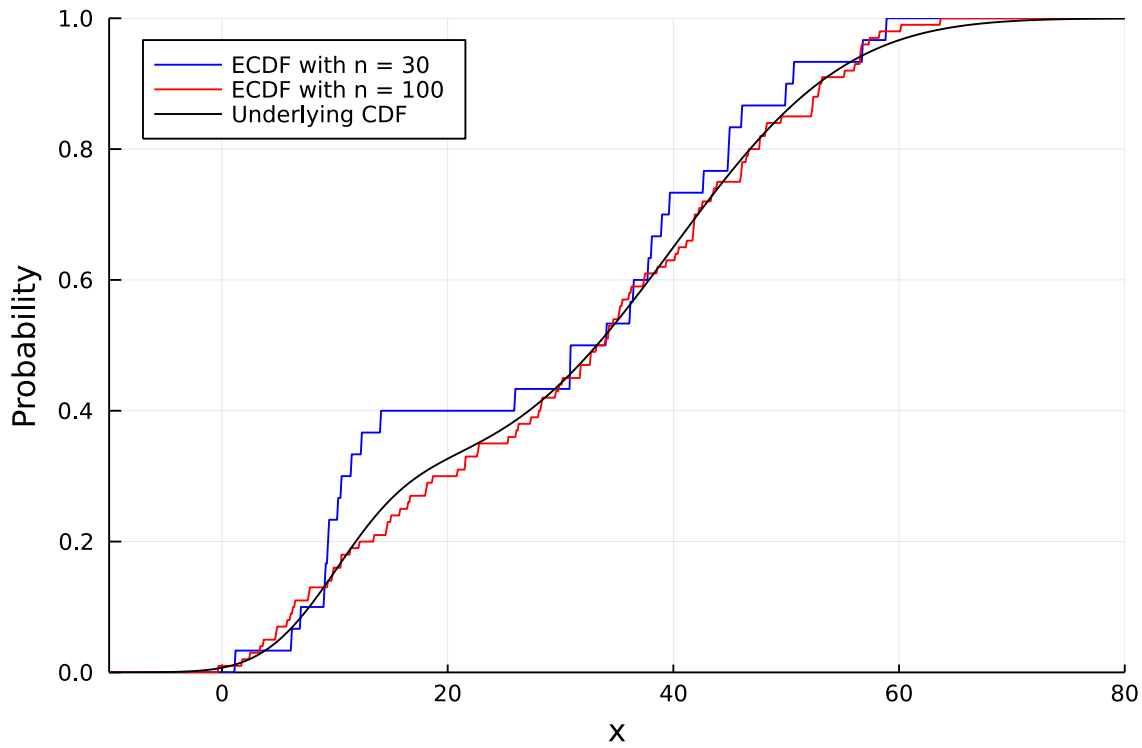
```
mixPDF(x) = p*pdf(dist1,x) + (1-p)*pdf(dist2,x)

kdeDist = kde(data)

xGrid = -20:0.1:80
pdfKDE = pdf(kdeDist,xGrid)

p1 = plot(xGrid, pdfKDE, c=:blue, label="KDE PDF")
stephist!(data, bins=50, c=:black, normed=:true, label="Histogram")
plot!(xGrid, mixPDF.(xGrid), c=:red, label="Underlying PDF",
    xlims=(-20,80), ylims=(0,0.035), legend=:topleft,
    xlabel="X", ylabel = "Density")

hVals = [0.5,2,10]
kdeS = [kde(data,bandwidth=h) for h in hVals]

p2 = plot(xGrid, pdf(kdeS[1],xGrid), c = :green, label= "h=$(hVals[1])")
plot!(xGrid, pdf(kdeS[2],xGrid), c = :blue, label= "h=$(hVals[2])")
plot!(xGrid, pdf(kdeS[3],xGrid), c = :purple, label= "h=$(hVals[3])",
    xlims=(-20,80), ylims=(0,0.035), legend=:topleft,
    xlabel="X", ylabel = "Density")

plot(p1,p2,size = (800,400))
```

Out[86]:



In [87]:
```
fix_seed!()

mixCDF(x) = p*cdf(dist1,x) + (1-p)*cdf(dist2,x)

n = [30, 100]
data1 = [mixRv() for _ in 1:n[1]]
data2 = [mixRv() for _ in 1:n[2]]

empiricalCDF1 = ecdf(data1)
empiricalCDF2 = ecdf(data2)

xGrid = -10:0.1:80
plot(xGrid,empiricalCDF1.(xGrid), c=:blue, label="ECDF with n = $(n[1])'
plot!(xGrid,empiricalCDF2.(xGrid), c=:red, label="ECDF with n = $(n[2])'
plot!(xGrid, mixCDF.(xGrid), c=:black, label="Underlying CDF",
    xlims=(-10,80), ylims=(0,1),
    xlabel="x", ylabel="Probability", legend=:topleft)
```

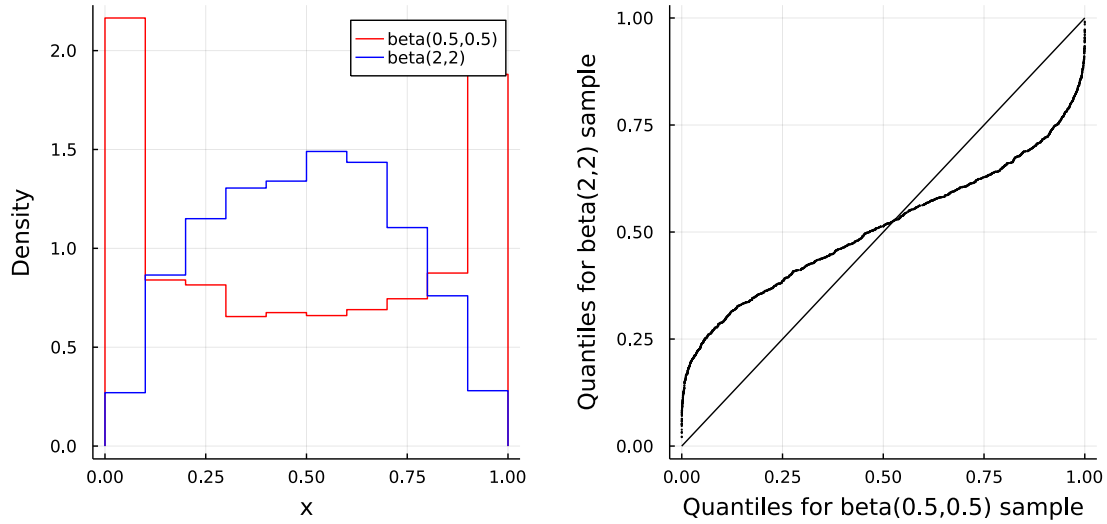
Out[87]:



In [88]:

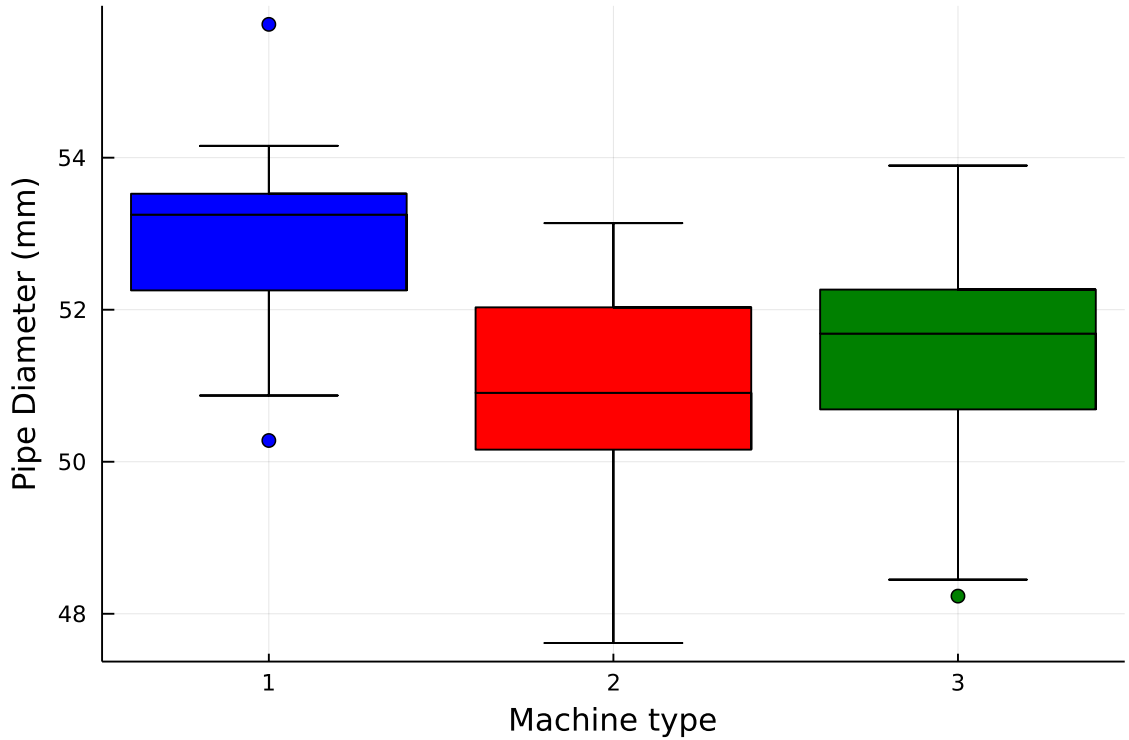
```
fix_seed!()

b1, b2 = 0.5 , 2
dist1, dist2, = Beta(b1,b1), Beta(b2,b2)

n = 2000
data1 = rand(dist1,n)
data2 = rand(dist2,n)

stephist(data1, bins=15, label = "beta($b1,$b1)", c = :red, normed = tru
p1 = stephist!(data2, bins=15, label = "beta($b2,$b2)",
        c = :blue, xlabel="x", ylabel="Density",normed = true)

p2 = qqplot(data1, data2, c=:black, ms=1, msw =0,
        xlabel="Quantiles for beta($b1,$b1) sample",
        ylabel="Quantiles for beta($b2,$b2) sample",
        legend=false)

plot(p1, p2, size=(800,400), margin = 5mm)
```

Out[88]:



In [89]:
```
data1 = parse.(Float64, readlines("./data/machine1.csv"))
data2 = parse.(Float64, readlines("./data/machine2.csv"))
data3 = parse.(Float64, readlines("./data/machine3.csv"))

boxplot([data1,data2,data3], c=[:blue :red :green], label="",
    xticks=([1:1:3;], ["1", "2", "3"]), xlabel="Machine type",
    ylabel="Pipe Diameter (mm)")
```

Out[89]:



Your favorite `Distribution`.

home

See Docs for `Distributions.jl` (https://juliastats.org/Distributions.jl/latest/)

In [90]:
```
dist = Normal(2.5,1.2)
```

Out[90]:
```
Normal{Float64}(μ=2.5, σ=1.2)
```

In [91]:
```
mean(dist), var(dist)
```

Out[91]:
```
(2.5, 1.44)
```

In [92]:
```
fix_seed!()
rand(dist,3,2,5) #a 3-tensor
```

Out[92]:
```
3×2×5 Array{Float64, 3}:
[:, :, 1] =
 3.31493  2.33818
 3.4941   3.20394
 2.07639  2.8568

[:, :, 2] =
 2.57794  4.3892
 2.36918  1.67331
 1.88295  1.58464

[:, :, 3] =
 2.97698   2.27491
 3.47396   0.571292
 2.08437  -0.476951

[:, :, 4] =
 5.23148  1.7785
 2.76363  3.87073
 2.35943  2.39366

[:, :, 5] =
 2.83536  3.06846
 2.63371  2.86028
 2.07054  1.58479
```

In [93]:
```
? NegativeBinomial
```

```
search: NegativeBinomial NegativeBinomialLink
```

Out[93]:
```
NegativeBinomial(r,p)
```

A *Negative binomial distribution* describes the number of failures before the `r` th success in a sequence of independent Bernoulli trials. It is parameterized by `r`, the number of successes, and `p`, the probability of success in an individual trial.

[Math Processing Error]

The distribution remains well-defined for any positive `r`, in which case

[Math Processing Error]

```
NegativeBinomial()        # Negative binomial distribution with r = 1 a
NegativeBinomial(r, p)    # Negative binomial distribution with r succe

params(d)       # Get the parameters, i.e. (r, p)
succprob(d)     # Get the success rate, i.e. p
failprob(d)     # Get the failure rate, i.e. 1 - p
```

External links:

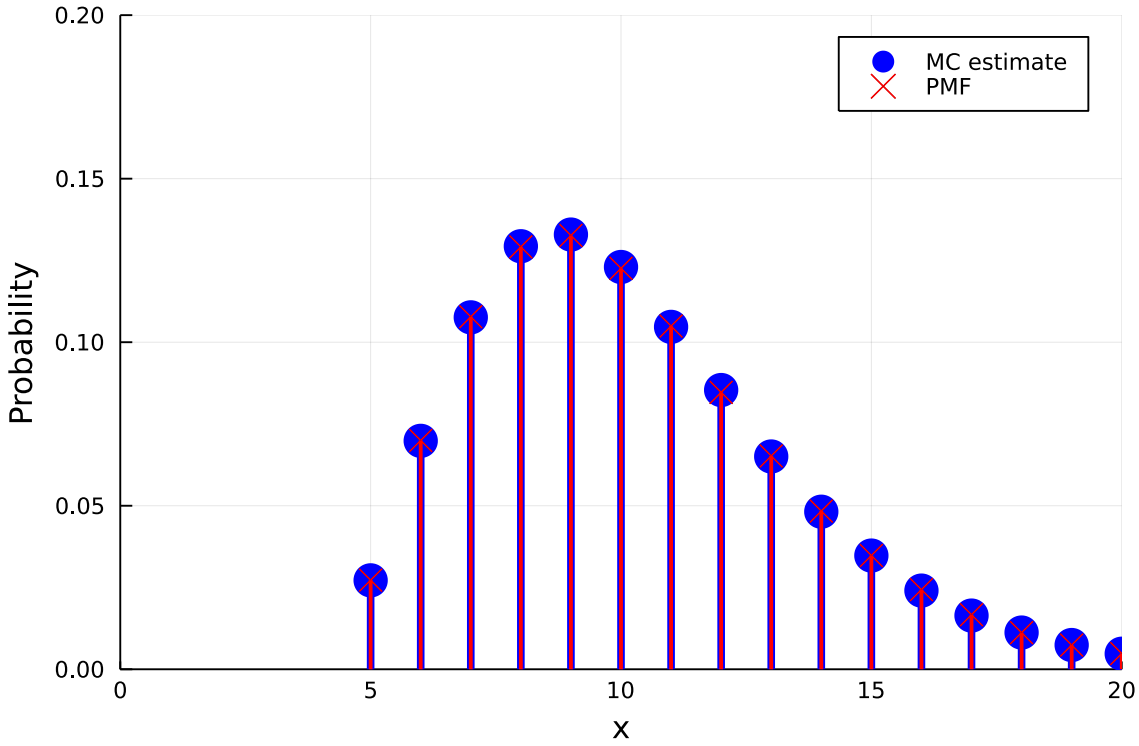
- Negative binomial distribution on Wolfram (https://reference.wolfram.com/language/ref/NegativeBinomialDistribution.html)

Note: The definition of the negative binomial distribution in Wolfram is different from the Wikipedia definition (https://en.wikipedia.org/wiki/Negative_binomial_distribution). In Wikipedia, `r` is the number of failures and `k` is the number of successes.

In [94]:

```
fix_seed!()

function rouletteSpins(r,p)
    x = 0
    wins = 0
    while true
        x += 1
        if rand() < p
            wins += 1
            if wins == r
                return x
            end
        end
    end
end

r, p, N = 5, 18/37,10^6
xGrid = r:r+15

mcEstimate = counts([rouletteSpins(r,p) for _ in 1:N],xGrid)/N

nbDist = NegativeBinomial(r,p)
nbPmf = [pdf(nbDist,x-r) for x in xGrid]

plot( xGrid, mcEstimate,
    line=:stem, marker=:circle, c=:blue,
    ms=10, msw=0, lw=4, label="MC estimate")
plot!( xGrid, nbPmf, line=:stem,
    marker=:xcross, c=:red, ms=6, msw=0, lw=2, label="PMF",
    xlims=(0,maximum(xGrid)), ylims=(0,0.2),
    xlabel="x", ylabel="Probability")
```

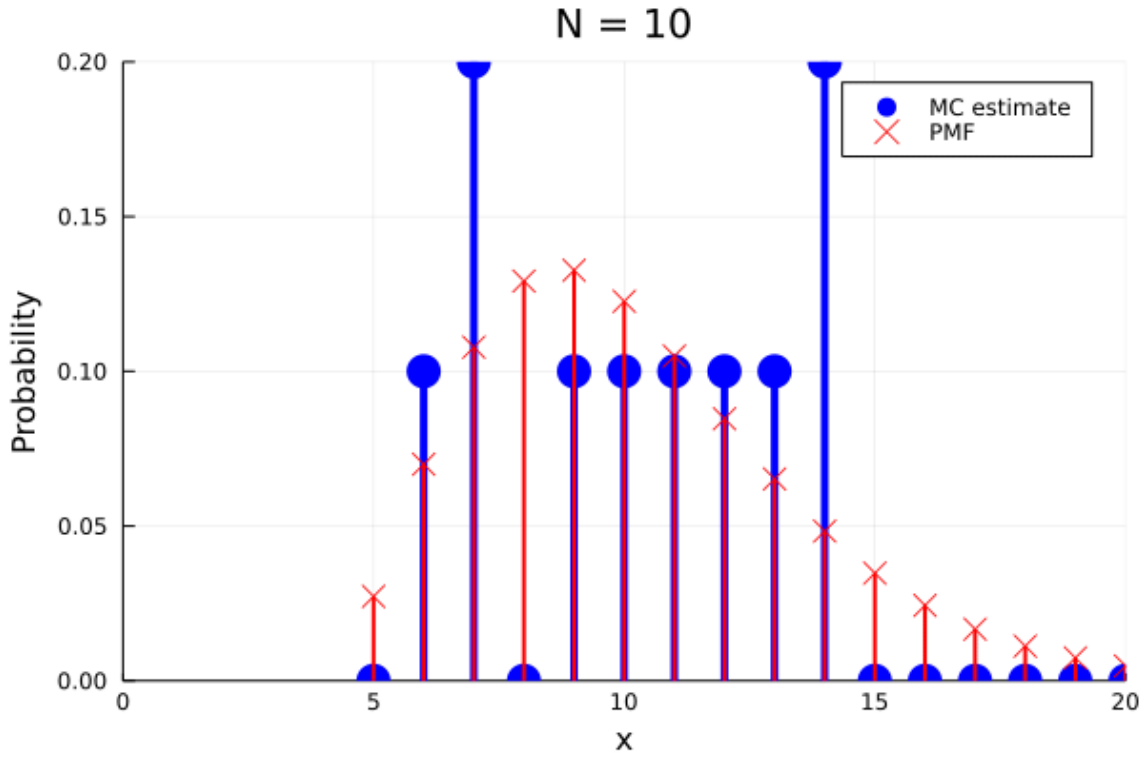
Out[94]:


0.20
0.15
0.10
0.05
0.00
Probability
0
5
10
15
20
x
MC estimate
PMF

In [95]:
```
function plot_it(N)
    fix_seed!()
    mcEstimate = counts([rouletteSpins(r,p) for _ in 1:N],xGrid)/N

    nbDist = NegativeBinomial(r,p)
    nbPmf = [pdf(nbDist,x-r) for x in xGrid]

    plot( xGrid, mcEstimate,
        line=:stem, marker=:circle, c=:blue,
        ms=10, msw=0, lw=4, label="MC estimate")
    plot!( xGrid, nbPmf, line=:stem,
        marker=:xcross, c=:red, ms=6, msw=0, lw=2, label="PMF",
        xlims=(0,maximum(xGrid)), ylims=(0,0.2),
        xlabel="x", ylabel="Probability",
        title="N = $N")
end

anim = Animation()

for N in union(10:10:100,200:100,1000,2000:1000:10^4)
    plot_it(N)
    frame(anim)
end

gif(anim,"sample_animation.gif",fps = 3)
```

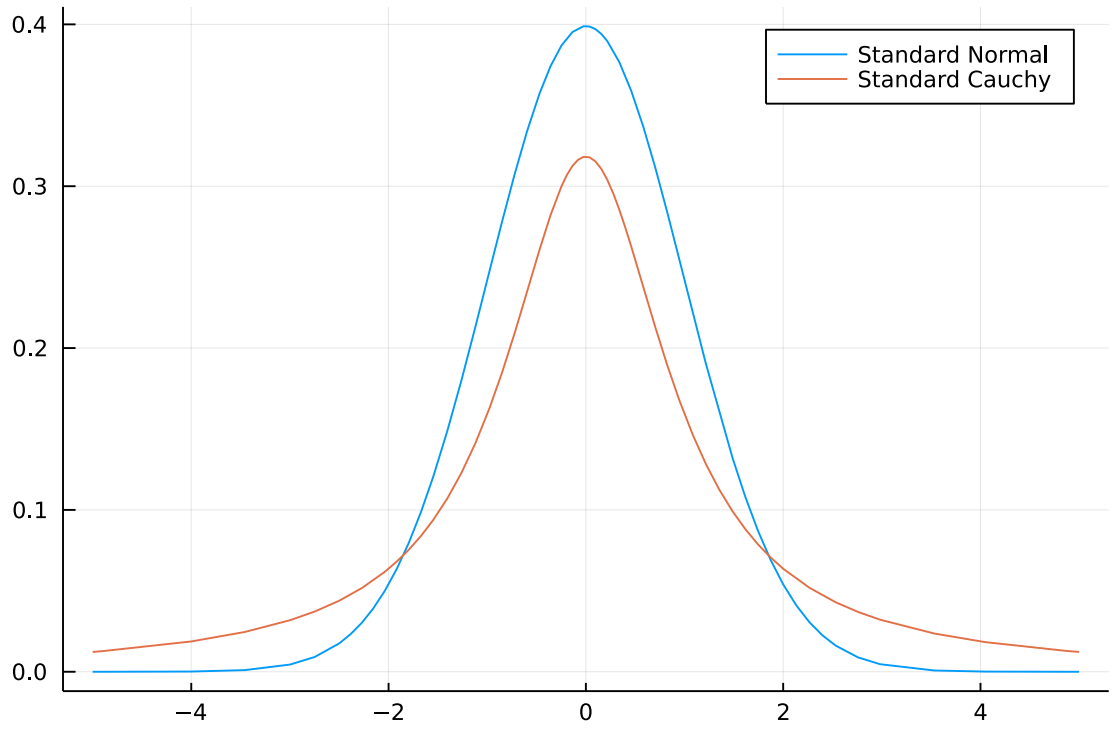
```
┌ Info: Saved animation to
│   fn = /Users/uqjnazar/git/mine/JuliaCon2021-StatisticsWithJuliaFromT
└ @ Plots /Users/uqjnazar/.julia/packages/Plots/S2aH5/src/animation.jl:
```

Out[95]:


N = 10
MC estimate
PMF
Probability
x
0.20
0.15
0.10
0.05
0.00
0
5
10
15
20

In [96]:
```
plot((x)->pdf(Normal(),x),label = "Standard Normal")
plot!((x)->pdf(Cauchy(),x),label = "Standard Cauchy")
```

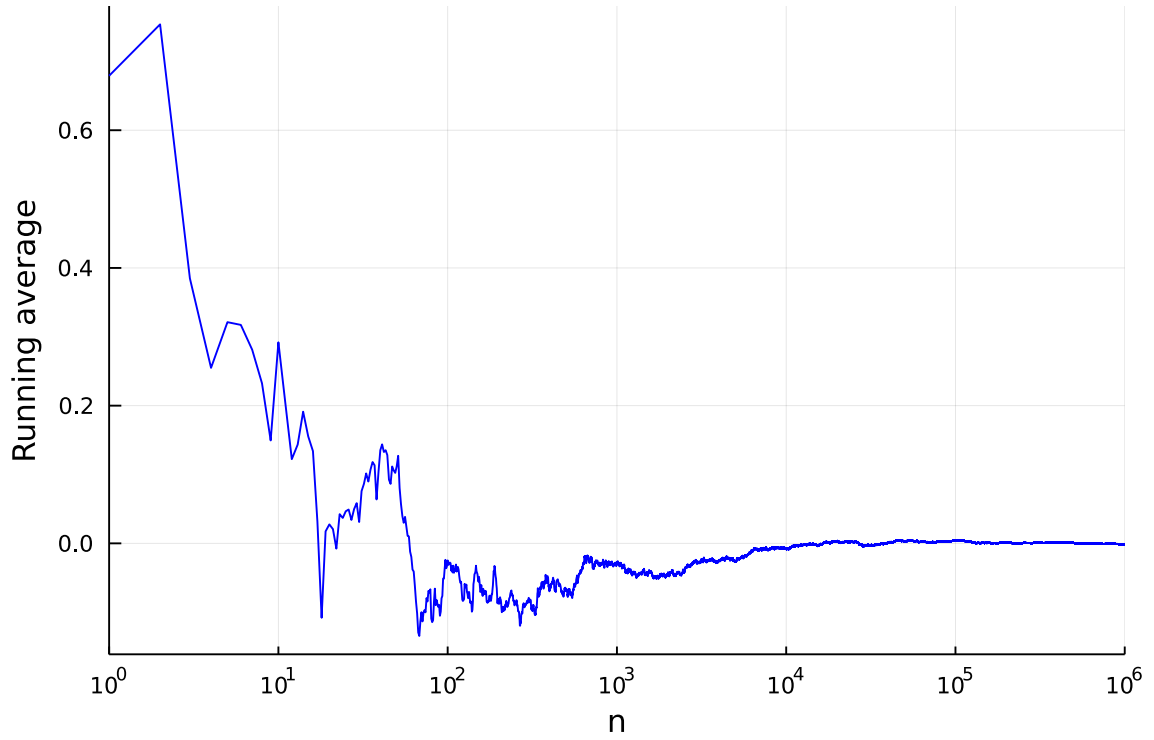
Out[96]:



In [97]:

```
fix_seed!()

n = 10^6
data = rand(Normal(), n)
#data = rand(Cauchy(),n) #Try this instead
averages = accumulate(+,data) ./ (1:n)

plot( 1:n, averages,
    c=:blue, legend=:none, xscale=:log10, xlims=(1,n), xlabel="n", ylabe
```

Out[97]:

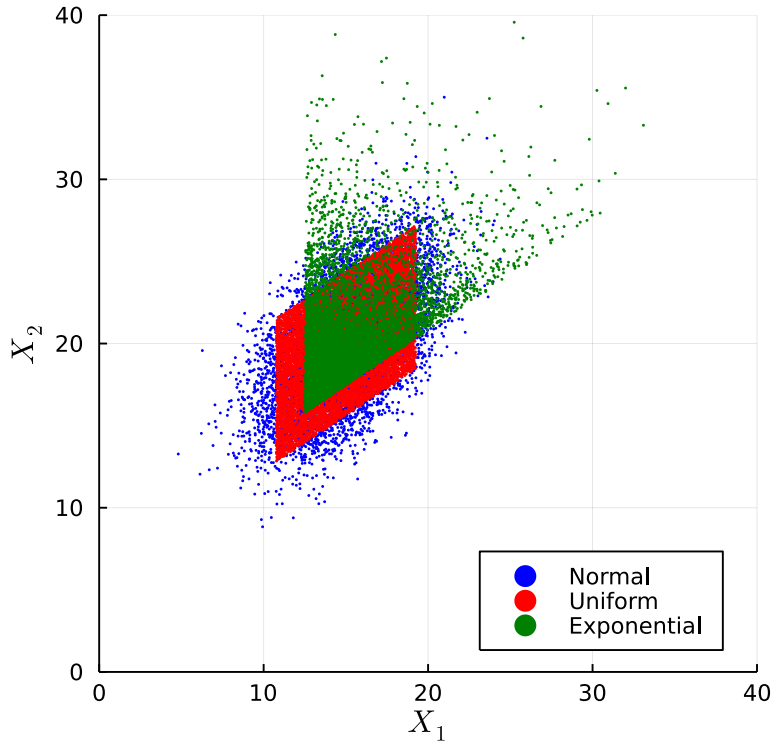


In [98]:

```
fix_seed!()

N = 10^4

Σ = [ 6 4 ;
      4 9]
μ = [15 ;
     20]
A = cholesky(Σ).L

rngGens = [()->rand(Normal()),
           ()->rand(Uniform(-sqrt(3),sqrt(3))),
           ()->rand(Exponential())-1]

rv(rg) = A*[rg(),rg()] + μ

data = [[rv(r) for _ in 1:N] for r in rngGens]

stats(data) = begin
    data1, data2 = first.(data),last.(data)
    println(round(mean(data1),digits=2), "\t",round(mean(data2),digits=2
            round(var(data1),digits=2), "\t", round(var(data2),digits=2)
            round(cov(data1,data2),digits=2))
end

println("Mean1\tMean2\tVar1\tVar2\tCov")
for d in data
    stats(d)
end

scatter(first.(data[1]), last.(data[1]), c=:blue, ms=1, msw=0, label="No
scatter!(first.(data[2]), last.(data[2]), c=:red, ms=1, msw=0, label="Un
scatter!(first.(data[3]), last.(data[3]), c=:green, ms=1, msw=0,label="E
    xlims=(0,40), ylims=(0,40), legend=:bottomright, ratio=:equal,
    xlabel=L"X_1", ylabel=L"X_2")
```

```
Mean1   Mean2   Var1    Var2    Cov
15.03   20.0    6.07    8.98    4.08
15.01   19.97   6.08    9.09    4.05
15.01   20.0    6.02    9.28    4.04
```

Out[98]:



In [99]:
```
dist = MvNormal([1,1],[2 2.3; 2.3 4])
```

Out[99]:
```
FullNormal(
dim: 2
μ: [1.0, 1.0]
Σ: [2.0 2.3; 2.3 4.0]
)
```

In [100]:
```
cor(dist)
```

Out[100]:
```
2×2 Matrix{Float64}:
 1.0       0.813173
 0.813173  1.0
```

```
In [101]: rand(dist,20)
```

```
Out[101]: 2×20 Matrix{Float64}:
 -1.14987  -0.000710292  0.734429  …  1.78702  2.84194  2.41636  1.1962
 -1.87928   0.0846214    1.79441      1.42384  4.54479  3.26142  1.5882
```

See also TruncatedDistributions.jl (https://github.com/yoninazarathy/TruncatedDistributions.jl) - still in progress as of July 2021.

We love `DataFrames.`

home

See also:

- Yesterday's (recorded) Dataframes tutorial (https://github.com/bkamins/JuliaCon2021-DataFrames-Tutorial) or here (https://pretalx.com/juliacon2021/talk/FXZXMB/) by Bogumił Kamiński.
- Docs for DataFrames.jl (https://dataframes.juliadata.org/stable/).

In [102]:
```
ENV["LINES"] = 10
data = CSV.File("./data/purchaseData.csv") |> DataFrame
```

Out[102]: 200 rows × 4 columns

	Name	Date	Grade	Price
	String?	String?	String?	Int64?
1	MARYANNA	14/09/2008	A	79700
2	REBECCA	11/03/2008	B	*missing*
3	ASHELY	5/08/2008	E	24311
4	KHADIJAH	2/09/2008	*missing*	38904
5	TANJA	1/12/2008	C	47052
6	JUDIE	17/05/2008	D	34365
7	NOE	15/08/2008	A	79344
8	JACALYN	7/10/2008	E	21474
9	SANDY	5/04/2008	D	*missing*
10	ALEJANDRA	16/12/2008	E	20916
⋮	⋮	⋮	⋮	⋮

In [103]:
```
size(data)
```

Out[103]:
```
(200, 4)
```

In [104]:
```
names(data)
```

Out[104]:
```
4-element Vector{String}:
 "Name"
 "Date"
 "Grade"
 "Price"
```

In [105]:
```
first(data, 6)
```

Out[105]: 6 rows × 4 columns

	Name String?	Date String?	Grade String?	Price Int64?
1	MARYANNA	14/09/2008	A	79700
2	REBECCA	11/03/2008	B	*missing*
3	ASHELY	5/08/2008	E	24311
4	KHADIJAH	2/09/2008	*missing*	38904
5	TANJA	1/12/2008	C	47052
6	JUDIE	17/05/2008	D	34365

In [106]:
```
describe(data)
```

Out[106]: 4 rows × 7 columns

	variable Symbol	mean Union…	min Any	median Union…	max Any	nmissing Int64	eltype Union
1	Name		ABBEY		ZACHARY	17	Union{Missing, String}
2	Date		1/07/2008		9/10/2008	4	Union{Missing, String}
3	Grade		A		E	13	Union{Missing, String}
4	Price	39702.0	8257	38045.5	79893	14	Union{Missing, Int64}

In [107]:
```
println("Grade of person 1: ", data[1, 3],
        ", ", data[1,:Grade],
        ", ", data.Grade[1])
```

```
Grade of person 1: A, A, A
```

In [108]:
```
data[[1,2,4], :]
```

Out[108]: 3 rows × 4 columns

	Name	Date	Grade	Price
	String?	String?	String?	Int64?
1	MARYANNA	14/09/2008	A	79700
2	REBECCA	11/03/2008	B	*missing*
3	KHADIJAH	2/09/2008	*missing*	38904

In [109]:
```
data[13:15, :Name]
```

Out[109]:
```
3-element Vector{Union{Missing, String}}:
 "SAMMIE"
 missing
 "STACEY"
```

In [110]:
```
data.Name[13:15]
```

Out[110]:
```
3-element Vector{Union{Missing, String}}:
 "SAMMIE"
 missing
 "STACEY"
```

In [111]:
```
data[13:15, [:Name]]
```

Out[111]: 3 rows × 1 columns

	Name String?
1	SAMMIE
2	*missing*
3	STACEY

Mising values

In [112]:
```
mean(data.Price)
```

Out[112]:
```
missing
```

In [113]:
```
ismissing.(data.Price) |> sum
```

Out[113]:
```
14
```

In [114]:
```
mean(skipmissing(data.Price))
```

Out[114]:
```
39702.01075268817
```

In [115]:
```
data.Grade[1:4]
```

Out[115]:
```
4-element PooledArrays.PooledVector{Union{Missing, String}, UInt32, Vec
 "A"
 "B"
 "E"
 missing
```

In [116]:
```
coalesce.(data.Grade, "QQ")[1:4]
```

Out[116]:
```
4-element Vector{String}:
 "A"
 "B"
 "E"
 "QQ"
```

In [117]:
```
dropmissing(data,:Price)
```

Out[117]: 186 rows × 4 columns

	Name	Date	Grade	Price
	String?	String?	String?	Int64
1	MARYANNA	14/09/2008	A	79700
2	ASHELY	5/08/2008	E	24311
3	KHADIJAH	2/09/2008	*missing*	38904
4	TANJA	1/12/2008	C	47052
5	JUDIE	17/05/2008	D	34365
6	NOE	15/08/2008	A	79344
7	JACALYN	7/10/2008	E	21474
8	ALEJANDRA	16/12/2008	E	20916
9	GARY	8/05/2008	C	40081
10	ROSELEE	29/06/2008	*missing*	37661
⋮	⋮	⋮	⋮	⋮

In [118]:
```
completecases(data) |> sum
```

Out[118]:
```
153
```

In [119]:
```
full_obs = completecases(data) |> findall
```

Out[119]:
```
153-element Vector{Int64}:
   1
   3
   5
   ⋮
 199
 200
```

In [120]:
```
data[setdiff(1:size(data,1), full_obs),:]
```

Out[120]: 47 rows × 4 columns

	Name String?	Date String?	Grade String?	Price Int64?
1	REBECCA	11/03/2008	B	*missing*
2	KHADIJAH	2/09/2008	*missing*	38904
3	SANDY	5/04/2008	D	*missing*
4	ROSELEE	29/06/2008	*missing*	37661
5	*missing*	12/12/2008	E	17328
6	RASHIDA	*missing*	C	44597
7	MINERVA	31/07/2008	*missing*	45751
8	JARRETT	30/11/2008	*missing*	41182
9	*missing*	1/10/2008	E	20229
10	*missing*	24/03/2008	E	24633
⋮	⋮	⋮	⋮	⋮

Gotta have some basic inference.

home

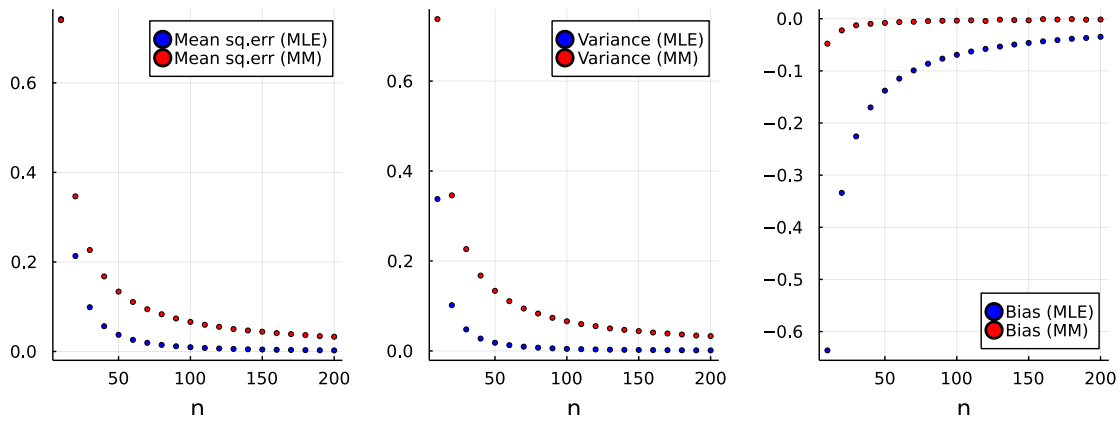
Point estimation

In [121]:

```
fix_seed!()

N = 10^5
nMin, nStep, nMax = 10, 10, 200
nn = Int(nMax/nStep)
sampleSizes = nMin:nStep:nMax
trueB = 5
trueDist = Uniform(-2, trueB)

#MLE for the upper bound
MLEest(data) = maximum(data)

#Method of moments estimator for the upper bound
MMest(data)  = mean(data) + sqrt(3)*std(data)

res = Dict{Symbol,Array{Float64}}( ((sym) -> sym => Array{Float64}(undef
                                  [:MSeMLE,:MSeMM, :VarMLE,:VarMM,:BiasMLE
for (i, n) in enumerate(sampleSizes)
    mleEst, mmEst = Array{Float64}(undef, N), Array{Float64}(undef, N)
    for j in 1:N
        sample    = rand(trueDist,n)
        mleEst[j] = MLEest(sample)
        mmEst[j]  = MMest(sample)
    end
    meanMLE, meanMM = mean(mleEst), mean(mmEst)
    varMLE, varMM = var(mleEst), var(mmEst)

    res[:MSeMLE][i] = varMLE + (meanMLE - trueB)^2
    res[:MSeMM][i] = varMM + (meanMM - trueB)^2
    res[:VarMLE][i] = varMLE
    res[:VarMM][i] = varMM
    res[:BiasMLE][i] = meanMLE - trueB
    res[:BiasMM][i] = meanMM - trueB
end

p1 = scatter(sampleSizes, [res[:MSeMLE] res[:MSeMM]], c=[:blue :red],
    label=["Mean sq.err (MLE)" "Mean sq.err (MM)"])
p2 = scatter(sampleSizes, [res[:VarMLE] res[:VarMM]], c=[:blue :red],
    label=["Variance (MLE)" "Variance (MM)"])
p3 = scatter(sampleSizes, [res[:BiasMLE] res[:BiasMM]], c=[:blue :red],
    label=["Bias (MLE)" "Bias (MM)"],legend = :bottomright)
```

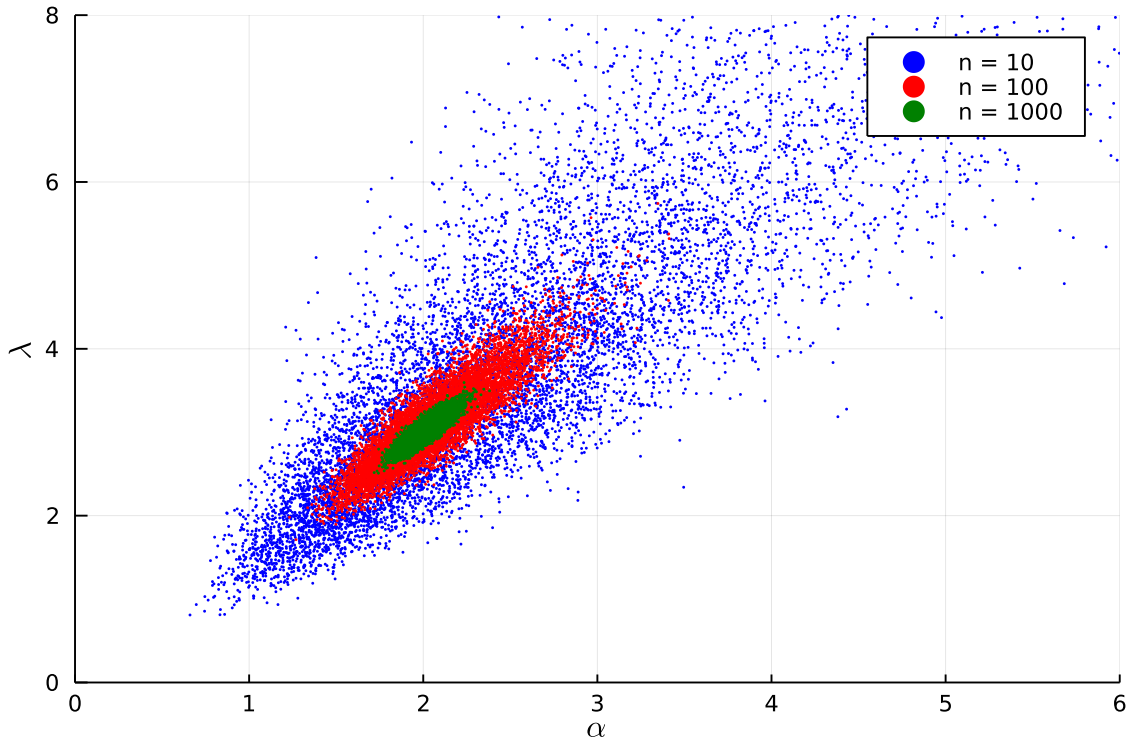
```
plot(p1, p2, p3, ms=2, shape=:circle, xlabel="n",
    layout=(1,3), size=(800, 300), margin = 2mm)
```

Out[121]:



In [122]:
```
fix_seed!()

eq(alpha, xb, xbl) = log(alpha) - digamma(alpha) - log(xb) + xbl

actualAlpha, actualLambda = 2, 3
gammaDist = Gamma(actualAlpha,1/actualLambda)

function mle(sample)
    alpha  = find_zero( (a)->eq(a,mean(sample),mean(log.(sample))), 1)
    lambda = alpha/mean(sample)
    return [alpha,lambda]
end

N = 10^4

mles10   = [mle(rand(gammaDist,10)) for _ in 1:N]
mles100  = [mle(rand(gammaDist,100)) for _ in 1:N]
mles1000 = [mle(rand(gammaDist,1000)) for _ in 1:N]

scatter(first.(mles10), last.(mles10),
    c=:blue, ms=1, msw=0, label="n = 10")
scatter!(first.(mles100), last.(mles100),
    c=:red, ms=1, msw=0, label="n = 100")
scatter!(first.(mles1000), last.(mles1000),
    c=:green, ms=1, msw=0, label="n = 1000",
    xlims=(0,6), ylims=(0,8), xlabel=L"\alpha", ylabel=L"\lambda")
```

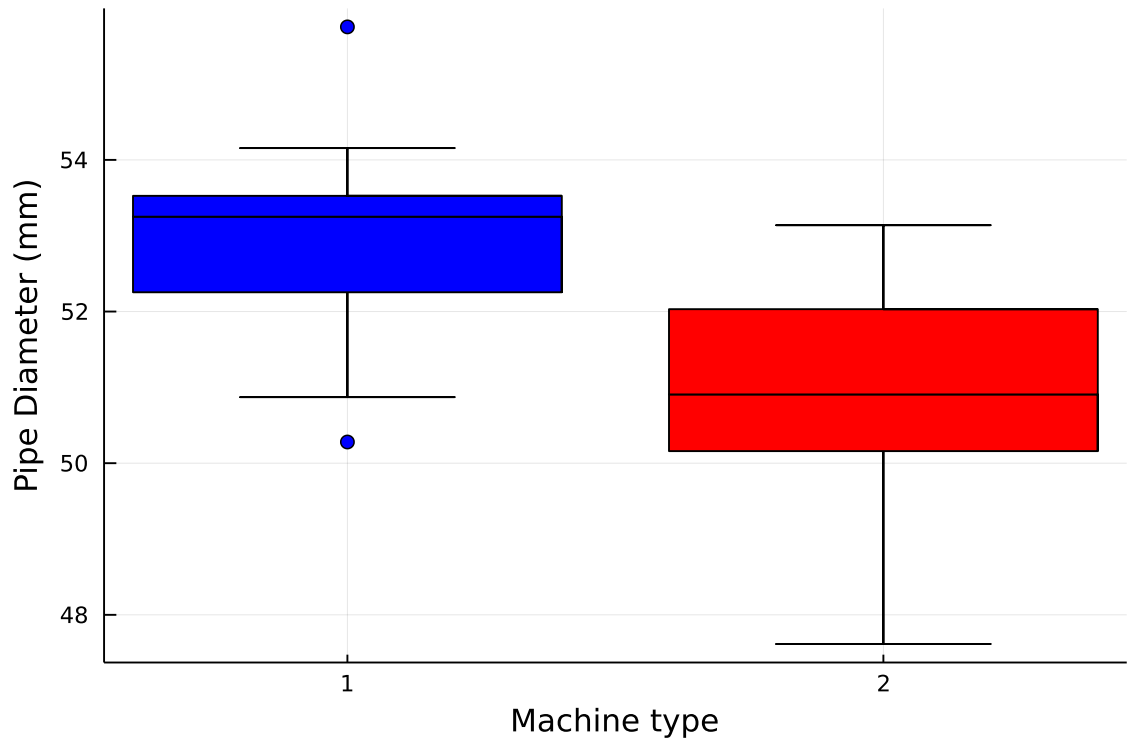
Out[122]:



Confidence Intervals

In [123]:
```
data1 = (CSV.File("./data/machine1.csv", header=false) |> DataFrame)[:,1
data2 = (CSV.File("./data/machine2.csv", header=false) |> DataFrame)[:,1
boxplot([data1,data2], c=[:blue :red], label="",
    xticks=([1:2;], ["1", "2"]), xlabel="Machine type",
    ylabel="Pipe Diameter (mm)")
```

Out[123]:



In [124]:
```
α = 0.05 #\alpha + [TAB]
confint(EqualVarianceTTest(data1,data2),α)
```

Out[124]:
```
(1.1127539574575822, 2.90486485574026)
```

In [161]:
```
#Doing it manually
xBar1, xBar2 = mean(data1), mean(data2)
n1, n2 = length(data1), length(data2)
t = quantile(TDist(n1+n2-2),1-α/2)

s1, s2 = std(data1), std(data2)
sP = sqrt(((n1-1)*s1^2 + (n2-1)*s2^2) / (n1+n2-2))

(xBar1 - xBar2 - t*sP* sqrt(1/n1 + 1/n2), xBar1 - xBar2 + t*sP* sqrt(1/r
```

Out[161]:
```
(1.1127539574575822, 2.90486485574026)
```

Hypothesis Tests

In [162]:
```
δ₀ = 0
test_result =UnequalVarianceTTest(data1, data2, δ₀)
```

Out[162]:
```
Two sample t-test (unequal variance)
------------------------------------
Population details:
    parameter of interest:   Mean difference
    value under h_0:         0
    point estimate:          2.00881
    95% confidence interval: (1.096, 2.9216)

Test summary:
    outcome with 95% confidence: reject h_0
    two-sided p-value:           <1e-04

Details:
    number of observations:   [20,18]
    t-statistic:              4.483705005611673
    degrees of freedom:       31.82453144280282
    empirical standard error: 0.4480244360600785
```

In [163]:
```
test_result.df
```

Out[163]:
```
31.82453144280282
```

In [164]:
```
pvalue(test_result)
```

Out[164]:
```
8.936189820682969e-5
```

In [165]:
```
methods(pvalue)
```

Out[165]: # 35 methods for generic function **pvalue**:

- pvalue(t::**HypothesisTests.VarianceEqualityTest{TD}**; *tail*) *where TD<: (ContinuousDistribution{F} where F<:VariateForm)* in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/var_equality.jl:30 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/var_equality.jl)
- pvalue(test::**CorrelationTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/correlation.jl:84 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/correlation.jl)
- pvalue(test::**WaldWolfowitzTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/wald_wolfowitz.jl:15 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/wald_wolfowitz.jl)
- pvalue(x::**ApproximateMannWhitneyUTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/mann_whitney.jl:225 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/mann_whitney.jl)
- pvalue(x::**ExactMannWhitneyUTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/mann_whitney.jl:136 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/mann_whitney.jl)
- pvalue(apt::**HypothesisTests.PermutationTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/permutation.jl:43 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/permutation.jl)
- pvalue(dist::**Distribution{Univariate, Continuous}**, x::**Number**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/HypothesisTests.jl:72 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/HypothesisTests.jl)
- pvalue(dist::**Distribution{Univariate, Discrete}**, x::**Number**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/HypothesisTests.jl:83 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/HypothesisTests.jl)
- pvalue(x::**HypothesisTests.ZTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/z.jl:31 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/z.jl)
- pvalue(x::**KSampleADTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/anderson_darling.jl:241 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/anderson_darling.jl)
- pvalue(T::**HypothesisTests.HotellingT2TestTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/hotelling.jl:8

(file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/hotelling.jl)

- pvalue(x::**ExactSignedRankTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/wilcoxon.jl:135 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/wilcoxon.jl)
- pvalue(x::**ApproximateSignedRankTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/wilcoxon.jl:225 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/wilcoxon.jl)
- pvalue(x::**FisherTLinearAssociation**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/circular.jl:138 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/circular.jl)
- pvalue(x::**HypothesisTests.TTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/t.jl:31 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/t.jl)
- pvalue(x::**HypothesisTests.ExactKSTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kolmogorov_smirnov.jl:80 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kolmogorov_smirnov.jl)
- pvalue(x::**LjungBoxTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/box_test.jl:123 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/box_test.jl)
- pvalue(x::**BreuschGodfreyTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/breusch_godfrey.jl:79 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/breusch_godfrey.jl)
- pvalue(x::**RayleighTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/circular.jl:63 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/circular.jl)
- pvalue(B::**BartlettTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/bartlett.jl:53 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/bartlett.jl)
- pvalue(x::**DurbinWatsonTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/durbin_watson.jl:180 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/durbin_watson.jl)
- pvalue(x::**ADFTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/augmented_dickey_fuller.jl:220 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/augmented_dickey_fuller.j
- pvalue(x::**BoxPierceTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/box_test.jl:74 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/box_test.jl)

- pvalue(t::**OneSampleADTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/anderson_darling.jl:55 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/anderson_darling.jl)
- pvalue(x::**ApproximateTwoSampleKSTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kolmogorov_smirnov.jl:181 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kolmogorov_smirnov.jl)
- pvalue(x::**FisherExactTest**; *tail, method*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/fisher.jl:123 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/fisher.jl)
- pvalue(t::**WhiteTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/white.jl:120 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/white.jl)
- pvalue(x::**ApproximateOneSampleKSTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kolmogorov_smirnov.jl:126 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kolmogorov_smirnov.jl)
- pvalue(x::**BinomialTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/binomial.jl:67 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/binomial.jl)
- pvalue(x::**VarianceFTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/f.jl:66 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/f.jl)
- pvalue(x::**KruskalWallisTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kruskal_wallis.jl:95 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/kruskal_wallis.jl)
- pvalue(x::**JammalamadakaCircularCorrelation**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/circular.jl:221 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/circular.jl)
- pvalue(x::**SignTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/binomial.jl:215 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/binomial.jl)
- pvalue(x::**PowerDivergenceTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/power_divergence.jl:44 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/power_divergence.jl)
- pvalue(x::**JarqueBeraTest**) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/jarque_bera.jl:87 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/jarque_bera.jl)

In [166]:
```
@which pvalue(test_result)
```

Out[166]: pvalue(x::**HypothesisTests.TTest**; *tail*) in HypothesisTests at /Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/t.jl:31 (file:///Users/uqjnazar/.julia/packages/HypothesisTests/V7PST/src/t.jl)

In [167]:
```
xBar1, s1, n1 = mean(data1), std(data1), length(data1)
xBar2, s2, n2 = mean(data2), std(data2), length(data2)

v = ( s1^2/n1 + s2^2/n2 )^2 / ( (s1^2/n1)^2/(n1-1) + (s2^2/n2)^2/(n2-1)
testStatistic = ( xBar1-xBar2 - δ₀ )  / sqrt( s1^2/n1 + s2^2/n2 )
pVal = 2*ccdf(TDist(v), abs(testStatistic))

println("Manually calculated degrees of freedom, v: ", v)
println("Manually calculated test statistic: ", testStatistic)
println("Manually calculated p-value: ", pVal, "\n")
```

```
Manually calculated degrees of freedom, v: 31.82453144280283
Manually calculated test statistic: 4.483705005611673
Manually calculated p-value: 8.936189820683007e-5
```

In [168]:
```
data = CSV.File("./data/fertilizer.csv") |> DataFrame

control = data.Control
fertilizer = data.FertilizerX

subGroups = collect(combinations([control;fertilizer],10))

meanFert = mean(fertilizer)
pVal = sum([mean(i) >= meanFert for i in subGroups])/length(subGroups)
println("p-value = ", pVal)
```

```
p-value = 0.023972157873086666
```

Linear models at our core.

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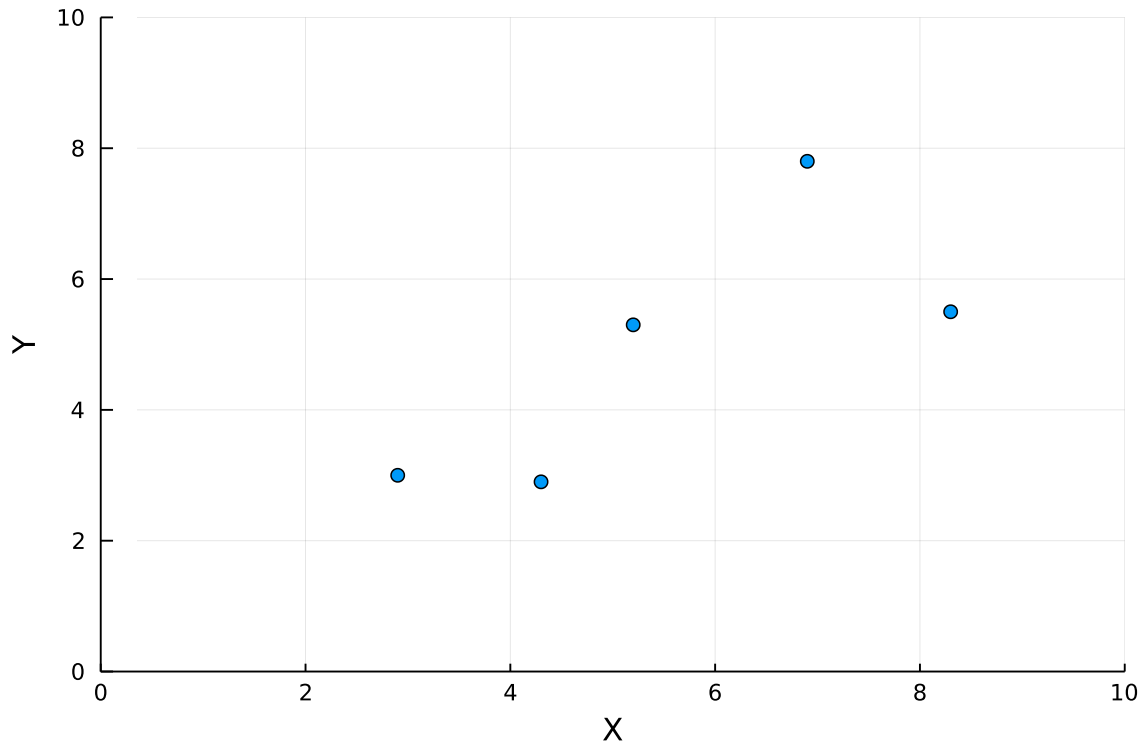
In [169]:
```
data = CSV.File("./data/L1L2data.csv") |> DataFrame
xVals, yVals = data[:,1], data[:,2]
scatter(xVals,yVals,label=false,xlim=(0,10),ylim=(0,10),xlabel="X", ylab
```

Out[169]:



In [170]:
```
n = length(xVals)
A = [ones(n) xVals] #The "design matrix"
```

Out[170]:
```
5×2 Matrix{Float64}:
 1.0  2.9
 1.0  4.3
 1.0  5.2
 1.0  6.9
 1.0  8.3
```

In [171]:

```
# Approach A
xBar, yBar = mean(xVals),mean(yVals)
sXX, sXY = ones(n)'*(xVals.-xBar).^2 , dot(xVals.-xBar,yVals.-yBar)
b1A = sXY/sXX
b0A = yBar - b1A*xBar

# Approach B
b1B = cor(xVals,yVals)*(std(yVals)/std(xVals))
b0B = yBar - b1B*xBar

# Approach C
b0C, b1C = A'A \ A'yVals

# Approach D
Adag = inv(A'*A)*A'
b0D, b1D = Adag*yVals

# Approach E
b0E, b1E = pinv(A)*yVals

# Approach F
b0F, b1F = A\yVals

# Approach G
F = qr(A)
Q, R = F.Q, F.R
b0G, b1G = (inv(R)*Q')*yVals

# Approach H
F = svd(A)
V, Sp, Us = F.V, Diagonal(1 ./ F.S), F.U'
b0H, b1H = (V*Sp*Us)*yVals

# Approach I
eta, eps = 0.002, 10^-6.
b, bPrev = [0,0], [1,1]
while norm(bPrev-b) >= eps
    global bPrev = b
    global b = b - eta*2*A'*(A*b - yVals)
end
b0I, b1I = b[1], b[2]

# Approach J
```

```
modelJ = lm(@formula(Y ~ X), data)
b0J, b1J = coef(modelJ)

# Approach K
modelK = glm(@formula(Y ~ X), data, Normal())
b0K, b1K = coef(modelK)

println(round.([b0A,b0B,b0C,b0D,b0E,b0F,b0G,b0H,b0I,b0J,b0K],digits=3))
println(round.([b1A,b1B,b1C,b1D,b1E,b1F,b1G,b1H,b1I,b1J,b1K],digits=3))
```

```
[0.945, 0.945, 0.945, 0.945, 0.945, 0.945, 0.945, 0.945, 0.944, 0.945, 
[0.716, 0.716, 0.716, 0.716, 0.716, 0.716, 0.716, 0.716, 0.717, 0.716, 
```

In [172]:
```
data = CSV.File("./data/weightHeight.csv") |> DataFrame

lm1 = lm(@formula(Height ~ Weight), data)
lm2 = fit(LinearModel,@formula(Height ~ Weight), data)

glm1 = glm(@formula(Height ~ Weight), data, Normal(), IdentityLink())
glm2 = fit(GeneralizedLinearModel,@formula(Height ~ Weight), data, Norma

println("***Output of LM Model:")
println(lm1)
println("\n***Output of GLM Model:")
println(glm1)

pred(x) = coef(lm1)'*[1, x]

println("\n***Individual methods applied to model output:")
println("Deviance: ",deviance(lm1))
println("Standard error: ",stderror(lm1))
println("Degrees of freedom: ",dof_residual(lm1))
println("Covariance matrix: ",vcov(lm1))

yVals = data.Height
SStotal = sum((yVals .- mean(yVals)).^2)

println("R squared (calculated in two ways):",r2(lm1),
        ",\t", 1 - deviance(lm1)/SStotal)

println("MSE (calculated in two ways: ",deviance(lm1)/dof_residual(lm1),
        ",\t",sum((pred.(data.Weight) - data.Height).^2)/(size(data)[1]

xlims = [minimum(data.Weight), maximum(data.Weight)]
scatter(data.Weight, data.Height, c=:blue, msw=0)
plot!(xlims, pred.(xlims),
    c=:red, xlims=(xlims),
    xlabel="Weight (kg)", ylabel="Height (cm)", legend=:none)
```

```
***Output of LM Model:
StatsModels.TableRegressionModel{LinearModel{GLM.LmResp{Vector{Float64}

Height ~ 1 + Weight

Coefficients:
─────────────────────────────────────────────────────────────────────────
```

	Coef.	Std. Error	t	Pr(>|t|)	Lower 95%	Upper
(Intercept)	135.793	1.95553	69.44	<1e-99	131.937	139.6
Weight	0.532299	0.0293556	18.13	<1e-43	0.474409	0.5

***Output of GLM Model:
StatsModels.TableRegressionModel{GeneralizedLinearModel{GLM.GlmResp{Vec

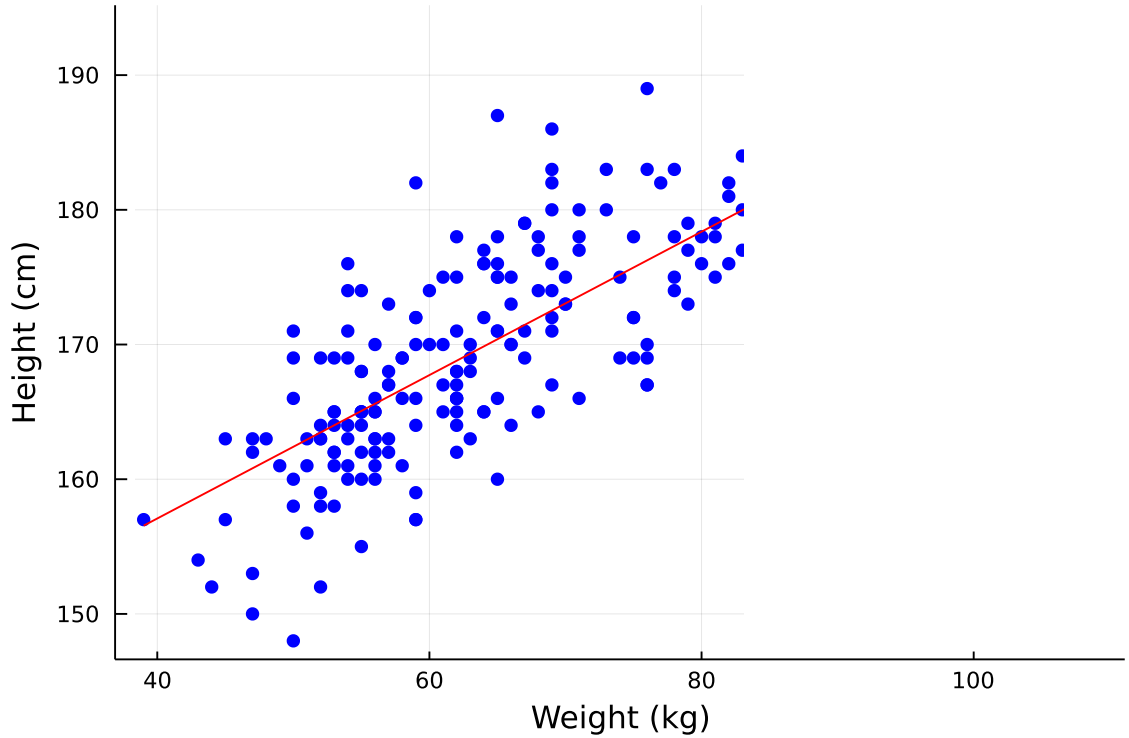
Height ~ 1 + Weight

Coefficients:

	Coef.	Std. Error	z	Pr(>|z|)	Lower 95%	Upper
(Intercept)	135.793	1.95553	69.44	<1e-99	131.96	139.6
Weight	0.532299	0.0293556	18.13	<1e-72	0.474763	0.5

***Individual methods applied to model output:
Deviance: 5854.05714276554
Standard error: [1.9555309971174046, 0.029355593998568793]
Degrees of freedom: 198.0
Covariance matrix: [3.824101480686991 -0.05628525996876029; -0.05628525
R squared (calculated in two ways):0.624144340084702, 0.624144340084
MSE (calculated in two ways: 29.56594516548253, 29.565945165482532

Out[172]:



Multiple variables

In [173]:
```
df = RDatasets.dataset("MASS", "cpus")
df.Freq = map( x->10^9/x , df.CycT)

model = lm(@formula(Perf ~ MMax + Cach + ChMax + Freq), df)
pred(x) = round(coef(model)'*vcat(1,x),digits = 3)

println("n = ", size(df)[1])
println("(Avg,Std) of observed performance: ", (mean(df.Perf),std(df.Per
println(model)
println("Estimated performance for computer A: ", pred([32000, 32, 32, 4
println("Estimated performance for computer B: ", pred([32000, 16, 32, 6
```

```
n = 209
(Avg,Std) of observed performance: (105.61722488038278, 160.83058719907
StatsModels.TableRegressionModel{LinearModel{GLM.LmResp{Vector{Float64}

Perf ~ 1 + MMax + Cach + ChMax + Freq

Coefficients:
```

	Coef.	Std. Error	t	Pr(>|t|)	Lower 95%	
(Intercept)	-46.5763	7.62382	-6.11	<1e-08	-61.6078	
MMax	0.00841457	0.000639214	13.16	<1e-28	0.00715426	
Cach	0.872508	0.152825	5.71	<1e-07	0.571189	
ChMax	0.96736	0.234847	4.12	<1e-04	0.504321	
Freq	9.74951e-7	5.5502e-7	1.76	0.0805	-1.1936e-7	

```
Estimated performance for computer A: 320.564
Estimated performance for computer B: 326.103
```

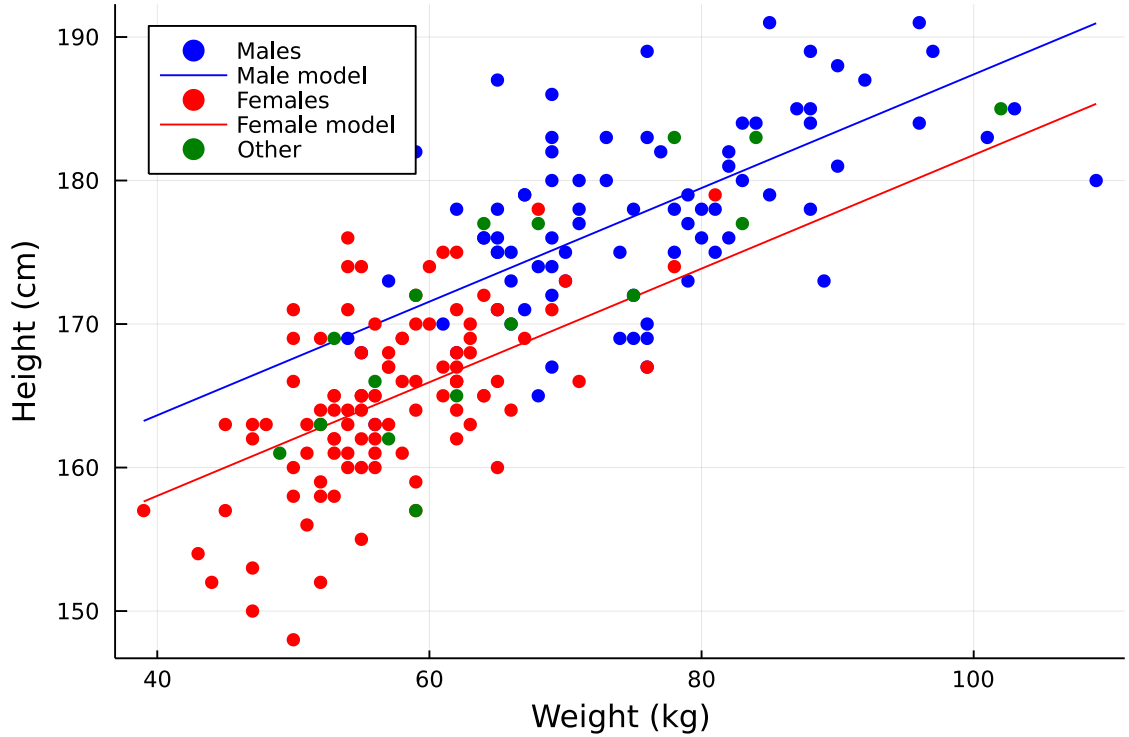
Categorical variables

In [174]:
```
df = CSV.File("./data/weightHeight.csv") |> DataFrame
n = size(df)[1]
df[shuffle(1:n),:] = df
df[[10,40,60,130,140,175,190,200],:Sex] .= "O1"
df[[9,44,63,132,138,172,192,199],:Sex] .= "O2"

model = lm(@formula(Height ~ Weight + Sex), df,
        contrasts=Dict(:Sex=>DummyCoding(base="F",levels=["M","O1","O2",
b0, b1, b2, b3, b4  = coef(model)
pred(weight,sex) = b0+b1*weight+b2*(sex=="M")+b3*(sex=="O1")+b3*(sex=="O
println(model)

males = df[df.Sex .== "M",:]
females = df[df.Sex .== "F",:]
other = df[(df.Sex .!= "M") .& (df.Sex .!= "F"),:]

xlim = [minimum(df.Weight), maximum(df.Weight)]
scatter(males.Weight, males.Height, c=:blue, msw=0, label="Males")
plot!(xlim, pred.(xlim,"M"), c=:blue, label="Male model")

scatter!(females.Weight, females.Height, c=:red, msw=0, label="Females")
plot!(xlim, pred.(xlim,"F"),
    c=:red, label="Female model", xlims=(xlim),
    xlabel="Weight (kg)", ylabel="Height (cm)", legend=:topleft)

scatter!(other.Weight, other.Height, c=:green, msw=0, label="Other")
```

```
StatsModels.TableRegressionModel{LinearModel{GLM.LmResp{Vector{Float64}

Height ~ 1 + Weight + Sex

Coefficients:
```

	Coef.	Std. Error	t	Pr(>|t|)	Lower 95%	Upper
(Intercept)	142.194	2.17705	65.32	<1e-99	137.901	146.48
Weight	0.395869	0.037162	10.65	<1e-20	0.322578	0.46
Sex: M	5.61479	1.01905	5.51	<1e-06	3.60501	7.62
Sex: O1	3.12116	1.88874	1.65	0.1000	-0.603814	6.84
Sex: O2	2.0663	1.92299	1.07	0.2839	-1.72624	5.85

Out[174]:

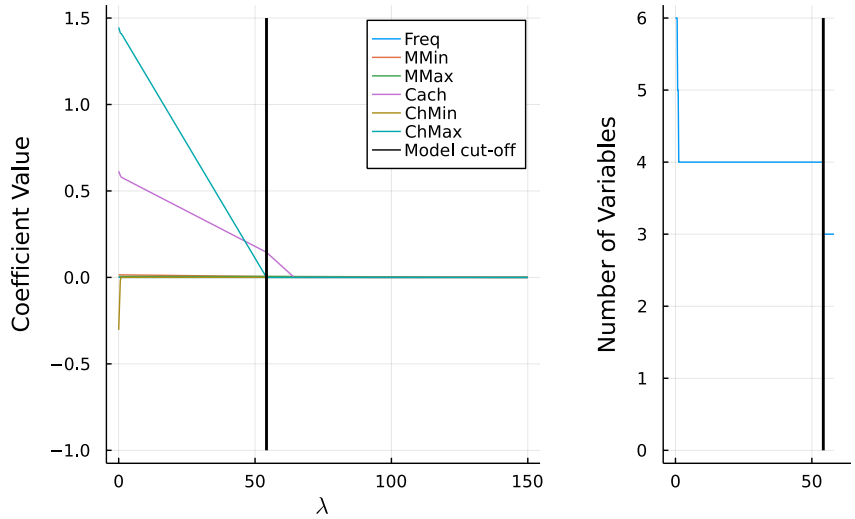


Variable selection with LASSO

In [177]:
```
df = RDatasets.dataset("MASS", "cpus")
df.Freq = map( x->10^9/x , df.CycT)
df = df[:, [:Perf, :Freq, :MMin, :MMax, :Cach, :ChMin, :ChMax]]
X = [df.Freq df.MMin df.MMax df.Cach df.ChMin df.ChMax]
Y = df.Perf

targetNumVars = 3

lambdaStep = 0.2
lamGrid = collect(0:lambdaStep:150)
lassoFit = fit(LassoPath,X, Y, λ = lamGrid);
dd = Array(lassoFit.coefs)'
nV = sum(dd .!= 0.0 ,dims=2)

goodLambda = lamGrid[findfirst((n)->n==targetNumVars,nV)]
newFit = fit(LassoPath,X, Y, λ = [goodLambda - lambdaStep, goodLambda])
println(newFit)
println("Coefficients: ", Array(newFit.coefs)'[2,:])

p1 = plot(lassoFit.λ, dd, label = ["Freq" "MMin" "MMax" "Cach" "ChMin" "
    ylabel = "Coefficient Value")
plot!([goodLambda,goodLambda],[-1,1.5],c=:black, lw=2, label = "Model cu

p2 = plot(lassoFit.λ,nV, ylabel = "Number of Variables",legend = false)
plot!([goodLambda,goodLambda],[0,6],c=:black, lw=2, label = "Model cut-o

plot(p1,p2,xlabel= L"\lambda", margin = 5mm, size = (800,400))
```

```
LassoPath (2) solutions for 7 predictors in 84 iterations):
──────────────────────────────
        λ   pct_dev  ncoefs
──────────────────────────────
[1]  54.0  0.673402       4
[2]  54.2  0.67203        3
──────────────────────────────
Coefficients: [0.0, 0.005294014227461394, 0.0056046927214597085, 0.1452
```

Out[177]:



Generalized linear models

In [178]:
```
df = RDatasets.dataset("MASS", "cpus")
n = size(df)[1]
df = df[shuffle(1:n),:]

pTest = 0.2
lastTindex = Int(floor(n*(1-pTest)))
numTest = n - lastTindex

train = df[1:lastTindex,:]
test = df[lastTindex+1:n,:]

form = @formula(Perf~CycT+MMin+MMax+Cach+ChMin+ChMax)
model1 = glm(form, train, Normal(),  IdentityLink())
model2 = glm(form, train, Poisson(), LogLink())
model3 = glm(form, train, Gamma(),  InverseLink())

invIdenityLink(x) = x
invLogLink(x) = exp(x)
invInverseLink(x) = 1/x

A = [ones(numTest) test.CycT test.MMin test.MMax test.Cach test.ChMin te
pred1 = invIdenityLink.(A*coef(model1))
pred2 = invLogLink.(A*coef(model2))
pred3 = invInverseLink.(A*coef(model3))

actual = test.Perf
lossModel1 = norm(pred1 - actual)
lossModel2 = norm(pred2 - actual)
lossModel3 = norm(pred3 - actual)

println("Model 1: ", coef(model1))
println("Model 2: ", coef(model2))
println("Model 3: ", coef(model3))
println("\nLoss of models 1,2,3: ",(lossModel1 ,lossModel2, lossModel3))
```

```
Model 1: [-64.40562610624185, 0.05281609072369714, 0.014630060511348993
Model 2: [4.022167583738135, -0.0019725826574401167, 1.2202420632006302
Model 3: [0.007807965416699389, 7.008659356350106e-5, 1.053052307550004

Loss of models 1,2,3: (281.9822744152483, 231.55851725675828, 397.19740
```

Basic Machine learning.

home

Some resources:

- Docs for MLJ.jl (https://alan-turing-institute.github.io/MLJ.jl/dev/) - not used here.
- Docs for Scikitlearn.jl (https://github.com/cstjean/ScikitLearn.jl) - not used here.
- Docs for Flux.jl (https://fluxml.ai/Flux.jl/stable/)
- The Mathematical Engineering of Deep Learning (https://deeplearningmath.org/amsi-summer-school-course-2021.html)
- ...

Applying off the shelf deep neural networks (see https://github.com/FluxML/Metalhead.jl (https://github.com/FluxML/Metalhead.jl))

In [179]:
```
vgg = VGG19(); #A neural network model VGG19 (about 500Mb of parameters,
vgg.layers
```

Out[179]:
```
Chain(
  Conv((3, 3), 3 => 64, relu, pad=1),   # 1_792 parameters
  Conv((3, 3), 64 => 64, relu, pad=1),  # 36_928 parameters
  MaxPool((2, 2)),
  Conv((3, 3), 64 => 128, relu, pad=1),  # 73_856 parameters
  Conv((3, 3), 128 => 128, relu, pad=1),  # 147_584 parameters
  MaxPool((2, 2)),
  Conv((3, 3), 128 => 256, relu, pad=1),  # 295_168 parameters
  Conv((3, 3), 256 => 256, relu, pad=1),  # 590_080 parameters
  Conv((3, 3), 256 => 256, relu, pad=1),  # 590_080 parameters
  Conv((3, 3), 256 => 256, relu, pad=1),  # 590_080 parameters
  MaxPool((2, 2)),
  Conv((3, 3), 256 => 512, relu, pad=1),  # 1_180_160 parameters
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  MaxPool((2, 2)),
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  Conv((3, 3), 512 => 512, relu, pad=1),  # 2_359_808 parameters
  MaxPool((2, 2)),
  Metalhead.var"#44#45"(),
  Dense(25088, 4096, relu),             # 102_764_544 parameters
  Dropout(0.5),
  Dense(4096, 4096, relu),              # 16_781_312 parameters
  Dropout(0.5),
  Dense(4096, 1000),                    # 4_097_000 parameters
  NNlib.softmax,
)                   # Total: 38 arrays, 143_667_240 parameters, 548.052
```

In [180]:
```
#download an arbitrary image and try to classify it
download("https://deeplearningmath.org/data/images/appleFruit.jpg","appl
img = load("appleFruit.jpg")
```

Out[180]:

In [181]:
```
@time begin
    classify(vgg,img) #vgg is the model
end
```

```
  0.348432 seconds (1.31 k allocations: 480.540 MiB)
```

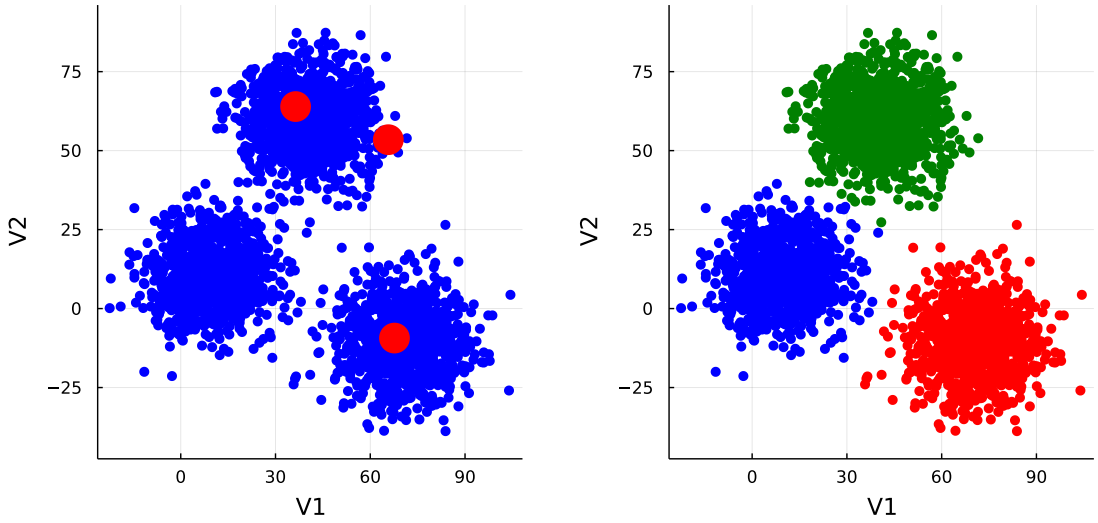
Out[181]:
```
"Granny Smith"
```

Clustering with *[Math Processing Error]*-means

In [182]:
```
fix_seed!()

K = 3
df = RDatasets.dataset("cluster", "xclara")
data = Matrix(df)'

seeds = initseeds(:rand, data, K)
xclaraKmeans = kmeans(data, K, init = seeds)

println("Number of clusters: ", nclusters(xclaraKmeans))
println("Counts of clusters: ", counts(xclaraKmeans))

df.Group  = assignments(xclaraKmeans)

p1 = scatter(df[:, :V1], df[:, :V2], c=:blue, msw=0)
     scatter!(df[seeds, :V1], df[seeds, :V2], markersize=12, c=:red, msv

p2 = scatter( df[df.Group .== 1, :V1], df[df.Group .== 1, :V2], c=:blue,
     scatter!( df[df.Group .== 2, :V1], df[df.Group .== 2, :V2], c=:red,
     scatter!( df[df.Group .== 3, :V1], df[df.Group .== 3, :V2], c=:gree

plot(p1,p2,legend=:none,ratio=:equal, size=(800,400), xlabel="V1", ylabe
```

```
Number of clusters: 3
Counts of clusters: [899, 952, 1149]
```

Out[182]:



The MNIST digits dataset

In [183]:
```
using MLDatasets
```

In [184]:
```
train_data = MLDatasets.MNIST.traindata(Float64)

imgs = train_data[1]
@show typeof(imgs)
@show size(imgs)

labels = train_data[2]
@show typeof(labels);
```

```
typeof(imgs) = Array{Float64, 3}
size(imgs) = (28, 28, 60000)
typeof(labels) = Vector{Int64}
```

In [185]:
```
test_data = MLDatasets.MNIST.testdata(Float64)
test_imgs = test_data[1]
test_labels = test_data[2]
@show size(test_imgs);
```

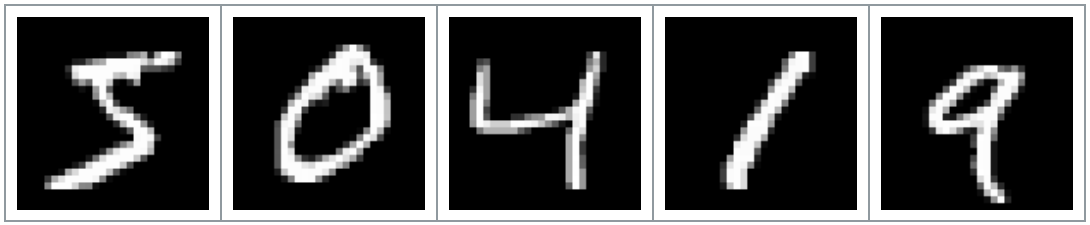
```
size(test_imgs) = (28, 28, 10000)
```

In [186]:
```
n_train, n_test = length(labels), length(test_labels)
```

Out[186]:
```
(60000, 10000)
```

In [187]:
```
[Gray.(train_data[1][:,:,k]') for k in 1:5]
```

Out[187]:

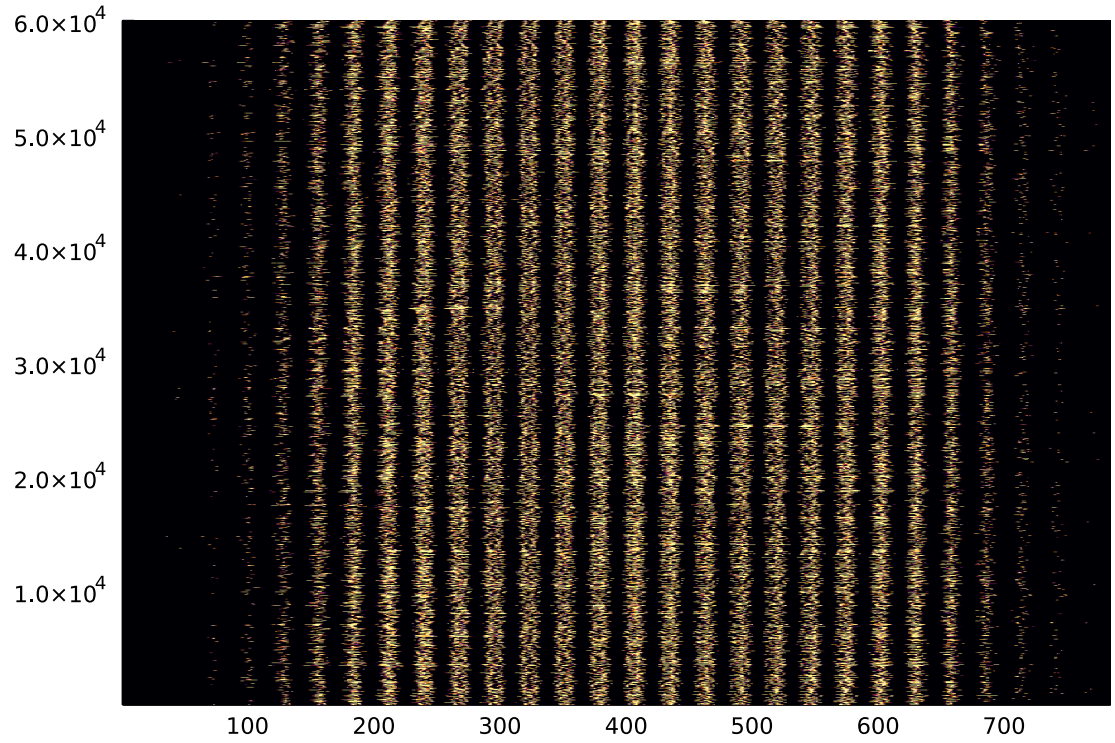


(a vector displayed as a row to save space)

In [188]:
```
X = vcat([vec(imgs[:,:,k])' for k in 1:last(size(imgs))]...)
@show size(X)
heatmap(X,legend=false)
```

```
size(X) = (60000, 784)
```

Out[188]:

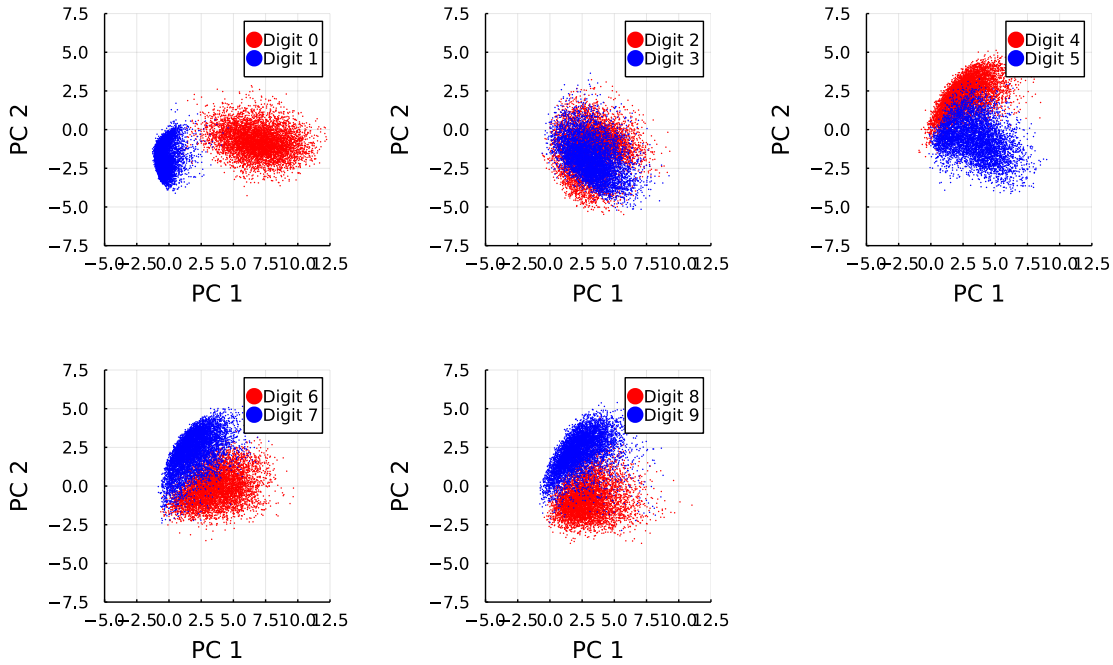


Principal component analysis (PCA)

In [189]:

```
pca = fit(PCA, X'; maxoutdim=2)
M = projection(pca)

args = (ms=0.8, msw=0, xlims=(-5,12.5), ylims=(-7.5,7.5),
            legend = :topright, xlabel="PC 1", ylabel="PC 2")

function compareDigits(dA,dB)
    xA = X[labels .== dA, :]'
    xB = X[labels .== dB, :]'
    zA, zB = M'*xA, M'*xB

    scatter(zA[1,:], zA[2,:], c=:red, label="Digit $(dA)"; args...)
    scatter!(zB[1,:], zB[2,:], c=:blue, label="Digit $(dB)"; args...)
end

plots = []
for k in 1:5
    push!(plots,compareDigits(2k-2,2k-1))
end
plot(plots...,size = (800, 500), margin = 5mm)
```

Out[189]:



A linear classifier

In [190]:
```
using Flux: onehotbatch

A = [ones(n_train) X]
Adag = pinv(A)
tfPM(x) = x ? +1 : -1
yDat(k) = tfPM.(onehotbatch(labels,0:9)'[:,k+1])
bets = [Adag*yDat(k) for k in 0:9]

linear_classify(square_image) = findmax([([1 ; vec(square_image)])'*bets
```

Out[190]:
```
linear_classify (generic function with 1 method)
```

In [191]:
```
predictions = [linear_classify(test_imgs[:,:,k]) for k in 1:n_test]
confusionMatrix = [sum((predictions .== i) .& (test_labels .== j)) for i
acc = sum(diag(confusionMatrix))/n_test

println("Accuracy: ", acc, "\nConfusion Matrix:")
show(stdout, "text/plain", confusionMatrix)
```

```
Accuracy: 0.8603
Confusion Matrix:
10×10 Matrix{Int64}:
 944     0   18    4    0   23   18    5   14   15
   0  1107   54   17   22   18   10   40   46   11
   1     2  813   23    6    3    9   16   11    2
   2     2   26  880    1   72    0    6   30   17
   2     3   15    5  881   24   22   26   27   80
   7     1    0   17    5  659   17    0   40    1
  14     5   42    9   10   23  875    1   15    1
   2     1   22   21    2   14    0  884   12   77
   7    14   37   22   11   39    7    0  759    4
   1     0    5   12   44   17    0   50   20  801
```

Training neural networks

In [192]:
```
train_range, validate_range = 1:5000, 5001:10000
batch_size = 1000

train_imgs = imgs[:,:,train_range]
train_labels = labels[train_range]
mb_idxs = Iterators.partition(1:length(train_range), batch_size)

validate_imgs = imgs[:,:,validate_range]
validate_labels = labels[validate_range];
```

In [193]:
```
function minibatch(x, y, index_range)
#     xBatch = Array{Float32}(undef, size(x[1])..., 1, length(indexRange
    x_batch = Array{Float32}(undef, 28,28, 1, length(index_range))
    for i in 1:length(index_range)
        x_batch[:, :, :, i] = Float32.(x[:,:,index_range[i]])
    end
    return (x_batch, onehotbatch(y[index_range], 0:9))
end

train_set = [minibatch(train_imgs, train_labels, bi) for bi in mb_idxs];
@show length(train_set)
validate_set = [minibatch(validate_imgs,validate_labels, 1:length(valida
@show length(validate_set);
```

```
length(train_set) = 5
length(validate_set) = 1
```

In [194]:
```
using Flux: onehotbatch, onecold, crossentropy, flatten, params
```

In [195]:
```
#A dense model
model1 = Chain(flatten,
               Dense(784, 200, relu),
               Dense(200, 100, tanh),
               Dense(100, 10, sigmoid),
               softmax)
```

Out[195]:
```
Chain(
  Flux.flatten,
  Dense(784, 200, relu),                # 157_000 parameters
  Dense(200, 100, tanh),                # 20_100 parameters
  Dense(100, 10, σ),                    # 1_010 parameters
  NNlib.softmax,
)                   # Total: 6 arrays, 178_110 parameters, 696.117 KiB.
```

In [196]:
```
#A convolutional model
model2 = Chain(
            Conv((5, 5), 1=>8, relu), MaxPool((2,2)),
            Conv((3, 3), 8=>16, relu), MaxPool((2,2)),
            flatten,
            Dense(400, 10),
            softmax)
```

Out[196]:
```
Chain(
  Conv((5, 5), 1 => 8, relu),           # 208 parameters
  MaxPool((2, 2)),
  Conv((3, 3), 8 => 16, relu),          # 1_168 parameters
  MaxPool((2, 2)),
  Flux.flatten,
  Dense(400, 10),                       # 4_010 parameters
  NNlib.softmax,
)                   # Total: 6 arrays, 5_386 parameters, 22.070 KiB.
```

In [197]:

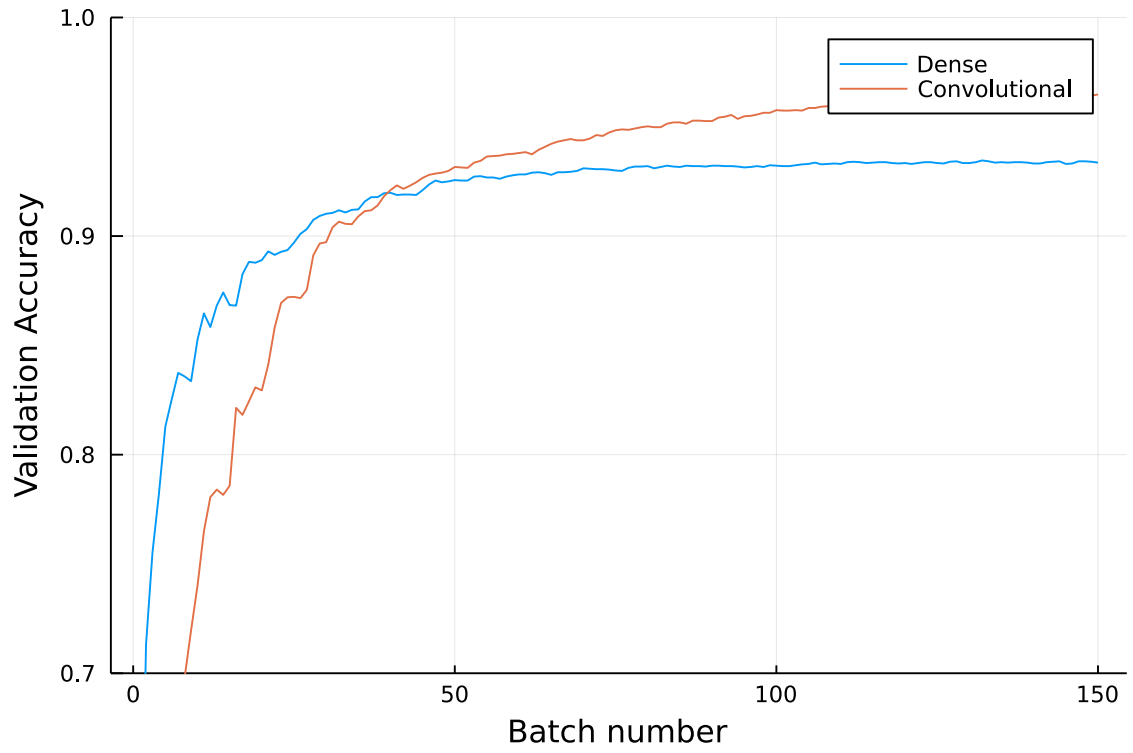
```
fix_seed!()

epochs = 30

η = 5e-3 #learning rate
opt1, opt2 = ADAM(η), ADAM(η)

accuracyPaths = [[],[]]

accuracy(x, y, model) = mean(onecold(model(x)) .== onecold(y))
loss(x, y, model) = crossentropy(model(x), y)

cbF1() = begin
            acc = accuracy(first(validate_set)..., model1)
            print("$acc, ")
            push!(accuracyPaths[1],acc)
        end

cbF2() = begin
            acc = accuracy(first(validate_set)..., model2)
            print("$acc, ")
            push!(accuracyPaths[2],acc)
        end

model1(train_set[1][1]); model2(train_set[1][1])

for ep in 1:epochs
    print("\nEpoch $ep \n  Model 1 accuracy: ")
    Flux.train!((x,y)->loss(x,y,model1), params(model1), train_set, opt1
    print("  Model 2 accuracy: ")
    Flux.train!((x,y)->loss(x,y,model2), params(model2), train_set, opt2
end

println("\n\nFinal Model1 (Dense) accuracy = ", accuracy(first(validate_
println("Final Model2 (Convolutional) accuracy = ", accuracy(first(valic

plot(accuracyPaths,label = ["Dense" "Convolutional"],
    ylim=(0.7,1.0), xlabel="Batch number", ylabel = "Validation Accuracy
```

```
Epoch 1
  Model 1 accuracy: 0.567, 0.7132, 0.7552, 0.782, 0.8128,   Model 2 accu
Epoch 2
  Model 1 accuracy: 0.8254, 0.8374, 0.8358, 0.8336, 0.8526,   Model 2 ac
```

```
Epoch 3
  Model 1 accuracy: 0.8646, 0.8584, 0.8682, 0.8742, 0.8684,   Model 2 ac
Epoch 4
  Model 1 accuracy: 0.8682, 0.8826, 0.8882, 0.8878, 0.889,   Model 2 acc
Epoch 5
  Model 1 accuracy: 0.893, 0.8914, 0.8928, 0.8936, 0.897,   Model 2 accu
Epoch 6
  Model 1 accuracy: 0.901, 0.9032, 0.9074, 0.9092, 0.9102,   Model 2 acc
Epoch 7
  Model 1 accuracy: 0.9106, 0.9118, 0.9108, 0.912, 0.9122,   Model 2 acc
Epoch 8
  Model 1 accuracy: 0.9158, 0.9178, 0.9178, 0.9196, 0.9198,   Model 2 ac
Epoch 9
  Model 1 accuracy: 0.9188, 0.919, 0.919, 0.9188, 0.921,   Model 2 accu
Epoch 10
  Model 1 accuracy: 0.9236, 0.9254, 0.9246, 0.925, 0.9256,   Model 2 acc
Epoch 11
  Model 1 accuracy: 0.9254, 0.9254, 0.9272, 0.9274, 0.9268,   Model 2 ac
Epoch 12
  Model 1 accuracy: 0.9268, 0.9262, 0.9272, 0.9278, 0.9282,   Model 2 ac
Epoch 13
  Model 1 accuracy: 0.9282, 0.929, 0.9292, 0.9288, 0.928,   Model 2 accu
Epoch 14
  Model 1 accuracy: 0.9292, 0.9292, 0.9294, 0.9298, 0.931,   Model 2 acc
Epoch 15
  Model 1 accuracy: 0.9308, 0.9306, 0.9306, 0.9304, 0.93,   Model 2 accu
Epoch 16
  Model 1 accuracy: 0.9298, 0.9312, 0.9318, 0.9318, 0.932,   Model 2 acc
Epoch 17
  Model 1 accuracy: 0.931, 0.9316, 0.9322, 0.9318, 0.9316,   Model 2 acc
Epoch 18
  Model 1 accuracy: 0.9322, 0.932, 0.932, 0.9318, 0.9322,   Model 2 accu
Epoch 19
  Model 1 accuracy: 0.9322, 0.932, 0.932, 0.9318, 0.9314,   Model 2 accu
Epoch 20
  Model 1 accuracy: 0.9316, 0.932, 0.9316, 0.9324, 0.9322,   Model 2 acc
Epoch 21
  Model 1 accuracy: 0.932, 0.932, 0.9324, 0.9328, 0.933,   Model 2 accu
Epoch 22
  Model 1 accuracy: 0.9336, 0.9328, 0.933, 0.9332, 0.933,   Model 2 accu
Epoch 23
  Model 1 accuracy: 0.9338, 0.934, 0.9338, 0.9334, 0.9336,   Model 2 acc
Epoch 24
  Model 1 accuracy: 0.9338, 0.9338, 0.9334, 0.9332, 0.9334,   Model 2 ac
```

```
Epoch 25
  Model 1 accuracy: 0.933, 0.9334, 0.9338, 0.9338, 0.9334,   Model 2 ac
Epoch 26
  Model 1 accuracy: 0.9332, 0.934, 0.9342, 0.9334, 0.9334,   Model 2 ac
Epoch 27
  Model 1 accuracy: 0.9338, 0.9346, 0.9342, 0.9336, 0.9338,   Model 2 a
Epoch 28
  Model 1 accuracy: 0.9336, 0.9338, 0.9338, 0.9336, 0.9332,   Model 2 a
Epoch 29
  Model 1 accuracy: 0.9332, 0.9338, 0.934, 0.9342, 0.933,   Model 2 acc
Epoch 30
  Model 1 accuracy: 0.9332, 0.9342, 0.9342, 0.934, 0.9336,   Model 2 ac

Final Model1 (Dense) accuracy = 0.9336
Final Model2 (Convolutional) accuracy = 0.9648
```

Out[197]:



The End

I hope you enjoyed the tutorial.

Many more of these types of examples + statistical background are at statisticswithjulia.org (https://statisticswithjulia.org/).