Introduction Rules | **Destruction Rules** | **Elimination Rules**

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⊤ Top

$$\frac{}{\top}\ \top I$$

TrueI: $True$

⊥ Bottom

$$\frac{\bot}{P}\ \bot E$$

FalsE: $False \Longrightarrow P$

→ Implication

$$\frac{\frac{P}{Q}}{P \to Q}\ \to I$$

impI: $(P \Longrightarrow Q) \Longrightarrow P \to Q$

$$\frac{P \to Q \qquad P}{Q}\ \to D$$

mp: $P \longrightarrow Q \Longrightarrow P \Longrightarrow Q$

(modus ponens)

$$\frac{P \to Q \qquad \frac{Q}{R} \qquad P}{R}\ \to E$$

impE: $P \longrightarrow Q \Longrightarrow P \Longrightarrow (Q \Longrightarrow R) \Longrightarrow R$

∧ Conjunction

$$\frac{P \qquad Q}{P \wedge Q}\ \wedge I$$

conjI: $P \Longrightarrow Q \Longrightarrow P \wedge Q$

$$\frac{P \wedge Q}{P}\ \wedge D1$$

conjunct1: $P \wedge Q \Longrightarrow P$

$$\frac{P \wedge Q}{Q}\ \wedge D2$$

conjunct2: $P \wedge Q \Longrightarrow Q$

$$\frac{P \wedge Q \qquad \frac{P \quad Q}{R}}{R}\ \wedge E$$

conjE: $P \wedge Q \Longrightarrow (P \Longrightarrow Q \Longrightarrow R) \Longrightarrow R$

∨ Disjunction

$$\frac{P}{P \vee Q}\ \vee I1$$

disjI1: $P \Longrightarrow P \vee Q$

$$\frac{Q}{P \vee Q}\ \vee I2$$

disjI2: $Q \Longrightarrow P \vee Q$

$$\frac{P \vee Q \qquad \frac{P}{R} \qquad \frac{Q}{R}}{R}\ \wedge E$$

disjE: $P \vee Q \Longrightarrow (P \Longrightarrow R) \Longrightarrow (Q \Longrightarrow R) \Longrightarrow R$

↔ Biconditional

$$\frac{\frac{P}{Q} \qquad \frac{Q}{P}}{P \leftrightarrow Q}\ \leftrightarrow I$$

iffI: $(P \Longrightarrow Q) \Longrightarrow (Q \Longrightarrow P) \Longrightarrow P = Q$

$$\frac{P \leftrightarrow Q \qquad P}{Q}\ \leftrightarrow D1$$

iffD1: $P = Q \Longrightarrow P \Longrightarrow Q$

$$\frac{P \leftrightarrow Q \qquad Q}{P}\ \leftrightarrow D2$$

iffD2: $P = Q \Longrightarrow Q \Longrightarrow P$

$$\frac{P \leftrightarrow Q \qquad \frac{P \to Q \quad Q \to P}{R}}{R}\ \leftrightarrow E$$

iffE: $P = Q \Longrightarrow (P \longrightarrow Q \Longrightarrow Q \longrightarrow P \Longrightarrow R) \Longrightarrow R$

∀ Universal quantifier

$$\frac{P\,d}{\forall x\,.\,P\,x}\ {*}\,\forall I$$

* *for fixed but arbitrary d*

allI: $\left(\bigwedge x\,.\,P\,x\right) \Longrightarrow \forall x\,.\,P\,x$

$$\frac{\forall x\,.\,P\,x}{P\,d}\ \forall D$$

* *for some specific d*

spec: $\forall x\,.\,P\,x \Longrightarrow P\,x$

$$\frac{\forall x\,.\,P\,x \qquad \frac{P\,d}{R}}{R}\ \forall E$$

* *for some specific d*

allE: $\forall x\,.\,P\,x \Longrightarrow (P\,x \Longrightarrow R) \Longrightarrow R$

∃ Existential quantifier

$$\frac{P\,d}{\exists x\,.\,P\,x}\ {*}\,\exists I$$

* *for some specific d*

exI: $P\,x \Longrightarrow \exists x\,.\,P\,x$

$$\frac{\exists x\,.\,P\,x \qquad \frac{P\,d}{R}}{R}\ \exists E$$

* *for fixed but arbitrary d*

exE: $\exists x\,.\,P\,x \Longrightarrow \left(\bigwedge x\,.\,P\,x \Longrightarrow R\right) \Longrightarrow R$

Read **ruleName:** $P \Longrightarrow (Q \Longrightarrow R) \Longrightarrow S$ as assuming P, and assuming that from Q you can deduce R, then you can conclude S.

$\neg$ Negation

$$\frac{\frac{P}{\bot}}{\neg P}\ \neg I$$

notI: $(P \Longrightarrow False) \Longrightarrow \neg P$

$$\frac{P \to Q}{\neg Q \to \neg P}$$

not_mono: $Q \longrightarrow P \Longrightarrow \neg P \longrightarrow \neg Q$

(contrapositive)

$$\frac{P \qquad \neg P}{R}\ \neg E$$

notE: $\neg P \Longrightarrow P \Longrightarrow R$

$=$ Equality

$$\frac{}{t = t}$$

refl: $t = t$

(reflexivity)

$$\frac{t_1 = t_2 \qquad t_2 = t_3}{t_1 = t_3}$$

trans: $r = s \Longrightarrow s = t \Longrightarrow r = t$

(transitivity)

$$\frac{t_1 = t_2}{t_2 = t_1}$$

sym: $s = t \Longrightarrow t = s$

(symmetry)

$$\frac{t_1 = t_2 \qquad P\ t_1}{P\ t_2}$$

subst: $s = t \Longrightarrow P\ s \Longrightarrow P\ t$

f Functions

$$\frac{}{f = g \leftrightarrow (\forall x\,.\, f\,x = g\,x)}$$

fun_eq_iff: $(f = g) = (\forall x\,.\, f\,x = g\,x)$

$$\frac{f\,d = g\,d}{f = g}$$

* for fixed but arbitrary d

ext: $\left(\bigwedge x\,.\, f\,x = g\,x\right) \Longrightarrow f = g$

$$\frac{x = y}{f\,x = f\,y}$$

arg_cong: $x = y \Longrightarrow f\,x = f\,y$

$\neg$ Classical Logic

$$\frac{}{P \vee \neg P}$$

excluded_middle: $\neg P \vee P$

$$\frac{\neg\neg P}{P}$$

notnotD: $\neg\neg P \Longrightarrow P$

$$\frac{\frac{\neg P}{\bot}}{P}$$

ccontr: $(\neg P \Longrightarrow False) \Longrightarrow P$

$$\frac{\neg Q \to \neg P}{P \to Q}$$

$$\frac{\frac{P}{Q} \qquad \frac{\neg P}{Q}}{Q}$$

case_split: $(P \Longrightarrow Q) \Longrightarrow (\neg P \Longrightarrow Q) \Longrightarrow Q$

Forward reading:
If we assume P
we can conclude Q

$$\frac{P}{Q}$$

Backward reading:
To prove Q
we need to show first P