**Solution to “The Impossible Puzzle” on 538**

I solved this puzzle by creating two grids—one for the set of possible products that Pete could have received on this piece of paper, and the second for possible sums that Susan could have received on her piece of paper—and systematically eliminating cells as possible candidates. Both grids have numbers 1 through 9 as row labels, and 1 through 9 as column labels. While for Pete each cell value is the product of row and column labels—for example, the cell (9,9) has value 81, (8,9) has 72, (7,9) has 63, etc.—the values of the cells in Susan’s grid are the sum of the row and column labels. For example, in Susan’s grid the value of cell (9,9) is 18, (8,9) has value 17, (7,9) has 16, etc. For simplicity’s sake, mirror-image cells that are duplicates can be immediately removed or ignored: cell (6,7), for instance, will have the same value as cell (7,6), so once can be disregarded entirely.

At the beginning of turn 1, because Barack asks Pete first if he knows the two numbers and Pete responds “no,” we can eliminate all cells on Pete’s grid whose values are unique to his entire grid (values only appearing once in the matrix). This eliminates 27 cells: those with values 81, 72, 63, 54, 45, 27, 64, 56, 48, 40, 32, 49, 42, 35, 28, 21, 14, 7, 30, 25, 20, 15, 10, 5, 3, 2, and 1. Because each of these numbers would allow Pete to identify two unique factors, we can also eliminate the 27 cells in Susan’s grid that correspond to the positions of these cells in Pete’s grid.

After doing so, we can then evaluate Susan’s first response of “no.” This indicates that we can eliminate cells whose values are unique to her entire grid. In addition to the eliminations corresponding to Pete’s grid, we can remove 3 addition cells on Susan’s grid: (4,9) with value 13, (6,6) with value 12, and (2,2) with value 4. This in turn allows us to return to Pete’s grid and eliminate the cells corresponding to these eliminations on Susan’s grid. On Pete’s grid, cells (4,9), (6,6) and (2,2)—with values of 36, 36 and 4, respectively—can also be eliminated. So at the end of round one, there only 15 cells remaining on both Pete’s and Sarah’s grids, which implies that there are only 15 remaining combinations of numbers as solutions to this puzzle.

At the beginning of turn 2, Pete again responds with a “no,” so we can again remove any cells with unique values, which in this case is only cell (1,4) with value 4. This implies that the same cell (1,4) with value 5 on Sarah’s grid can also be eliminated. Sarah also responds during turn 2 with a “no,” meaning we can also eliminate cell (2,3) on her grid, with value 5, and the corresponding cell (2,3) on Pete’s grid, which has value 6. So, Pete and Sarah now each have 13 cells remaining on their grids.

We proceed with a similar process of sequential elimination during turns 3 and 4.

At the beginning of turn 5, we know Pete replies with a “yes,” which indicates the product of two numbers on his piece of paper must be a unique value on his grid. At this point, the values remaining on his grid are 18, 9, 24, 16, 8, 24, 18, 8 and 9—so the unique value is 16. This corresponds to cell (2,8), so the two factors are 2 and 8!

However, had Pete responded on the 5th turn with a “no,” this would have allowed us to remove the unique 16 in cell (2,8) from his grid. After eliminating the corresponding cell on Susan’s grid, she would still have these values remaining: 11, 10, 11, 10, 9, 10, 9, 6 and 6. Therefore, she would have had to respond to Barack’s fifth question with a “no” because at that point there would have been no unique values on her grid.