Fubini's theorem

1. What are the differences between double integrals and iterated integrals?

   - What is Fubini's theorem?

$$\iint_{[0,1]\times[0,1]} \frac{y}{1+xy}dxdy$$

   - ~~When does Fubini's theorem fails? Compute~~ $\cancel{\int_0^1 \int_0^1 f(x,y)dydx}$ ~~for~~

$$f(x,y) = \begin{cases} \dfrac{x^2-y^2}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

2. Change of limits

   - Compute $\iint_D e^{-y^2} dxdy$ for

$$D = \{(x,y) \,|\, 0 \le x \le 1,\, x \le y \le 1\}$$

   - Compute the triple integral: region V is bounded by $2x = y$, $2x = y + 2$, $y = 0$, $y = 4$, $z = 0$, $z = 3$.

$$\iiint_V \frac{2x-y}{2} + \frac{z}{3} dxdydz$$

Surface integrals on planar surfaces

1. How to define a surface integral

   - How to parametrize a surface?
   - How do we subdivide planar surfaces?
   - How to generalize double integrals to surface integrals?

2. What is a flux? What does it mean?

   - How do we find a flux of a vector field?
   - How to find normal vector to the surface?

Homework

- Reading assignment
  - Chapter §2.1 ~§2.2
- Writing assignment (due **Sep. 21th, 11:59pm**)

1. Explain why the double integral $\iint_D f(x,y)dxdy$ computes the mass of D provided that $f(x,y)$ is the density at (x,y).
2. Prove the following statement if it is true, or give a counter example if false.
   (a) If $f(x,y)$ is continuous on the domain $[a,b]\times[c,d]$, then
   $$\int_a^b \int_c^d f(x,y)dxdy = \int_c^d \int_a^b f(x,y)dydx$$
   (b) If a function $f(x,y,z)$ is continuous on $V=\{0\le z\le 2, 0\le y\le x, 0\le z\le y\}$, then
   $$\int_0^2 \int_0^x \int_0^y f(x,y,z)dzdydx = \int_0^2 \int_0^y \int_0^x f(x,y,z)dzdxdy$$
3. Let P be bounded plane in $\mathbf{R}^3$. Let $z=f(x,y,z)$ be a real-valued function and $\mathbf{F}(x,y,z)$ be a vector field on.
   (a) Although f is a 3-variable function, the surface integral
   $$\iint_P f dA$$
   can be computed by double integral instead of triple integral. Explain why.
   (b) Explain the difference between
   $$\iint_P f dA \text{ and } \iint_P \mathbf{F}\cdot d\mathbf{A}$$