

Integration by substitution

- Why do we need to substitute?
 - How can we use spherical, cylindrical coordinates to parametrize surface or volume?
- What is a Jacobian?
 - What is a transformation?
 - What does a differential of a transformation does to an infinitesimal volume.
 - What is the meaning of the determinant of a differential?
- What is the formula of integration by substitution ...
 - for double integrals?
 - for triple integrals?

Example

- Find the volumes of the unit sphere divided by the saddle surface $z = x^2 - y^2$.
- Verify the divergence theorem holds for the vector field

$$\mathbf{F}(x, y, z) = (x^2, y, \sin(z))$$

on each volume above.

Homework

- Reading assignment
 - Chapter §6.1 – §6.2.
- Writing assignment (due **Nov. 9nd, 11:59pm**)
 1. Explain how to compute the Jacobian of a transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ carefully.
 2. Explain the integration by substitution carefully.
 3. Let V be the ellipsoid $\frac{x^2}{2} + \frac{y^2}{3} + \frac{z^2}{4} \leq 1$.
 - (a) Find the transformation $T : [0, 1] \times [0, \pi] \times [0, 2 * \pi] \rightarrow V$
 - (b) Using the above transformation, find the volume of V .