Integration by substition

- Why do we need to substitute?
  - How can we use spherical, cylinderical coordinates to parametrize surface or volume?
- What is a Jacobian?
  - What is a transformation?
  - What does a differential of a transformation does to an infinitesimal volume.
  - What is the meaning of the determinant of a differential?
- What is the formula of integration by subsitution ...
  - for double integrals?
  - for triple integrals?

Example

- Find the volumes of the unit sphere divided by the saddle surface $z = x^2 - y^2$.
- Verify the divergence theorem holds for the vector field

$$\mathbf{F}(x, y, z) = (x^2, y, \sin(z))$$

on each volume above.

Homework

- Reading assignment
  - Chapter §6.1 – §6.2.
- Writing assignment (due **Nov. 9nd, 11:59pm**)

1. Explain how to compute the Jacobian of a transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ carefully.
2. Explain the integration by substitution carefully.
3. Let V be the ellipsoid $\frac{x^2}{2} + \frac{y^2}{3} + \frac{z^2}{4} \leq 1$.
   (a) Find the transformation $T : [0, 1] \times [0, \pi] \times [0, 2\pi] \to V$
   (b) Using the above transformation, find the volume of V.