

Fubini's theorem

1. What are the differences between double integrals and iterated integrals?

- What is Fubini's theorem?

$$\iint_{[0,1] \times [0,1]} \frac{y}{1+xy} dx dy$$

- When does Fubini's theorem fails? Compute $\int_0^1 \int_0^1 f(x,y) dy dx$ for

$$f(x,y) = \begin{cases} \frac{x^2 - y^2}{(x^2 + y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

2. Change of limits

- Compute $\iint_D e^{-y^2} dx dy$ for

$$D = \{(x,y) \mid 0 \leq x \leq 1, x \leq y \leq 1\}$$

- Compute the triple integral: region V is bounded by $2x = y$, $2x = y + 2$, $y = 0$, $y = 4$, $z = 0$, $z = 3$.

$$\iiint_V \frac{2x - y}{2} + \frac{z}{3} dx dy dz$$

Surface integrals on planar surfaces

1. How to define a surface integral

- How to parametrize a surface?
- How do we subdivide planar surfaces?
- How to generalize double integrals to surface integrals?

2. What is a flux? What does it mean?

- How do we find a flux of a vector field?
- How to find normal vector to the surface?

Homework

- Reading assignment

- Chapter §2.1 ~§2.2

- Writing assignment (due **Sep. 21th, 11:59pm**)

1. Explain why the double integral $\iint_D f(x,y)dx dy$ computes the *mass* of D provided that $f(x,y)$ is the *density* at (x,y) .

2. Prove the following statement if it is true, or give a counter example if false.

- (a) If $f(x,y)$ is continuous on the domain $[a,b] \times [c,d]$, then

$$\int_a^b \int_c^d f(x,y)dx dy = \int_c^d \int_a^b f(x,y)dy dx$$

- (b) If a function $f(x,y,z)$ is continuous on $V = \{0 \leq z \leq 2, 0 \leq y \leq x, 0 \leq z \leq y\}$, then

$$\int_0^2 \int_0^x \int_0^y f(x,y,z)dz dy dx = \int_0^2 \int_0^y \int_0^x f(x,y,z)dz dx dy$$

3. Let P be bounded plane in \mathbf{R}^3 . Let $z = f(x,y,z)$ be a real-valued function and $\mathbf{F}(x,y,z)$ be a vector field on.

- (a) Although f is a 3-variable function, the surface integral

$$\iint_P f dA$$

can be computed by double integral instead of triple integral. Explain why.

- (b) Explain the difference between

$$\iint_P f dA \text{ and } \iint_P \mathbf{F} \cdot d\mathbf{A}$$