Week 09 (19.10.28-11.1) SE102, Fall 2019 DGIST

## Divergence theorem

- $\nabla$ (nabla) operator
  - How to define divergence?
  - What are the properties of  $\nabla$  operators?

## Divergence theorem

- What is a divergence of a vector field?
- What is a diverence theorem?
- What is a meaning of diverence of a vector field?

## Homework

- Reading assignment
  - Chapter §3.4 §4.3.
- Writing assignment (due **Nov. 2nd, 11:59pm**) Let  $\mathbf{F}(x,y,z) = (P(x,y,z), Q(x,y,z), R(x,y,z))$ ,  $\mathbf{G} = (U(x,y,z), V(x,y,z), W(x,y,z))$  are vector fields on  $\mathbf{R}^3$ .
  - 1. Write the following formula explicitly in terms of *P*, *Q*, *R*, *U*, *V*, *W*.
    - (a)  $\nabla \cdot (F+G)$
    - (b)  $\nabla \times (F+G)$
    - (c)  $\nabla \cdot (F \times G)$
    - (d)  $\nabla \times (F \times G)$
  - 2. Let  $f: \mathbb{R}^3 \to \mathbb{R}$  be a differentiable function. Prove the following identity.
    - (a)  $\nabla \cdot (f\mathbf{F}) = f\nabla \cdot \mathbf{F} + \nabla f \cdot \mathbf{F}$
    - (b)  $\nabla \times (f\mathbf{F}) = f\nabla \times \mathbf{F} + \nabla f \times \mathbf{F}$
  - 3. Explain the meaning of the following statement.
    - (a) A surface is closed.
    - (b) A surface is oriented.
  - 4. Explain the divergence theorem explicitly. (No need to prove.)