Chain rule and gradient

- Chain rule (revisited)
  - What is a partial derivative?
  - What is a differential?
  - What is a chain rule?
- Gradient ∇f
  - What is a gradient?
  - What does gradient ∇f represent on the surface $z = f(x, y)$? (What is a level set?)
  - How to compute partial derivative of a function with a given direction?
  - What happens when $\nabla f = \vec{0}$?

Homework

- Reading assignment
  - Chapter §3.2, §5.2 – 5.3.
- Writing assignment (due **Nov. 23nd, 11:59pm**)

1. Let $f(x_1, \cdots, x_n)$ be a real-valued multivariable function, i.e. $w = f(x_1, \cdots, x_n)$ is a real number. Let $c(t) = (x_1(t), \cdots, x_n(t))$ be a n-dimensional differentiable curve. From the chain rule, we have

$$(f \circ c)'(t) = df'(t) \cdot dc(t)$$

   Is this result different from the inner product $\nabla f(t) \circ c'(t)$? Explain your answer.
2. Let $c(t)$ be a level curve of $z = f(x, y)$ at $z = c_0$. Explain why ∇f is always perpendicular to the curve c at $c(t)$.
3. Find an example of $f(x, y)$ where $\nabla f(1, \sqrt{2}) = \vec{0}$ and draw the graph of $z = f(x, y)$ near $(1, \sqrt{2})$. (You must write the explicit formula of $f(x, y)$. You may use matlab for drawing the surface. Use `surf` command for instance.)