Partial derivative

- For 2-variable function $f(x, y)$, how do we define partial derivatives $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$?
- At the point $(x_0, y_0, f(x_0, y_0))$ on a surface S of the graph of $z = f(x, y)$, how to compute the normal vector to the surface S?

Surface integrals

- What is a parametrization of a surface?
  - How can we parametrize $z = x^2 - y^2$ for $(x, y) \in [0, 1] \times [0, 1]$?
  - How can we parametrize the sphere $x^2 + y^2 + z^2 = 1$?
- How do we compute the surface integral $\iint_S f dS$ when ...
  - S **is the graph of** $z = f(x, y)$ **on** $(x, y) \in D \subset \mathbf{R}^2$. For example, if $f(x, y) = x^2 - y^2$ and $D = [0, 1] \times [0, 1]$, what is
  $$\iint_S x + z dS = ?$$
  - S **is given by the parametrization** $X : D \to S$. For example,
  $$X(u, v) = (\sin u \cos v, \sin u \sin v, \cos u), \quad (u, v) \in [0, 2\pi] \times [0, \pi]$$
  is a parametrization of a surface $S \subset \mathbf{R}^3$, what is
  $$\iint_S dS = ?$$
- How do we compute the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$?
  - Compute when S is the unit sphere $x^2 + y^2 + z^2 = 1$ and
  $$\mathbf{F} = \left(\frac{x}{(x^2 + y^2 + z^2)^{3/2}}, \frac{y}{(x^2 + y^2 + z^2)^{3/2}}, \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right)$$
  - Deduce that
  $$\iint_S \mathbf{E}(q) d\mathbf{S} = \frac{q}{\epsilon_0}$$
  when
  $$\mathbf{E}(q) = \frac{q}{4\pi\epsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3}$$
  where $\mathbf{r}(x, y, z) = (x, y, z)$.

Homework

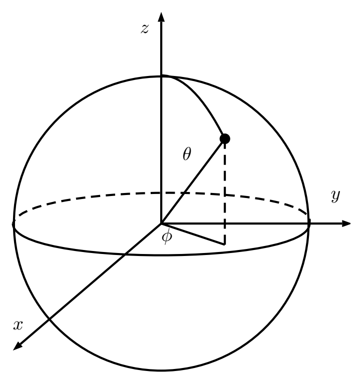
- Reading assignment
  - Chapter §2.4, §3.2, and §4.1
- Writing assignment (due **Sep. 28th, 11:59pm**)
  1. Compute the partial derivative $f_x(0,0)$ for

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{x^4+y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

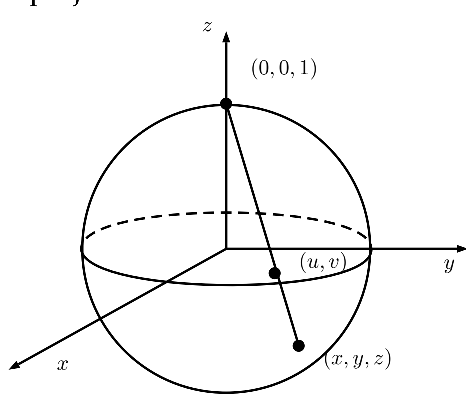
     Note that you cannot obtain a proper value by taking derivative on x by fixing y as a constant. Use the definition of partial derivative to obtain $f_x(0,0)$.
  2. There are three ways to parametrize the sphere $\mathbf{S}^2 = \{x^2+y^2+z^2=1\}$.
     - Split the sphere into two hemispheres:

$$z = \sqrt{1-x^2-y^2}, \quad z = -\sqrt{1-x^2-y^2}$$

     - Use the spherical coordinate θ, ϕ:



     - Using the stereographic projection:



     Write the parametrization function explicitly $X : D \to \mathbf{S}^2$ for each parametrizations.