Verifying Divergence theorem

- Recall that *divergence theorem* states that

$$\iiint_V \nabla \mathbf{F} dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$

We will verify that this for the following situation:

- Let S_1 be the unit sphere and S_2 be the saddle surface defined by

$$S_1 = \{(x,y,z) \mid x^2 + y^2 + z^2 = 1\}, \quad S_2 = \{(x,y,z) \mid z = x^2 - y^2\}$$

Let C be the intersection between S_1 and S_2. The curve C splits the sphere S_1 into two regions $S_1 = S_1^+ \cup S_1^-$ where S_1^+ lies above S_1^-. Let $S_2' \subset S_2$ be the region in S_2 bounded by C. Let V the volume bounded by S_1^+ and S_2'.

- Let $\mathbf{F}$ be a 3-dimensional vector field defined by

$$\mathbf{F}(x,y,z) = (x^2, y, \sin(z))$$

1. Draw the surfaces S_1, S_2, and the curve C together in $\mathbf{R}^3$.
2. Let the orientations of S_1 and S_2 being the *upward*, i.e. the normal vector $\mathbf{n}$ satisfies $\mathbf{n} \cdot \mathbf{k} \geq 0$ for $\mathbf{k} = (0,0,1)$.
   (a) Compute the flux $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$ and $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$.
   (b) Compute the triple integral $\iiint_V \nabla \mathbf{F} dV$
3. Compare the values obtained above and explain why this verifies the divergence theorem.

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