

Chain rule

- What is a meaning of a derivative
 - of a single-variable function $f(x)$?
 - of a parametric curve $c(t) = (x(t), y(t))$?
 - of a multivariable function $f(x, y)$?
- How to take a derivative of $F(t) = f(x(t), y(t))$?
- How to generalize to the derivative of $f(u, v) = (x(u, v), y(u, v))$?

Derivative of multivariable function

- What is a linear transformation?
 - How do we understand a matrix as a linear transformation?
- How do we write the derivative of a multivariable function $f(x_1, \dots, x_n) = (y_1, \dots, y_m)$ as a matrix?
- What is the geometric meaning of the derivative?

Homework

- Reading assignment
 - Chapter §5.2 – §5.4.
- Writing assignment (due **Oct. 5th, 11:59pm**)

1. Answer the following questions for

$$f(x, y) = e^x + x^2 + \sin y, \quad c(t) = (\cos t, 2 \sin t)$$

(a) Write $F(t) = (f \circ c)(t)$ as a single-variable function on t , and find $F'(t)$.

(b) Use chain rule to find $\left. \frac{d}{dt} \right|_{t=0} f(c(t))$.

2. Answer the following questions for

$$g(x, y) = (x^2 - y + 1, xy + y^3)$$

(a) Write the derivative $dg(1, 2)$ as 2×2 matrix and explain the meaning of this matrix.

(b) Let $c(t)$ be the curve in Problem 1, and let $d(t) = (g \circ c)(t)$. Find $d'(0)$.

(c) If $\alpha(t)$ is a curve in \mathbf{R}^2 such that $\alpha(0) = (1, 1)$ and $\alpha'(0) = (-1, 2)$, then what is $\beta(0)$ and $\beta'(0)$ for $\beta(t) = (g \circ \alpha)(t)$?