More on double integrals

- Green's theorem as a special case of Stokes' theorem
- Various tranformation on 2-dimensional domains.
  - Linear transformation.
  - Polar coordinate change.
  - Triangluar domain to rectangular domain.
- Indefinite integrals
  - $\int_{-\infty}^{\infty} e^{-x^2} dx$
  - Gamma function: $\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx$
  - Beta function: $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$
- Computing the volume of 4-dimensional sphere... what?

Homework

- Reading assignment
  - Chapter §6.3.
- Writing assignment (due **Nov. 16nd, 11:59pm**)

1. Let D be a convex n-gon on $\mathbf{R}^2$ with vertices $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ in counter-clockwise order.
   (a) Using Green's theorem on the vector field $F(x, y) = (-y/2, x/2)$, show that
   $$\text{area}(D) = \frac{1}{2}\left(\det \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} + \cdots + \det \begin{bmatrix} x_n & y_n \\ x_1 & y_1 \end{bmatrix}\right)$$
   (b) Given a two-variable function $f(x, y)$ defined on D, explain how to compute the double integral $\iint_D f(x, y) dx dy$.
   (c) Using the method describe above, compute
   $$\iint_D \sin(x)e^y + x^2y^2 dx dy$$
   where D is a regular pentagon whose vertices are
   $$(1, 0), (\cos \frac{2\pi}{5}, \sin \frac{2\pi}{5}), \cdots, (\cos \frac{8\pi}{5}, \sin \frac{8\pi}{5}).$$