May 27

Conditional Expectation : f_X given. $f(y|x)$.

Def: Let X and Y be 2 r.v. (continuous).

The conditional expectation of Y given X is the r.v. denoted $\mathbb{E}(Y|X)$ and is defined as following:

conditional pdf of Y given $X=x$

STEP 1: Define the function $G(x) = \mathbb{E}(Y|X=x)$. $G(x) = \int_{\mathbb{R}} y\, \underline{f(y|x)}\, d$

STEP 2: set $\mathbb{E}(Y|X) = G(x)$

Remark:

① If X and Y are discrete, the definition of $\mathbb{E}(Y|X)$ is the same:

In step 1, write $G(x) = \sum_{y} y\, \underline{P(y|x)}$ conditional p.f of Y given $X=x$.

② We can define more generally $\mathbb{E}(F(Y)|X)$ where $F: \mathbb{R} \to \mathbb{R}$ is a function: In step 1 we will write $G(x) = \mathbb{E}(F(Y)|X=x) = \int_{y_2} F(y) f(y|x)\, dy$ ... and set $\mathbb{E}(F(Y)|X) = G(x)$