투빅스 16기 정규세션 6주차

# Neural Network Basic Assignment

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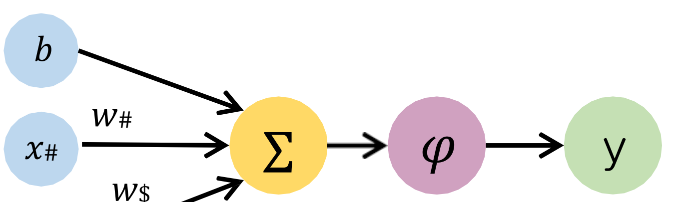
1. Sigmoid Function을 $z$에 대해 미분하세요.

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

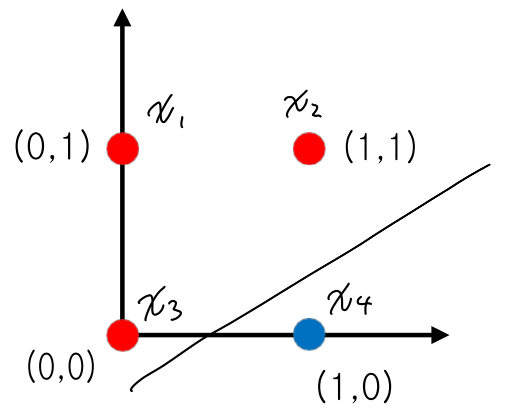
$$\sigma'(z) = \sigma(z)\cdot(1-\sigma(z))$$

$$= \frac{1}{1+e^{-z}}\cdot\left(1-\frac{1}{1+e^{-z}}\right) = \frac{1}{1+e^{-z}}\left(\frac{e^{-z}}{1+e^{-z}}\right) = \boxed{\frac{e^{-z}}{(1+e^{-z})^2}}$$

2. 다음과 같은 구조의 Perceptron과 ●(=1), ●(=0)을 평면좌표상에 나타낸 그림이 있습니다.




$$\varphi(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$




$$\begin{cases} \text{임의의 } b : 0.5 \\ \quad〃 \; x\# : 0.3 \\ \quad〃 \; x\$ : 0.7 \end{cases}$$

2-1. ●, ●를 분류하는 임의의 $b, w$를 선정하고 분류해보세요.

$\hat{z} = 0.3x\# + 0.7x\$ + 0.5$

$\hat{z}_1 = 0.3\cdot 0 + 0.7\cdot 1 + 0.5 = 1.2 > 0$ $\quad \hat{y}_1 = 1$

$\hat{z}_2 = 0.3\cdot 1 + 0.7\cdot 1 + 0.5 = 1.7 > 0$ $\quad \hat{y}_2 = 1$

$\hat{z}_3 = 0.3\cdot 0 + 0.7\cdot 0 + 0.5 = 0.5 > 0$ $\quad \hat{y}_3 = 1$

$\hat{z}_4 = 0.3\cdot 1 + 0.7\cdot 0 + 0.5 = 0.8 > 0$ $\quad \hat{y}_4 = 1$

| $x_1$ | $x_2$ | S | $\hat{Y}$ | Y |
|---|---|---|---|---|
| 0 | 0 | 0.5 | 1 | 1 |
| 0 | 1 | 1.2 | 1 | 1 |
| 1 | 0 | 0.8 | 1 | 0 |
| 1 | 1 | 1.7 | 1 | 1 |

2-2. Perceptron 학습 규칙에 따라 임의의 학습률을 정하고 $b, w$를 1회 업데이트 해주세요.

임의의 학습률 : 0.1 , $b = 0.5$, $w\# = 0.3$, $w\$ = 0.7$

$w_i \leftarrow w_i + \eta(y-\hat{y})x_i$

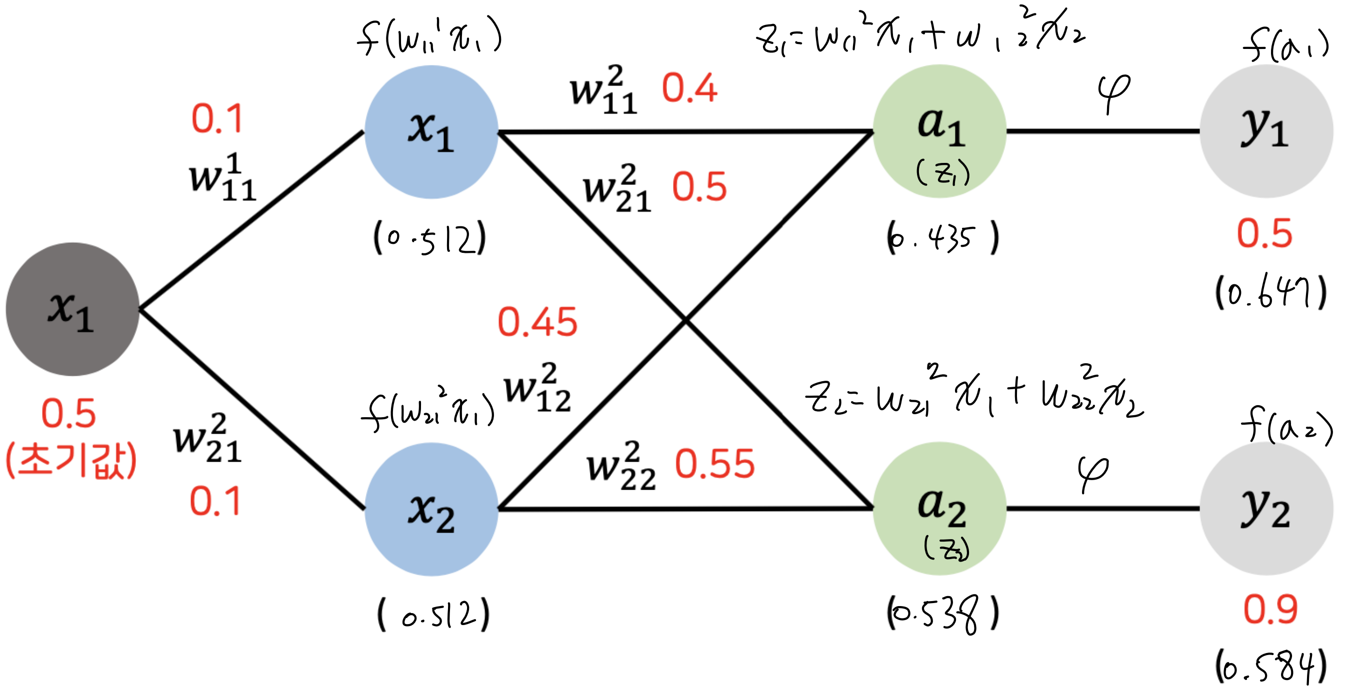
$w\# \leftarrow w\# + 0.1\times(0-1)\times 1 = 0.3 - 0.1 = 0.2$

$w\$ \leftarrow w\$ + 0.1\times(0-1)\times 0 = 0.7$

$b \leftarrow b + 0.1\times(0-1)\times 1 = 0.5 - 0.1 = 0.4$

$b = 0.4, \; w\# = 0.2, \; w\$ = 0.7$

3. 다음과 같은 구조와 초기값을 가진 Multilayer Perceptron이 있습니다.




3-1. Forward Propagation이 일어날 때, 각 노드는 어떤 값을 갖게 되는지 빈 칸을 채워주세요.
(Sigmoid Function 사용)

3-2. output layer에 있는 노드들의 Mean Squared Error을 구해주세요.

$$MSE = \frac{1}{2}\sum_{1}^{2}\frac{1}{2}(y_i - \hat{y}_i)^2$$
$$= \frac{1}{2}\left\{\frac{1}{2}(0.5-0.647)^2 + \frac{1}{2}(0.9-0.584)^2\right\}$$
$$= \frac{1}{2}(0.0108045 + 0.49928) = 0.255\ldots$$

3-3. 3-2에서 구한 답을 토대로, Back Propagation이 일어날 때 가중치 $w_{11}^1$과 $w_{11}^2$의 조정된 값을 구해주세요. (learning rate : 0.4)

조정된 $w_{11}^1$ 값 : 0.104...
조정된 $w_{11}^2$ 값 : 0.403...

수고하셨습니다.

## 3-3 풀이과정

1) $w_{11}^1$

| |
|---|
| * $z_{10} = w_{11}^1 x_1 = 0.05$ |
| $a_{10} = f(w_{11}^1 x_1) = 0.512$ |
| $z_{20} = 0.435$ |
| $a_{20} = 0.647$ |
| $z_{21} = 0.538$ |
| $a_{21} = 0.584$ |

$$\frac{\partial E}{\partial w_{11}^1} = \left(\frac{\partial E_1}{\partial a_{10}} + \frac{\partial E_2}{\partial a_{10}}\right) \cdot \frac{\partial a_{10}}{\partial z_{10}} \cdot \frac{\partial z_{10}}{\partial w_{11}^1}$$

* $\frac{\partial E_1}{\partial a_{10}} = \frac{\partial E_1}{\partial a_{20}} \cdot \frac{\partial a_{20}}{\partial z_{20}} \cdot \frac{\partial z_{20}}{\partial a_{10}} = 0.074 \times 0.228 \times 0.4 = 0.0067$

- $\frac{\partial E_1}{\partial a_{20}} = 0.074$

$$\frac{1}{4}\left((y_1 - a_{20})^2 + \overset{\emptyset}{\cancel{(y_2 - a_{21})^2}}\right)$$

$$= -\frac{1}{2}(y_1 - a_{20}) = -\frac{1}{2}(0.5 - 0.647) = 0.074$$

- $\frac{\partial a_{20}}{\partial z_{20}} = 0.228$

$$\sigma(z) \cdot (1 - \sigma(z)) = 0.647 \cdot (1 - 0.647)$$
$$= 0.228$$

- $\frac{\partial z_{20}}{\partial a_{10}} = w_{11}^2 = 0.4$

* $\frac{\partial E_2}{\partial a_{10}} = \frac{\partial E_2}{\partial a_{21}} \cdot \frac{\partial a_{21}}{\partial z_{21}} \cdot \frac{\partial z_{21}}{\partial a_{10}} = 0.074$

- $\frac{\partial E_2}{\partial a_{21}} = 0.608$

$$\frac{1}{4}\left((y_2 - a_{21})^2 + \overset{\emptyset}{\cancel{\quad}}\right)$$

$$= -\frac{1}{2}(y_2 - a_{21}) = -\frac{1}{2}(0.9 - 0.584) = -0.608$$

- $\frac{a_{21}}{\partial z_{21}} = 0.243$

$$0.584 \cdot (1 - 0.584) = 0.243$$

- $\frac{\partial z_{21}}{\partial a_{10}} = w_{21}^2 = 0.5$

$$* \frac{\partial a_{10}}{\partial z_{10}} = \sigma(z) \cdot (1-\sigma(z)) = 0.512 \cdot (1-0.512) = 0.25$$

$$* \frac{\partial z_{10}}{\partial w_{11}'} = x_1 = 0.5$$

$$\frac{\partial E}{\partial w_{11}'} = \left(\frac{\partial E_1}{\partial a_{10}} + \frac{\partial E_2}{\partial a_{10}}\right) \cdot \frac{\partial a_{10}}{\partial z_{10}} \cdot \frac{\partial z_{10}}{\partial w_{11}'}$$

$$= (0.0067 + 0.074) \cdot 0.25 \cdot 0.5$$

$$\fallingdotseq 0.01$$

$$\therefore w_{11}' \leftarrow w_{11}' + 0.4 \cdot \frac{\partial E_t}{\partial w_{11}'} = 0.1 + 0.4 \cdot 0.01$$

$$\fallingdotseq 0.104$$

조정 전 $w_{11}'$ : 0.1

조정 후 $w_{11}'$ : 0.104..

2) $w_{11}^2$ 조정

$$\frac{\partial E}{\partial w_{11}^2} = \frac{\partial E_1}{\partial a_{20}} \cdot \frac{\partial a_{20}}{\partial z_{20}} \cdot \frac{\partial z_{20}}{\partial w_{11}^2} \fallingdotseq 0.009$$

$$* \frac{\partial E_1}{\partial a_{20}} = 0.074$$

$$* \frac{\partial a_{20}}{\partial z_{20}} = 0.228$$

$$* \frac{\partial z_{20}}{\partial w_{11}^2} = 0.512$$

$$\therefore w_{11}^2 \leftarrow w_{11}^2 + 0.4 \cdot \frac{\partial E}{\partial w_{11}^2} = 0.4 + 0.4 \cdot 0.009$$

$$\fallingdotseq 0.4034\ldots$$

조정 전 $w_{11}^2$ : 0.4

조정 후 $w_{11}^2$ : 0.4034...