Visualization of topological edge modes in mechanical graphene

Hokkaido University

Yoon Gun

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# Introduction

A system that violates time-reversal symmetry (TRS) with non-trivial topologies is known to propagate waves without backscattering ensured with bulk-boundary correspondence [1]. In other words, it is possible to introduce topologically protected modes on an arbitrary phononic system by intentionally breaking TRS. For example, breaking TRS on a 2D material such as graphene causes the energy gap to open at the Dirac point [2]. And these opened gap causes topologically protected modes on the edge of a system. These are the most critical factors for the unique quantum phenomena: a topological magneto-electric effect, an image magnetic monopole effect, topological Kerr and Faraday rotation, and the quantum anomalous Hall effect (AHE) [2].

Although many topologically protected modes appear on the quantum mechanical system, Raghu and Haldane proposed that non-trivial topological modes are instead a wave phenomenon than quantum effect by demonstrating optical analog of quantum Hall effect (QHE) with periodically arranged gyromagnetic rods [3, 4]. Many theoretical proofs have been proposed in this field [5, 4, 6].

With these theoretical models, there has been a lot of research to demonstrate TRS breaking on various phononic systems [7]. Examples are with mechanical graphene by applying Coriolis force on the non-inertial reference frame [8] or implementing on steadily rotating gyroscopes [9], with acoustic lattice by using steady background airflow [10], with optomechanical array by applying optomechanical interaction of light and sound [11], with acoustic media on time-dependent modulation [12].

This research will verify the theoretical model of breaking the TRS on mechanical graphene by applying Coriolis force on a non-inertial reference frame of a rotating system with experiment [13, 6]. As an experimental device, we introduce two devices. The first is a 1D spring-mass type chain in which masses are placed on the edge of a circle. The second is a 2D graphene shape mechanical graphene with a spring-mass system.

# Formulation

Before investigating the effect of Coriolis force on mechanical graphene, I’ll start with a 1-dimensional 2-periodic spring-mass mechanical lattice system for simplicity. To check the effect of breaking TRS on a mechanical lattice, I’ll start from the inertial frame of reference.

## 1D mechanical lattice on the inertial frame of reference

A screenshot of a video game

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Fig. 1. Schematic of the 1-dimensional 2-periodic spring-mass mechanical lattice system and represent as a displacement of each sublattice. Every sublattice shares the same mass and every spring shares the spring constant . Spring constant has described as to avoid confusion with wavenumber . Although this figure contains a y-axis, I only considered the wavenumber , which propagates through the x-axis for the calculation.

By applying Newton’s second law of motion to the system given by Fig. 1, the motion of each sublattice can be described by the following set of equations.

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |

Here means the index of each lattice. To solve this system equation, by letting the characteristic frequency of spring-mass system , eq (1) and (2) can be deformed as below.

|  |  |
| --- | --- |
|  | (3) |
|  | (4) |

Additionally, by letting displacement with with phase factor yields the following equations.

|  |  |
| --- | --- |
|  | (5) |
|  | (6) |

Here represents a complex conjugate of . eq (5) and (6) can be reduced into an eigenvalue equation of .

|  |  |
| --- | --- |
|  | (7) |

with Hamiltonian   defined as

|  |  |
| --- | --- |
|  | (8) |

then the eigenvalue equation becomes

|  |  |
| --- | --- |
|  | (9) |

with   represents identity matrix.

Numerical solution for is depicted in Fig. 2.

|  |  |
| --- | --- |
| Diagram, venn diagram  Description automatically generated  (a) Dispersion relation of Fig. 1 | |
| A picture containing icon  Description automatically generated  (b) Acoustic mode with on (a) | (c) Optical mode with on (a) |

Fig. 2. Result of eigenvalue equation of from eq (7).

|  |  |
| --- | --- |
| (a) | |
| (b) | (c) |

Fig. 3

Zak phase

## 1D mechanical lattice on the non-inertial reference frame

### with Coriolis force

Shape, arrow

Description automatically generated

Fig. 4. Visual illustration of broken time-reversal symmetry with a Coriolis force.

### with Coriolis force and centrifugal force

### with Coriolis force, centrifugal force, and transverse force

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## 2D Mechanical graphene on the inertial frame of reference

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Fig. 5

## 2D Mechanical graphene on the non-inertial reference frame

Change of Dirac point in mechanical graphene

Calculation of Brillouin zone of graphene

# Experiment

# Result

# Conclusion

# Further research

# References

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