Visualization of topological edge modes in mechanical graphene

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# Introduction

A system that violates time-reversal symmetry (TRS) with non-trivial topologies is known to propagate waves without backscattering ensured with bulk-boundary correspondence [1]. In other words, it is possible to introduce topologically protected modes on an arbitrary phononic system by intentionally breaking TRS. For example, breaking TRS on a 2D material such as graphene causes the energy gap to open at the Dirac point [2]. And these opened gap causes topologically protected modes on the edge of a system. These are the most critical factors for the unique quantum phenomena: a topological magneto-electric effect, an image magnetic monopole effect, topological Kerr and Faraday rotation, and the quantum anomalous Hall effect (AHE) [2].

Although many topologically protected modes appear on the quantum mechanical system, Raghu and Haldane proposed that non-trivial topological modes are instead a wave phenomenon than quantum effect by demonstrating optical analog of quantum Hall effect (QHE) with periodically arranged gyromagnetic rods [3, 4]. Many theoretical proofs have been proposed in this field [5, 4, 6].

With these theoretical models, there has been a lot of research to demonstrate TRS breaking on various phononic systems [7]. Examples are with mechanical graphene by applying Coriolis force on the non-inertial reference frame [8] or implementing on steadily rotating gyroscopes [9], with acoustic lattice by using steady background airflow [10], with optomechanical array by applying optomechanical interaction of light and sound [11], with acoustic media on time-dependent modulation [12].

This research will verify the theoretical model of breaking the TRS on mechanical graphene by applying Coriolis force on a non-inertial reference frame of a rotating system with experiment [13, 6]. As an experimental device, we introduce two devices. The first is a 1D spring-mass type chain in which masses are placed on the edge of a circle. The second is a 2D graphene shape mechanical graphene with a spring-mass system.

# Formulation

Before investigating the effect of Coriolis force on mechanical graphene, I’ll start with a 1-dimensional 2-periodic spring-mass mechanical lattice system for simplicity. To check the effect of breaking TRS on a mechanical lattice, I’ll start from the inertial frame of reference.

Assumption: springs are linear

## 1D mechanical lattice on the inertial frame of reference

A screenshot of a video game

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Fig. . Schematic of the 1-dimensional 2-periodic spring-mass mechanical lattice system. and represent as a displacement of each sublattice. Every sublattice shares the same mass and every spring shares the spring constant . Spring constant has described as to avoid confusion with wavenumber . Although this figure contains a y-axis, I only considered the wavenumber , which propagates through the x-axis for the calculation.

By applying Newton’s second law of motion to the system given by Fig. 1, the motion of nth sublattice can be described by the following set of equations.

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |

Here means the index of each lattice. To solve this system equation, by letting the characteristic frequency of spring-mass system , eq (1) and (2) can be deformed as below.

|  |  |
| --- | --- |
|  | (3) |
|  | (4) |

Additionally, by letting displacement with with phase factor yields the following equations.

|  |  |
| --- | --- |
|  | (5) |
|  | (6) |

Here represents a complex conjugate of . eq (5) and (6) can be reduced into an eigenvalue equation of .

|  |  |
| --- | --- |
|  | (7) |

with Hamiltonian   defined as

|  |  |
| --- | --- |
|  | (8) |

then the eigenvalue equation becomes

|  |  |
| --- | --- |
|  | (9) |

where   denotes identity matrix. Numerical solution for is depicted in Fig. 2. I have used parameter values of N/m and kg for the simplicity. For the eigenvalue, eigenvector calculation, python alongside with NumPy library has used.

|  |  |
| --- | --- |
| (a) Dispersion relation of Fig. 1 | |
| (b) Acoustic mode with on (a) | (c) Optical mode with on (a) |

Fig. . Result of eigenvalue equation of from eq (9), N=21. (a) Dispersion relation from the first Brillouin zone of mechanical lattice. Band gap is closed on a default setting. (b) Acoustic mode eigenvector visualization with . (c) Optical mode eigenvector visualization with . We can see that Acoustic mode and Optical mode is different from its consisting number of waves. 1 for Acoustic mode and 2 for Optical mode.

|  |  |
| --- | --- |
| (a) Schematic of experimental setup with N=12 | (b) Expanded sublattice |
| (c) Dispersion relation of (b) | (d) Dispersion relation (c) projected on XZ-plane. |

Fig. . Rationale for approximation with only . We can see that propagation of wavenumber have no significant effect on the resulting dispersion relation. Parameter values of N/m and kg for the simplicity of calculation.

Although Fig. 2 provides straightforward result of each mechanical sublattice motions, the actual experimental setup cannot implement the exact system since the model requires infinite series of periodic chain. To avoid the infinite chain, one must build a ring-shaped chain as with Fig. 3a. In this paper, we will assume there is 12-unit cells with sublattice consist of 2 mass-spring system respectively. The resulting sublattice becomes Fig. 3b

As we can check from Fig. 3c and Fig. 3d, there is no significant difference due to added wavenumber and a distortion in the connection of string except the translation of dispersion relation. This gives us a confidence to use a approximated model suggested on Fig. 1.

**Zak phase**

## 1D mechanical lattice on the non-inertial reference frame

### with Coriolis force

A picture containing lit

Description automatically generated

Fig. . Visual illustration of broken TRS with a Coriolis force. When the time of a moving particle is reversed on the rotating non-inertial reference frame, its direction of velocity change which results in the reversion of the direction of Coriolis force. Due to this property, the particle does not move on the same trace when it has passed through, which breaks the TRS.

To add Coriolis force into the consideration, we need to add y axis on and respectively by making them a vector . This is because Coriolis force applied to the orthogonal direction of the velocity of a mass. With Coriolis force added on the modified variables, eq (**5**) and eq (**6**) becomes as below.

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |

This equation cannot be solved in a straightforward way as we have done on 2.1 since there are and on different vectors and . There are two ways to get around of this issue. The first is to define the determinant equation. For example, one can define the quadratic equation of given by equation below.

|  |  |
| --- | --- |
|  | (12) |

where is the Hamiltonian defined by the right-hand side of eq (**10**) and eq(**11**), is an identity matrix, and is given by

|  |  |
| --- | --- |
|  | (13) |

But this method grows very quickly with the number of sublattices on the system, which makes it hard to explore the effect of Coriolis force on arbitrary mechanical lattice. Therefore, in this paper, we will use more general method to transform eq (10), eq (11) into an eigenvalue equation by expanding its eigenvector with and [1, 8]. The modified eigenvector becomes

|  |  |
| --- | --- |
|  | (14) |

First, we define a matrix representing eq (**10**) and eq (**11**)

|  |  |
| --- | --- |
|  | (15) |

And define another matrix of identity operation for and .

|  |  |
| --- | --- |
|  | (16) |

Combining eq (15) and eq (16) we can obtain the eigenvalue equation.

|  |  |
| --- | --- |
|  | (17) |

where

|  |  |
| --- | --- |
|  | (18) |

Since the matrix is its own inverse , eq (17) becomes

|  |  |
| --- | --- |
|  | (19) |

The results are as depicted in Fig. 5.

|  |  |
| --- | --- |
| (a) /s | (b) /s |
| (c) /s | (d) /s |

Fig. 5. Dispersion relation of 1D mechanical lattice on various angular velocity. As the angular velocity increases, a gap come up below the acoustic mode. Its amplitude is exactly two times of the angular velocity of the system.

## Simulation

## 2D Mechanical graphene on the inertial frame of reference

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Fig. .

### Bulk mode

### Graphene ribbon

## 2D Mechanical graphene on the non-inertial reference frame

Change of Dirac point in mechanical graphene

Calculation of Brillouin zone of graphene

# Experiment

# Result

# Conclusion

# Further research

# References

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