1번

$\sigma(z) = \frac{1}{1+e^{-z}} = (1+e^{-z})^{-1}$ 이고,

$$\sigma(z)\cdot\frac{d}{dz} = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1+e^{-z}}{(1+e^{-z})^2} - \frac{1}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2}$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right) = \sigma(z)(1-\sigma(z))$$

2번

#1 $w_1 = 1.0$, $w_2 = 1.0$, $b = 1.5$ 로 가정하자. 이때, $\varphi(w_1x_1 + w_2x_2 + b) = y$, $\varphi(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$ 이다.

	x_1	x_2	s	y
①	0	1	1	1
②	0	0	1	1
③	1	1	1	1
④	1	0	1	0

① $0\times1 + 1\times1 + 1.5 = 2.5$, $\varphi(2.5) = 1$

② $0\times1 + 0\times1 + 1.5 = 1.5$, $\varphi(1.5) = 1$

③ $1\times1 + 1\times1 + 1.5 = 3.5$, $\varphi(3.5) = 1$

④ $1\times1 + 0\times1 + 1.5 = 2.5$, $\varphi(2.5) = 1$

∴ 임의로 정한 w_i와 b로는 제대로 분류가 되지 않았다.

#2

학습률 $\eta = 0.05$, 가중치 갱신식 $w_i \leftarrow w_i + \eta(y-o)x_i$

④ $b \leftarrow b + 0.05(1-0)\cdot1$

$w_1 \leftarrow w_1 + 0.05(1-0)\cdot1$

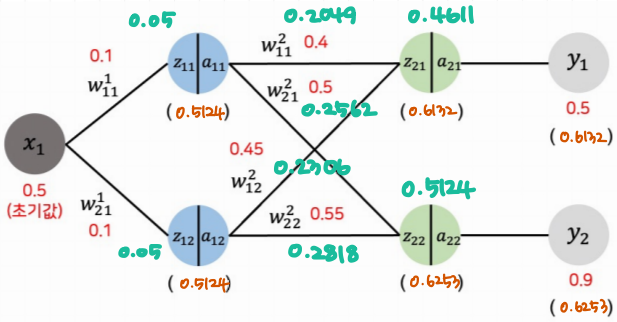
$w_2 \leftarrow w_2 + 0.05(1-0)\cdot0$

학습 전 : $w_1 = 1$, $w_2 = 1$, $b = 1.5$

학습 후 : $w_1 = 1.05$, $w_2 = 1$, $b = 1.55$

3번

#1



#2

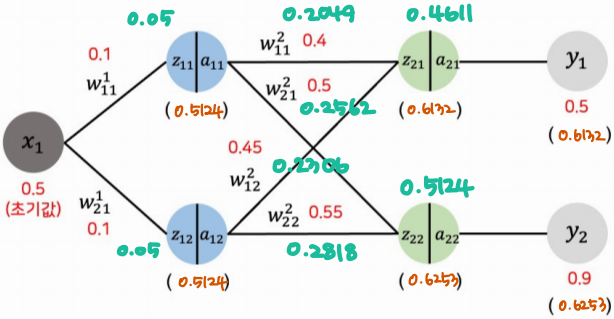
역전파 학습 전 전체 Error : $E_{tot} = \frac{1}{2}\Sigma(target - output)^2$

$= \frac{1}{2}((0.5 - 0.6132)^2 + (0.9 - 0.6253)^2) = 0.044$

#3

$\eta = 0.5$

〈순전파 결과〉



* w_{11}^1이 Etot (MSE)에 미치는 영향

$$\frac{\partial E_{tot}}{\partial w_{11}^1} = \frac{\partial E_{tot}}{\partial a_{21}} \cdot \frac{\partial a_{21}}{\partial z_{21}} \cdot \frac{\partial z_{21}}{\partial a_{11}} \cdot \frac{\partial a_{11}}{\partial z_{11}} \cdot \frac{\partial z_{11}}{\partial w_{11}^1}$$

- $E_{tot} = \frac{1}{2}((target_{y_1} - a_{21})^2 + (target_{y_2} - a_{22})^2)$ 이므로 $\frac{\partial E_{tot}}{\partial a_{21}} = a_{21} - target_{y_1}$
- $a_{21} = sigmoid(z_{21})$ 이므로 $\frac{\partial a_{21}}{\partial z_{21}} = sigmoid(z_{21})(1 - sigmoid(z_{21}))$
- $z_{21} = a_{11} \cdot w_{11}^2 + a_{12} \cdot w_{12}^2$ 이므로 $\frac{\partial z_{21}}{\partial a_{11}} = w_{11}^2$
- $a_{11} = sigmoid(z_{11})$ 이므로 $\frac{\partial a_{11}}{\partial z_{11}} = sigmoid(z_{11})(1 - sigmoid(z_{11}))$
- $z_{11} = x_1 w_{11}^1$ 이므로 $\frac{\partial z_{11}}{\partial w_{11}^1} = x_1$

따라서, $\frac{\partial E_{tot}}{\partial w_{11}^1} = (a_{21} - target_{y_1}) \cdot sigmoid(z_{21})(1 - sigmoid(z_{21})) \cdot sigmoid(z_{11})(1 - sigmoid(z_{11})) \cdot w_{11}^2 \cdot x_1$

$= (0.6132 - 0.5) \cdot 0.6132 \cdot (1 - 0.6132) \cdot 0.5124 \cdot (1 - 0.5124) \cdot 0.4 \cdot 0.1$

$= 0.000268$

$w_{11}^{1+} = w - \eta \cdot \frac{\partial E_{tot}}{\partial w_{11}^1} = 0.1 - 0.5 \cdot 0.000268 = 0.099866$

* w_{11}^2이 E_{tot}(MSE)에 미치는 영향

$$\frac{\partial E_{tot}}{\partial w_{11}^2} = \frac{\partial E_{tot}}{\partial a_{21}} \cdot \frac{\partial a_{21}}{\partial z_{21}} \cdot \frac{\partial z_{21}}{\partial w_{11}^2}$$

- $E_{tot} = \frac{1}{2}((target_{y_1} - a_{21})^2 + (target_{y_2} - a_{22})^2)$ 이므로 $\frac{\partial E_{tot}}{\partial a_{21}} = a_{21} - target_{y_1}$
- $a_{21} = sigmoid(z_{21}) = \frac{1}{1+e^{-z_{21}}}$ 이므로 $\frac{\partial a_{21}}{\partial z_{21}} = sigmoid(z_{21})(1 - sigmoid(z_{21}))$
- $z_{21} = a_{11} \cdot w_{11}^2 + a_{12} \cdot w_{12}^2$ 이므로 $\frac{\partial z_{21}}{\partial w_{11}^2} = a_{11}$

따라서, $\frac{\partial E_{tot}}{\partial w_{11}^2} = (a_{21} - target_{y_1}) \cdot sigmoid(z_{21})(1 - sigmoid(z_{21})) \cdot a_{11}$

$= (0.61372 - 0.5) \cdot 0.61372 \cdot (1 - 0.61372) \cdot 0.4$

$= 0.01017$

$w_{11}^{2+} = w - \eta \cdot \frac{\partial E_{tot}}{\partial w_{11}} = 0.4 - 0.5 \cdot 0.01017 = 0.39465$