MC 20-02

Ball and Beam Control

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1-1. Motor Modeling

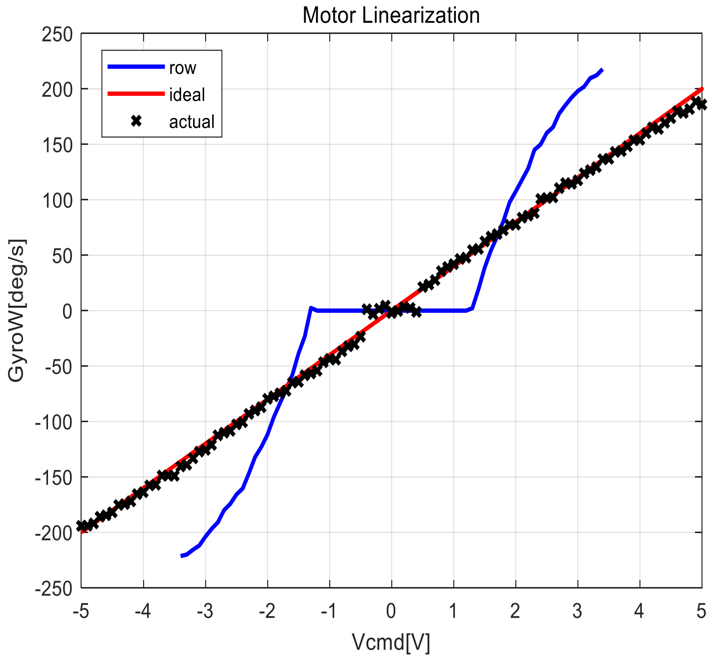


Figure 1. Static Characteristic and Linearization of Motor

Table 1. Static Characteristic

$Vccw^{deadzone}$	$Vcw^{deadzone}$	$Vccw^{saturation}$	$Vcw^{saturation}$
$-1.4[V]$	$1.4[V]$	$-3.5[V]$	$3.5084[V]$

Table 2. Linearized Characteristic

$Vccw^{deadzone}$	$Vcw^{deadzone}$	$Vccw^{saturation}$	$Vcw^{saturation}$
$-0.585[V]$	$0.5[V]$	$-2.2609[V]$	$2.27[V]$

Linearization process is done with 2^{nd} order mapping function with adding additional mid point in determining mapping function.

1. Classical Control

1-1. Motor Modeling

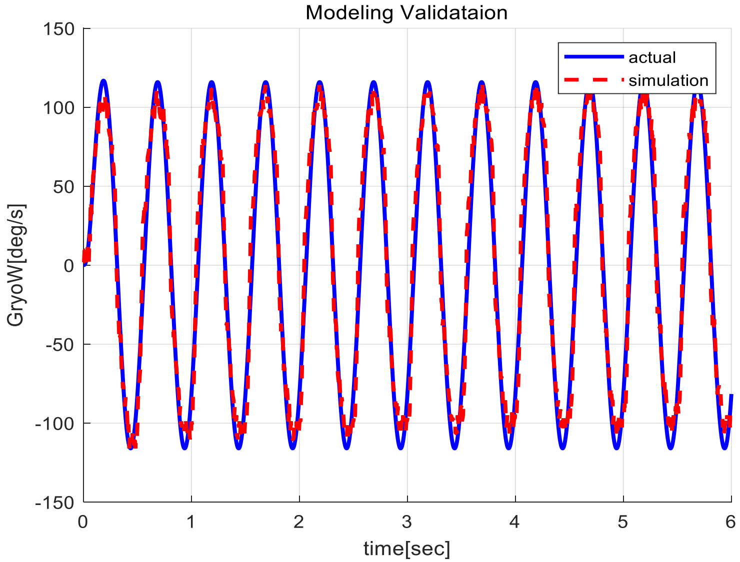


Figure 2. Motor Modeling Validation

With estimated bandwidth of 6.5Hz, modeled motor using frequency sweep method sweeping frequency from

$$\frac{1}{10} f_{est} \sim \frac{1}{3} f_{est}$$

$$\frac{W(s)}{V(s)} = \frac{48390}{s^2 + 58.28s + 899.8} [deg/s \cdot V]$$

➢ System approximation

$$\frac{W(s)}{V(s)} \approx \frac{53.7786}{0.0333s + 1}$$

Table 3. 2nd order Motor Modeling parameters

Motor Bandwidth	DC gain
$4.7741\ [Hz]$	$53.7786\ [deg/s \cdot V]$

Table 4. 1st order Motor Modeling parameters

Time Constant	DC gain
$0.0333[s]$	$53.7786\ [deg/s \cdot V]$

1. Classical Control

1-2. System Control Loop and Controller Design

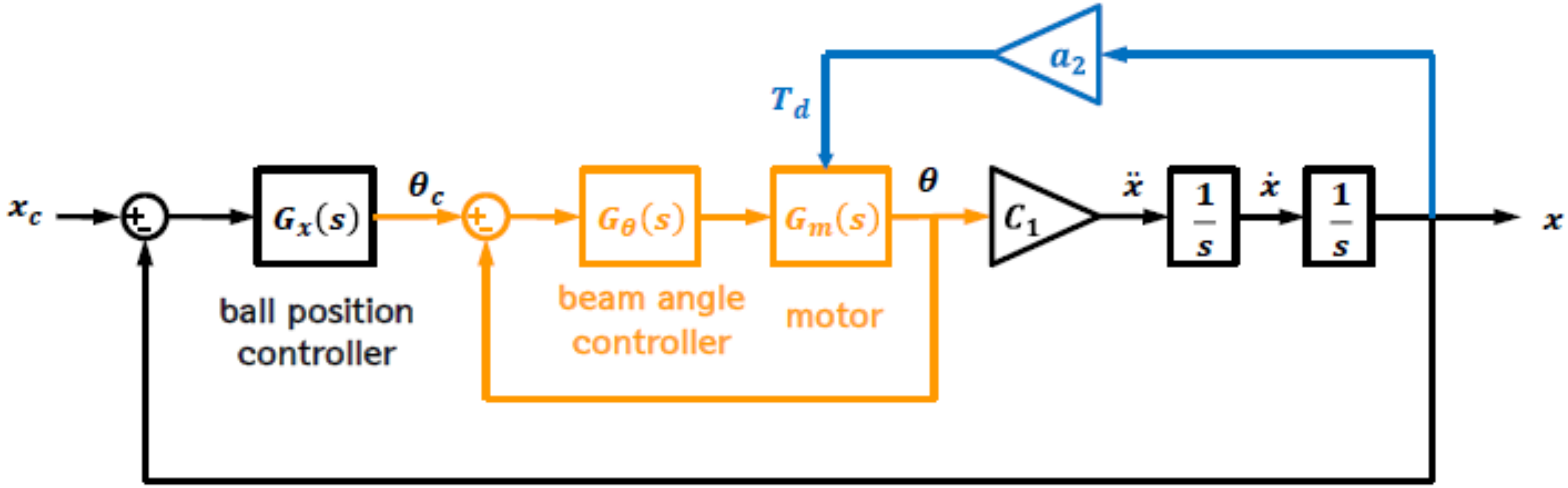


Figure 3. Classical Control loop Structure

1. Classical Control

1-2. System Control Loop and Controller Design

➢ **Inner Loop Controller Design**

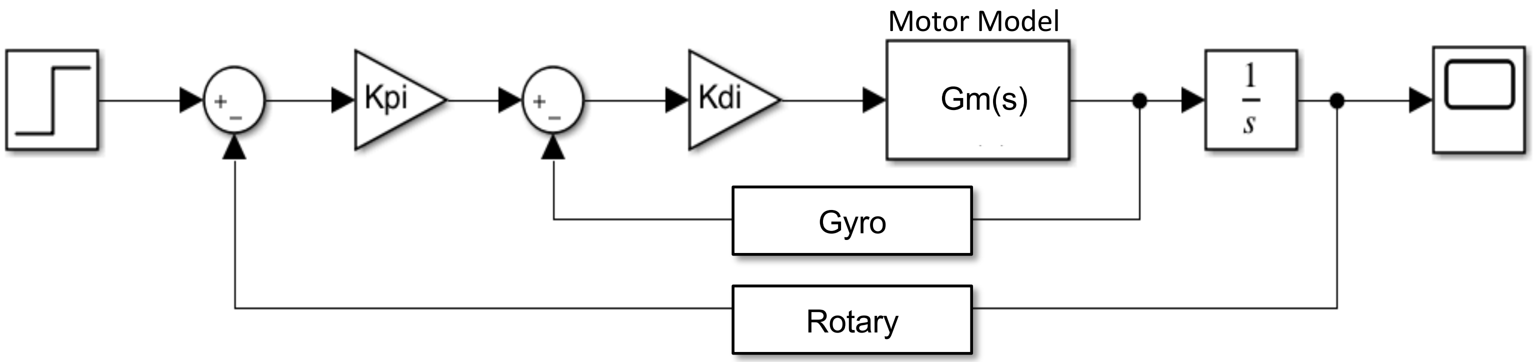


Figure 4. Inner Loop Control loop Structure

$$G_{cl_i}(s) = \frac{\theta(s)}{\theta_c(s)} = \frac{K_m K_{di} K_{pi}}{\tau_m s^2 + (1 + K_m K_{di})s + K_m K_{di} K_{pi}} \, [-] \quad \rightarrow \quad Eq\ (1)$$

K_m : $motor\ gain$

K_{di} : $p\ controller\ gain\ of\ \omega$

K_{pi} : $p\ controller\ gain\ of\ \theta$

1. Classical Control

1-2. System Control Loop and Controller Design

➢ **Inner Loop Controller Design**

Table 5. Inner Loop Specifications

Motor Specification		
Step response	T_r	$2.2441[sec]$
	$\%OS$	$0.0[\%]$
margin	GM	$Inf[dB]$
	PM	$47.3[deg]$
Desired Specification		
Step response	$T_r \leq 0.15[sec]$	
	$\%OS \leq 20[\%]$	
margin	$GM \geq 6[dB]$	
	$PM \geq 45[deg]$	
calculate ζ, ω_n with step response specification		
$\zeta \geq 0.4559[-]$	$\omega_n \geq 20.1695[rad/s]$	

Table 6. Design Parameters

Determined ζ, ω_n	
ζ	$0.7000[-]$
ω_{ci}	$38.0000[rad/s]$
Determined K_p, K_d	
K_{pi}	$62.2324[-]$
K_{di}	$0.8241[-]$

$from\ Eq\ (1),$

$$\Delta_{in} = s^2 + 2\zeta\omega_{ci}s + \omega_{ci}^2$$
$$= \tau_m s^2 + (1 + K_m K_{di})s + K_m K_{di} K_{pi}$$
$$K_{di} = \frac{2\zeta\omega_{ci}\tau_m - 1}{K_m}$$
$$K_{pi} = \omega_{ci}^2\tau_m / K_m K_{di}$$

1. Classical Control

1-2. System Control Loop and Controller Design

➢ **Inner Loop Controller Design**

$$p = -26.6 \pm j\,27.1$$

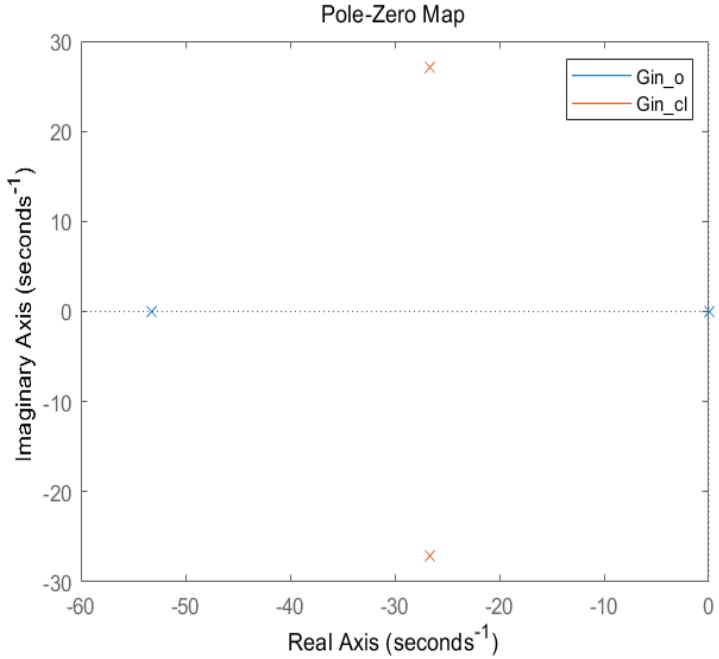


Figure 5. pole-zero map of inner loop

$$GM = inf[dB]\,, PM = 65.2[deg]$$

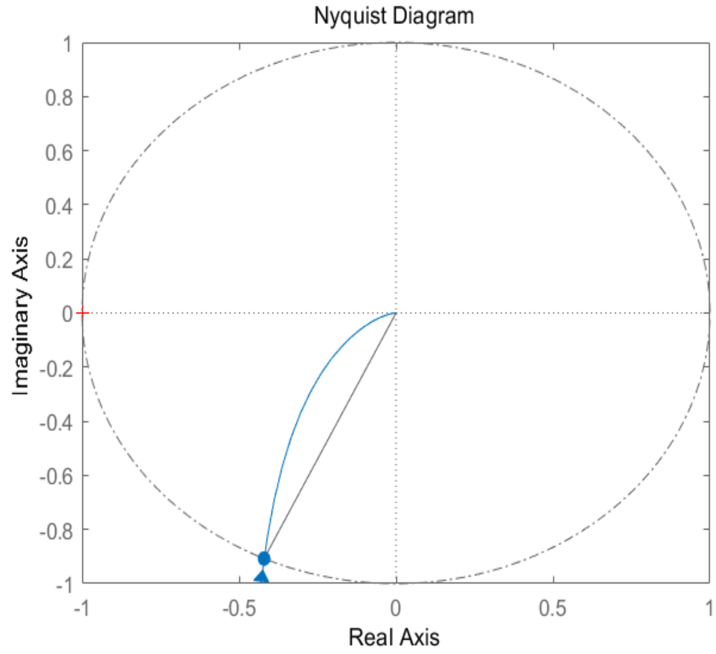


Figure 6. Nyquist of inner loop

1. Classical Control

1-2. System Control Loop and Controller Design

➢ **Outer Loop Controller Design**

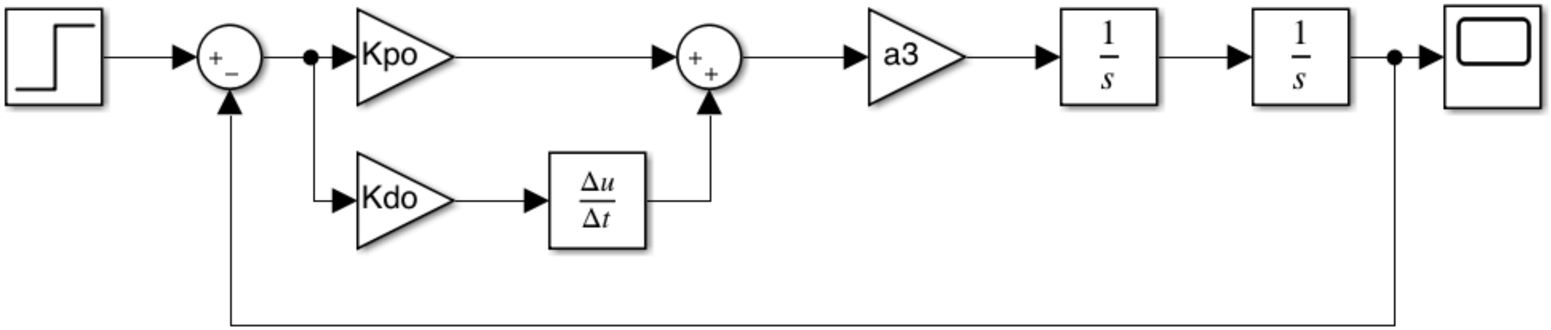


Figure 7. Outer Loop Control loop Structure

$$G_{cl_o} = \frac{x(s)}{x_c(s)} = \frac{K_{do}s + K_{po}}{\frac{1}{a_3}s^2 + K_{do}s + K_{po}} [-] \quad \rightarrow \quad Eq\ (2)$$

1. Classical Control

1-2. System Control Loop and Controller Design

- **Outer Loop Controller Design**

Table 7. Outer Loop Specifications

Desired Specification	
Step response	$T_r \leq 0.15[sec]$
	$\%OS \leq 20[\%]$
margin	$GM \geq 6[dB]$
	$PM \geq 60[deg]$
calculate ζ, ω_n with step response specification	
$\zeta \geq 0.4559[-]$	$\omega_n \geq 11.7809[rad/s]$

Table 8. Design Parameters

Determined ζ, ω_n	
ζ	$0.7000[-]$
ω_{co}	$3.5000[rad/s]$
Determined $K_p,\ K_d$	
K_{po}	$3.5710[-]$
K_{do}	$1.0000[-]$

$from\ Eq\ (2),$

$$\Delta_{out} = s^2 + 2\zeta\omega_{co}s + \omega_{co}^2 = \frac{1}{a_3}s^2 + K_{do}s + K_{po}$$
$$K_{do} = 2\zeta\omega_{co}/a3$$
$$K_{po} = \omega_{co}^2/a3$$

1. Classical Control

1-2. System Control Loop and Controller Design

- **Outer Loop Controller Design**

$GM = 16.3[dB], PM = 48.6[deg]$

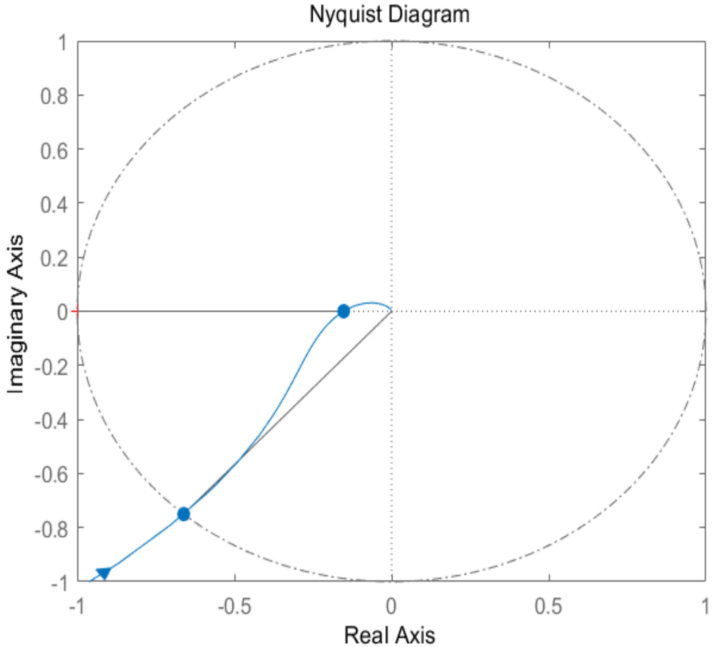


Figure 8. a) Nyquist of Outer loop (Inner Loop with 1st order motor)

$GM = inf[dB], PM = 65.2[deg]$

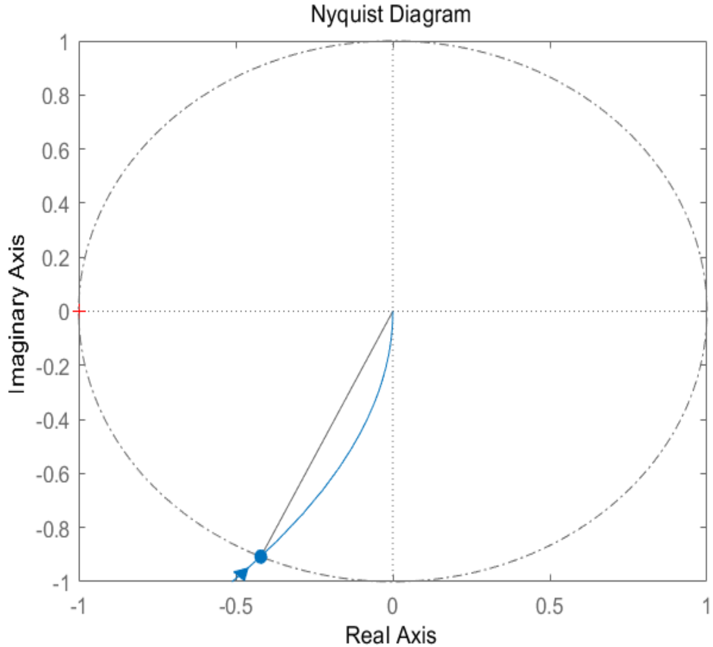


Figure 8. b) Nyquist of Outer loop (inner loop $\cong$ 1)

1. Classical Control

1-2. System Control Loop and Controller Design

➢ **Outer Loop Controller Design**

$$p = -23.9 \pm j\,24.5, -2.72 \pm 2.78, z = -2.5$$

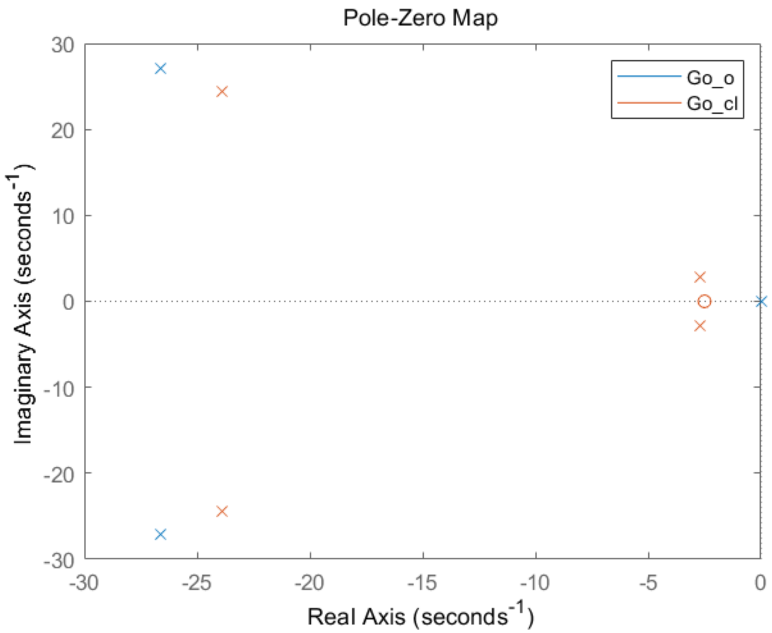


Figure 9. Nyquist of Outer loop

1. Classical Control

1-3. Experimental Results

➢ Inner Loop

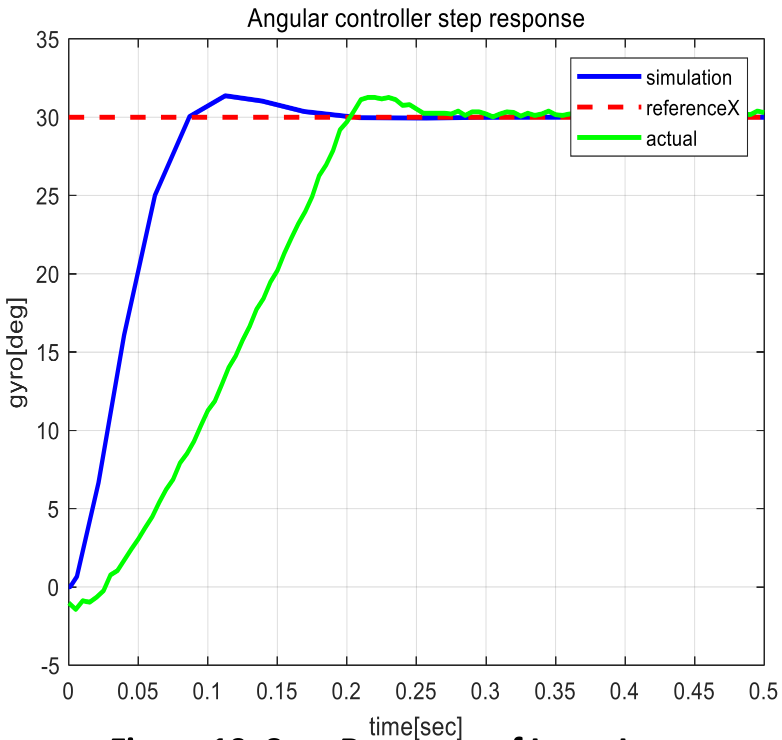


Figure 10. Step Response of Inner Loop

Table 9. Performance of Inner Loop

	simulation	actual
t_r $(rising\ time)$	$0.0718[sec]$	$0.1853[sec]$
$\%OS$ $(overshoot)$	$4.5730[\%]$	$4.2120[\%]$
e_{ss} $(steady - state\ error)$	$0.0000[deg]$	$0.2547[deg]$

1. Classical Control

1-3. Experimental Results

➢ Outer Loop

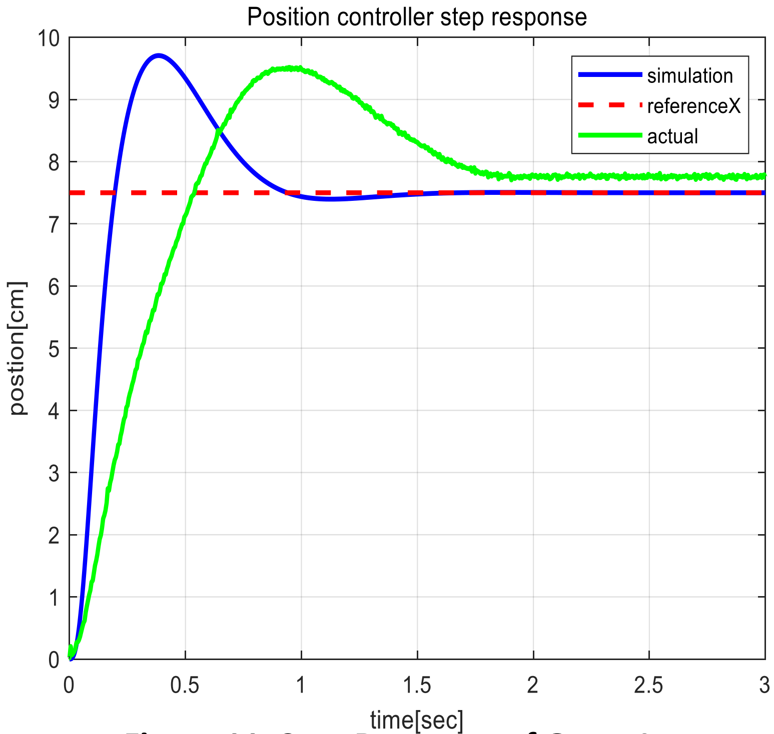


Figure 11. Step Response of Outer Loop

Table 10. Performance of Outer Loop

	simulation	actual
t_r $(rising\ time)$	$0.2502[sec]$	$0.4655[sec]$
$\%OS$ $(overshoot)$	$26.0480[\%]$	$26.0173[\%]$
e_{ss} $(steady-state\ error)$	$0.0000[cm]$	$0.2845[cm]$

1. Classical Control

1-3. Experimental Results

➢ **Outer Loop**

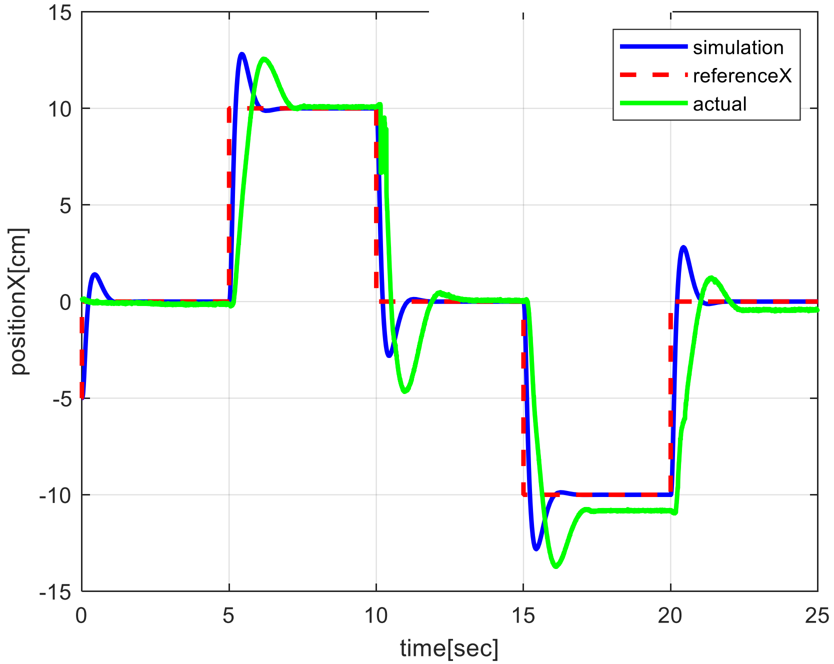


Figure 12. Position controller simulation result

About Sim-real gap

- Simulation model = design model (ideal)
- In simulation, we didn't consider about a2 term from figure 3. (disturbance torque derived from inertia)
- Effect of a2 gets bigger when position of ball is far from operating point

2. Modern Control

2-1. System Modeling with State Space Equation

- **Motor Characteristic Linearization**

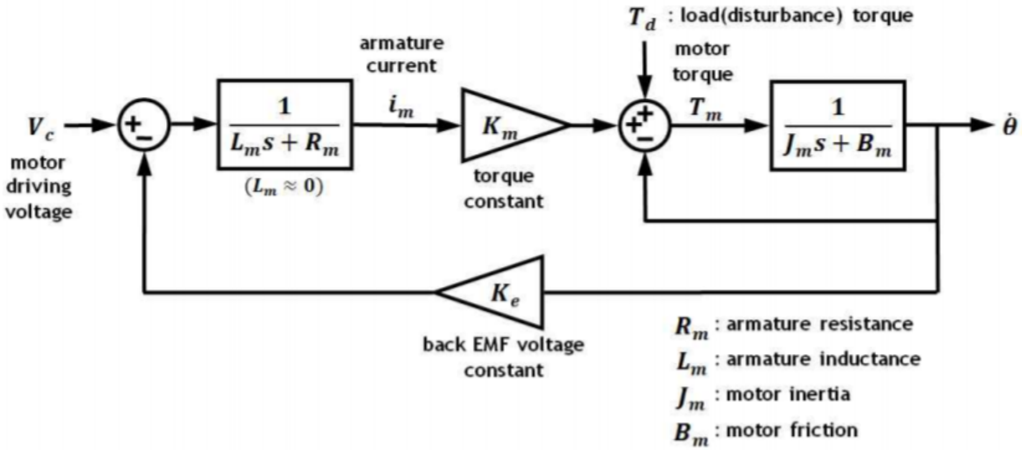


Figure 13. Motor Block Diagram

$$\frac{W(s)}{V(s)} = \frac{\frac{K_m}{L_m s + R_m} \cdot \frac{1}{Js + B_m}}{1 + \frac{K_m}{L_m s + R_m} \cdot \frac{1}{J_m s + B_m} \cdot K_e}$$

$$= \frac{K_m}{\cancel{L_m} J s^2 + (R_m J + \cancel{L_m B_m})s + (\cancel{R_m B_m} + K_m K_e)}$$

$$= \frac{K_m}{R_m J s + K_m K_e}$$

$$= \frac{\frac{K_m}{K_m K_e}}{\frac{R_m J}{K_m K_e} s + 1}, K_e \cong K_m$$

$$\boxed{K_{m1} = \frac{1}{K_e} \to K_e = \frac{1}{K_{m1}}} \longrightarrow Eq\ (3)$$

$$\boxed{\tau_{m1} = \frac{R_m J}{K_e^2} \to R_m = \frac{\tau_{m1} K_e^2}{J}} \longrightarrow Eq\ (4)$$

2. Modern Control

2-1. System Modeling with State Space Equation

➢ Ball and Beam Dynamics

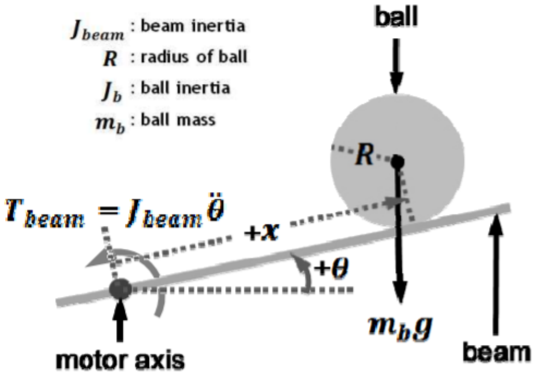


Figure 14. Ball and Beam Dynamics

Two major equation in ball and beam system can be driven

$$\Sigma F_x = mg\ sin\theta = F\ translation + F\ ball\ rotation = F_T + F_T$$

$$mg \sin\theta = F_T + F_R = \frac{7}{5} m\ddot{x}$$

$$\therefore \ddot{x} = \frac{5}{7} g \sin\theta$$

$$\Sigma M = T_{motor} + T_{ball} = T_m + T_b$$

$$J_{beam}\,\ddot{\theta} = T_m + T_b = K_m I - J_m\ddot{\theta} - b_m\dot{\theta} - m_b g\ cos\theta$$

$$\ddot{\theta} = \frac{\frac{K_m}{R_m} V_{cmd} - \left(\frac{K_m K_e}{R_m} + b_m\right)\dot{\theta} - x m_b g cos\theta}{J_{beam\_}J_m}$$

2. Modern Control

2-1. System Modeling with State Space Equation

➢ State Space Equation

By small angle approximation
$(\cos\theta \approx 1, sin\theta \approx \theta)$,

$$\dot{X} = AX + Bu, \qquad u = V$$

$$X = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a1 & a2 & 0 \\ 0 & 0 & 0 & 1 \\ a3 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ b1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{a1 = -\left(\frac{K_m K_e}{JR_m}\right), \qquad a2 = -\frac{m_b g}{J}, \qquad a3 = \frac{5g}{7}, \qquad b1 = \frac{K_m}{JR_m}} \longrightarrow \quad Eq\ (5)$$

2. Modern Control

2-1. System Modeling with State Space Equation

Table 11. Parameters

Motor parameters	value
L_m	$0\ [H]$
B_m	$0\ [N \cdot m \cdot s/rad]$
K_e	$1.0654\ [V \cdot s/rad]$
K_m	$1.0654\ [N \cdot m/A]$
R_m	$15.3775[\Omega]$
J	$0.0025[\mathrm{kg} \cdot m^2]$
SSE parameters	**value**
$a1$	-29.9967
$a2$	-44.7628
$a3$	7.0071
$b1$	28.1553

L_m: *approximation to zero*

B_m: *approximation to zero*

K_e: *calculated by Eq* (3)

K_m: $K_e \cong K_m$

R_m: *calculated by Eq* (4)

superposition of Moments of Inertia

$$J = \frac{1}{12} m_{beam} L_{beam}^2 + \frac{2}{5} m_{ball} R_{ball}^2 + \boxed{m_{ball}(L_{beam} + R_{ball})}$$

drived by parallel axis theorem
Assume the ball is at the end of the beam

$a1, a2, a3, b1$: *calculated by Eq* (5)

2. Modern Control

2-2. LQ Servo Controller Loop and Design

- **Fading Memory Filter**

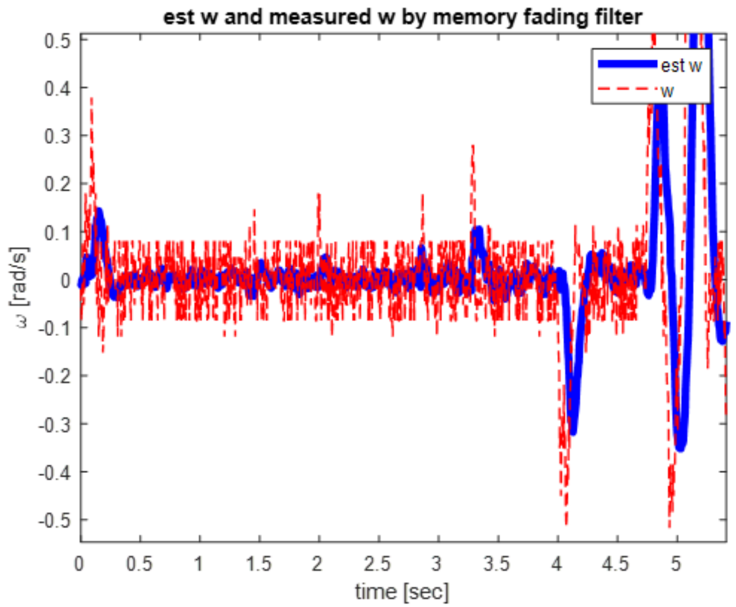


Figure 16. a) Angular Velocity Estimation from FMF

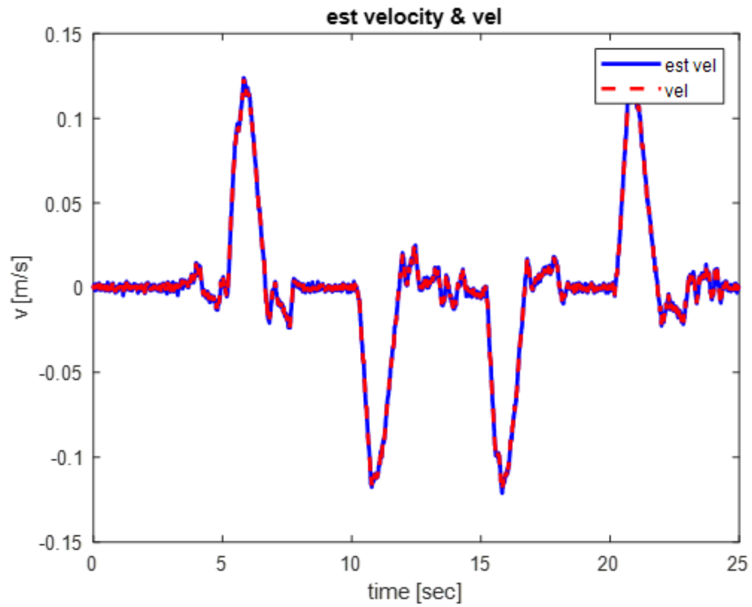


Figure 16. b) Velocity Estimation from FMF

2. Modern Control

2-2. LQ Servo Controller Loop and Design

➢ **Fading Memory Filter**

beta $\downarrow \rightarrow$ gain $\uparrow \rightarrow$ *bandwidth* $\uparrow \rightarrow$ *noise amplification* $\uparrow$
beta $\uparrow \rightarrow$ gain $\downarrow \rightarrow$ *bandwidth* $\downarrow \rightarrow$ *noise amplification* $\downarrow$

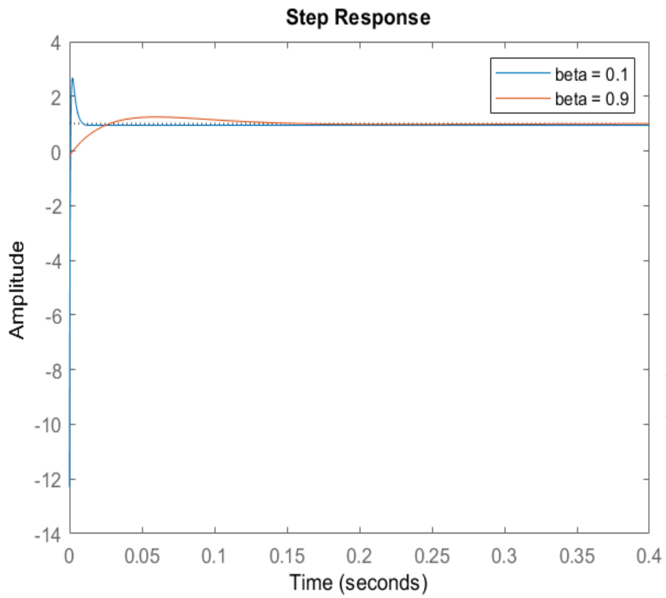


Figure 17. Transient Response of FMF Depending on Beta

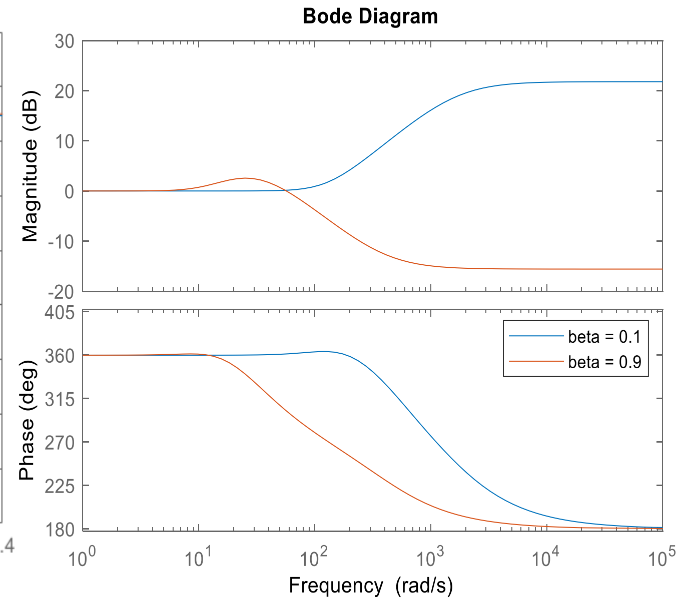


Figure 18. Bode Diagram of FMF depending on Beta

Bandwidth of fading memory filter should be larger than system bandwidth about 5 times

Even if we take high beta, bandwidth of it is fast enough compared to system bandwidth.

When we take low beta, it amplifies noise of gyro, which leads oscillation

Therefore, we choose beta = 0.9 to minimize effect of noise amplification

2. Modern Control

2-2. LQ Servo Controller Loop and Design

- **LQ Servo Controller Loop**

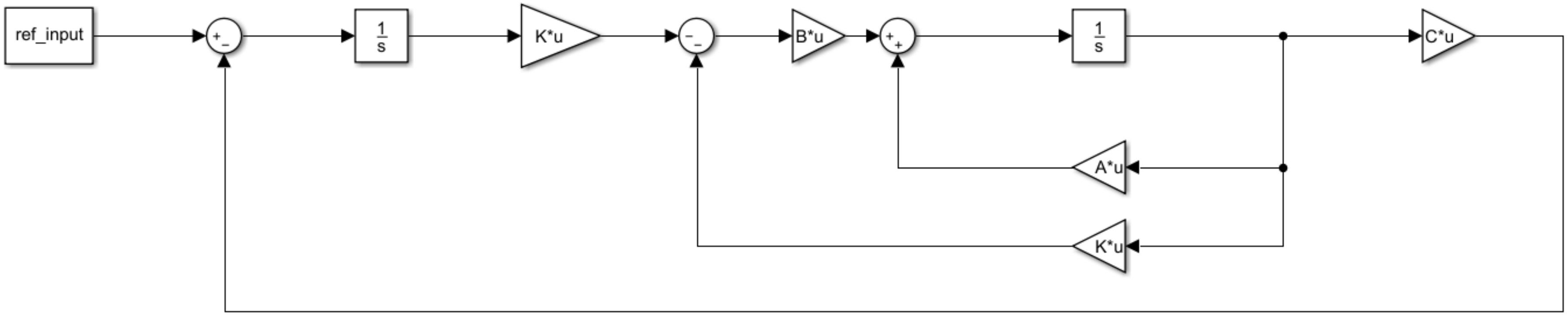


Figure 19. LQ Servo Controller Loop

2. Modern Control

2-2. LQ Servo Controller Loop and Design

➢ Chosen Parameters

$$cost\ function = J_{LQR} = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

$$Q = \begin{bmatrix} 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 70 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$R = 0.01, \qquad R_I = 200$$

- Values of Q matrix and R , $\mathrm{R_I}$ are chosen with considering system state, input, and dissipating energy performance which is represented in the cost function. Also, designer's heuristic of the system is added in the consideration.
- We have put more weight on the x state for tracking performance when deciding weighting value of Q. Weight on θ and $\dot{x}$ is slightly added to consider angle and velocity of ball. Also ,R and R_I is chosen in the concept of penalty.

2. Modern Control

2-2. LQ Servo Controller Loop and Design

➢ **Relative Margin Check**

$$GMR = -12.6[dB]\,, PM = 62.7[deg]$$

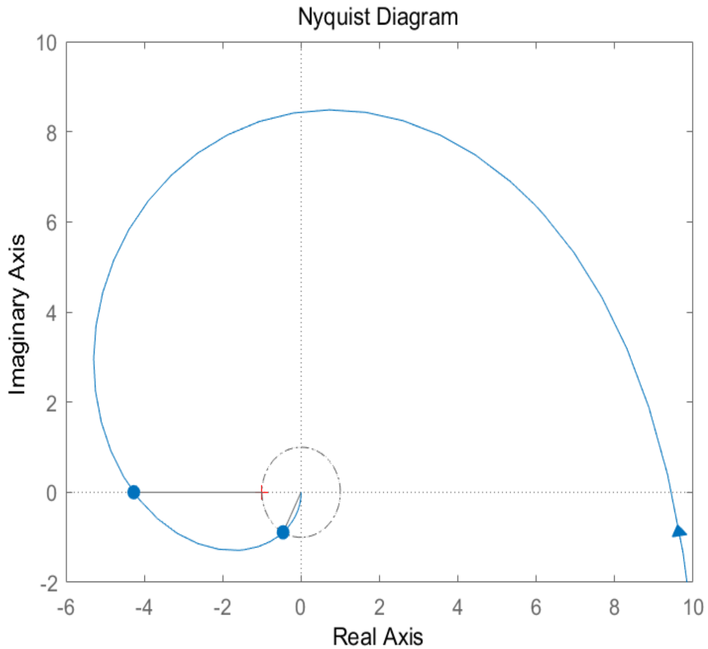


Figure 20. Nyquist of LQ Servo Controller

2. Modern Control

2-2. LQ Servo Controller Loop and Design

➢ **Experimental Result**

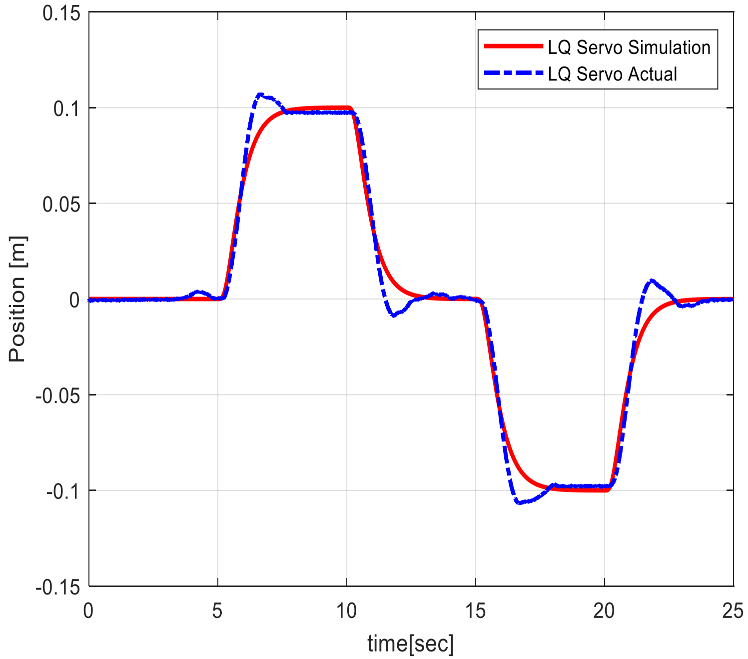


Figure 21. Result of LQ Servo controller

Table 12. Performance of LQ Servo Controller

	Simulation	Actual
t_r $(rising\ time)$	$1.76\ [sec]$	$1.24\ [sec]$
$\%OS\ (overshoot)$	$0[\%]$	$7.7[\%]$
$e_{ss}\ (steady-state\ error)$	$0.000[cm]$	$0.024[cm]$

Sim-real gap :

- *About overshoot : effect of model reduction of motor (2nd motor -> 1st order), break of small angle approximation*

3. Conclusion

- This experiment was held to compare classical method and modern method of control in ball and beam system.

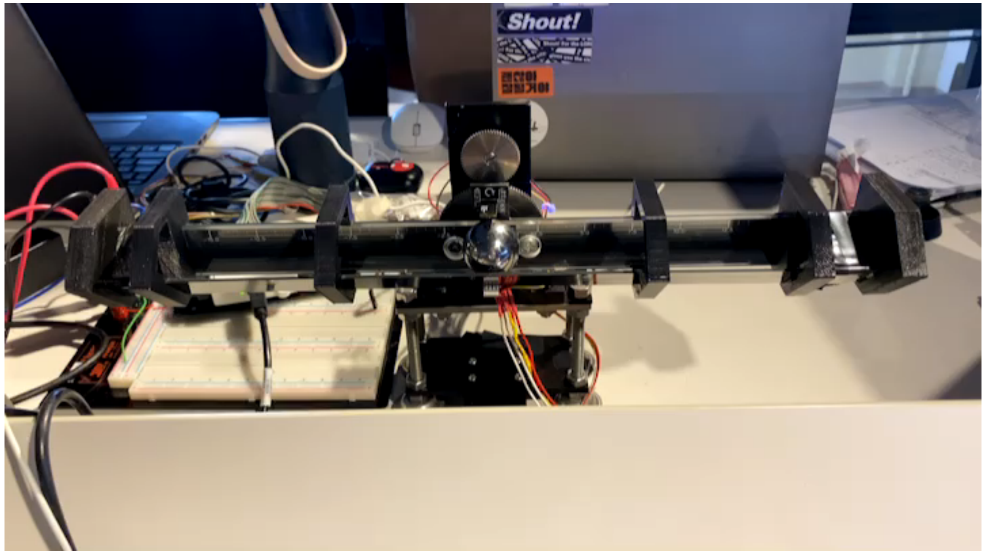
- Main difference between classical control and modern control method is that in classical control , the frequency separation, which can be observed through inner loop and outer loop control of ball and beam system, is necessary to control the system properly. Which is means, classical control method is more conservative control method.

- In classical method control system, multi control loop and PD controller was used to control ball and beam system.
  → Step response result : t_r = $0.4655[sec]$, $\%OS = 26.0173[\%]$, $e_{ss} = 0.2845[cm]$

- For modern control method, LQ servo was used to control system to control ball and beam system.
  → Step response result : t_r =1.24 $[sec]$, $\%OS = 7.7[\%]$, $e_{ss} = 0.024[cm]$

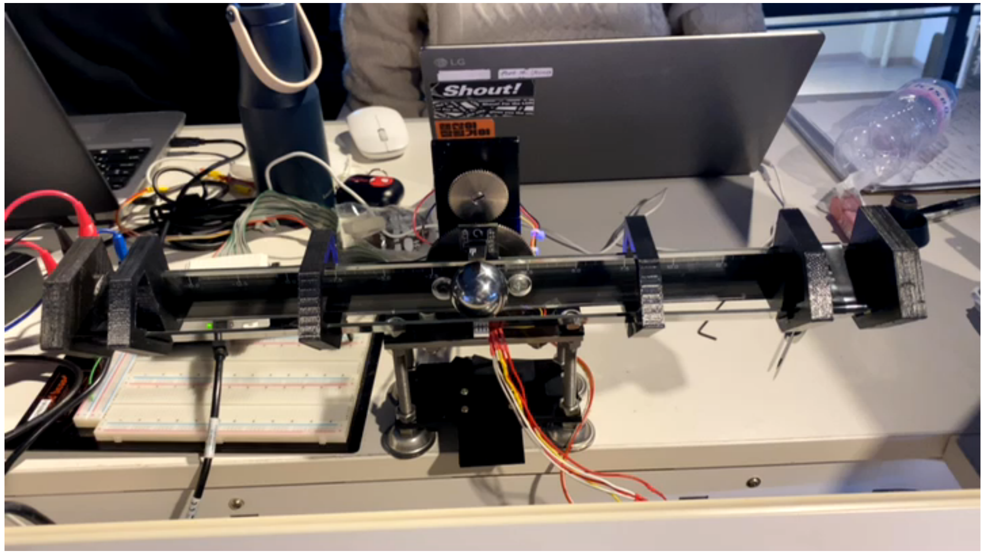
- Analyzing time response of experiment results, classical method using PD controller showed $0.7844[\text{sec}]$ faster rising time than LQ servo. But in a sense of percent overshoot and steady state error, LQ servo showed $18.3173[\%]$, $0.2605[\text{cm}]$ better performance respectively.

4. Appendix



Video 1. Classical Control (PD Controller) Result Video

4. Appendix



Video 1. Modern Control (LQ Servo) Result Video

THANK YOU!