

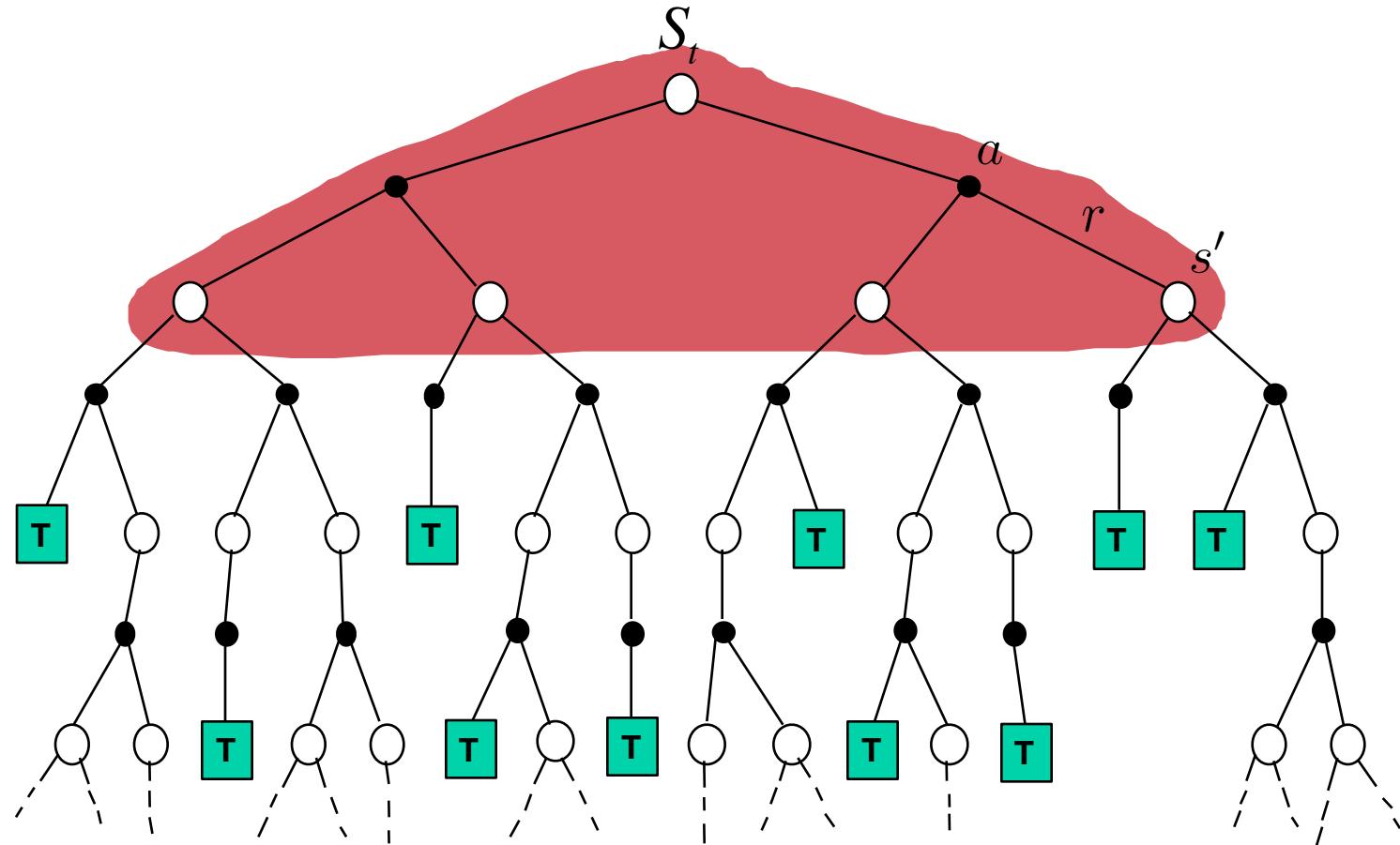
Chapter 6: Temporal Difference Learning

Objectives of this chapter:

- Introduce Temporal Difference (TD) learning
- Focus first on policy evaluation, or prediction, methods
- Compare efficiency of TD learning with MC learning
- Then extend to control methods

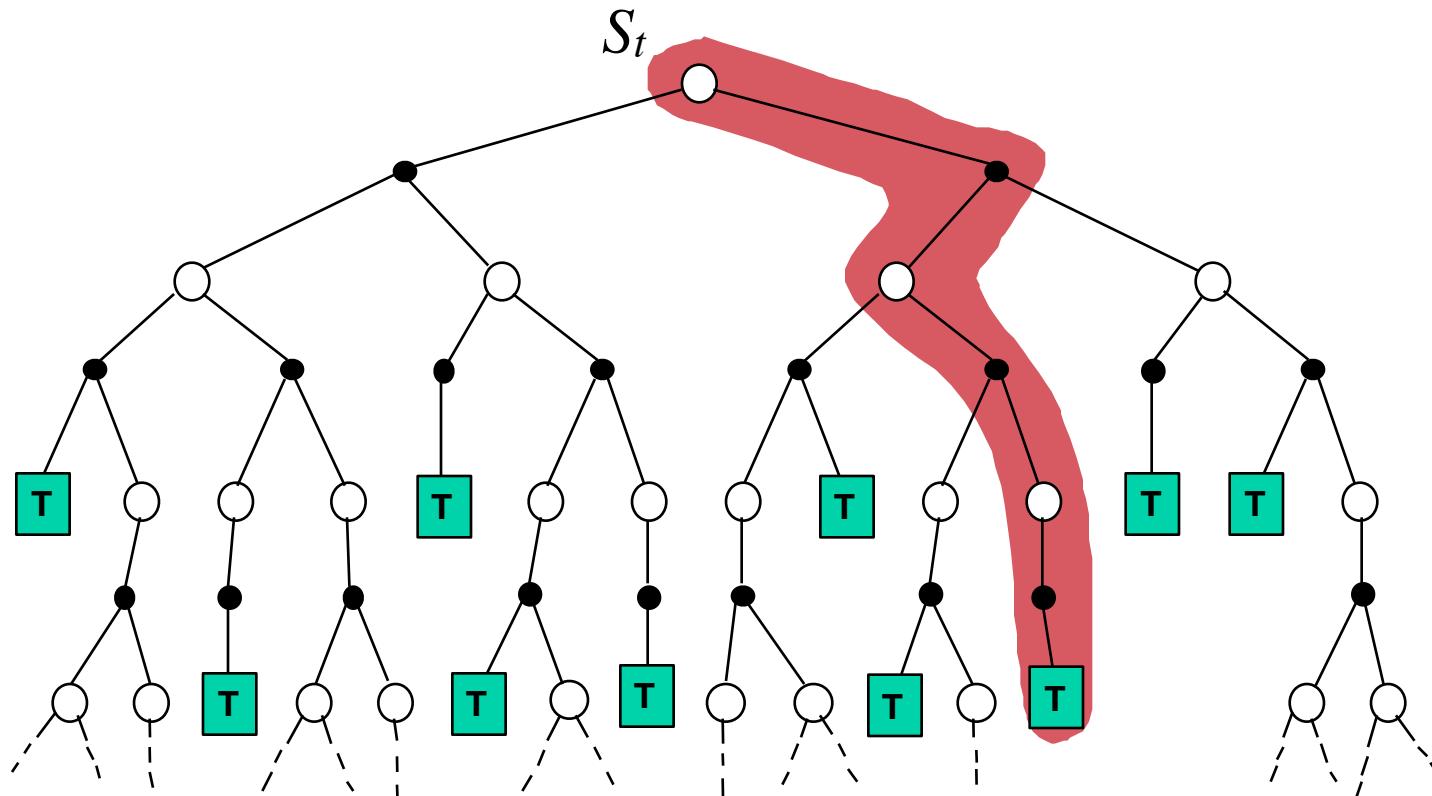
cf. Dynamic Programming

$$V(S_t) \leftarrow E_{\pi} [R_{t+1} + \gamma V(S_{t+1})] = \sum_a \pi(a|S_t) \sum_{s',r} p(s',r|S_t,a) [r + \gamma V(s')]$$



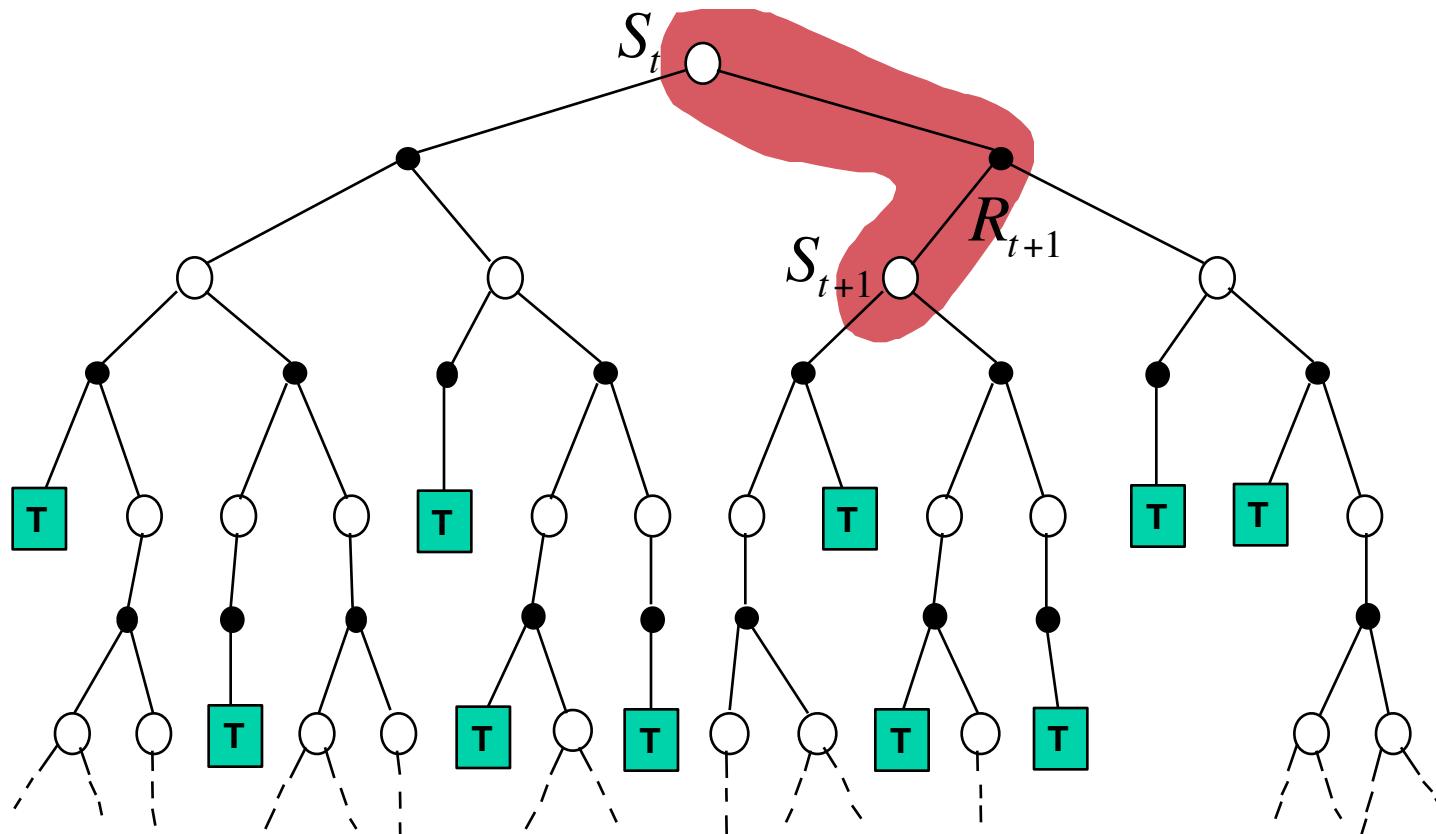
Simple Monte Carlo

$$V(S_t) \leftarrow V(S_t) + \alpha [G_t - V(S_t)]$$



Simplest TD Method

$$V(S_t) \leftarrow V(S_t) + \alpha [R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$$



TD methods bootstrap and sample

- **Bootstrapping:** update involves an *estimate*
 - MC does not bootstrap
 - DP bootstraps
 - TD bootstraps
- **Sampling:** update does not involve an *expected value*
 - MC samples
 - DP does not sample
 - TD samples

TD Prediction

Policy Evaluation (the prediction problem):

for a given policy π , compute the state-value function v_π

Recall: Simple every-visit Monte Carlo method:

$$V(S_t) \leftarrow V(S_t) + \alpha [G_t - V(S_t)]$$

target: the actual return after time t

The simplest temporal-difference method TD(0):

one-step TD
also called
“TD update”

$$V(S_t) \leftarrow V(S_t) + \alpha [R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$$

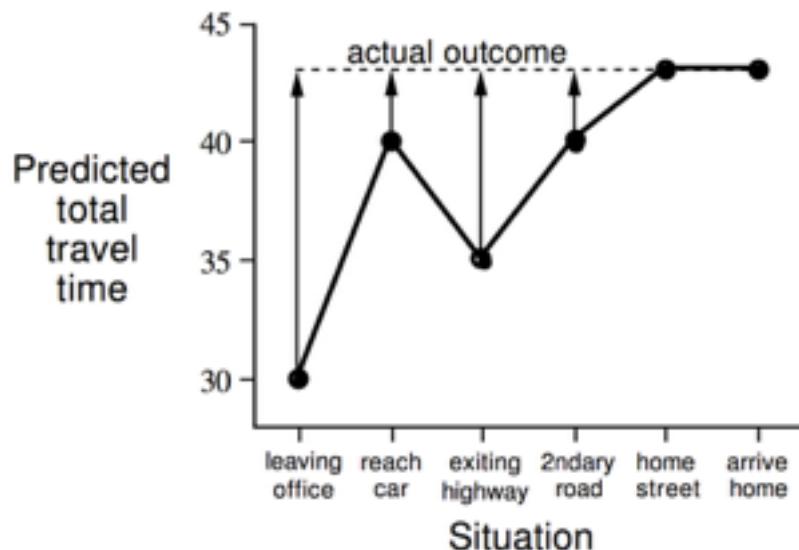
target: an estimate of the return

Example: Driving Home

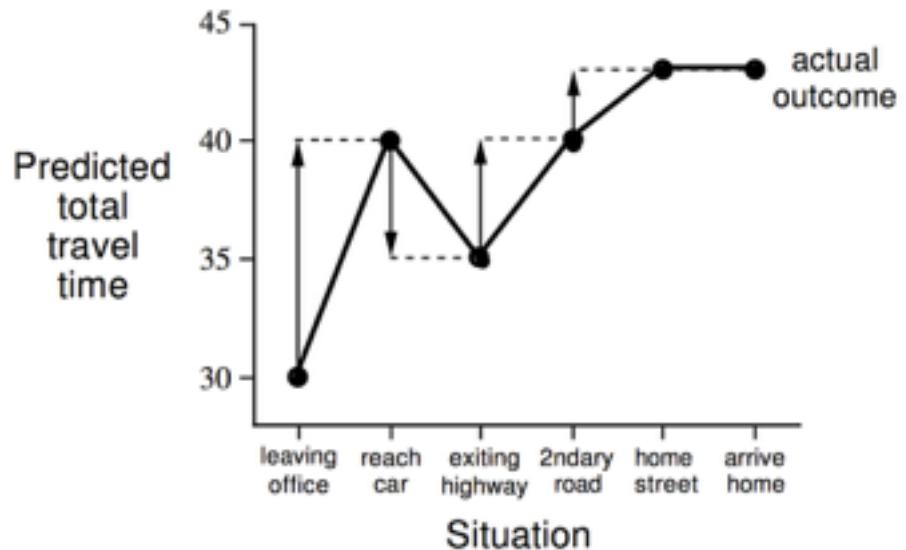
<i>State</i>	<i>Elapsed Time (minutes)</i>	<i>Predicted Time to Go</i>	<i>Predicted Total Time</i>
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Driving Home

Changes recommended by
Monte Carlo methods ($\alpha=1$)



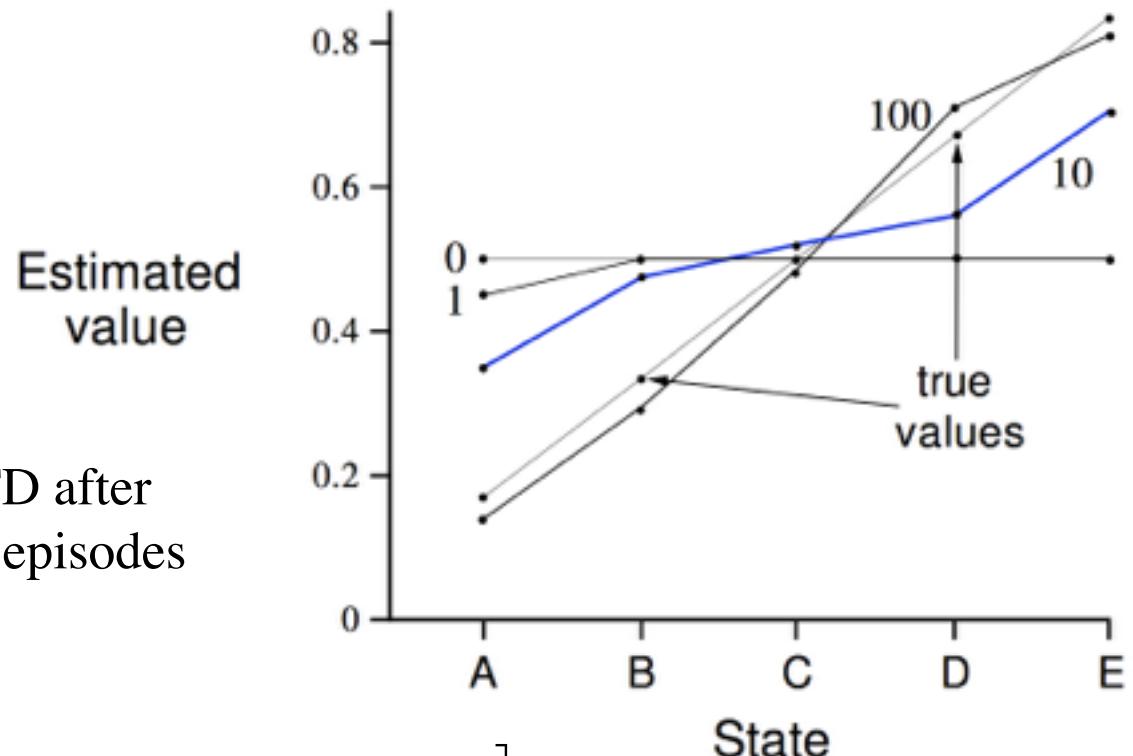
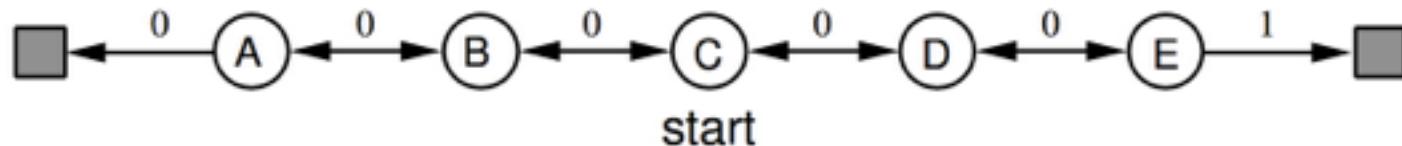
Changes recommended
by TD methods ($\alpha=1$)



Advantages of TD Learning

- TD methods do not require a model of the environment, only experience
- TD, but not MC, methods can be fully incremental
 - You can learn **before** knowing the final outcome
 - Less memory
 - Less peak computation
 - You can learn **without** the final outcome
 - From incomplete sequences
- Both MC and TD converge (under certain assumptions to be detailed later), but which is faster?

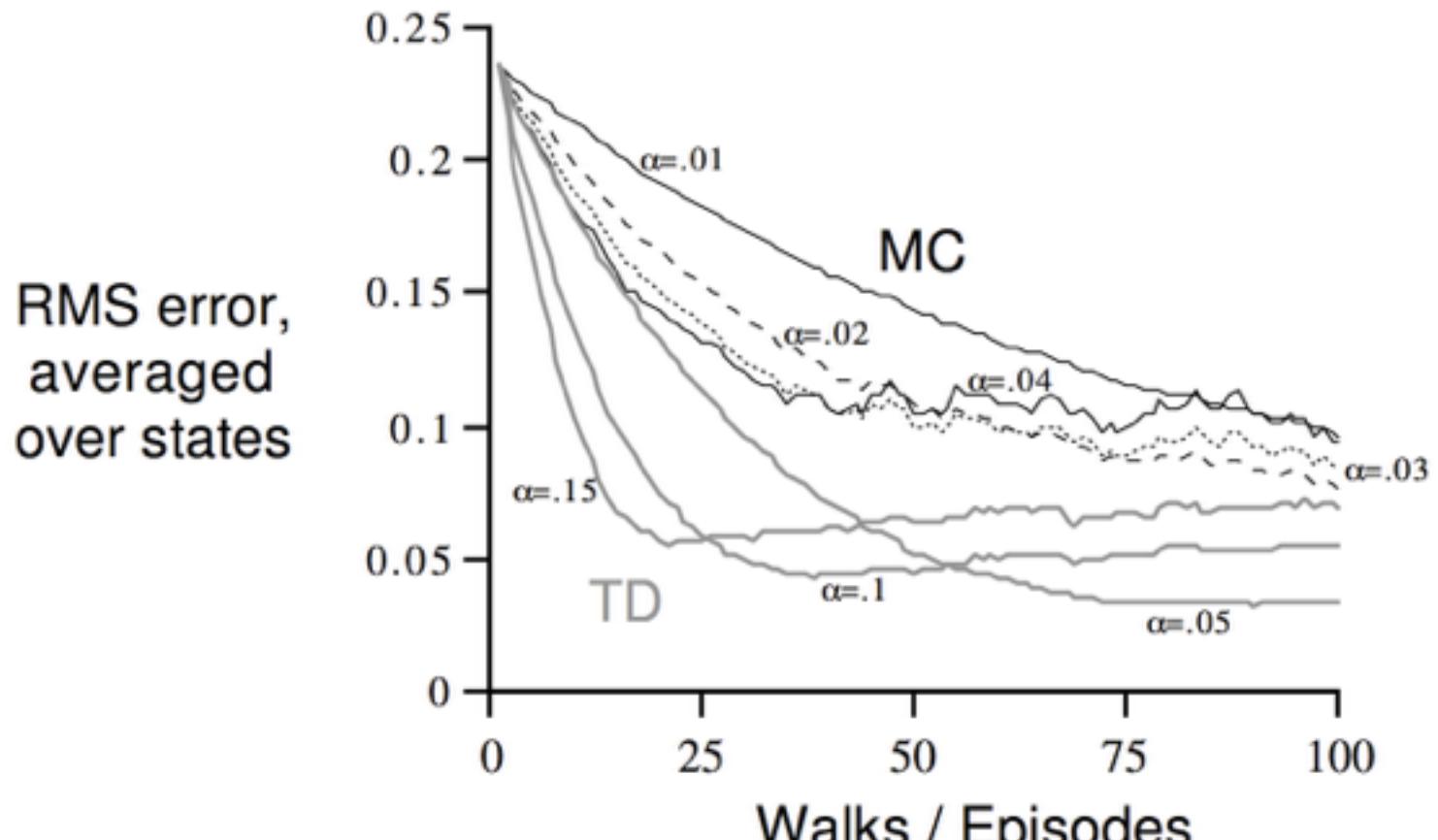
Random Walk Example



Values learned by TD after various numbers of episodes

$$V(S_t) \leftarrow V(S_t) + \alpha [R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$$

TD and MC on the Random Walk



Data averaged over
100 sequences of episodes

Batch Updating in TD and MC methods

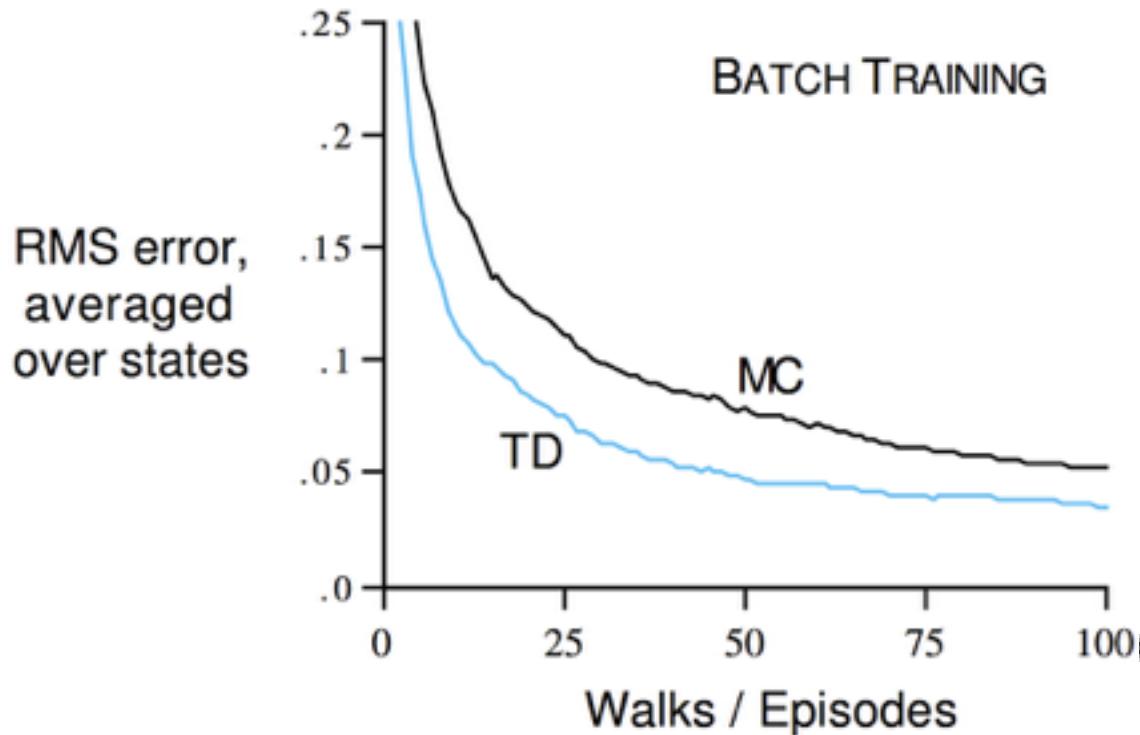
Batch Updating: train completely on a finite amount of data,
e.g., train repeatedly on 10 episodes until convergence.

Compute updates according to TD or MC, but only update
estimates after each complete pass through the data.

For any finite Markov prediction task, under batch updating,
TD converges for sufficiently small α .

Constant- α MC also converges under these conditions, **but to
a difference answer!**

Random Walk under Batch Updating



After each new episode, all previous episodes were treated as a batch, and algorithm was trained until convergence. All repeated 100 times.

You are the Predictor

Suppose you observe the following 8 episodes:

A, 0, B, 0

B, 1

B, 1

$V(B)? \quad 0.75$

B, 1

$V(A)? \quad 0?$

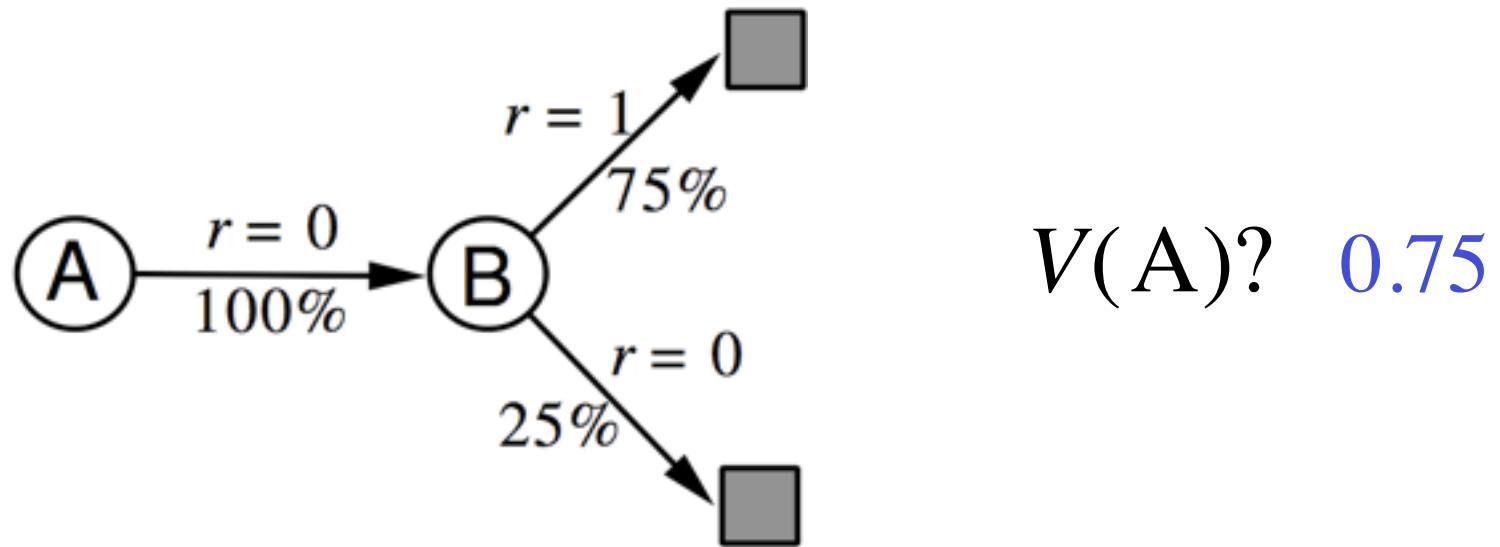
B, 1

B, 1

B, 0

Assume Markov states, no discounting ($\gamma = 1$)

You are the Predictor



You are the Predictor

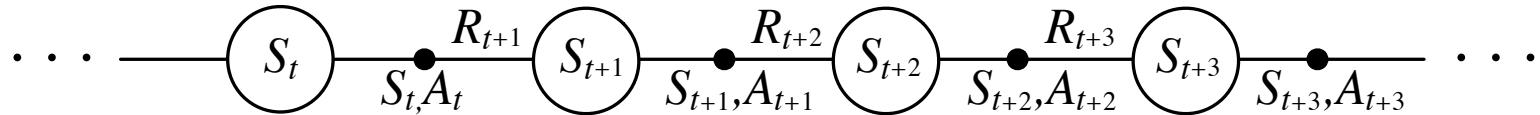
- The prediction that best matches the training data is $V(A)=0$
 - This **minimizes the mean-square-error** on the training set
 - This is what a batch Monte Carlo method gets
- If we consider the sequentiality of the problem, then we would set $V(A)=.75$
 - This is correct for the **maximum likelihood** estimate of a Markov model generating the data
 - i.e, if we do a best fit Markov model, and assume it is exactly correct, and then compute what it predicts (how?)
 - This is called the **certainty-equivalence estimate**
 - This is what TD gets

Summary so far

- Introduced *one-step tabular model-free TD methods*
- These methods bootstrap and sample, combining aspects of DP and MC methods
- TD methods are *computationally congenial*
- If the world is truly Markov, then TD methods will learn faster than MC methods
- MC methods have lower error on past data, but higher error on future data

Learning An Action-Value Function

Estimate q_π for the current policy π



After every transition from a nonterminal state, S_t , do this:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha [R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)]$$

If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

Sarsa: On-Policy TD Control

Turn this into a control method by always updating the policy to be greedy with respect to the current estimate:

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\text{terminal-state}, \cdot) = 0$

Repeat (for each episode):

 Initialize S

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Repeat (for each step of episode):

 Take action A , observe R, S'

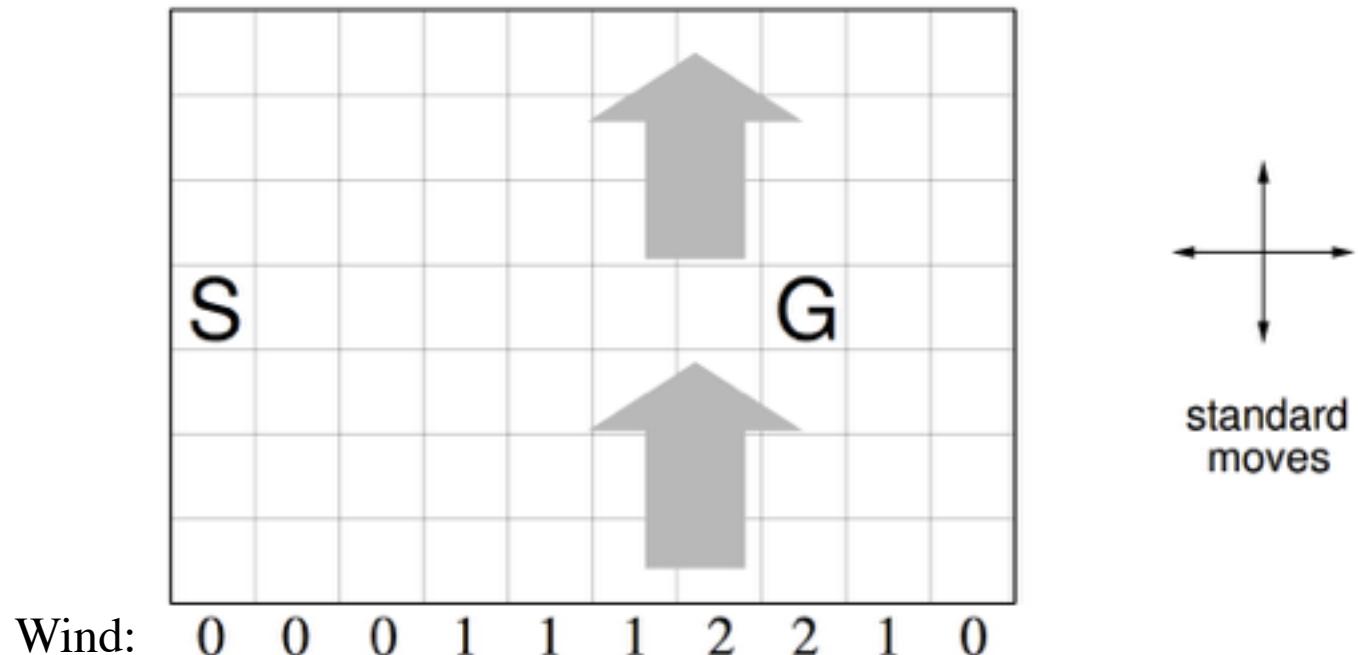
 Choose A' from S' using policy derived from Q (e.g., ε -greedy)

$$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma Q(S', A') - Q(S, A)]$$

$$S \leftarrow S'; A \leftarrow A';$$

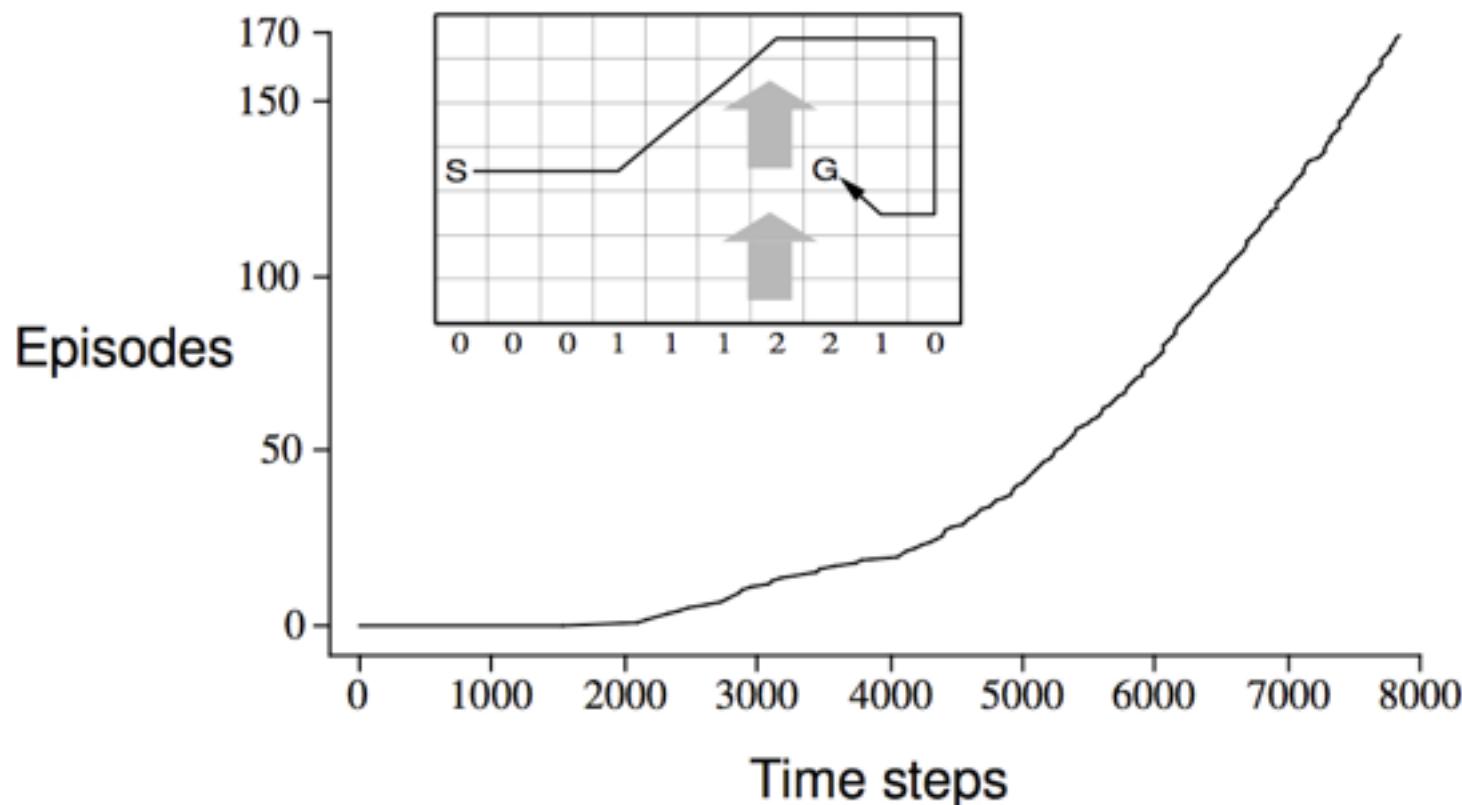
 until S is terminal

Windy Gridworld



undiscounted, episodic, reward = -1 until goal

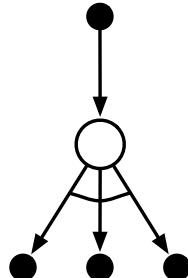
Results of Sarsa on the Windy Gridworld



Q-Learning: Off-Policy TD Control

One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$



Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\text{terminal-state}, \cdot) = 0$

Repeat (for each episode):

 Initialize S

 Repeat (for each step of episode):

 Choose A from S using policy derived from Q (e.g., ε -greedy)

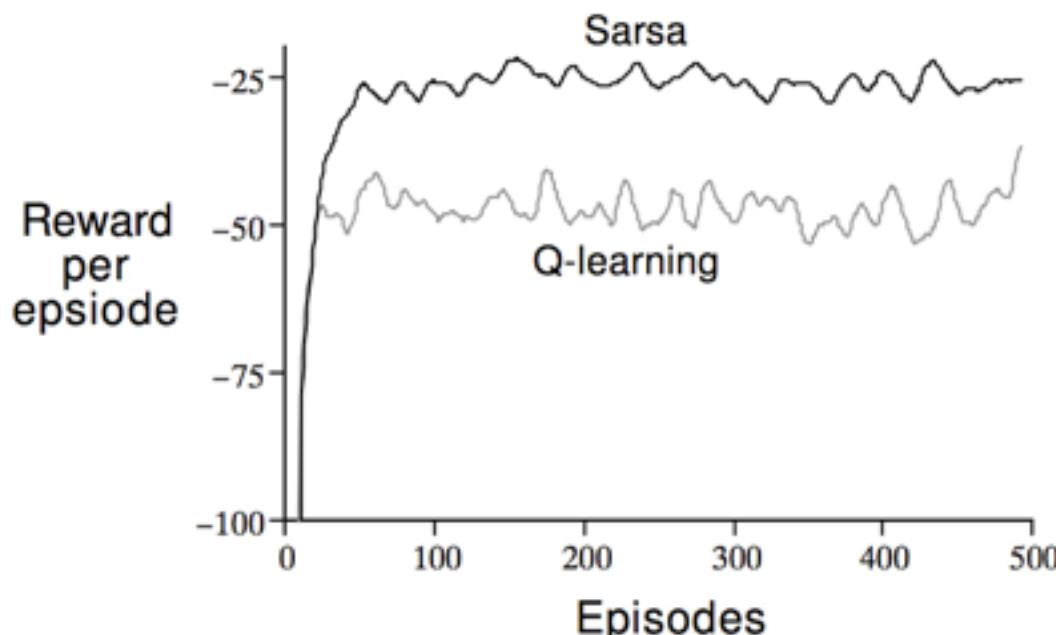
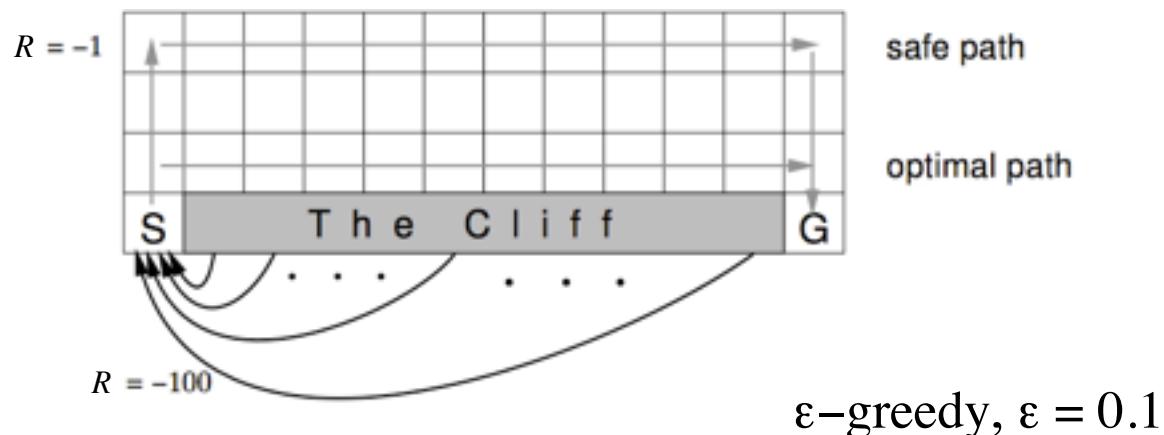
 Take action A , observe R, S'

$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$

$S \leftarrow S'$;

 until S is terminal

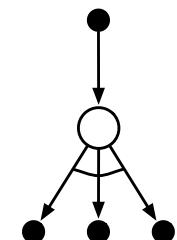
Cliffwalking



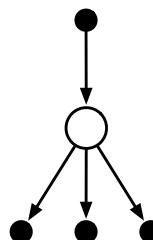
Expected Sarsa

- Instead of the *sample* value-of-next-state, use the expectation!

$$\begin{aligned} Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t) \right] \\ &\leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \sum_a \pi(a|S_{t+1})Q(S_{t+1}, a) - Q(S_t, A_t) \right] \end{aligned}$$



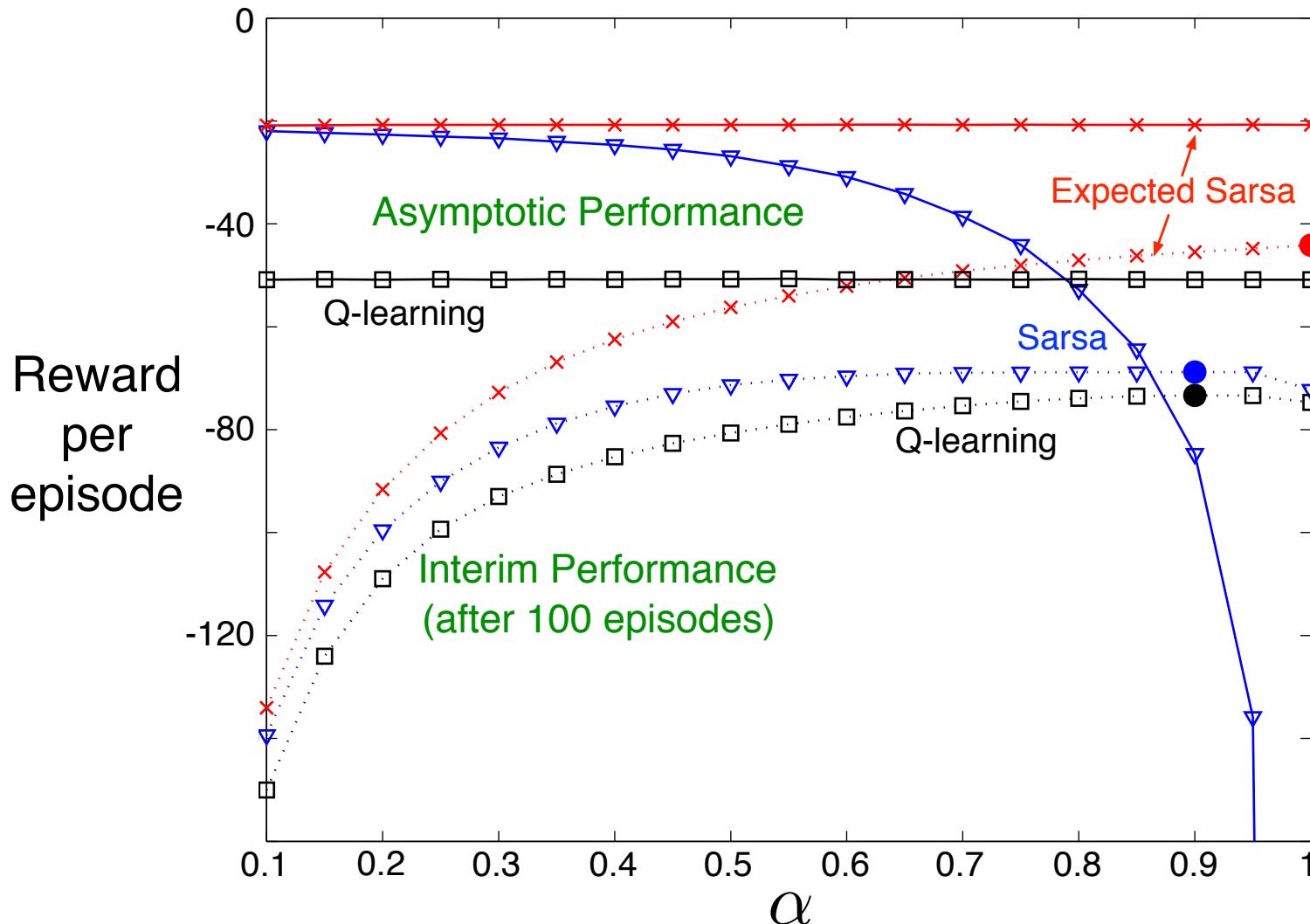
Q-learning



Expected Sarsa

- Expected Sarsa's performs better than Sarsa (but costs more)

Performance on the Cliff-walking Task

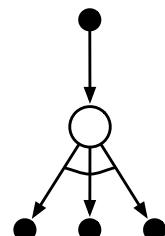


Off-policy Expected Sarsa

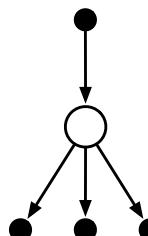
- Expected Sarsa generalizes to arbitrary behavior policies μ
 - in which case it includes Q-learning as the special case in which π is the greedy policy

$$\begin{aligned} Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t) \right] \\ &\leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \sum_a \pi(a|S_{t+1})Q(S_{t+1}, a) - Q(S_t, A_t) \right] \end{aligned}$$

Nothing
changes
here



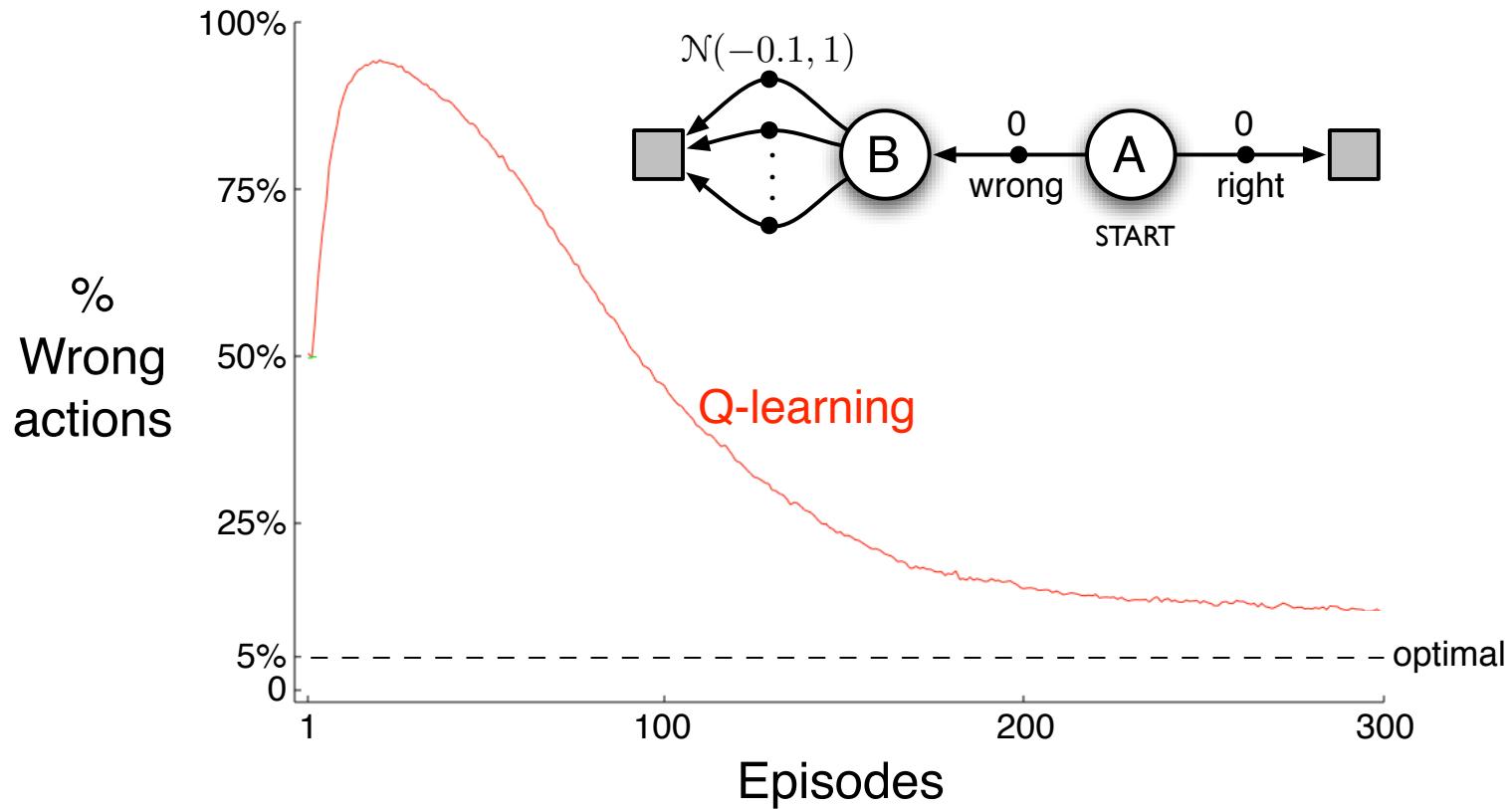
Q-learning



Expected Sarsa

- This idea seems to be new

Maximization Bias Example



Tabular Q-learning:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$

Double Q-Learning

- Train 2 action-value functions, Q_1 and Q_2
- Do Q-learning on both, but
 - never on the same time steps (Q_1 and Q_2 are indep.)
 - pick Q_1 or Q_2 at random to be updated on each step
- If updating Q_1 , use Q_2 for the value of the next state:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha \left(R_{t+1} + Q_2\left(S_{t+1}, \arg \max_a Q_1(S_{t+1}, a)\right) - Q_1(S_t, A_t) \right)$$

- Action selections are (say) ε -greedy with respect to the sum of Q_1 and Q_2

Double Q-Learning

Initialize $Q_1(s, a)$ and $Q_2(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily

Initialize $Q_1(\text{terminal-state}, \cdot) = Q_2(\text{terminal-state}, \cdot) = 0$

Repeat (for each episode):

 Initialize S

 Repeat (for each step of episode):

 Choose A from S using policy derived from Q_1 and Q_2 (e.g., ε -greedy in $Q_1 + Q_2$)

 Take action A , observe R, S'

 With 0.5 probability:

$$Q_1(S, A) \leftarrow Q_1(S, A) + \alpha \left(R + \gamma Q_2(S', \arg\max_a Q_1(S', a)) - Q_1(S, A) \right)$$

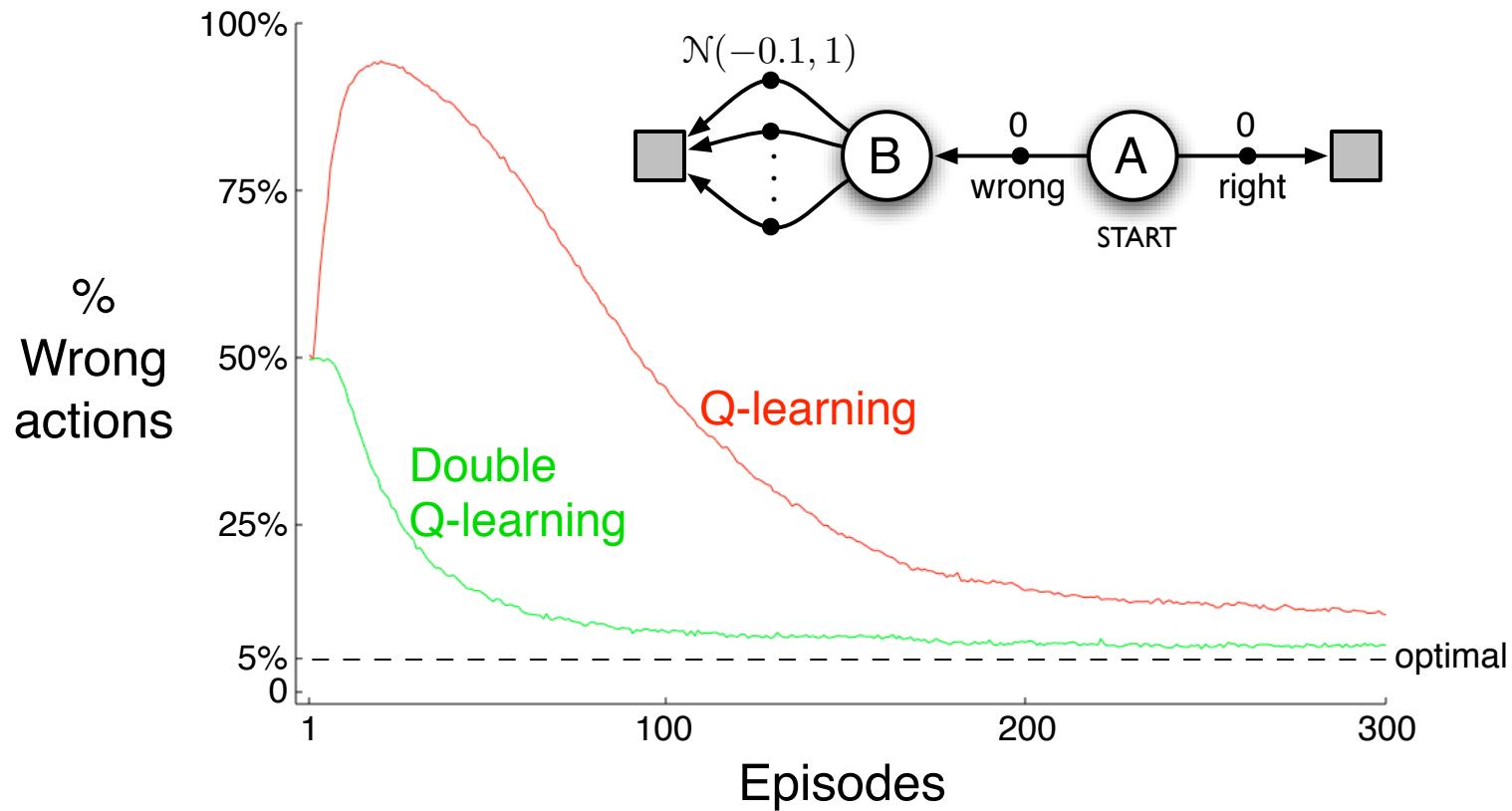
 else:

$$Q_2(S, A) \leftarrow Q_2(S, A) + \alpha \left(R + \gamma Q_1(S', \arg\max_a Q_2(S', a)) - Q_2(S, A) \right)$$

$S \leftarrow S'$;

 until S is terminal

Example of Maximization Bias

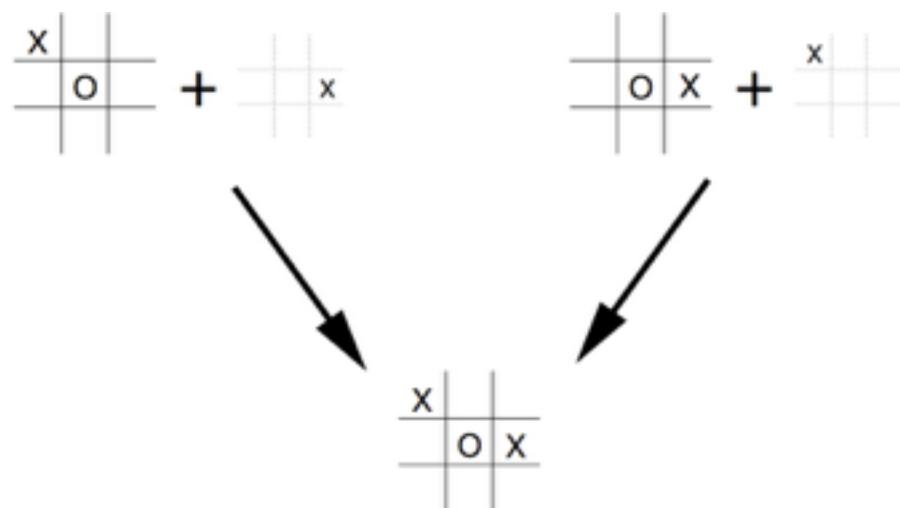


Double Q-learning:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha \left(R_{t+1} + Q_2(S_{t+1}, \arg \max_a Q_1(S_{t+1}, a)) - Q_1(S_t, A_t) \right)$$

Afterstates

- Usually, a state-value function evaluates states in which the agent can take an action.
- But sometimes it is useful to evaluate states **after** agent has acted, as in tic-tac-toe.
- Why is this useful?



- What is this in general?

Summary

- Introduced *one-step tabular model-free TD methods*
- These methods bootstrap and sample, combining aspects of DP and MC methods
- TD methods are *computationally congenial*
- If the world is truly Markov, then TD methods will learn faster than MC methods
- MC methods have lower error on past data, but higher error on future data
- Extend prediction to control by employing some form of GPI
 - On-policy control: *Sarsa, Expected Sarsa*
 - Off-policy control: *Q-learning, Expected Sarsa*
- Avoiding maximization bias with Double Q-learning

Unified View

