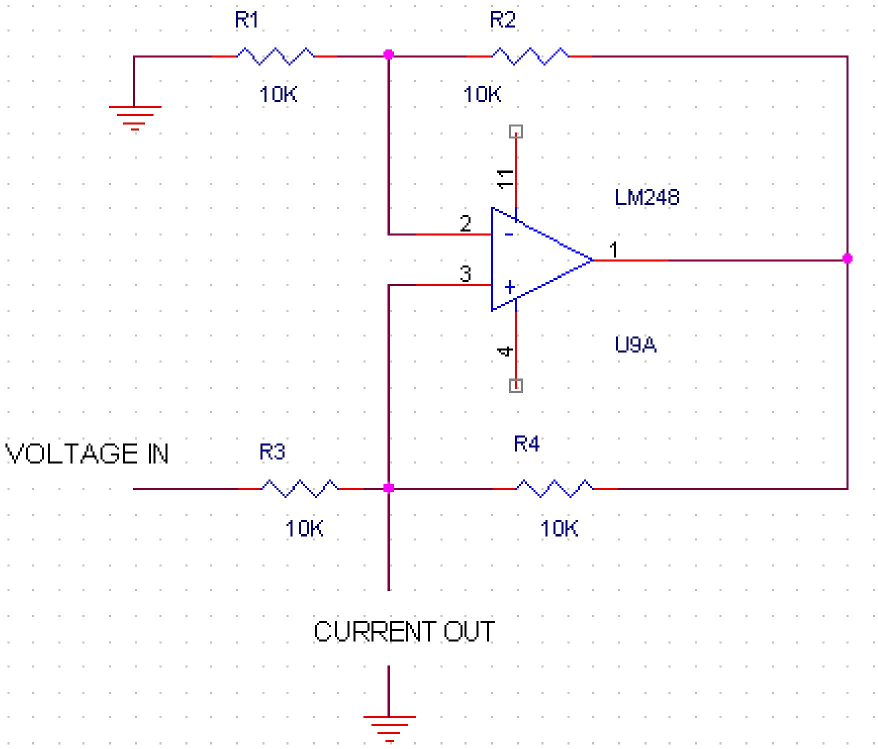
The Howland Current Pump

The figure below is one configuration of the famous Howland Current Pump. It is used to convert voltage to current, independent of the load. Operation of this circuit is not obvious. It doesn't have the standard operational amplifier configuration which is one input to the op amp of a known, or fixed value. The reason for this is that the output is fed back to both the positive input and the negative input. So there is no single input that is fixed. The same rules apply, that is, the inputs to the op amp always want to be the same. A bit of algebra is required in order to determine how this circuit works.

O.K.! What do we know from looking at this circuit? Because of the feedback from the output to the "-" input of the op amp and because R1 and R2 are of equal value, the "-" input will always have ½ the voltage value of the output. The "+" input of the op amp will also have this voltage value (inputs always want to be equal). Notice that R1 and R2 do not have to be 10KOhms. They can be any reasonable value (the higher the better, without being affected by the op-amp input) as long as R1 = R2.



How do we approach the problem? Let's start with what we know. First, the voltage at the "-" input is equal to half of the output voltage (call the voltage at the "-" input Va);

$$Va = \frac{Vout}{2} \qquad \text{or} \qquad Vout = 2Va$$

Now lets look at some currents. The current through R3 is;

$$\frac{\text{VOLTAGE IN} - Va}{R3} = i1$$

Call the output of the op amp "Vout". The current through R4 is;

$$\frac{Vout - Va}{R4} = i2$$

(Notice that we are assuming that the current through R4 is flowing from Vout toward Va).

We know that the CURRENT OUT is equal to i1 plus i2 (the "+" input to the op amp doesn't draw any current). So we get the formulas;

$$\text{CURRENT OUT} = i1 + i2$$

Substituting for i1 and i2:

$$\text{CURRENT OUT} = \frac{\text{VOLTAGE IN} - Va}{R3} + \frac{Vout - Va}{R4}$$

Since R3 and R4 are equal we can just call them "R".

Remember that Va is equal to $\frac{Vout}{2}$

Substituting this into the above equation yields;

$$\text{CURRENT OUT} = \frac{\text{VOLTAGE IN} - \frac{Vout}{2}}{R} + \frac{Vout - \frac{Vout}{2}}{R}$$

The $\text{Vout} - \frac{\text{Vout}}{2}$ term becomes just $\frac{\text{Vout}}{2}$

Since both terms are divided by R, simply add the $\frac{+\text{Vout}}{2}$ to the $\frac{-\text{Vout}}{2}$ terms, leaving nothing but

the "VOLTAGE IN" term, divided by R. The result is;

$$\text{CURRENT OUT} = \frac{\text{VOLTAGE IN}}{R}$$

Holy cow! What a simple relationship. But is it true? Lets substitute real values again and see what happens.

Howland Current Pump Example:

In the above circuit let Vin equal 10 Volts.
Then the Current Out will be 10V divided by 10KOhms or 1 milliAmp.
Take a situation where the load resistance is greater than zero Ohms and a value of 1Kohm for the load resistor.

The current through the 1Kohm load resistor must be 1mA (as long as the circuit is not in saturation). This means that the voltage drop across the load resistor is 1 Volt. Therefore, the voltage at pin 3 of the LM248 is also 1 Volt. Since the voltage on pin 2 is the same as the voltage on pin 3, the voltage on pin 2 will be ½ Vout. So Vout will be equal to 2 Volts. Now lets figure out the current through R3 and then R4;

$$\text{Current through R3} = \frac{\text{VOLTAGE IN} - \text{V(pin 2)}}{\text{10KOhms}} = \frac{\text{10Volts} - \text{1 Volt}}{\text{10KOhms}} = \text{0.9mA}$$

$$\text{Current through R4} = \frac{\text{Vout} - \text{Va}}{\text{10KOhms}} = \frac{\text{2Volts} - \text{1 Volt}}{\text{10KOhms}} = \text{0.1mA}$$

The sum of these current is 1mA, the desired current.

So the darned thing works! At least in this instance. You may wish to try a few more variations just to be sure. One thing to note is that for any particular input, there is no particular output from the op amp. When load resistance changes, the current through the load remains constant (or directly related to the input voltage). The only thing that can vary in order to accommodate these load changes is the op amp output voltage.

The simpler method:

Knowing what this circuit does leads to one of the easiest circuit solutions yet. The "current out" has to be the same, whatever the load. So let's consider the load to be zero Ohms. Simply ground the junction of R3, R4 and pin3 of the LM248. Now, the "voltage in" is applied entirely across R3. Pin 3 of the LM248 is held firmly at ground. Therefore pin 2 of of the LM248 will also try to be at ground potential. The only way this can occur is if there is no current through the resistor R1 which is connected to ground. No current through R1 means no current through R2 which means no voltage (ground potential) at the output of the LM248 and thus no contribution to the output current from the LM248. All of the load current will pass through R3. This leads to the formula:

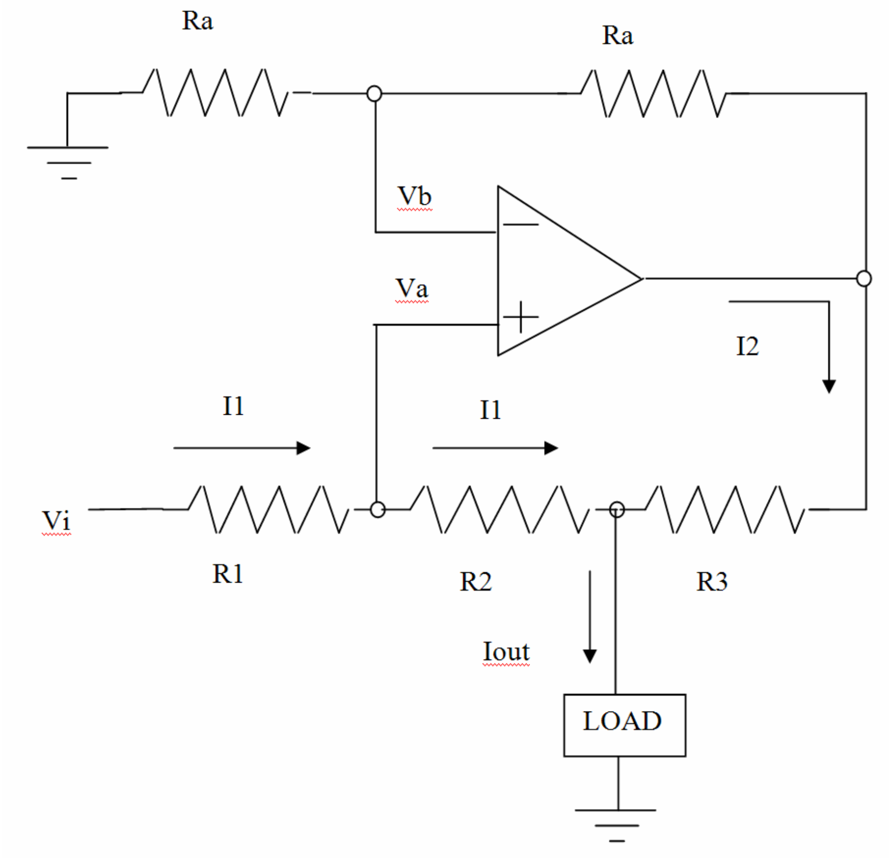
$$\text{Current Out} = \frac{\text{Voltage In}}{\text{R3}} \qquad \text{Look Familiar?}$$

The only short-coming of this circuit is that the output currents are quite small. Our example used 10KOhm resistors but if we were to use 1KOhm resistors, the CURRENT OUT would be try to be 10 milliamps with 10 Volts in. With a 1KOhm load the voltage drop across the load would be 10 Volts. Ten volts in (VOLTAGE IN) and ten volts for Va means zero current through R3. This means that all of the 10 milliamps has to come from the output of the op amp. But the op amp would have to produce 20 volts in order for this to happen. This is well above most op amp positive power supply voltages. Then too, most standard op amps can't source or sink 10 milliamps, but that's another story.

Being limited to low currents is a serious circuit drawback for the Howland current pump. But help is on the way!

The Improved Howland Current Pump

There is a circuit that at least helps the op amp output voltage from trying to exceed the op amp supply voltage (even though the op amp is still limited in the amount of current it can source and sink).   This circuit is known as the 'Improved Howland Current Pump'.



With the addition of one resistor, R3, the output voltage requirement of the LM248 is reduced significantly from that of the original Howland Current Pump.

Using the schematic above, a determination of the General Solution for this circuit can be made:

Lets use the same trick that we learned while examining the Howland Current Pump. Since this is a current source it should not matter what the load is so, Make the Load be zero Ohms that is, ground the output.

With the output (junction of R2 and R3) grounded: $I1 = \frac{Vin}{R1 + R2}$

The voltage at Va is, $Va = I1 * R2$

The voltage at Vb is the same as at Va, therefore Vout must equal 2Va.

The current I2 that passes through R3 is: $I2 = \frac{Vout}{R3}$

Since Iout = I1 + I2, $Iout = \frac{Vin}{R1 + R2} + \frac{Vout}{R3}$

Another way of expressing Vout is to say that $Vout = 2Va, = 2 * I1 * R2$

and substituting $\frac{Vin}{R1 + R2}$ for I1 in the above equation, $Vout = \frac{2 * Vin * R2}{R1 + R2}$

From the equation, $Iout = \frac{Vin}{R1 + R2} + \frac{Vout}{R3}$ we now get; $Iout = \frac{Vin}{R1+R2} + \frac{\;\frac{2 * Vin * R2}{R1+R2}\;}{R3}$

Multiplying the denominator of the second term by R3 we get:

$$Iout = \frac{Vin}{R1 + R2} + \frac{2 * Vin * R2}{R3 (R1 + R2)}$$

Factoring out $\frac{Vin}{R1 + R2}$ from each term we get $Iout = \frac{Vin}{R1 + R2}\left(1 + \frac{2 * R2}{R3}\right)$

This is the General form of the Vin/Iout transfer function of the Improved Howland Current Pump.

Let’s test it out.
If you’re convinced that grounding the output is totally valid then we’ll use this configuration once again. If you’re not convinced, then too bad! we’re grounding it in this example.

Given the situation in which $R1 = R2 + R3$

Select some values for R1, R2 and R3 based upon this relationship.

If R1 = 10KOhms and R2 = 9.9KOhms then R3 must equal 100 Ohms.

For simplicity let Vin equal 10 Volts

Remember, Ra can be any value (preferably a high resistance value) as long as it is not affected (loaded) by input Vb of the op-amp. 100KOhms sounds good to me.

So, what is the current contributed by the R1-R2 path? It will be the input voltage (10V) divided by the sum of R1 and R2, or 19.9KOhms. This current is approximately 5.025 X10e-4 or 502.5 micro-Amps.

The voltage drop across R2 will be 9.9KOhms times 502.5 micro-Amps or approximately 4.975 Volts.
This will also be the voltage value at Va, the input to the op-amp. Vb will also be at this potential. This can only happen if Vout is 2 * Vb. So Vout is 2 * 4.975V or approximately 9.95 Volts.

This 9.95 Volts is also across R3, the other end of which is to ground. Therefore the current contribution of R3 is approx. 0.0995 Amps.

The total current through the load is the 502.5 micro-Amps and the 0.0995Amps or approximately 0.1A (100mA).

So we have achieved a 100mA output without saturating the voltage output of the op-amp (less than 10V out) using this configuration. I realize that this is some special op-amp because that's a lot of current for most op-amps.

HEY! Notice one more thing. The output voltage is directly proportional to the numeric value of the output current. 10 Volts in, 100mA out. This makes for convenience in design.

Anyway, does our GENERAL FORMULA work in this case?

$$Iout = \frac{Vin}{R1 + R2}\left(1 + \frac{2 * R2}{R3}\right), \quad = \quad 0.1 \text{ Amps, or } 100\text{mA}$$

Simplifying the General Form of the Equation:

The last example used a specific relationship between the values of R1, R2 and R3. If we do a little mathematical hocus-pocus we may be able to tailor the General Form of the Improved Howland Current Pump equation to this specific situation.

Here goes!

$$Iout = \frac{Vin}{R1 + R2}\left(1 + \frac{2 * R2}{R3}\right)$$

Expand the exression:

$$= \frac{Vin}{R1 + R2} + \frac{2 * R2 * Vin}{R3\,(R1 + R2)}$$

Multiply the top and bottom of the first term by R3 and combine the two terms:

$$= \frac{R3 * Vin + 2 * R2 * Vin}{R3\,(R1 + R2)}$$

Remembering that this special case requires that R2 + R3 equals R1, substitute 'R1 minus R2' for R3 in the numerator only:

$$= \frac{R1 * Vin - R2 * Vin + 2 * R2 * Vin}{R3\,(R1 + R2)}$$

Simplifying the numerator

$$= \frac{R1 * Vin + R2 * Vin}{R3\,(R1 + R2)}$$

Combining numerator terms:

$$= \frac{Vin\,(R1 + R2)}{R3\,(R1 + R2)}$$

Cancelling terms:

$$Iout = \frac{Vin}{R3}$$

How about that! What a simple transfer function. Remember, it's only for this special case.

Minimizing Unique Resistor Values

Let's take one more case. Suppose you don't care about the convenience of calculating voltage-to-current but that you want to minimize the number of unique resistor values in the circuit. To do this, let's set R1 equal to R2. We can also set the Ra's equal to R1 as long as they don't take too much current away from the op-amp output.

We'll do the mathematical gymnastics first and try to get a new formula for this situation.

One more time, with feeling.

$$Iout = \frac{Vin}{R1 + R2}\left(1 + \frac{2 * R2}{R3}\right)$$

Expand the exression:

$$= \frac{Vin}{R1 + R2} + \frac{2 * R2 * Vin}{R3\,(R1 + R2)}$$

Since R1 = R2, substitute R1 for R2:

$$= \frac{Vin}{2 * R1} + \frac{2 * R1 * Vin}{R3\,(\,2 * R1)}$$

Cancel:

$$= \frac{Vin}{2 * R1} + \frac{Vin}{R3}$$

Factor:

$$Iout = Vin\left(\frac{1}{2 * R1} + \frac{1}{R3}\right)$$

That's it. So if you want to find the best common resistor values you'll have to solve for R3, set up a spreadsheet or use Mathcad and select common values.

Good luck.