

Lecture 6:

Training Neural Networks,

Part I

Administrative

Assignment 1 due **Thursday (today)**, 11:59pm on Canvas

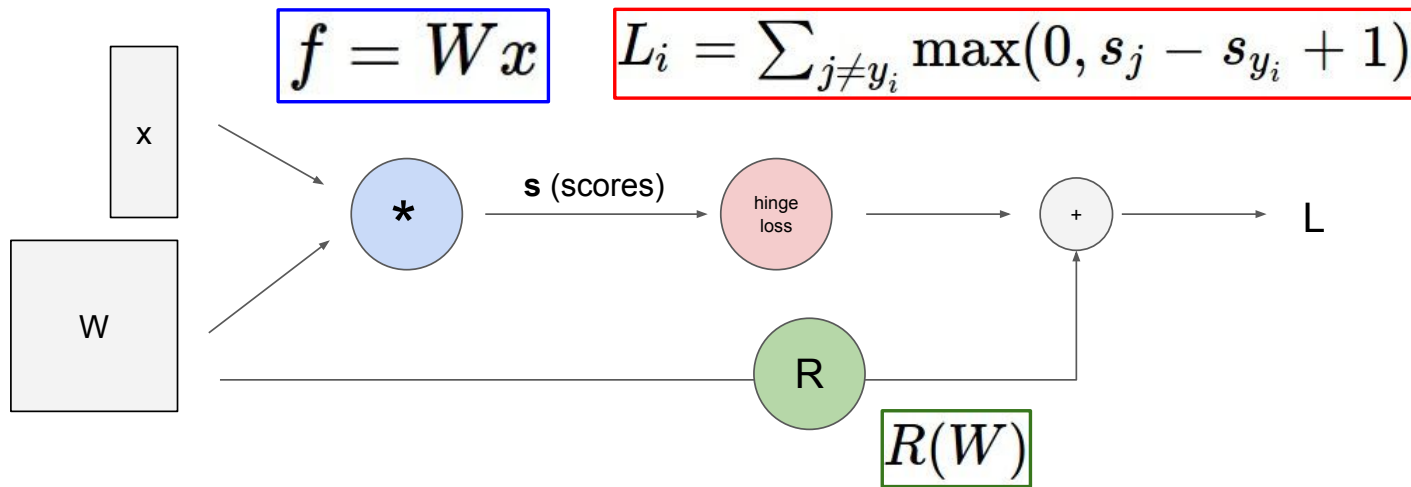
Assignment 2 out today

Project proposal due Tuesday April 25

Notes on backprop for a linear layer and vector/tensor derivatives linked to Lecture 4 on syllabus

Where we are now...

Computational graphs



Where we are now...

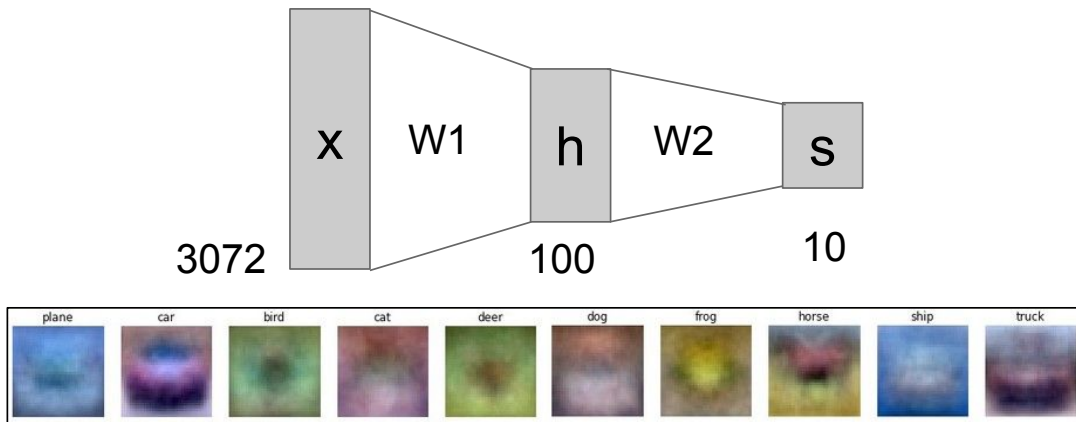
Neural Networks

Linear score function:

$$f = Wx$$

2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$



Where we are now...

Convolutional Neural Networks

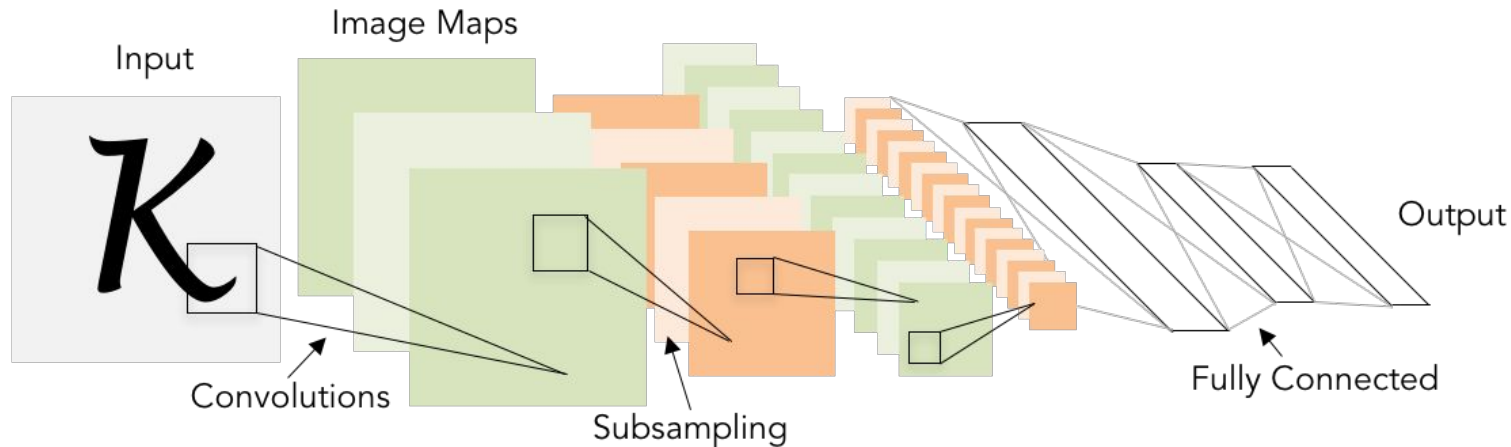
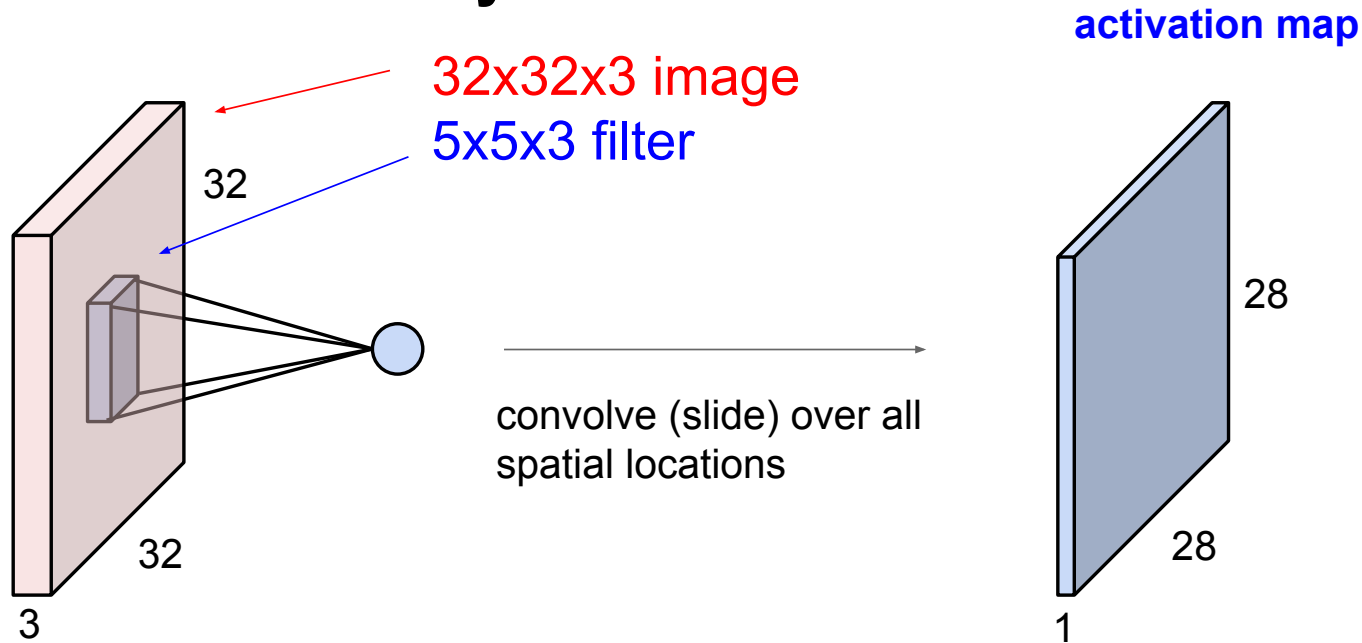


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

Where we are now...

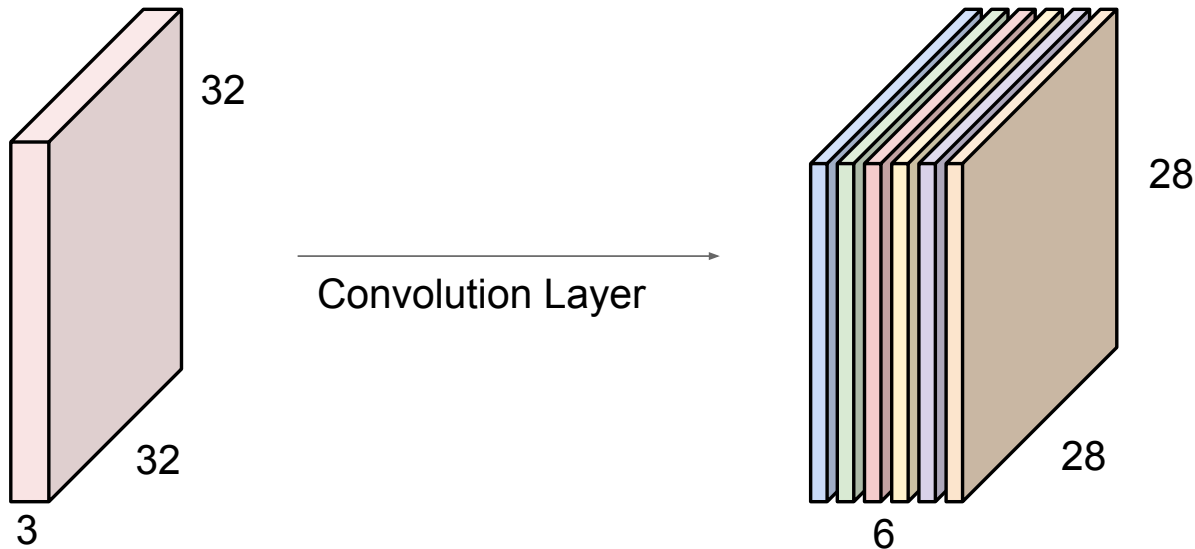
Convolutional Layer



Where we are now...

Convolutional Layer

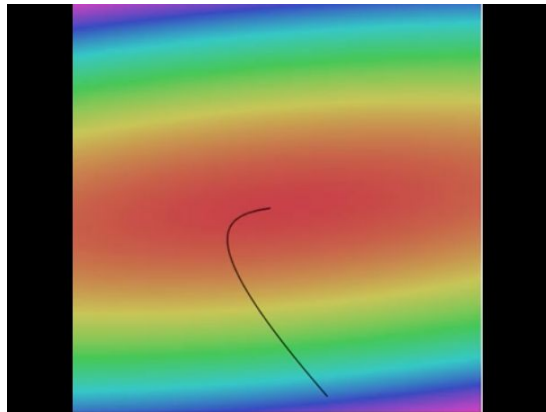
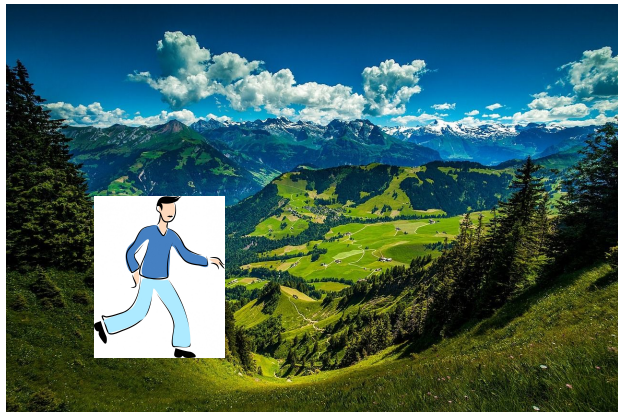
For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a “new image” of size 28x28x6!

Where we are now...

Learning network parameters through optimization



```
# Vanilla Gradient Descent
```

```
while True:
```

```
    weights_grad = evaluate_gradient(loss_fun, data, weights)
```

```
    weights += - step_size * weights_grad # perform parameter update
```

[Landscape image](#) is [CC0 1.0](#) public domain

[Walking man image](#) is [CC0 1.0](#) public domain

Where we are now...

Mini-batch SGD

Loop:

1. **Sample** a batch of data
2. **Forward** prop it through the graph (network), get loss
3. **Backprop** to calculate the gradients
4. **Update** the parameters using the gradient

Next: Training Neural Networks

Overview

1. One time setup

activation functions, preprocessing, weight initialization, regularization, gradient checking

2. Training dynamics

babysitting the learning process, parameter updates, hyperparameter optimization

3. Evaluation

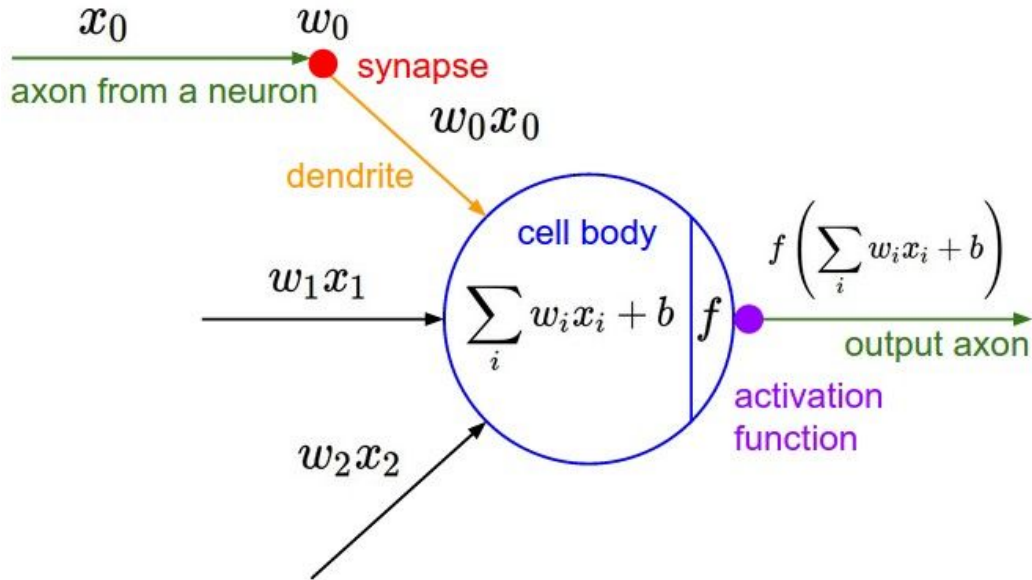
model ensembles

Part 1

- Activation Functions
- Data Preprocessing
- Weight Initialization
- Batch Normalization
- Babysitting the Learning Process
- Hyperparameter Optimization

Activation Functions

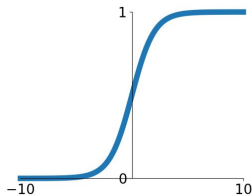
Activation Functions



Activation Functions

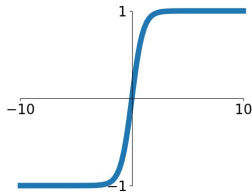
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



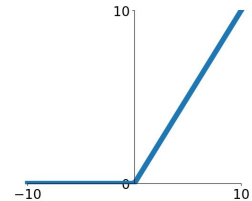
tanh

$$\tanh(x)$$



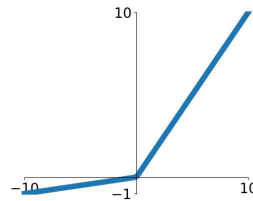
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

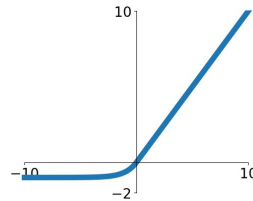


Maxout

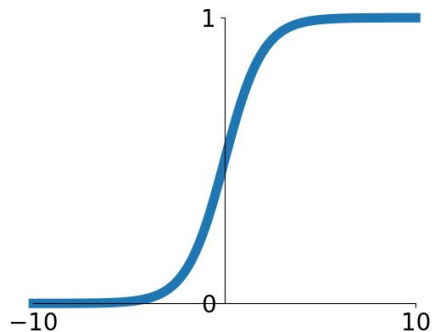
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Activation Functions

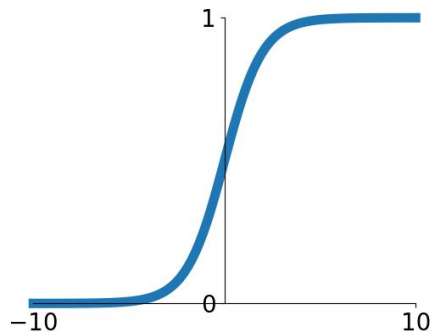


Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range $[0, 1]$
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

Activation Functions



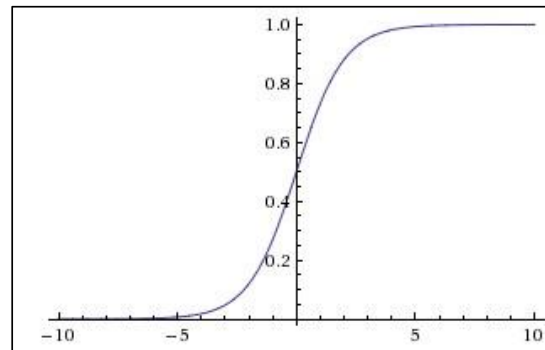
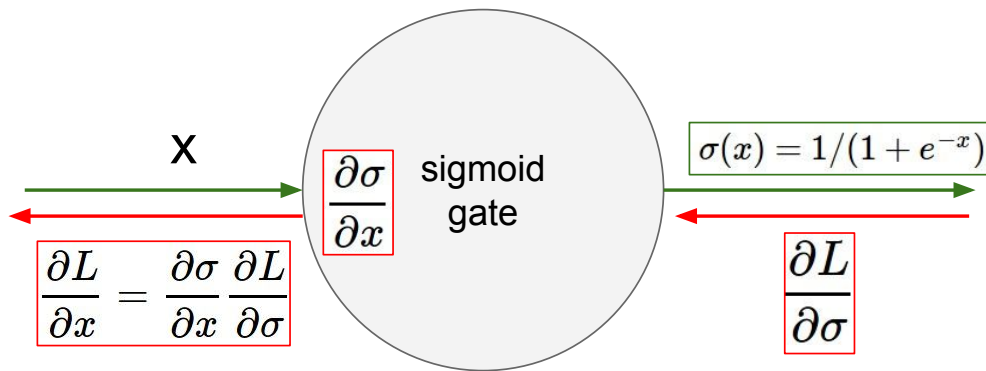
Sigmoid

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3 problems:

1. Saturated neurons “kill” the gradients

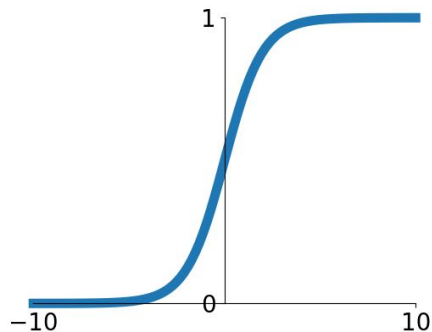


What happens when $x = -10$? $\rightarrow \frac{\partial \sigma}{\partial x} \approx 0$ very small gradient passed down

What happens when $x = 0$? $\rightarrow \text{ok}$

What happens when $x = 10$? $\rightarrow \frac{\partial \sigma}{\partial x} \approx 0$ "

Activation Functions



Sigmoid

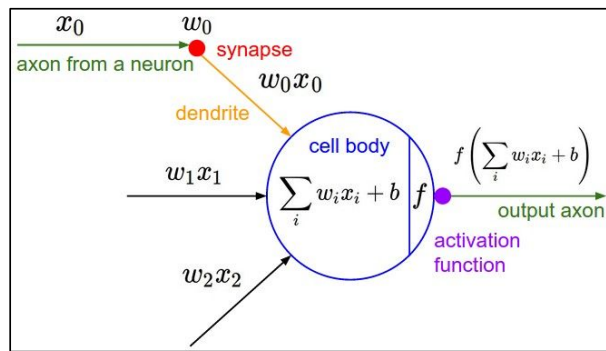
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range $[0, 1]$
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron (x) is always positive:



$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on \mathbf{w} ?

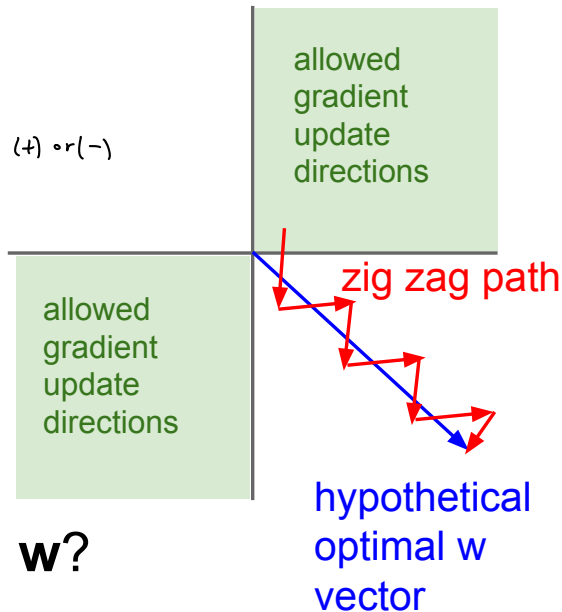
Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

$$\frac{\partial f}{\partial w_i} = x_i > 0$$

$$\frac{\partial L}{\partial f} \cdot \frac{\partial f}{\partial w} \rightarrow \text{all } (+) \text{ or } (-)$$

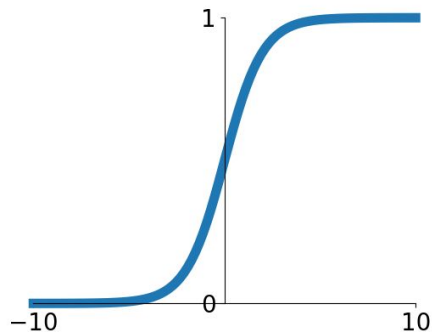
↓
same
number



What can we say about the gradients on **w**?

Always all positive or all negative :(
(this is also why you want zero-mean data!)

Activation Functions



Sigmoid

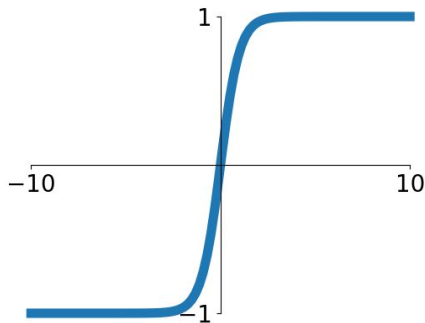
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range $[0, 1]$
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered
3. $\exp()$ is a bit compute expensive

Activation Functions

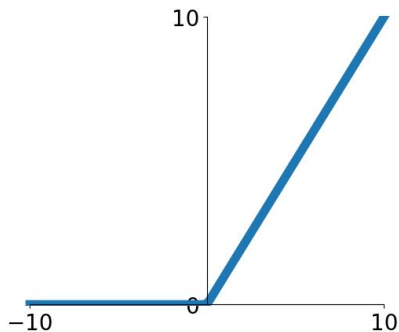


$\tanh(x)$

- Squashes numbers to range $[-1,1]$
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

Activation Functions



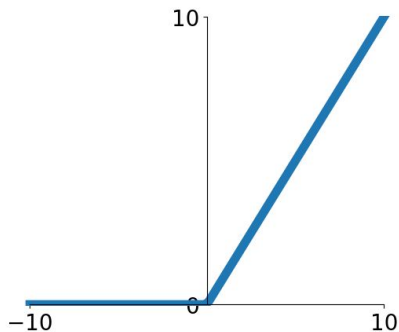
ReLU

(Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient → ^{0 or} identity
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid

[Krizhevsky et al., 2012]

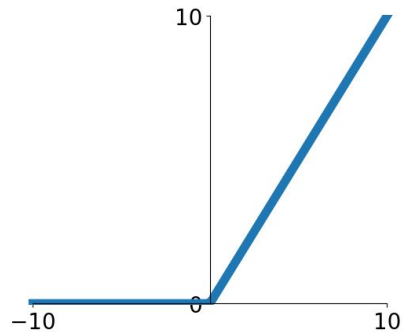
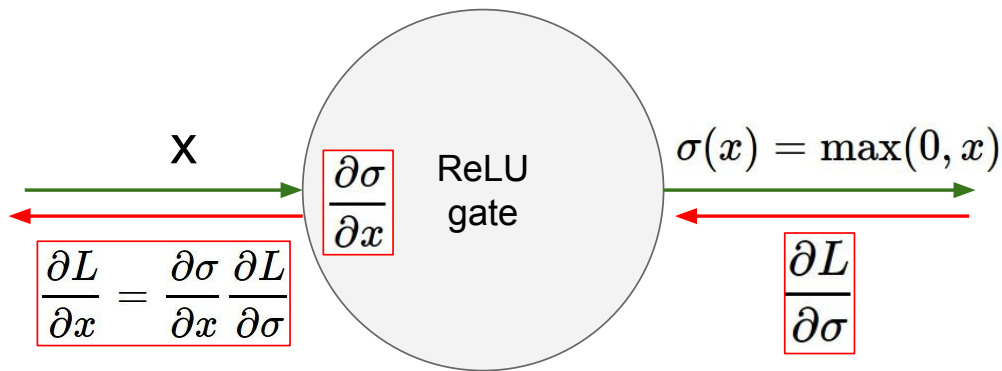
Activation Functions



ReLU (Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid
- Not zero-centered output
- An annoyance:

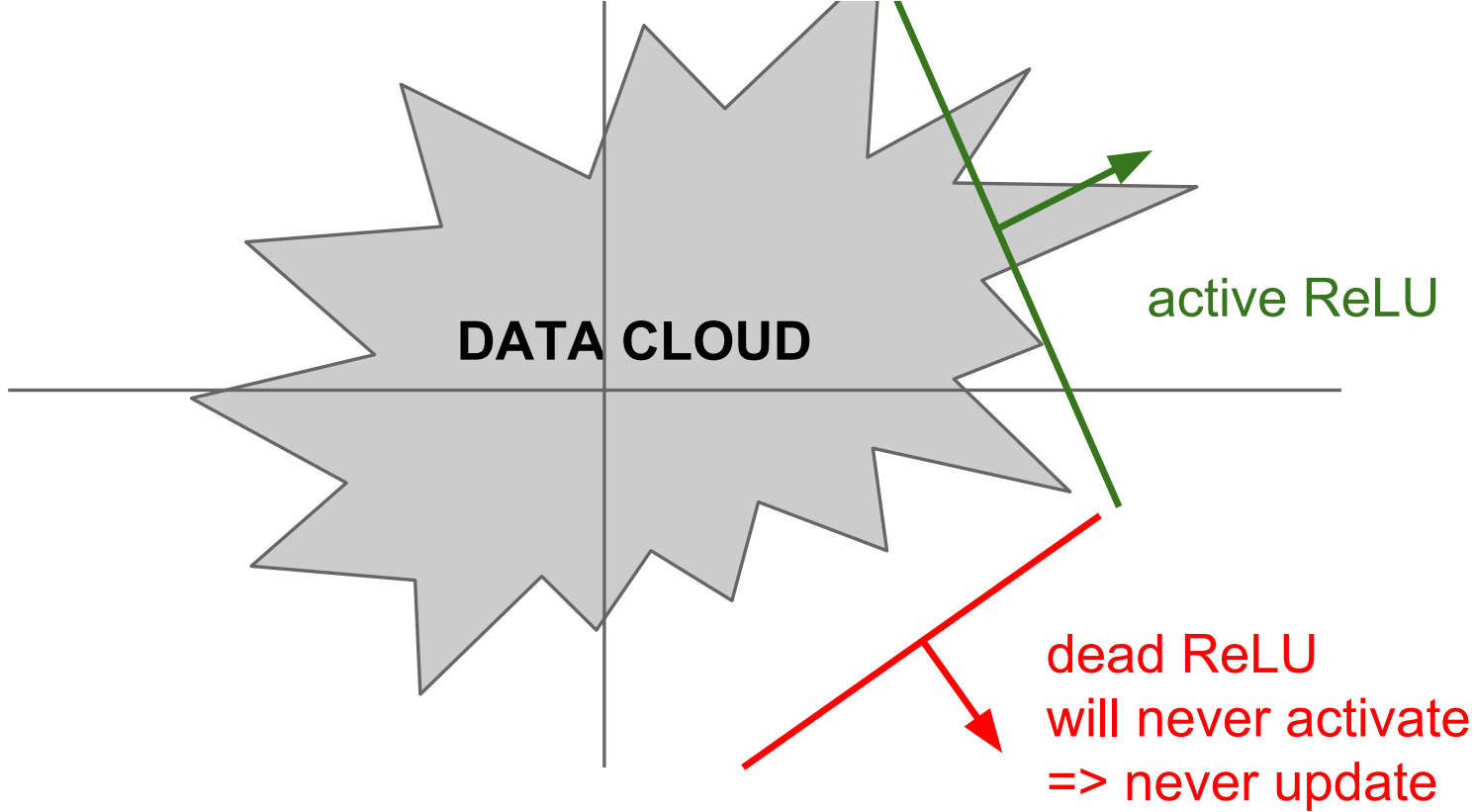
hint: what is the gradient when $x < 0$?

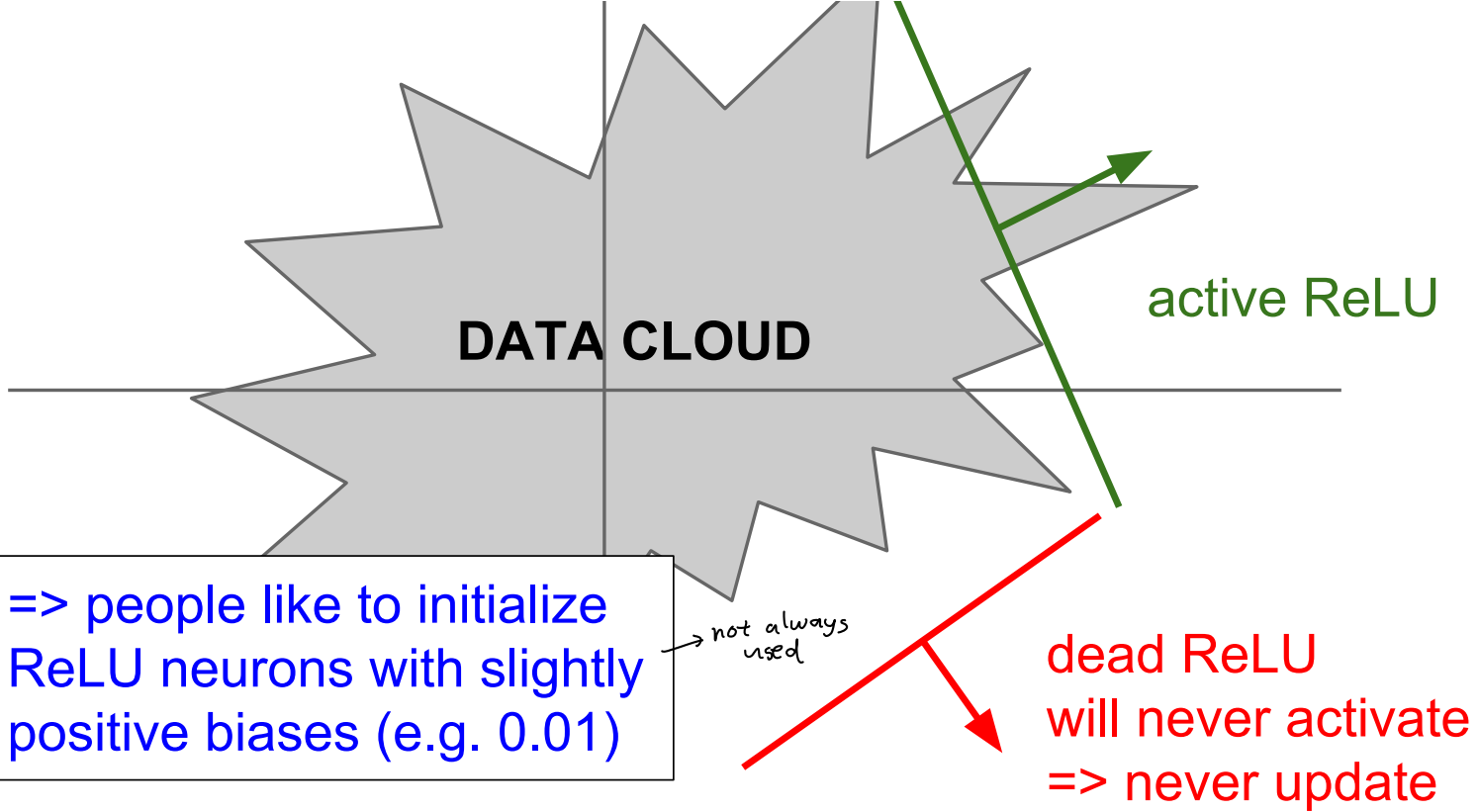


What happens when $x = -10$? \rightarrow gradient is 0 (relu kills the gradient)

What happens when $x = 0$? \rightarrow undefined (0 in practice)

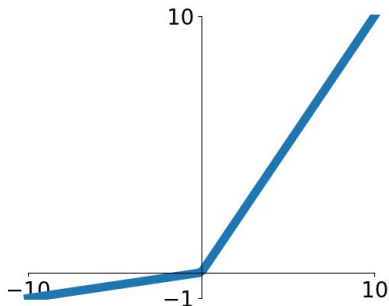
What happens when $x = 10$? \rightarrow ok





Activation Functions

[Mass et al., 2013]
[He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

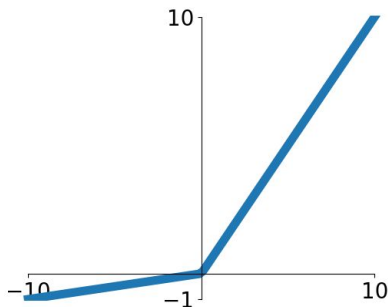
Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Activation Functions

[Mass et al., 2013]

[He et al., 2015]



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

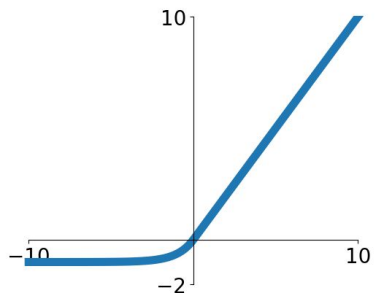
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into α
(parameter)

Exponential Linear Units (ELU)



- All benefits of ReLU
- Closer to zero mean outputs
- Negative saturation regime compared with Leaky ReLU adds some robustness to noise
- Computation requires $\exp()$

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$

Maxout “Neuron”

[Goodfellow et al., 2013]

- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

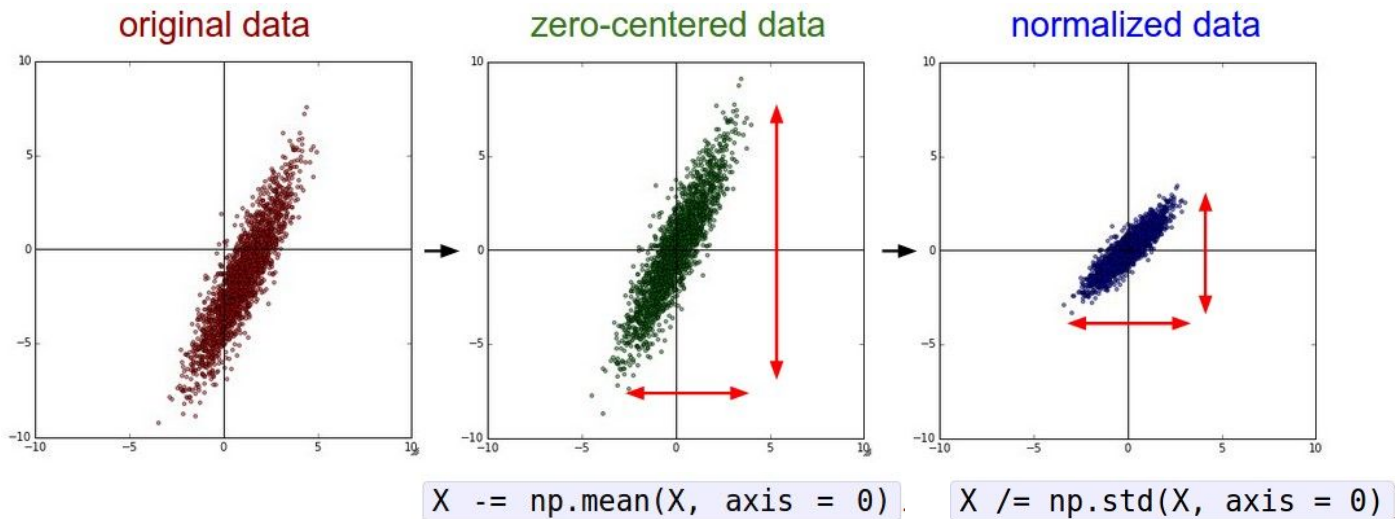
Problem: doubles the number of parameters/neuron :(

TLDR: In practice:

- Use **ReLU**. Be careful with your learning rates
- Try out **Leaky ReLU / Maxout / ELU**
- Try out **tanh** but don't expect much
- **Don't use sigmoid**

Data Preprocessing

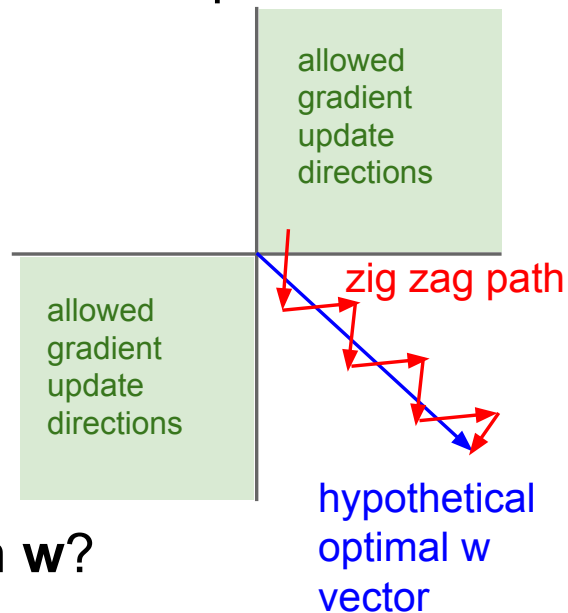
Step 1: Preprocess the data



(Assume X [NxD] is data matrix,
each example in a row)

Remember: Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

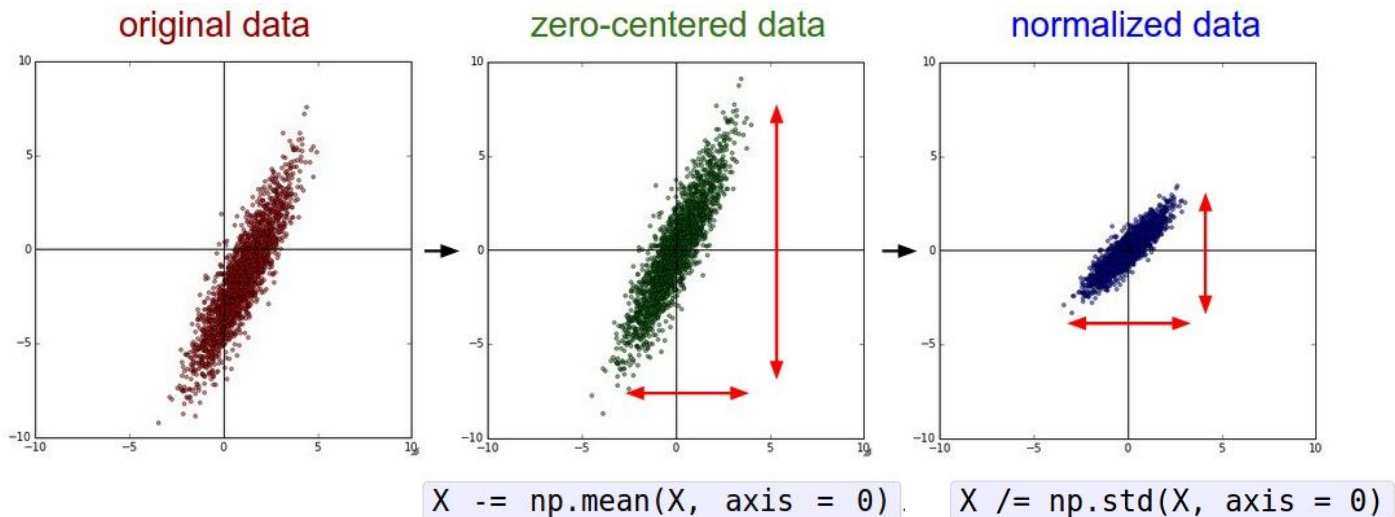


What can we say about the gradients on \mathbf{w} ?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

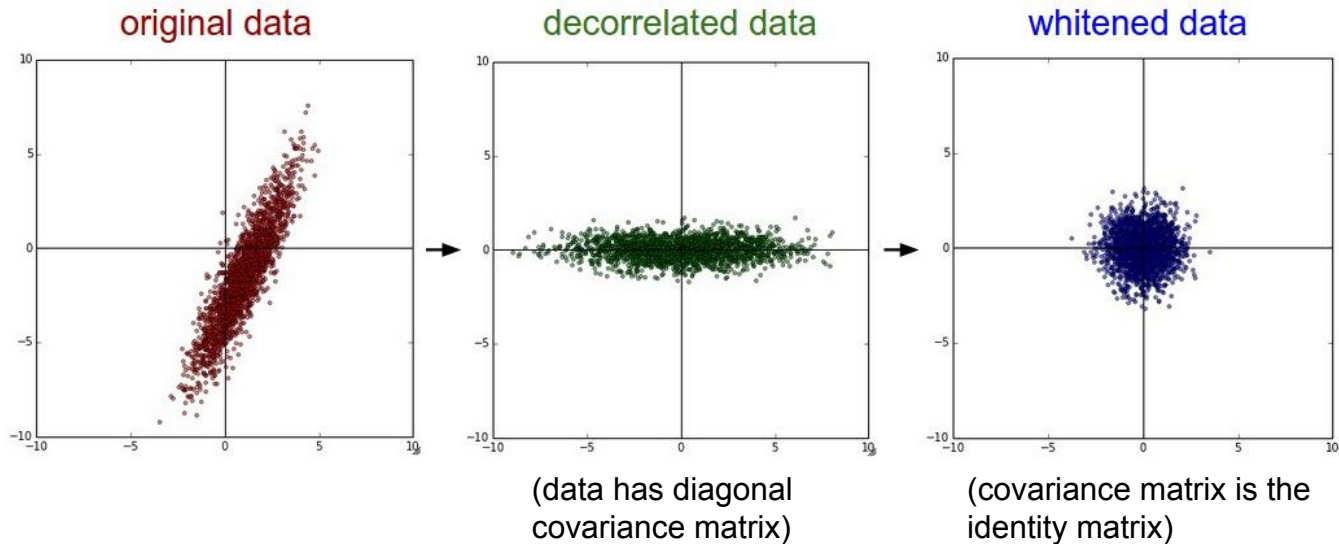
Step 1: Preprocess the data



(Assume X [NxD] is data matrix,
each example in a row)

Step 1: Preprocess the data

In practice, you may also see **PCA** and **Whitening** of the data



TLDR: In practice for Images: center only

e.g. consider CIFAR-10 example with [32,32,3] images

Two ways
to center
data

- Subtract the mean image (e.g. AlexNet)

(mean image = [32,32,3] array)

- Subtract per-channel mean (e.g. VGGNet)

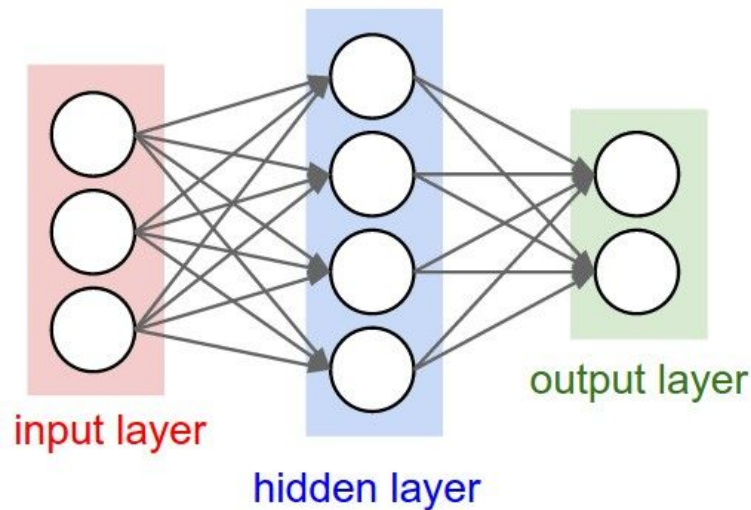
(mean along each channel = 3 numbers)

↪
easier because
similar across
whole image

Not common to normalize
variance, to do PCA or
whitening

Weight Initialization

- Q: what happens when $W=0$ init is used?



- First idea: **Small random numbers**
(gaussian with zero mean and $1e-2$ standard deviation)

```
W = 0.01* np.random.randn(D,H)
```

- First idea: **Small random numbers**
(gaussian with zero mean and $1e-2$ standard deviation)

```
W = 0.01* np.random.randn(D,H)
```

Works ~okay for small networks, but problems with deeper networks.

Lets look at some activation statistics

E.g. 10-layer net with 500 neurons on each layer, using tanh non-linearities, and initializing as described in last slide.

```
# assume some unit gaussian 10-D input data
D = np.random.randn(1000, 500)
hidden_layer_sizes = [500]*10
nonlinearities = ['tanh']*len(hidden_layer_sizes)
```

```
act = {'relu':lambda x:np.maximum(0,x), 'tanh':lambda x:np.tanh(x)}
Hs = {}
for i in xrange(len(hidden_layer_sizes)):
    X = D if i == 0 else Hs[i-1] # input at this layer
    fan_in = X.shape[1]
    fan_out = hidden_layer_sizes[i]
    W = np.random.randn(fan_in, fan_out) * 0.01 # layer initialization

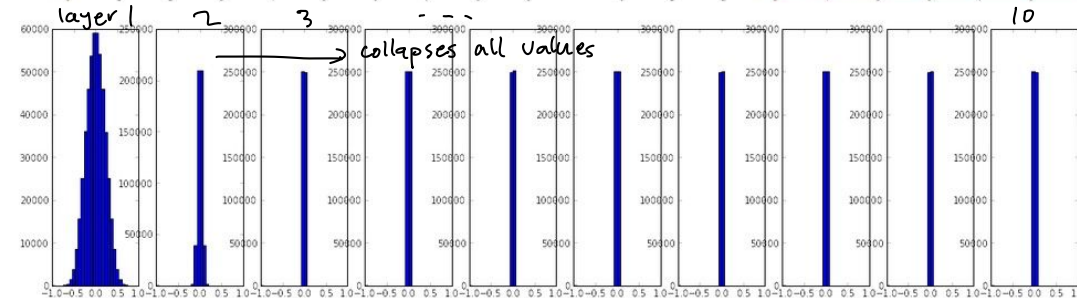
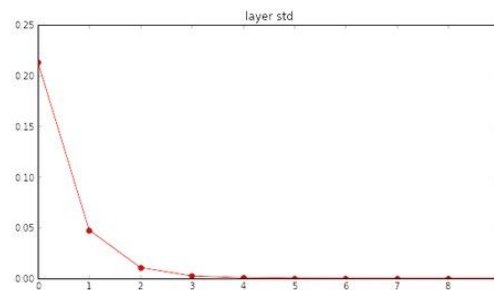
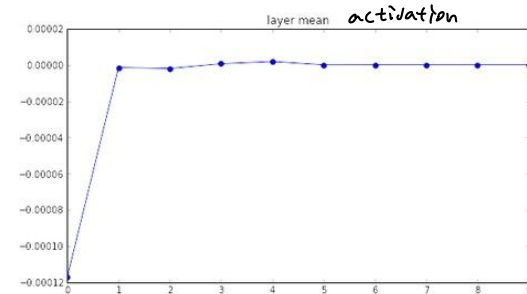
    H = np.dot(X, W) # matrix multiply
    H = act[nonlinearities[i]](H) # nonlinearity
    Hs[i] = H # cache result on this layer
```

```
# look at distributions at each layer
print 'input layer had mean %f and std %f' % (np.mean(D), np.std(D))
layer_means = [np.mean(H) for i,H in Hs.iteritems()]
layer_stds = [np.std(H) for i,H in Hs.iteritems()]
for i,H in Hs.iteritems():
    print 'hidden layer %d had mean %f and std %f' % (i+1, layer_means[i], layer_stds[i])
```

```
# plot the means and standard deviations
plt.figure()
plt.subplot(121)
plt.plot(Hs.keys(), layer_means, 'ob-')
plt.title('layer mean')
plt.subplot(122)
plt.plot(Hs.keys(), layer_stds, 'or-')
plt.title('layer std')
```

```
# plot the raw distributions
plt.figure()
for i,H in Hs.iteritems():
    plt.subplot(1,len(Hs),i+1)
    plt.hist(H.ravel(), 30, range=(-1,1))
```

input layer had mean 0.000927 and std 0.998388
 hidden layer 1 had mean -0.000117 and std 0.213081
 hidden layer 2 had mean -0.000001 and std 0.047551
 hidden layer 3 had mean -0.000002 and std 0.010630
 hidden layer 4 had mean 0.000001 and std 0.002378
 hidden layer 5 had mean 0.000002 and std 0.000532
 hidden layer 6 had mean -0.000000 and std 0.000119
 hidden layer 7 had mean 0.000000 and std 0.000026
 hidden layer 8 had mean -0.000000 and std 0.000006
 hidden layer 9 had mean 0.000000 and std 0.000001
 hidden layer 10 had mean -0.000000 and std 0.000000



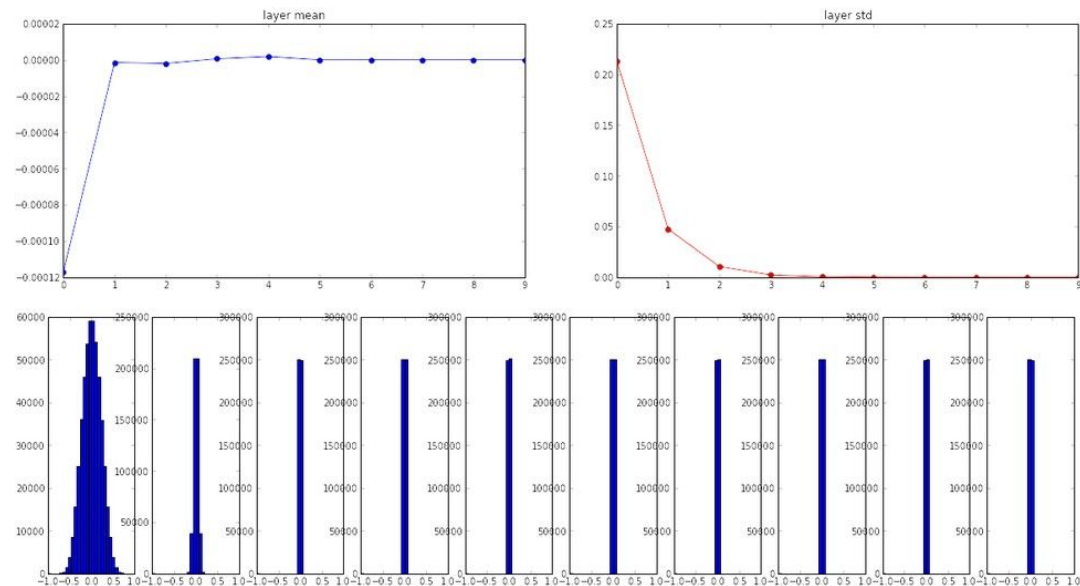
all the activations become 0

input layer had mean 0.000927 and std 0.998388
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 hidden layer 7 had mean 0.000000 and std 0.000026
 hidden layer 8 had mean -0.000000 and std 0.000006
 hidden layer 9 had mean 0.000000 and std 0.000001
 hidden layer 10 had mean -0.000000 and std 0.000000

All activations become zero!

Q: think about the backward pass.
 What do the gradients look like?

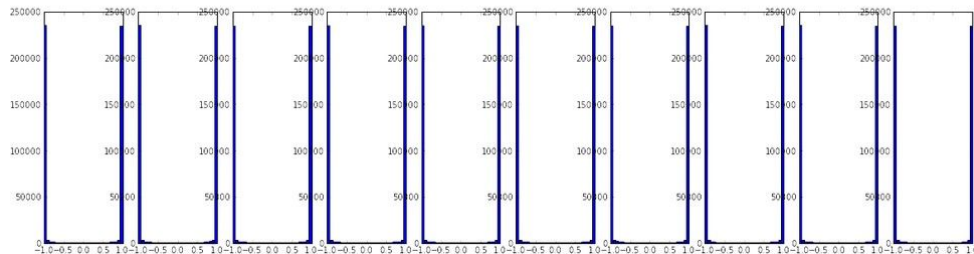
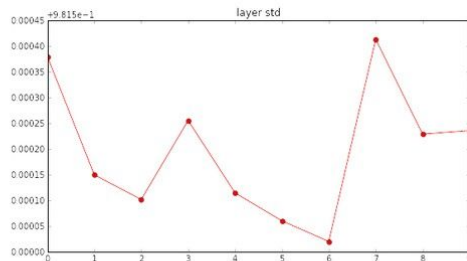
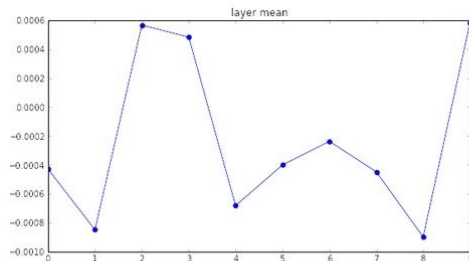
Hint: think about backward pass for a $W \times X$ gate.



```
W = np.random.randn(fan_in, fan_out) * 1.0 # layer initialization
```

input layer had mean 0.001800 and std 1.001311
 hidden layer 1 had mean -0.000430 and std 0.981879
 hidden layer 2 had mean -0.000849 and std 0.981649
 hidden layer 3 had mean 0.000566 and std 0.981601
 hidden layer 4 had mean 0.000483 and std 0.981755
 hidden layer 5 had mean -0.000682 and std 0.981614
 hidden layer 6 had mean -0.000401 and std 0.981560
 hidden layer 7 had mean -0.000237 and std 0.981520
 hidden layer 8 had mean -0.000448 and std 0.981913
 hidden layer 9 had mean -0.000899 and std 0.981728
 hidden layer 10 had mean 0.000584 and std 0.981736

*1.0 instead of *0.01



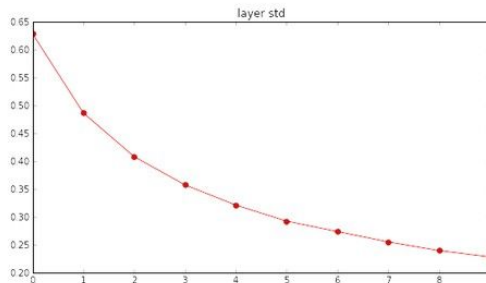
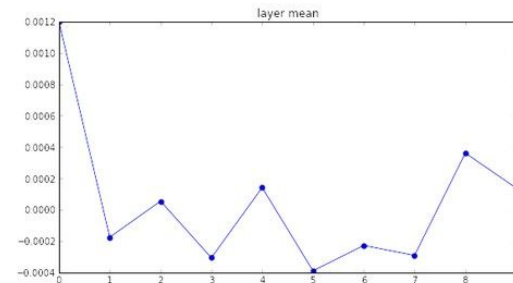
Almost all neurons completely saturated, either -1 and 1. Gradients will be all zero.

input layer had mean 0.001800 and std 1.001311
 hidden layer 1 had mean 0.001198 and std 0.627953
 hidden layer 2 had mean -0.000175 and std 0.486051
 hidden layer 3 had mean 0.000055 and std 0.407723
 hidden layer 4 had mean -0.000306 and std 0.357108
 hidden layer 5 had mean 0.000142 and std 0.320917
 hidden layer 6 had mean -0.000389 and std 0.292116
 hidden layer 7 had mean -0.000228 and std 0.273387
 hidden layer 8 had mean -0.000291 and std 0.254935
 hidden layer 9 had mean 0.000361 and std 0.239266
 hidden layer 10 had mean 0.000139 and std 0.228008

`W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization`

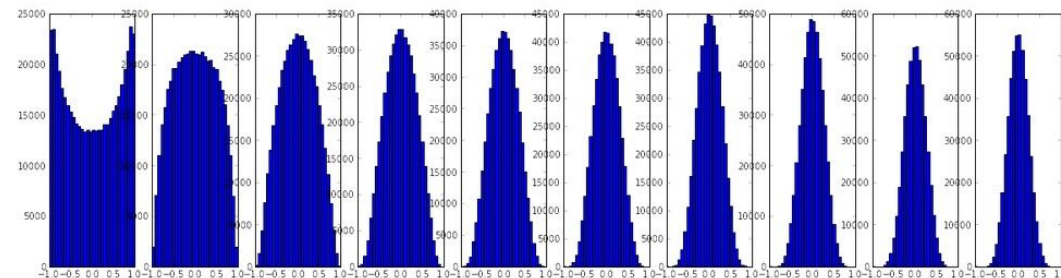
scale by # of
input we have

“Xavier initialization”
[Glorot et al., 2010]



Reasonable initialization.
 (Mathematical derivation
 assumes linear activations)

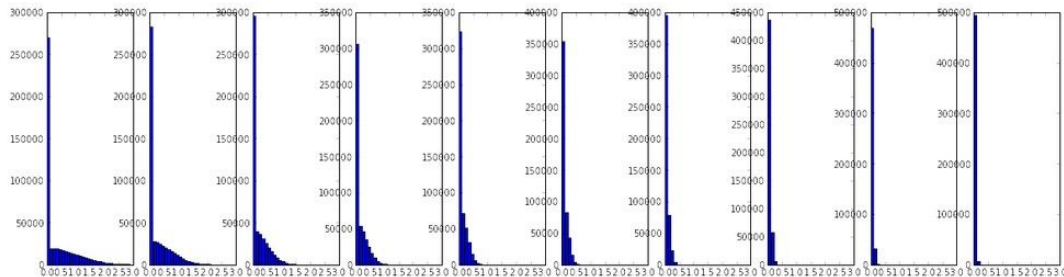
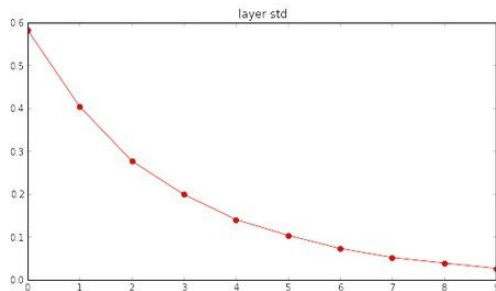
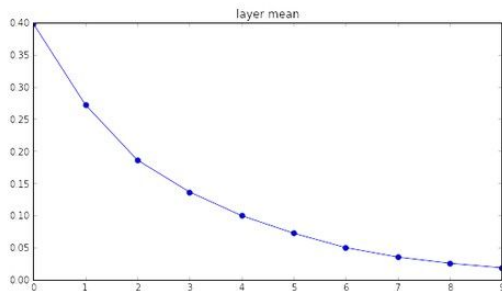
↓
assumes we are in the
active region of tanh



input layer had mean 0.000501 and std 0.999444
 hidden layer 1 had mean 0.398623 and std 0.582273
 hidden layer 2 had mean 0.272352 and std 0.403795
 hidden layer 3 had mean 0.186076 and std 0.276912
 hidden layer 4 had mean 0.136442 and std 0.198685
 hidden layer 5 had mean 0.099568 and std 0.140299
 hidden layer 6 had mean 0.072234 and std 0.103280
 hidden layer 7 had mean 0.049775 and std 0.072748
 hidden layer 8 had mean 0.035138 and std 0.051572
 hidden layer 9 had mean 0.025404 and std 0.038583
 hidden layer 10 had mean 0.018408 and std 0.026076

`W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization`

but when using the ReLU nonlinearity it breaks.

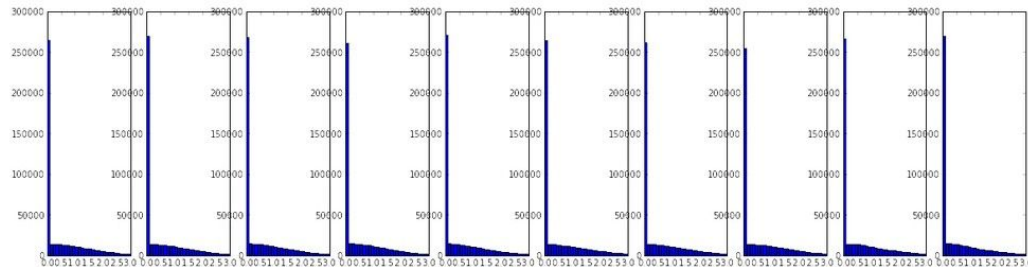
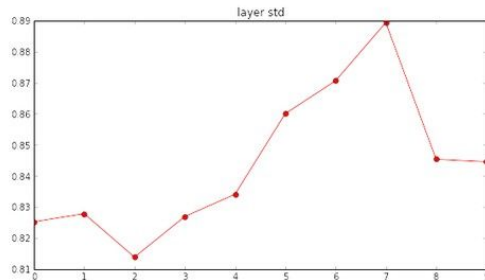
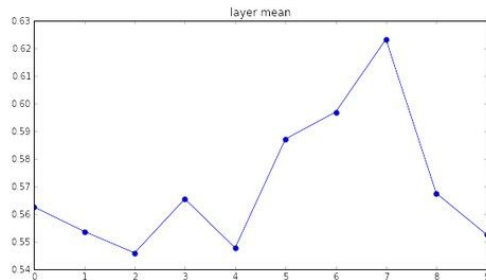


input layer had mean 0.000501 and std 0.999444
 hidden layer 1 had mean 0.562488 and std 0.825232
 hidden layer 2 had mean 0.553614 and std 0.827835
 hidden layer 3 had mean 0.545867 and std 0.813855
 hidden layer 4 had mean 0.565396 and std 0.826902
 hidden layer 5 had mean 0.547678 and std 0.834092
 hidden layer 6 had mean 0.587103 and std 0.860035
 hidden layer 7 had mean 0.596867 and std 0.870610
 hidden layer 8 had mean 0.623214 and std 0.889348
 hidden layer 9 had mean 0.567498 and std 0.845357
 hidden layer 10 had mean 0.552531 and std 0.844523

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

for relu

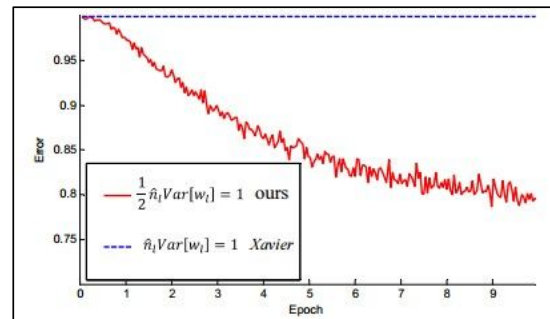
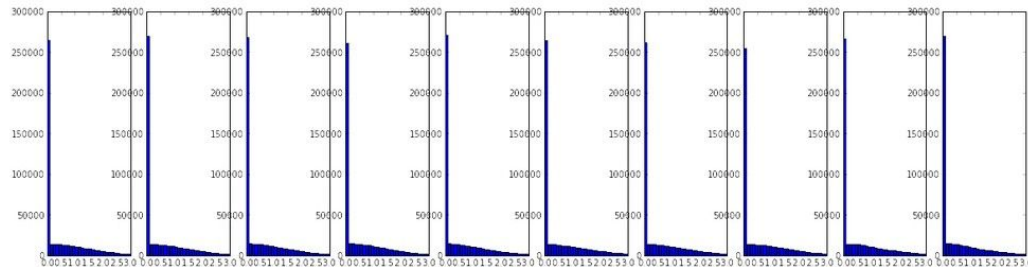
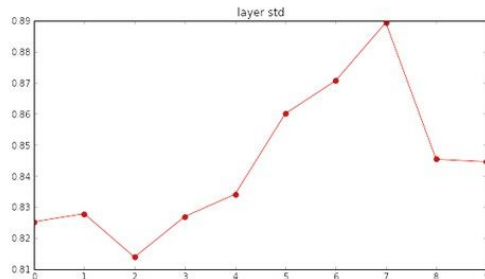
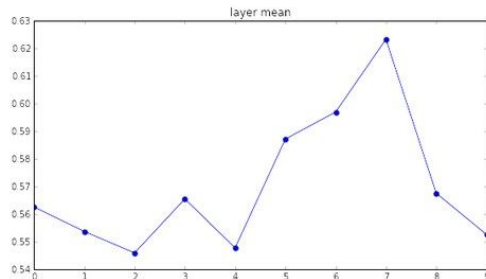
He et al., 2015
 (note additional /2)



input layer had mean 0.000501 and std 0.999444
 hidden layer 1 had mean 0.562488 and std 0.825232
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 hidden layer 9 had mean 0.567498 and std 0.845357
 hidden layer 10 had mean 0.552531 and std 0.844523

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

He et al., 2015
 (note additional /2)



Proper initialization is an active area of research...

Understanding the difficulty of training deep feedforward neural networks

by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by

Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and

Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015

...

Batch Normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want unit gaussian activations? just make them so.”

consider a batch of activations at some layer.
To make each dimension unit gaussian, apply:

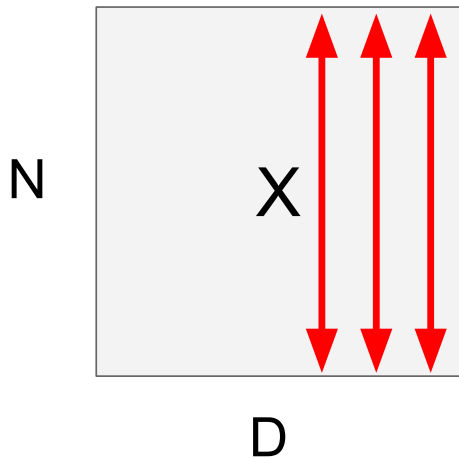
$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

this is a vanilla
differentiable function...

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want unit gaussian activations?
just make them so.”



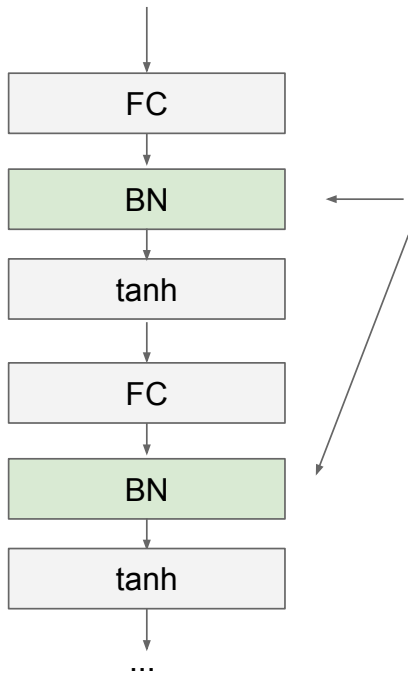
1. compute the empirical mean and variance independently for each dimension.

2. Normalize

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]

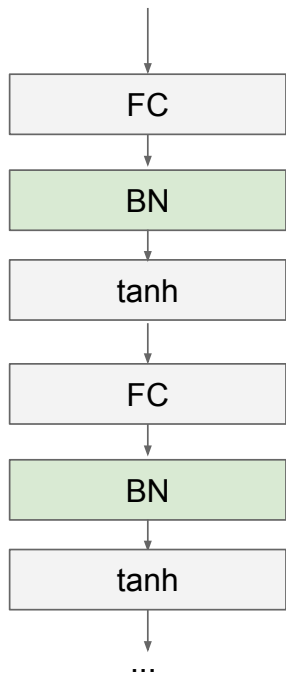


Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]



Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

Problem: do we necessarily want a unit gaussian input to a tanh layer?

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]

Normalize:

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

And then allow the network to squash the range if it wants to:

$$y^{(k)} = \gamma^{(k)} \hat{x}^{(k)} + \beta^{(k)}$$

Note, the network can learn:

$$\gamma^{(k)} = \sqrt{\text{Var}[x^{(k)}]}$$

$$\beta^{(k)} = \mathbb{E}[x^{(k)}]$$

to recover the identity mapping.

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_1 \dots x_m\}$;

Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma, \beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \quad // \text{ mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2 \quad // \text{ mini-batch variance}$$

$$\hat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad // \text{ normalize}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i) \quad // \text{ scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;

Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma, \beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \quad // \text{ mini-batch mean}$$

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$$\hat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad // \text{ normalize}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i) \quad // \text{ scale and shift}$$

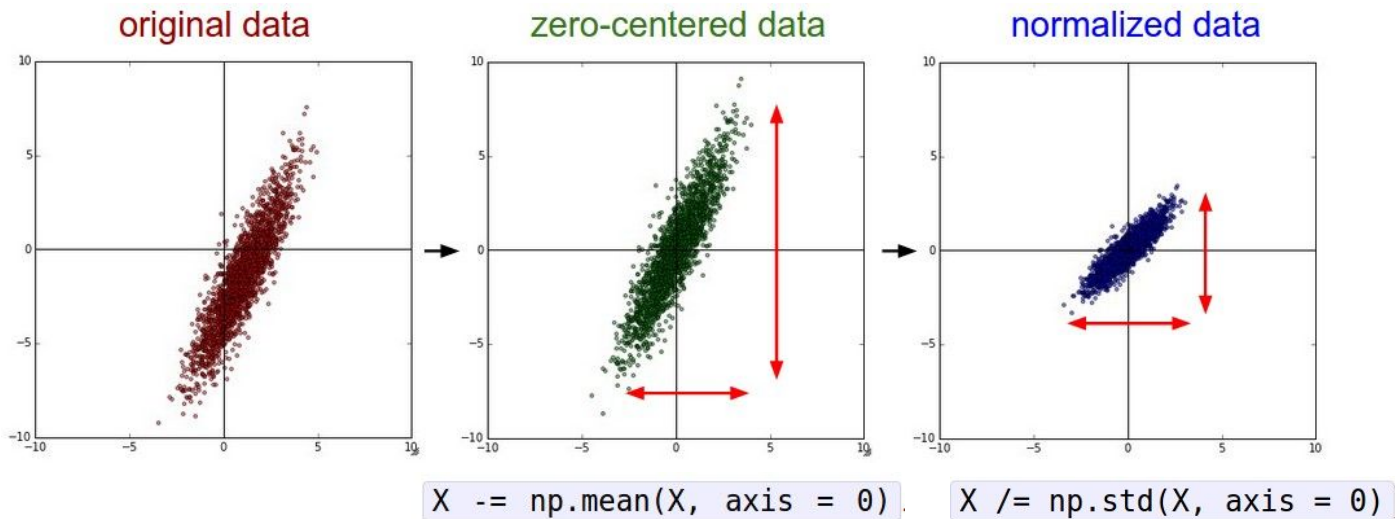
Note: at test time BatchNorm layer functions differently:

The mean/std are not computed based on the batch. Instead, a single fixed empirical mean of activations during training is used.

(e.g. can be estimated during training with running averages)

Babysitting the Learning Process

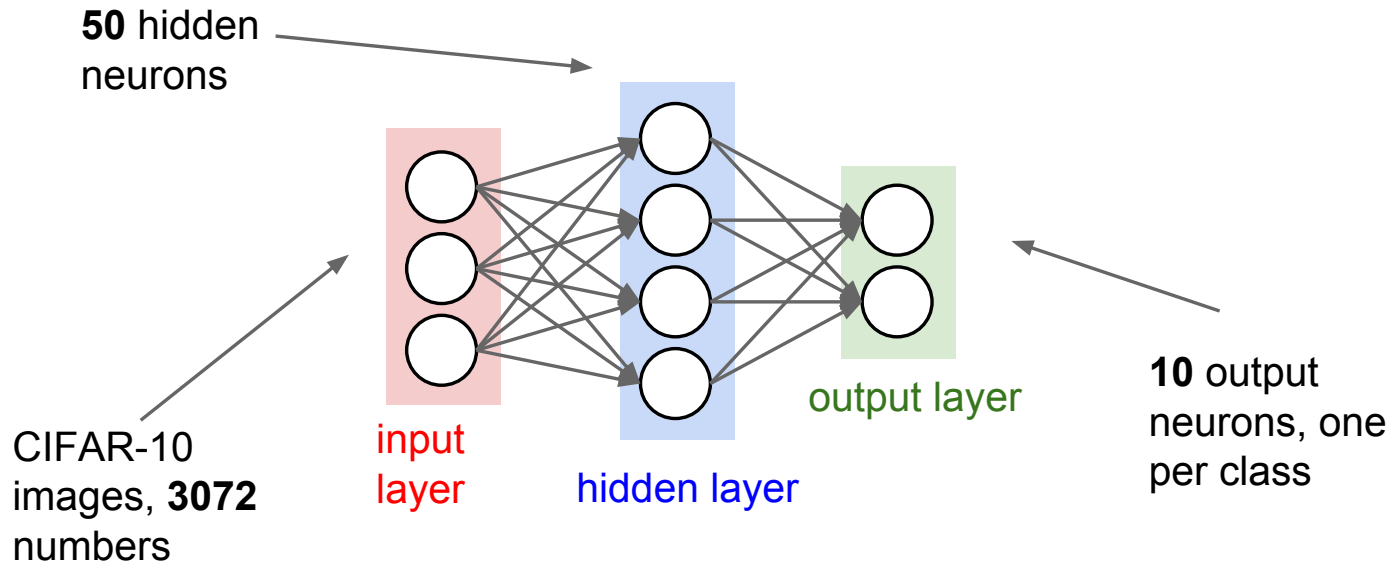
Step 1: Preprocess the data



(Assume X [NxD] is data matrix,
each example in a row)

Step 2: Choose the architecture:

say we start with one hidden layer of 50 neurons:



Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):  
    # initialize a model  
    model = {}  
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)  
    model['b1'] = np.zeros(hidden_size)  
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)  
    model['b2'] = np.zeros(output_size)  
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes  
loss, grad = two_layer_net(X_train, model, y_train, 0.0)  
print loss
```

disable regularization

2.30261216167

loss ~2.3. $\rightarrow -\log_{10}$

“correct” for
10 classes

returns the loss and the
gradient for all parameters

Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):  
    # initialize a model  
    model = {}  
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)  
    model['b1'] = np.zeros(hidden_size)  
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)  
    model['b2'] = np.zeros(output_size)  
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input_size, hidden_size, number of classes  
loss, grad = two_layer_net(X_train, model, y_train, 1e3) # crank up regularization  
print loss
```

3.06859716482

loss went up, good. (sanity check)

Lets try to train now...

Tip: Make sure that you can overfit very small portion of the training data

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

The above code:

- take the first 20 examples from CIFAR-10 *very small amount of data so we can overfit*
- turn off regularization (reg = 0.0)
- use simple vanilla 'sgd'

Lets try to train now...

Tip: Make sure that you can overfit very small portion of the training data

Very small loss, train accuracy 1.00, nice!

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

```
Finished epoch 1 / 200: cost 2.302603, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 2 / 200: cost 2.302258, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 3 / 200: cost 2.301849, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 4 / 200: cost 2.301196, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 5 / 200: cost 2.300044, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 6 / 200: cost 2.297864, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 7 / 200: cost 2.293595, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 8 / 200: cost 2.285096, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 9 / 200: cost 2.268094, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 10 / 200: cost 2.234787, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 11 / 200: cost 2.173187, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 12 / 200: cost 2.076862, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 13 / 200: cost 1.974090, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 14 / 200: cost 1.895885, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 15 / 200: cost 1.820876, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 16 / 200: cost 1.737430, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 17 / 200: cost 1.642356, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 18 / 200: cost 1.535239, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 19 / 200: cost 1.421527, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 20 / 200: cost 1.305760, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 195 / 200: cost 0.002694, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 196 / 200: cost 0.002674, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 197 / 200: cost 0.002655, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 198 / 200: cost 0.002635, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 199 / 200: cost 0.002617, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 200 / 200: cost 0.002597, train: 1.000000, val 1.000000, lr 1.000000e-03
finished optimization. best validation accuracy: 1.000000
```

Lets try to train now...

Start with small
regularization and find
learning rate that
makes the loss go
down.

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)
```

Lets try to train now...

Start with small regularization and find learning rate that makes the loss go down.

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches=True,
                                  learning_rate=1e-6, verbose=True)
```

Epoch	Cost	Train Loss	Val Loss	Learning Rate
1 / 10	2.302576	0.080000	0.103000	1.000000e-06
2 / 10	2.302582	0.121000	0.124000	1.000000e-06
3 / 10	2.302558	0.119000	0.138000	1.000000e-06
4 / 10	2.302519	0.127000	0.151000	1.000000e-06
5 / 10	2.302517	0.158000	0.171000	1.000000e-06
6 / 10	2.302518	0.179000	0.172000	1.000000e-06
7 / 10	2.302466	0.180000	0.176000	1.000000e-06
8 / 10	2.302452	0.175000	0.185000	1.000000e-06
9 / 10	2.302459	0.206000	0.192000	1.000000e-06
10 / 10	2.302420	0.190000	0.192000	1.000000e-06

finished optimization. best validation accuracy: 0.192000

Loss barely changing

Lets try to train now...

Start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches=True,
                                  learning_rate=1e-6, verbose=True)
```

```
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best validation accuracy: 0.192000
```

Loss barely changing: Learning rate is probably too low

Lets try to train now...

Start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches=True,
                                  learning_rate=1e-6, verbose=True)
```

```
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best validation accuracy: 0.192000
```

Loss barely changing: Learning rate is probably too low

Notice train/val accuracy goes to 20% though, what's up with that? (remember this is softmax)

Lets try to train now...

Start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
```

Now let's try learning rate 1e6.



Lets try to train now...

Start with small
regularization and find
learning rate that
makes the loss go
down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e6, verbose=True)
```

```
/home/karpathy/cs231n/code/cs231n/classifiers/neural_net.py:50: RuntimeWarning: divide by zero en
countered in log
  data_loss = -np.sum(np.log(probs[range(N), y])) / N
/home/karpathy/cs231n/code/cs231n/classifiers/neural_net.py:48: RuntimeWarning: invalid value enc
ountered in subtract
  probs = np.exp(scores - np.max(scores, axis=1, keepdims=True))
Finished epoch 1 / 10: cost nan, train: 0.091000, val 0.087000, lr 1.000000e+06
Finished epoch 2 / 10: cost nan, train: 0.095000, val 0.087000, lr 1.000000e+06
Finished epoch 3 / 10: cost nan, train: 0.100000, val 0.087000, lr 1.000000e+06
```

cost: NaN almost
always means high
learning rate...

Lets try to train now...

Start with small
regularization and find
learning rate that
makes the loss go
down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=3e-3, verbose=True)
```

```
Finished epoch 1 / 10: cost 2.186654, train: 0.308000, val 0.306000, lr 3.000000e-03
Finished epoch 2 / 10: cost 2.176230, train: 0.330000, val 0.350000, lr 3.000000e-03
Finished epoch 3 / 10: cost 1.942257, train: 0.376000, val 0.352000, lr 3.000000e-03
Finished epoch 4 / 10: cost 1.827868, train: 0.329000, val 0.310000, lr 3.000000e-03
Finished epoch 5 / 10: cost inf, train: 0.128000, val 0.128000, lr 3.000000e-03
Finished epoch 6 / 10: cost inf, train: 0.144000, val 0.147000, lr 3.000000e-03
```

3e-3 is still too high. Cost explodes....

=> Rough range for learning rate we
should be cross-validating is
somewhere [1e-3 ... 1e-5]

Hyperparameter Optimization

Cross-validation strategy

coarse -> **fine** cross-validation in stages

First stage: only a few epochs to get rough idea of what params work

Second stage: longer running time, finer search

... (repeat as necessary)

Tip for detecting explosions in the solver:

If the cost is ever $> 3 \times$ original cost, break out early

For example: run coarse search for 5 epochs

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)

    trainer = ClassifierTrainer()
    model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
    trainer = ClassifierTrainer()
    best_model_local, stats = trainer.train(X_train, y_train, X_val, y_val,
                                           model, two_layer_net,
                                           num_epochs=5, reg=reg,
                                           update='momentum', learning_rate_decay=0.9,
                                           sample_batches = True, batch_size = 100,
                                           learning_rate=lr, verbose=False)
```

note it's best to optimize
in log space!

```
val_acc: 0.412000, lr: 1.405206e-04, reg: 4.793564e-01, (1 / 100)
val_acc: 0.214000, lr: 7.231888e-06, reg: 2.321281e-04, (2 / 100)
val_acc: 0.208000, lr: 2.119571e-06, reg: 8.011857e+01, (3 / 100)
val_acc: 0.196000, lr: 1.551131e-05, reg: 4.374936e-05, (4 / 100)
val_acc: 0.079000, lr: 1.753300e-05, reg: 1.200424e+03, (5 / 100)
val_acc: 0.223000, lr: 4.215128e-05, reg: 4.196174e+01, (6 / 100)
val_acc: 0.441000, lr: 1.750259e-04, reg: 2.110807e-04, (7 / 100)
val_acc: 0.241000, lr: 6.749231e-05, reg: 4.226413e+01, (8 / 100)
val_acc: 0.482000, lr: 4.296863e-04, reg: 6.642555e-01, (9 / 100)
val_acc: 0.079000, lr: 5.401602e-06, reg: 1.599828e+04, (10 / 100)
val_acc: 0.154000, lr: 1.618508e-06, reg: 4.925252e-01, (11 / 100)
```

nice

Now run finer search...

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)
```

adjust range

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-4, 0)
    lr = 10**uniform(-3, -4)
```

val_acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
val_acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
val_acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100)
val_acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100)
val_acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
val_acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
val_acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
val_acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
val_acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
val_acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
val_acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
val_acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
val_acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
val_acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
val_acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
val_acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
val_acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
val_acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
val_acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
val_acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
val_acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
val_acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)

53% - relatively good
for a 2-layer neural net
with 50 hidden neurons.

Now run finer search...

```
max_count = 100
for count in xrange(max_count):
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```

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```

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val_acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
val_acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
val_acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
val_acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
val_acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
val_acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
val_acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
val_acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
val_acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
val_acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
val_acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
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val_acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
val_acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)

53% - relatively good
for a 2-layer neural net
with 50 hidden neurons.

But this best
cross-validation result is
worrying. Why?

best learning rates were
at the edge of range
we were sampling

Random Search vs. Grid Search

*Random Search for
Hyper-Parameter Optimization*
Bergstra and Bengio, 2012

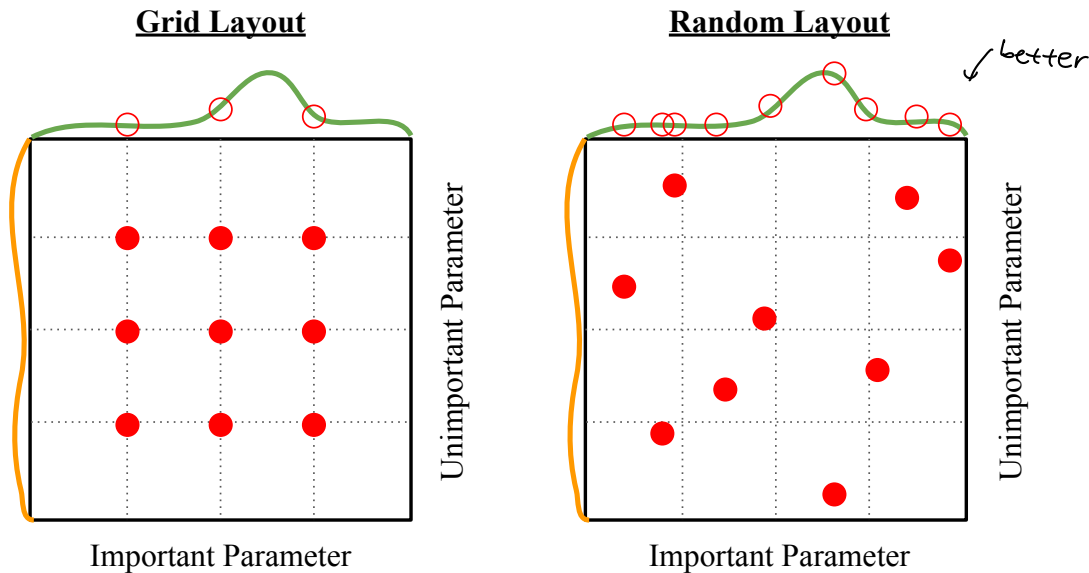


Illustration of Bergstra et al., 2012 by Shayne
Longpre, copyright CS231n 2017

Hyperparameters to play with:

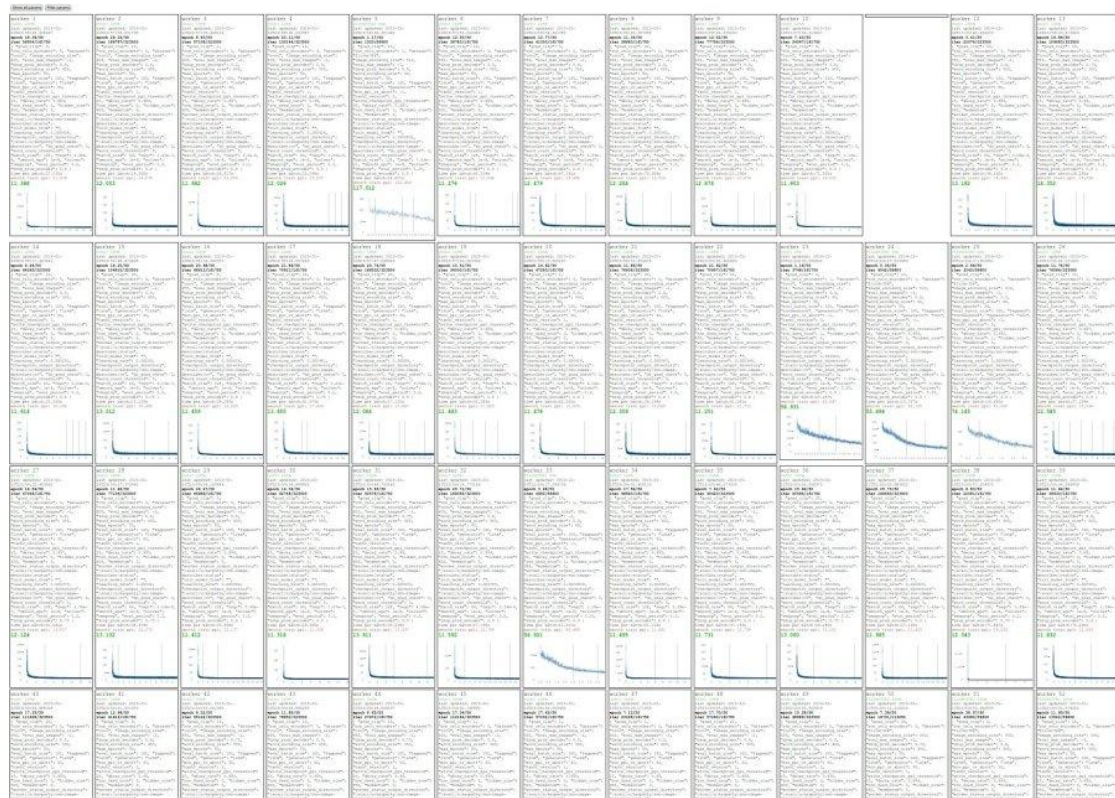
- network architecture
- learning rate, its decay schedule, update type
- regularization (L2/Dropout strength)

neural networks practitioner
music = loss function

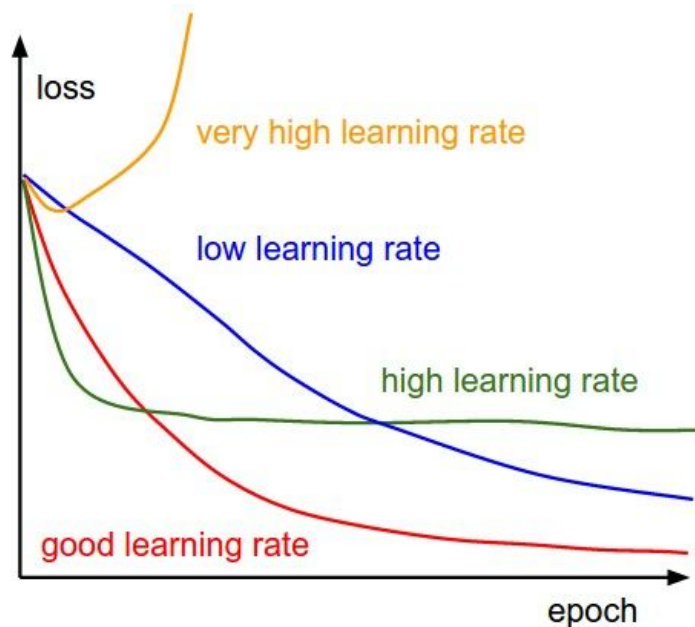
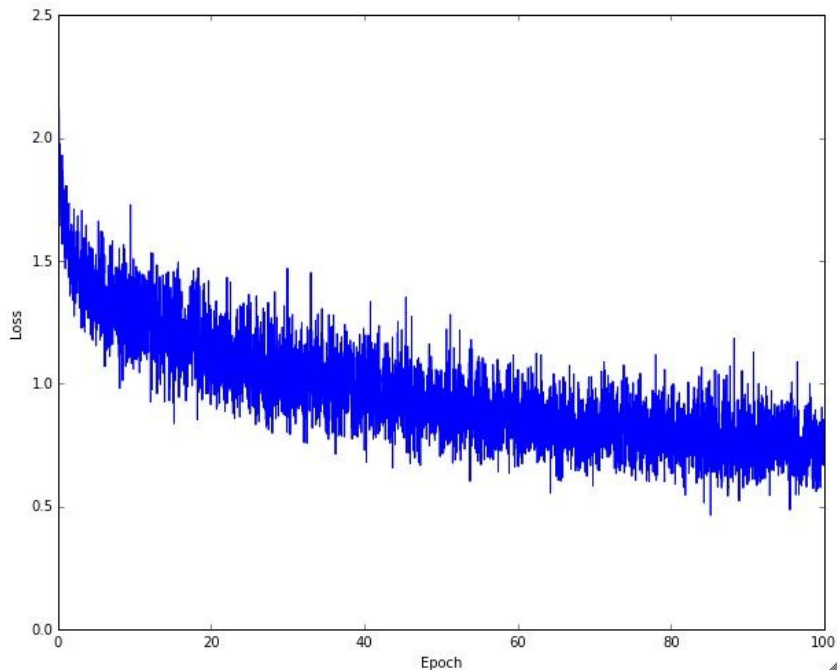


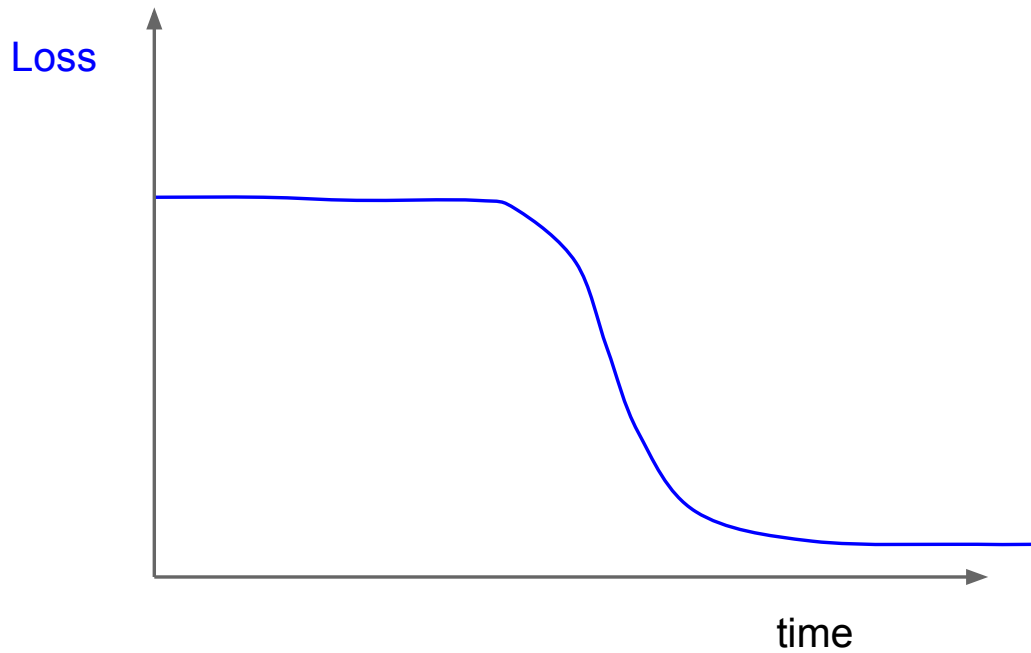
[This image](#) by Paolo Guereta is licensed under [CC-BY 2.0](#)

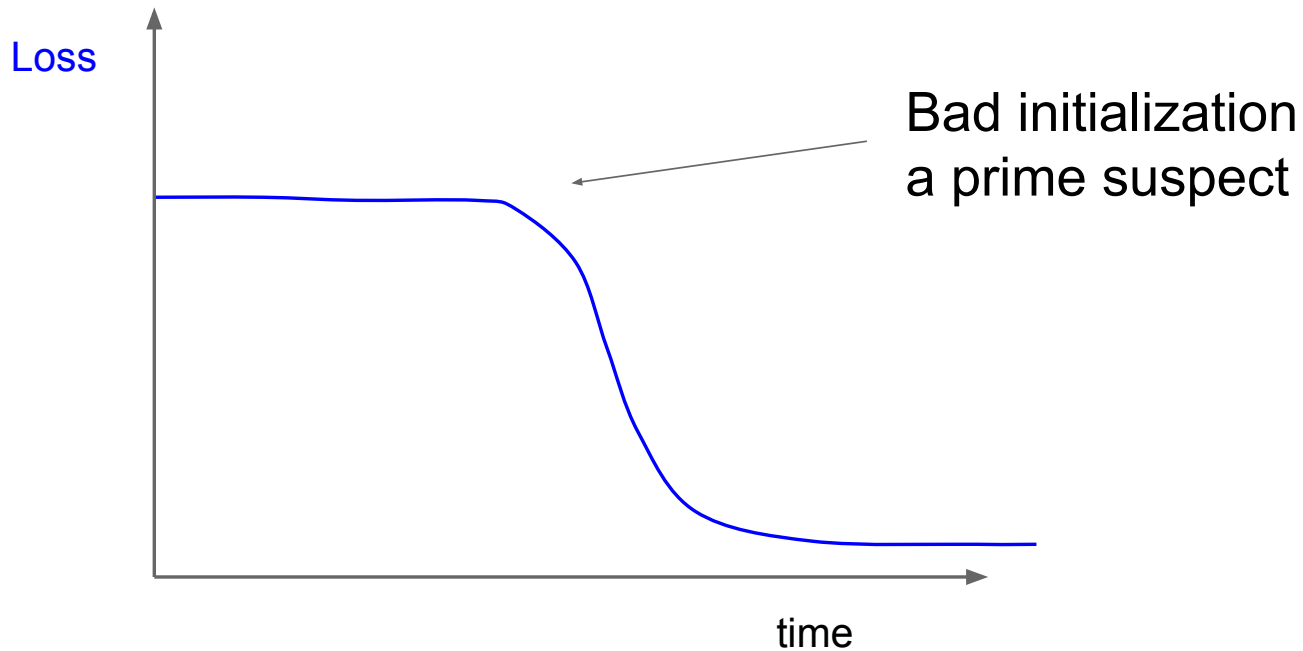
Cross-validation “command center”



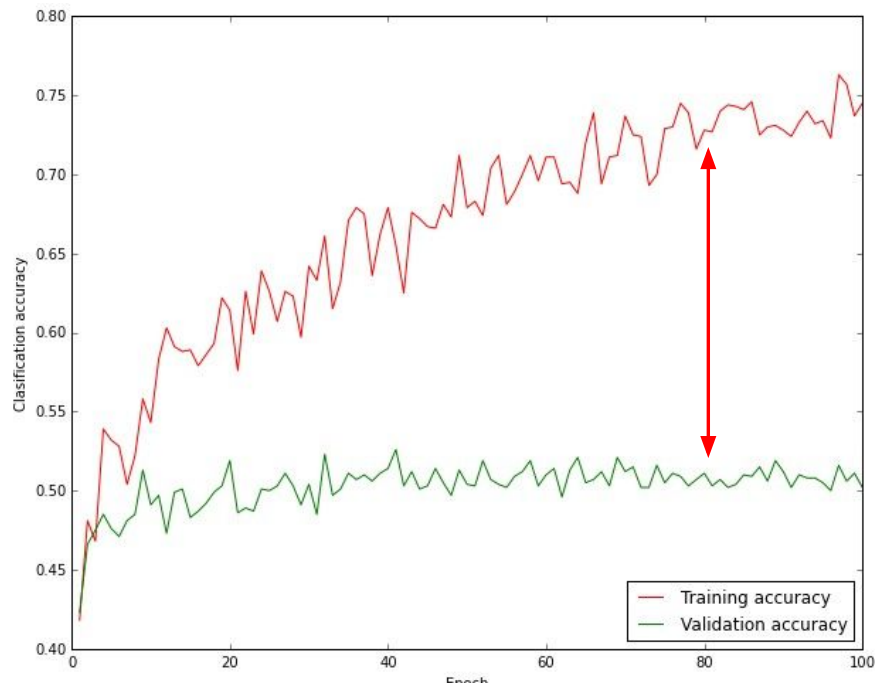
Monitor and visualize the loss curve







Monitor and visualize the accuracy:



big gap = overfitting

=> increase regularization strength?

no gap

=> increase model capacity?

Track the ratio of weight updates / weight magnitudes:

```
# assume parameter vector W and its gradient vector dW
param_scale = np.linalg.norm(W.ravel())
update = -learning_rate*dW # simple SGD update
update_scale = np.linalg.norm(update.ravel())
W += update # the actual update
print update_scale / param_scale # want ~1e-3
```

ratio between the updates and values: $\sim 0.0002 / 0.02 = 0.01$ (about okay)
want this to be somewhere around 0.001 or so

Summary

TLDRs

We looked in detail at:

- Activation Functions (use ReLU)
- Data Preprocessing (images: subtract mean)
- Weight Initialization (use Xavier init)
- Batch Normalization (use)
- Babysitting the Learning process
- Hyperparameter Optimization
(random sample hyperparams, in log space when appropriate)

Next time:

Training Neural Networks, Part 2

- Parameter update schemes
- Learning rate schedules
- Gradient checking
- Regularization (Dropout etc.)
- Evaluation (Ensembles etc.)
- Transfer learning / fine-tuning