4172 Final

YoonSeok Kim

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1 Abstruct

This report is written to illustrate how we reproduce the graphs from the G.L. He & R. Litterman's paper.

2 Chart 1

2.1 Chart 1A

The starting expected return is set to be μ and we apply traditional Mean-Variance method without constraints to obtain the optimal portfolio:

$$\max\ w'\mu - \frac{\delta w'\Sigma w}{2} \tag{1}$$

$$\text{where } \delta = 2.5$$

Suppose w^* is the unconstrained optimal portfolio. Then

$$w^* = (\delta\sigma)^{-1}\mu \tag{2}$$

If we set

$$\mu_0 = (7\%, 7\%, 7\%, 7\%, 7\%, 7\%, 7\%)$$

and

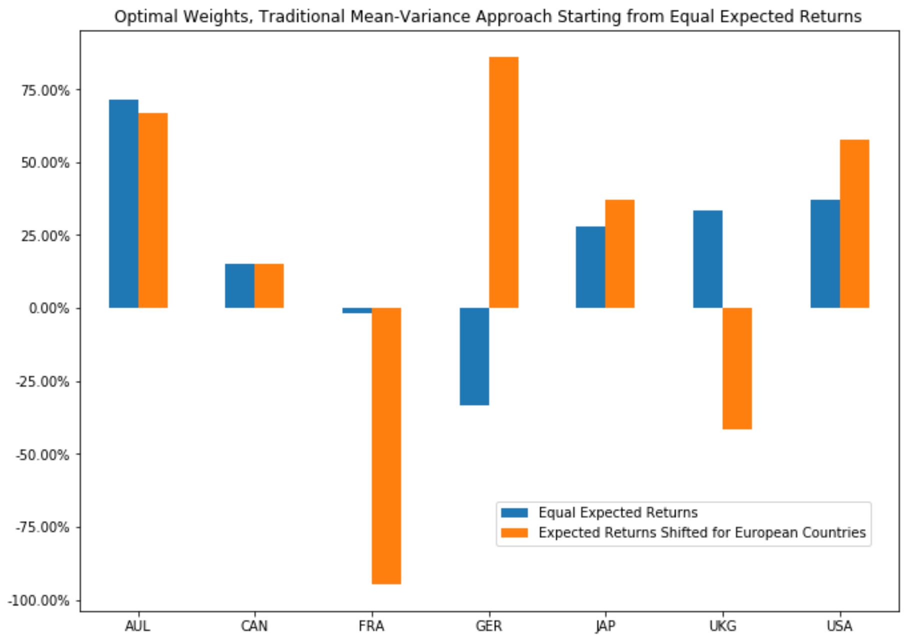
$$\mu_{\text{shift}} = (7\%, 7\%, 4.5\%, 9.5\%, 7\%, 4.5\%, 7\%)$$

and plug it to the formula(2), then we could obtain two sets of optimal weights w_0, w_{shift} and corresponding w_{diff}, respectively. Corresponding plot is also shown as follow[1][2]

[1]E.E.R represents Equal Expected Return. same for entire article

[2]E.R. represents Expected Return. same for entire article

Country Name	Weight for E.E.R.	Weight for E.R. Shifted	Weight Difference
AUL	0.714	0.671	-0.04
CAN	0.152	0.150	0.00
FRA	-0.017	-0.947	-0.93
GER	-0.335	0.862	1.20
JAP	0.279	0.372	0.09
UKG	0.336	-0.416	-0.75
USA	0.373	0.577	0.20



2.2 Chart 1B

For this chart, we do not need to solve any models but system of linear equations. Suppose we have $r_{F_0}, r_{G_0}, r_{U_0}$ as equilibrium means for France, Germany and UK, respectively. New views are going to update the return to $(r_F, r_G, r_U)$with such conditions:

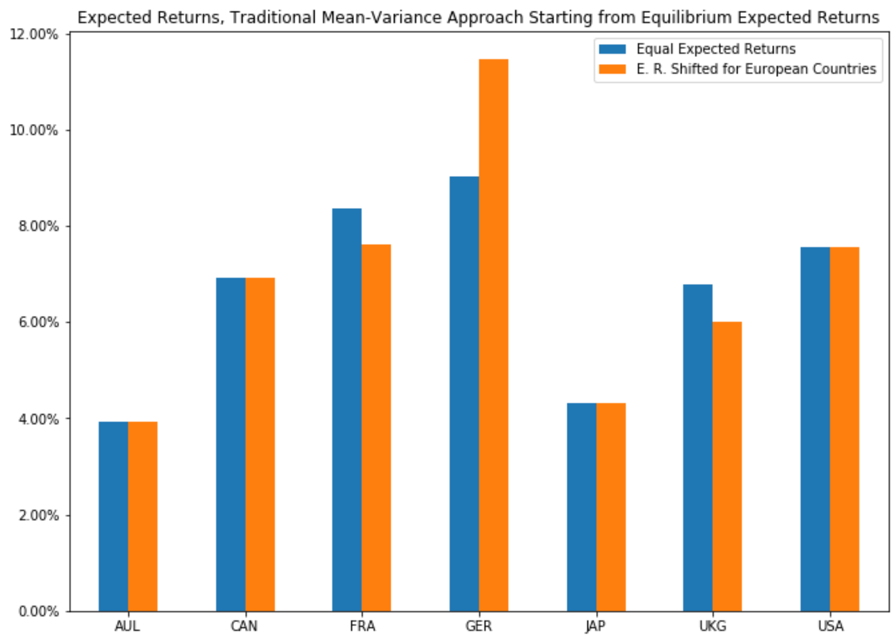
$$r_G = \frac{w_F}{w_F + w_U} r_F + \frac{w_U}{w_F + w_U} r_U + 5\% \tag{3}$$

$$w_G r_G + w_F r_F + w_U r_U = w_G r_{G_0} + w_F r_{F_0} + w_U r_{U_0} \tag{4}$$

$$r_F - r_U = r_{F_0} - r_{U_0} \tag{5}$$

where w_F, w_G and w_U are equilibrium weights for France, Germany and United Kingdom, respectively. Solved weights and corresponding plot are[3]

Country Name	E.E.R	E.R Shifted	Difference
AUL	3.9	3.9	0.00
CAN	6.9	6.9	0.00
FRA	8.4	7.6	-0.8
GER	9.0	11.5	2.5
JAP	4.3	4.3	0.00
UKG	6.8	6.0	-0.8
USA	7.6	7.6	0.00

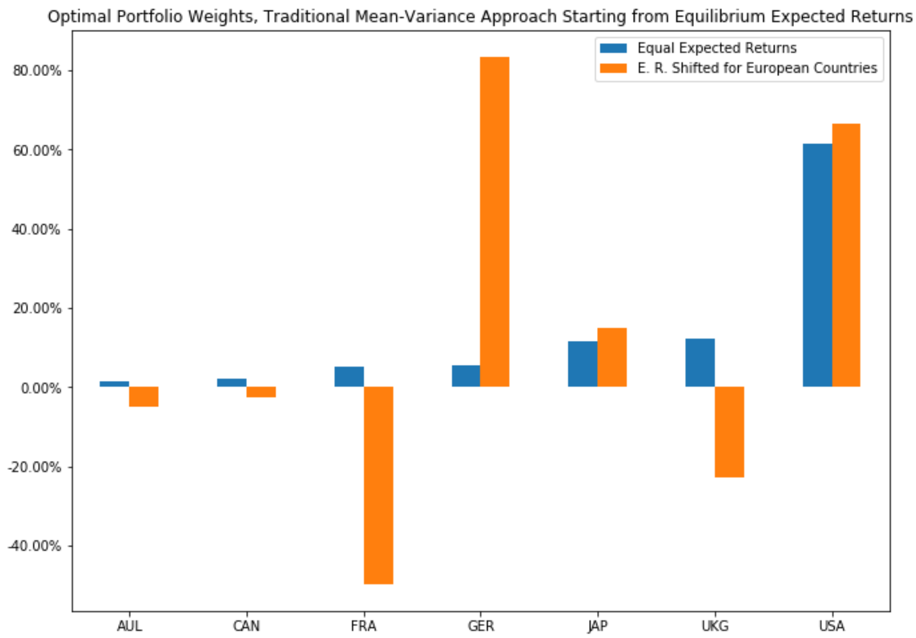


2.3 Chart 1C

After getting equilibrium expected returns and shifted expected returns, we could obtain optimal portfolio weights(O.P.W) for two sets of returns by using equation (2). The corresponding results and graph are:

[3]Solutions we obtained are different from given paper. However, we found another paper which is written by the same authors with the same data and the example. The results they obtained are the SAME as we obtained. See [1] table 3 for that paper for more details

Country Name	O.P.W for E.E.R	O.P.W for E.R Shifted	Difference
AUL	0.006	-0.061	-0.068
CAN	0.0116	-0.036	-0.048
FRA	0.064	-0.498	-0.56
GER	0.04	0.837	0.796
JAP	0.116	0.150	0.0332
UKG	0.131	-0.227	-0.359
USA	0.629	0.682	0.053



3 Chart 2

3.1 Chart 2A

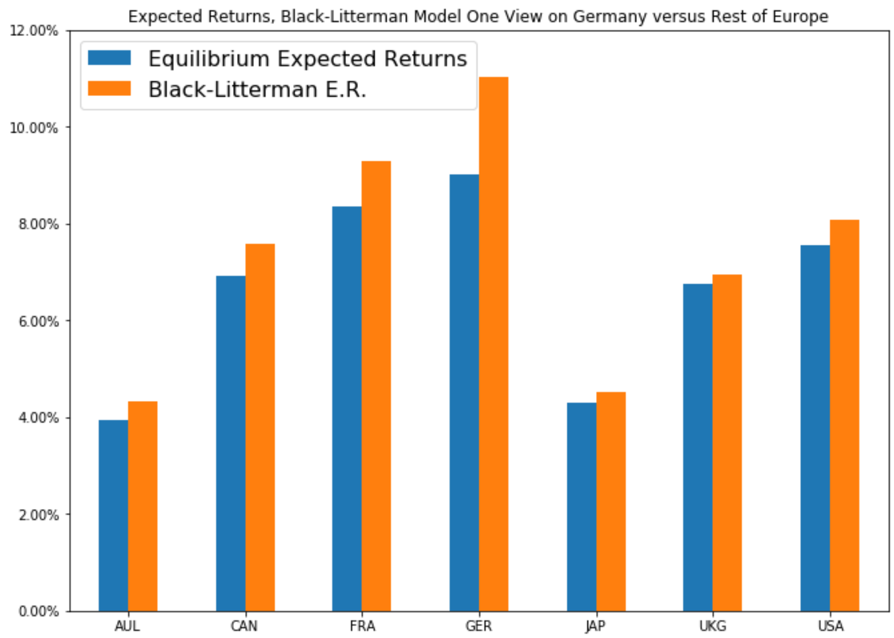
Suppose we hold the same view that Germany will outperform the rest of Europe by 5% and we use Black-Litterman model to update mean by formula

$$\mu_{BL} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \tag{6}$$

where P is the matrix of view which equal to $[0, 0, -0.295, 1, 0, -0.705, 0]$. Σ is the variance-covariance matrix. Ω is the confidence matrix which equal to $\text{diag}(\tau P\Sigma P')$ and $\Pi = \delta\Sigma w_{eq}$.

Updated means are as follows:

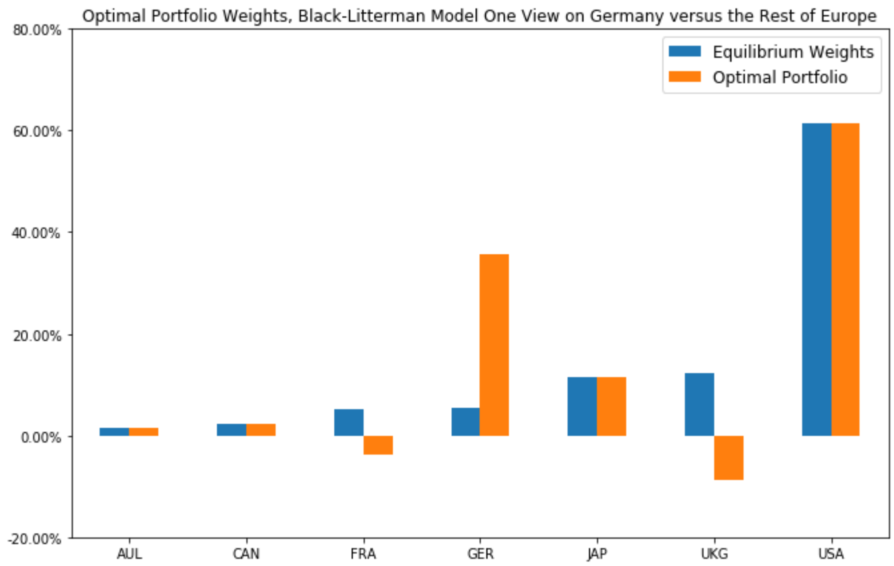
Country Name	E.E.R.	Black-Litterman E.R.
AUL	0.039	0.0433
CAN	0.069	0.0758
FRA	0.083	0.0929
GER	0.090	0.1104
JAP	0.043	0.0451
UKG	0.068	0.0695
USA	0.076	0.0807



3.2 Chart 2B

Optimal weights could be obtained by equation(2). Results and graphs are shown as following:

Country Name	Equal Weights	Optimal Portfolio
AUL	0.016	0.016
CAN	0.022	0.022
FRA	0.052	-0.037
GER	0.055	0.356
JAP	0.116	0.116
UKG	0.124	-0.0883
USA	0.615	0.615



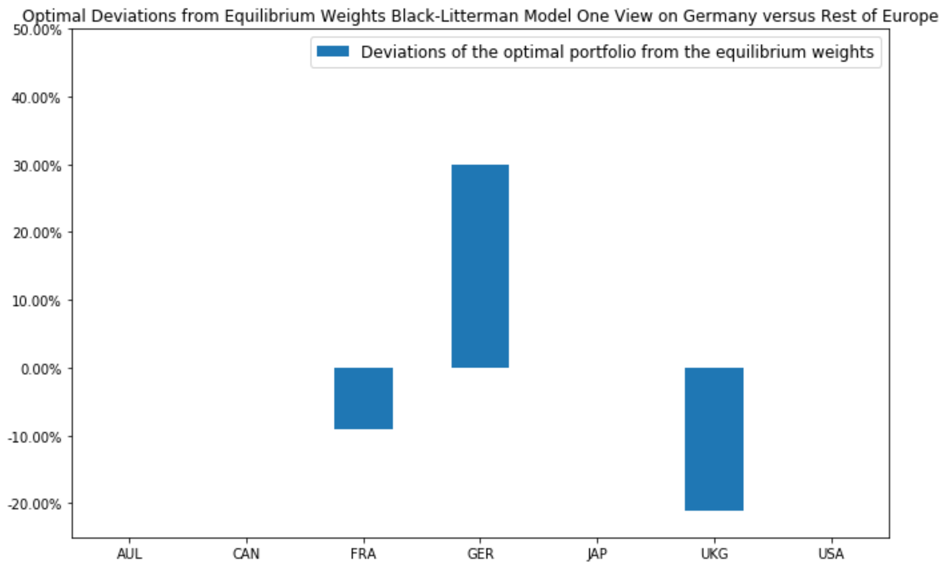
3.3 Chart 2C

The deviations of the optimal portfolio from the equilibrium weights is given by

$$\Delta = w_{opt} - w_{equ} \tag{7}$$

The results are

Country Name	Deviations
AUL	0
CAN	0
FRA	-0.09
GER	0.3
JAP	0
UKG	-0.21
USA	0



4 Chart 3

4.1 Chart 3A

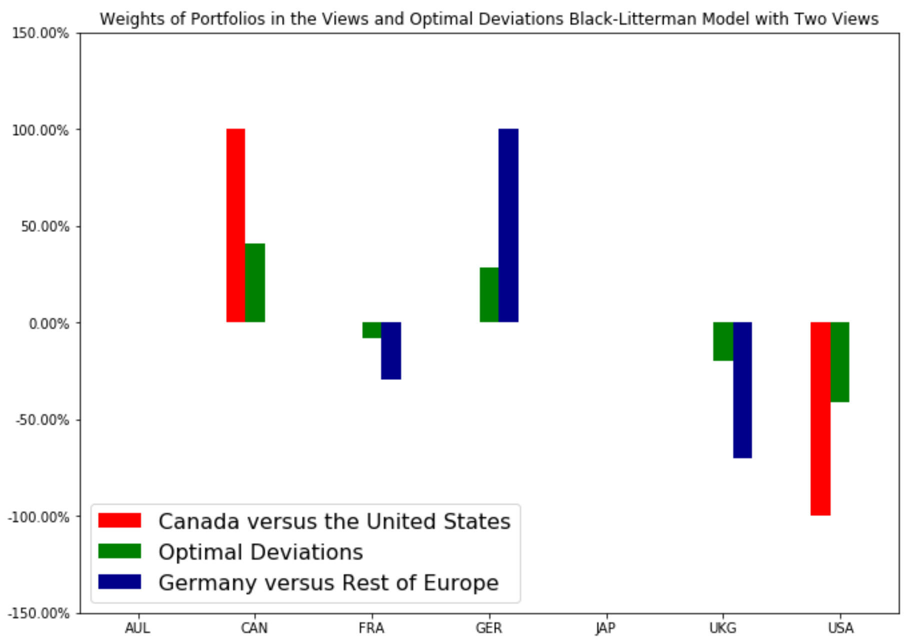
In addition to the original view, the investor has another view that the Canadian equity market will outperform the US equity market by 3%. That is, the P and Q matrices become to

$$\mathbf{P} = \begin{pmatrix} 0 & 0 & -0.295 & 1 & 0 & -0.705 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} 0.05 \\ 0.03 \end{pmatrix}$$

Substitute new $\mathbf{P}$ and $\mathbf{Q}$ into formula(6) and we could obtain

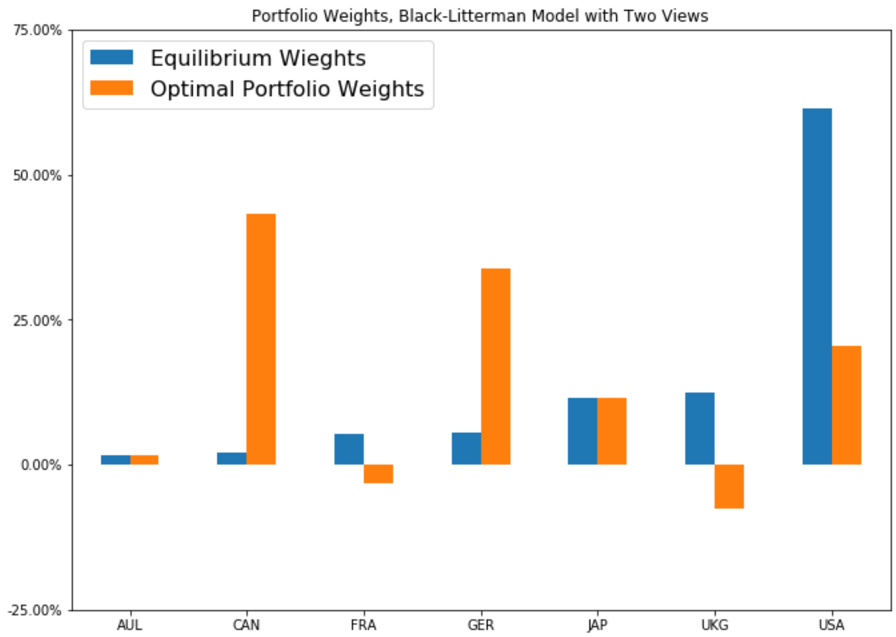
Country Name	GER vs Rest of EU	CAN vs USA	Optimal Deviations
AUL	0.00	0.00	0.00
CAN	0.00	1.00	0.4113
FRA	-0.295	0.00	-0.0833
GER	1.00	0.00	0.2921
JAP	0.00	0.00	0 .00
UKG	-0.705	0.00	-0.1987
USA	0.00	-1.00	-0.4113



4.2 Chart 3B

Applying formula(1) by updated mean calculated from Black-Litterman model, we could also obtain the optimal weights and its graphs, which are

Country Name	Equilibrium Weight	Optimal Portfolio weight
AUL	0.016	0.016
CAN	0.022	0.433
FRA	0.052	-0.031
GER	0.055	0.337
JAP	0.116	0.116
UKG	0.124	-0.075
USA	0.615	0.204



5 Chart 4

One advantage of Black-Litterman model is that investors could adjust their confidence level and expected return easily. Suppose we have three cases. First case is the one we did in Chart 3A. Second case is that the second view about Canada and USA changes where Canada will outperform USA by 4%. Third case is first view confidence level reduces half. i.e. In the second case, we would have a new **Q** matrix, which is

$$\mathbf{Q} = \begin{pmatrix} 0.05 \\ 0.04 \end{pmatrix}$$

Third case is $\Omega_3 = \Omega C$, where **C** is the confidence matrix given by

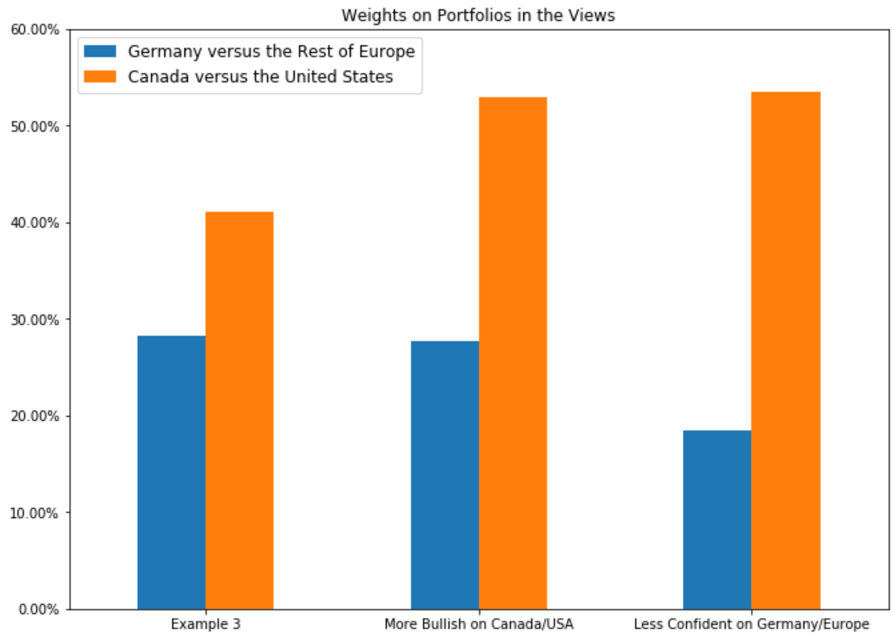
$$\mathbf{C} = \begin{pmatrix} 2.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$$

Then we could obtain weights portfolios, which is given by

$$\Lambda = \tau\Omega^{-1}Q/\delta - [\Omega/\tau + P\Sigma P']^{-1}P\Sigma w_{eq} - [\Omega/\tau + P\Sigma P']^{-1}P\Sigma P'\tau\Sigma^{-1}Q/\delta \quad (8)$$

Results are

View	GER vs EUR	CAN vs USA
Example	0.282	0.411
More Bullish on CA/USA	0.277	0.529
Less Confident on GER/EUR	0.184	0.434



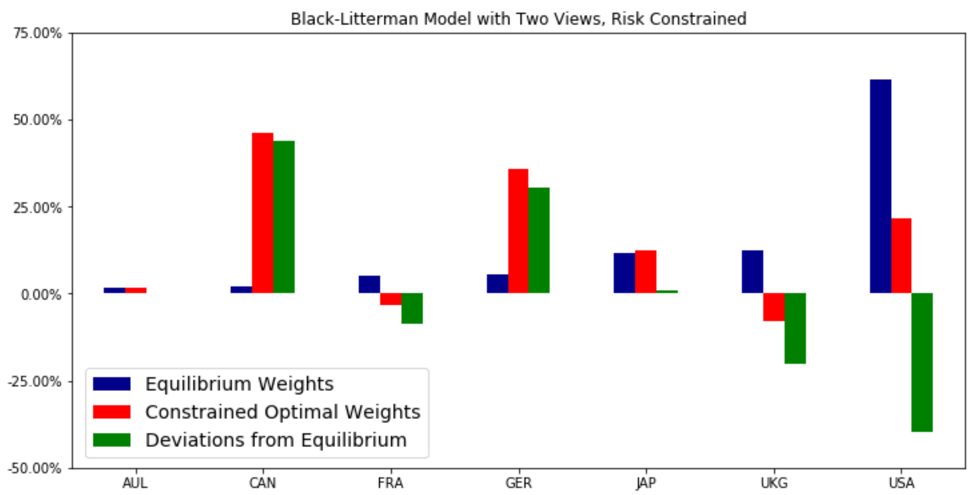
6 Chart 5

More constrains could be added to the Mean-Variance model. Suppose we have risk constrain here. i.e.

$$\begin{aligned} &\max\ w'\mu \\ &\text{subject to } w'\Sigma w \leq \sigma^2 \end{aligned} \tag{9}$$

Suppose we have $\sigma = 0.2$, it could by solved by Pyomo that

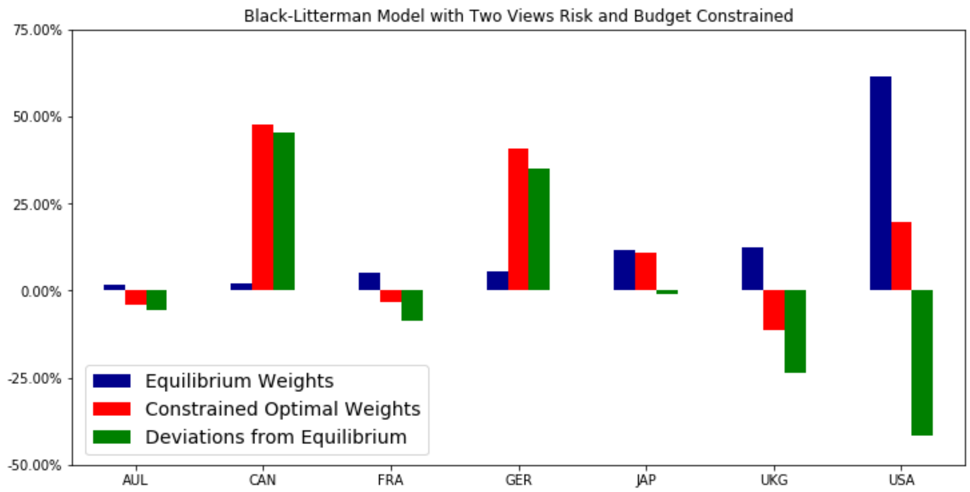
Country Name	Equilibrum Weights	Constrain Optimal Weight	Deviation
AUL	0.016	0.017	0.00098
CAN	0.022	0.460	0.43794
FRA	0.052	-0.033	-0.08527
GER	0.055	0.358	0.30284
JAP	0.116	0.123	0.00714
UKG	0.124	-0.079	-0.20334
USA	0.615	0.216	-0.39873



7 Chart 6

Adding budget constrain into the model. i.e. Besides from the constrain condition, $w'\iota = 1$ needs to be satisfied. The result is as follows:

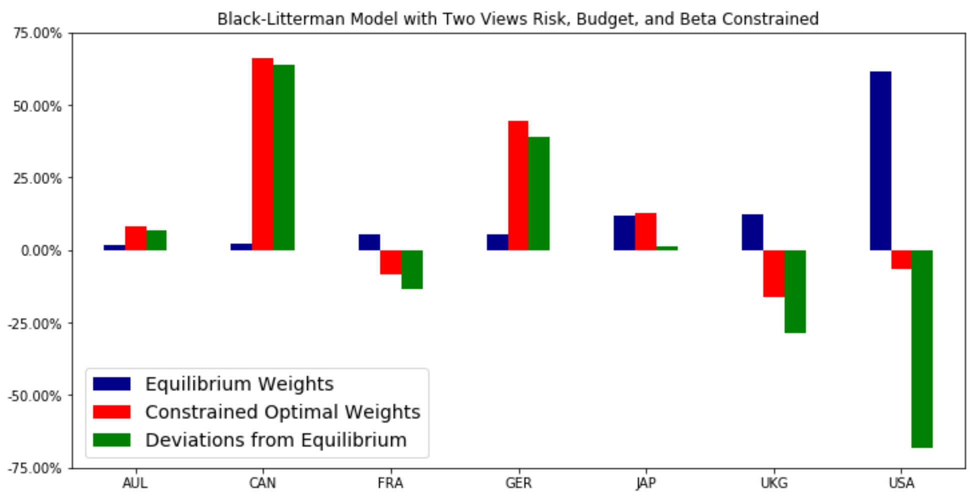
Country Name	Equilibrum Weights	Constrain Optimal Weight	Deviation
AUL	0.016	-0.0416	-0.0575
CAN	0.022	0.475	0.453
FRA	0.052	-0.033	-0.085
GER	0.055	0.407	0.352
JAP	0.116	0.107	-0.009
UKG	0.124	-0.112	-0.237
USA	0.615	0.198	-0.417



8 Chart 7

Beta constrain is added to the model. i.e. $w'\Sigma w_{eq} = w'_{eq}\Sigma w_{eq}$. Results are

Country Name	Equilibrum Weights	Constrain Optimal Weight	Deviation
AUL	0.016	0.084	0.068
CAN	0.022	0.660	0.638
FRA	0.052	-0.083	-0.135
GER	0.055	0.443	0.388
JAP	0.116	0.126	0.010
UKG	0.124	-0.164	-0.288
USA	0.615	-0.065	-0.680



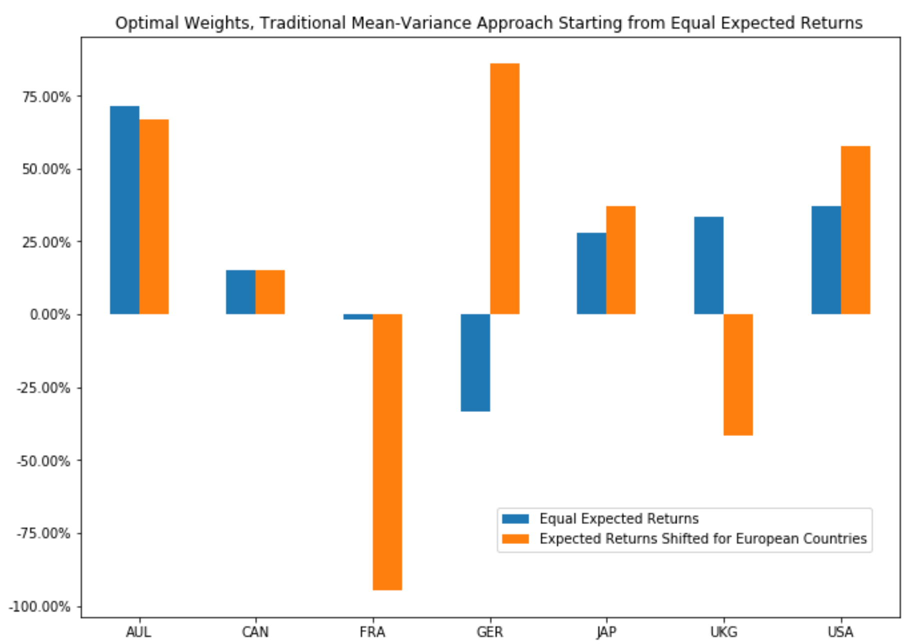
9 Modified Data Part

If increase the weight of each of the first three countries by 0.5% decrease weight of each of the next three countries by 0.5% and keep the USA weight unchanged, we would have $w_{eq} = [0.021, 0.027, 0.057, 0.05, 0.111, 0.119, 0.615]$. Procedure to produce the required graphs are almost same as we did previous. Results will be directly shown without further explanation.

9.1 Char1

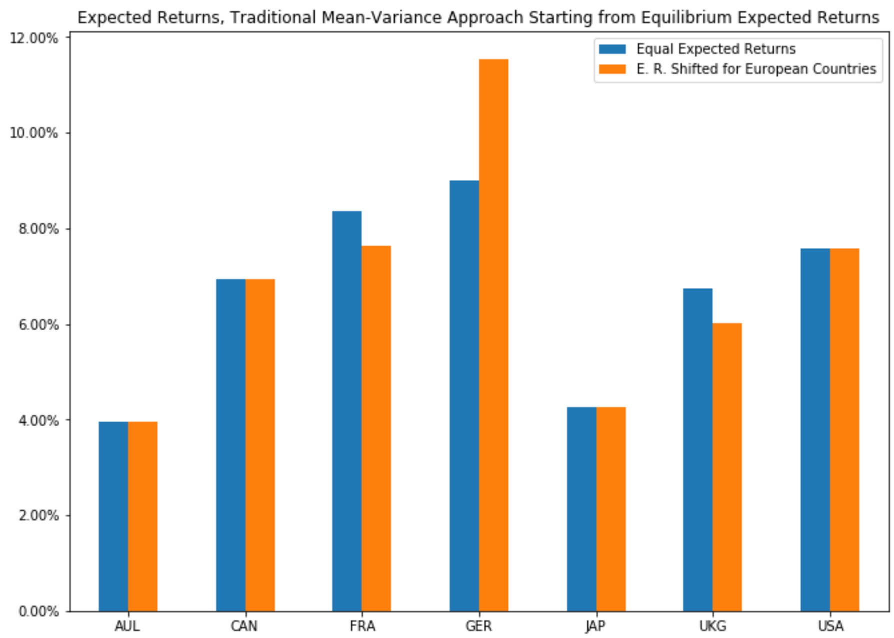
9.1.1 Chart 1A

Country Name	Weight for E.E.R.	Weight for E.R. Shifted	Weight Difference
AUL	0.714	0.671	-0.04
CAN	0.152	0.150	-0.00
FRA	-0.017	-0.947	-0.93
GER	-0.335	0.862	1.20
JAP	0.279	0.372	0.09
UKG	0.336	-0.416	-0.75
USA	0.373	0.577	0.20



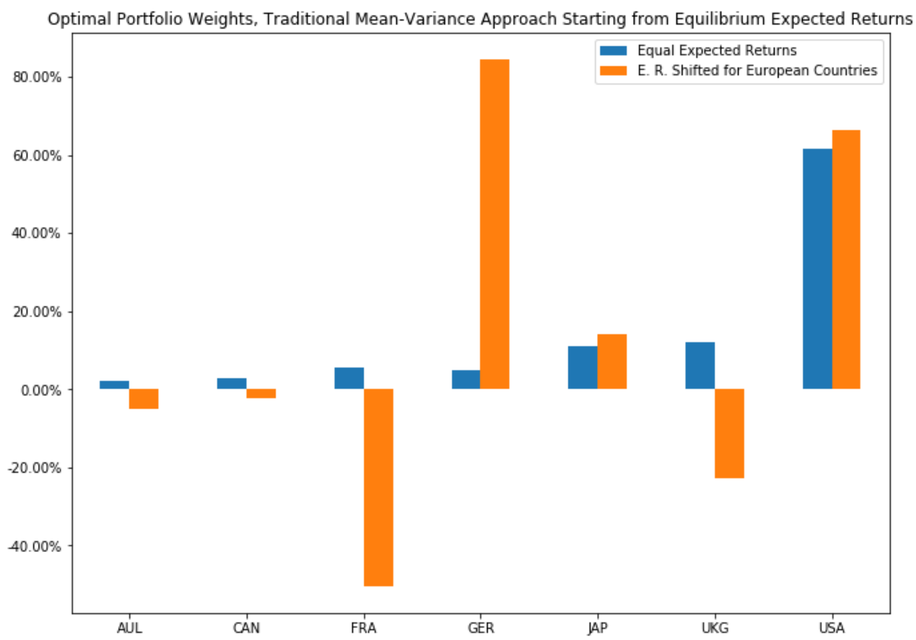
9.1.2 Chart 1B

Country Name	Equilibrium E.R	E.R Shifted	Difference
AUL	0.039	0.039	0.00
CAN	0.069	0.069	0.00
FRA	0.084	0.076	-0.007
GER	0.090	0.115	0.025
JAP	0.043	0.043	0.00
UKG	0.067	0.060	-0.007
USA	0.076	0.076	0.00



9.1.3 Chart 1C

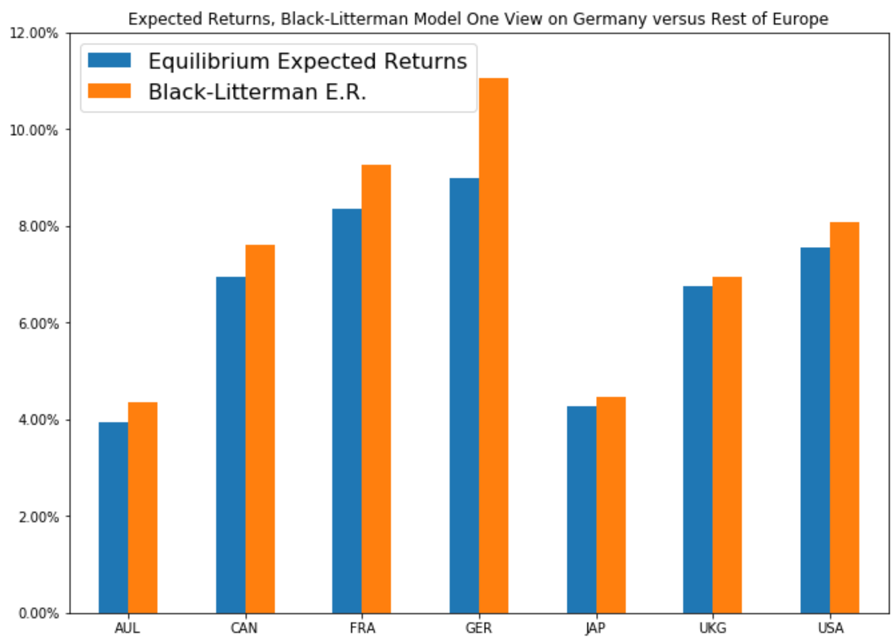
Country Name	O.P.W for E.E.R	O.P.W for E.R Shifted	Difference
AUL	0.021	-0.049	-0.07
CAN	0.027	-0.023	-0.05
FRA	0.057	-0.505	-0.56
GER	0.05	0.846	0.80
JAP	0.111	0.142	0.031
UKG	0.119	-0.228	-0.34
USA	0.615	0.663	0.048



9.2 Chart 2

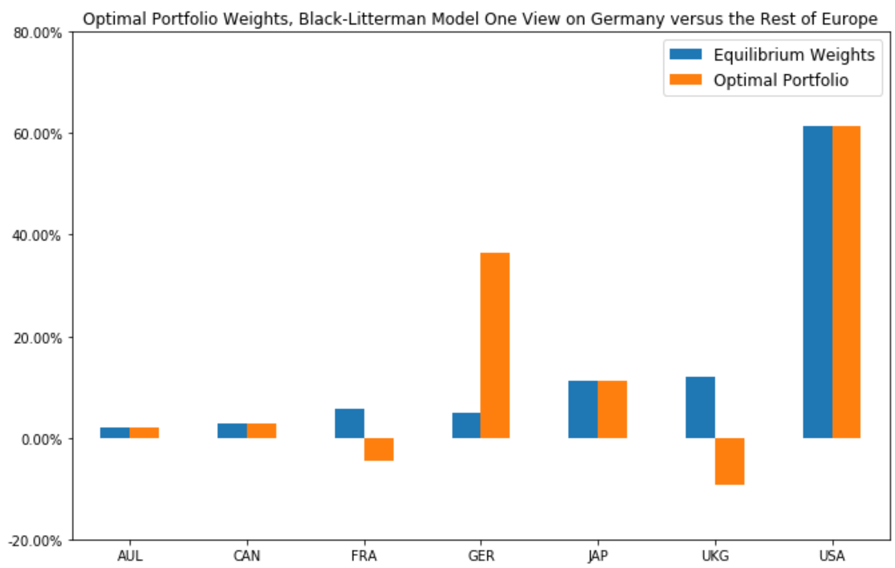
9.2.1 Chart 2A

Country Name	E.E.R	BLack-Litterman E.R
AUL	0.039	0.043
CAN	0.069	0.076
FRA	0.084	0.093
GER	0.090	0.111
JAP	0.043	0.045
UKG	0.067	0.069
USA	0.075	0.081



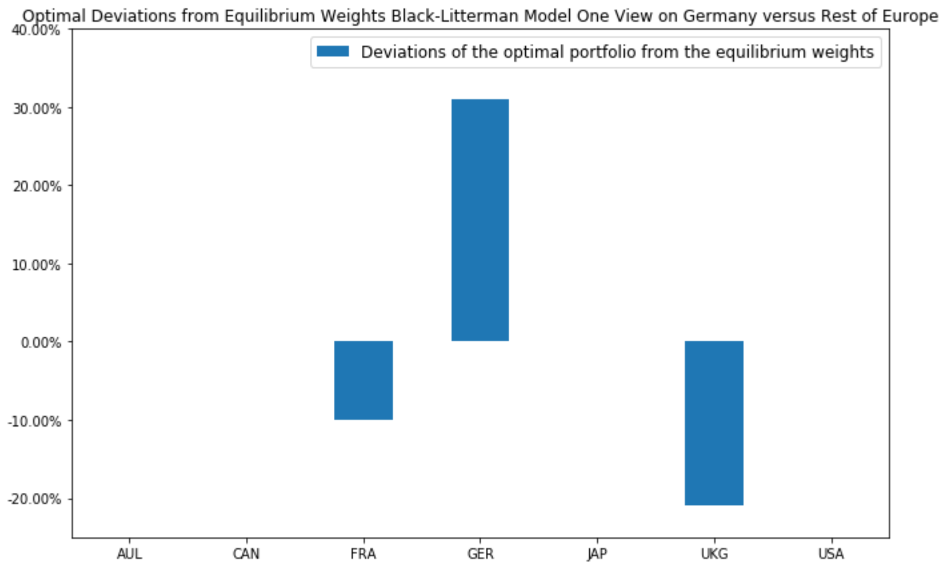
9.2.2 Chart 2B

Country Name	Equilibrium Weights	Optimal Portfolio
AUL	0.021	0.021
CAN	0.027	0.027
FRA	0.057	-0.045
GER	0.05	0.364
JAP	0.111	0.111
UKG	0.119	-0.09
USA	0.615	0.615



9.2.3 Chart 2C

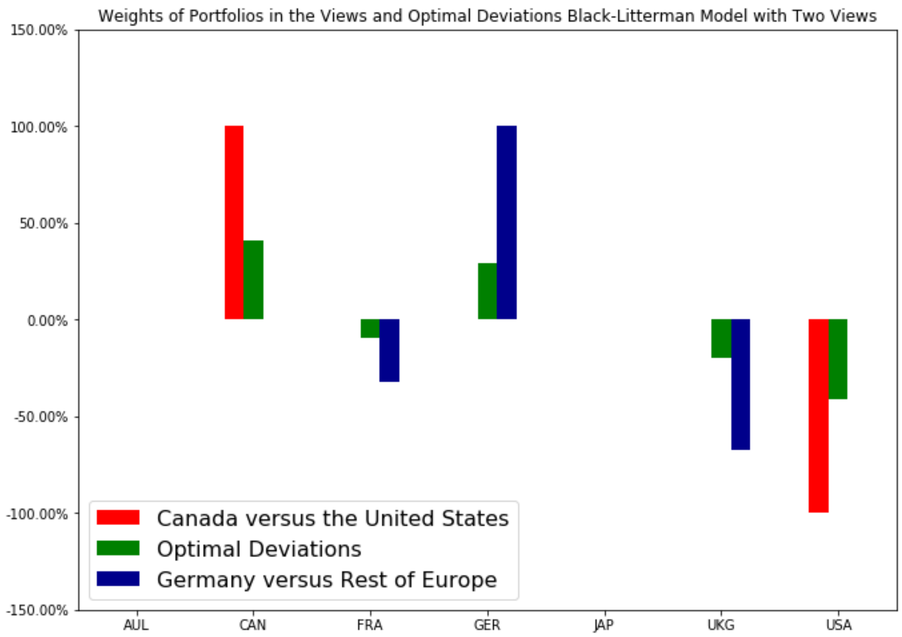
Country Name	Optimal Deviations
AUL	0
CAN	0
FRA	-0.10
GER	0.31
JAP	0
UKG	-0.21
USA	0



9.3 Chart 3

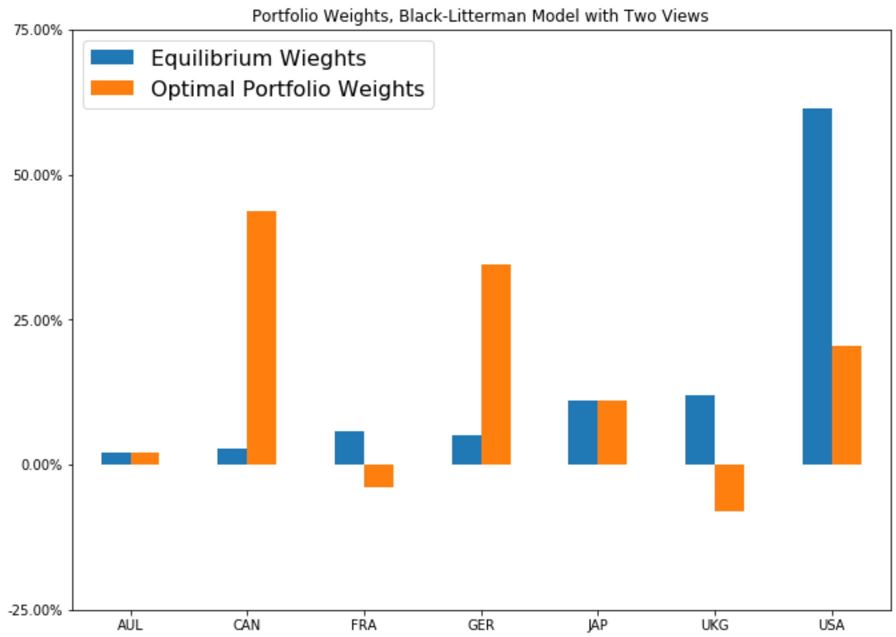
9.3.1 Chart 3A

Country Name	Germany vs Europe	Canada vs USA	Optimal Deviations
AUL	0.000	0.00	0.000
CAN	0.000	1.00	0.409
FRA	-0.324	0.00	-0.096
GER	1.000	0.00	0.295
JAP	0.000	0.00	0 .000
UKG	-0.676	0.00	-0.200
USA	-0.000	-1.00	-0.410



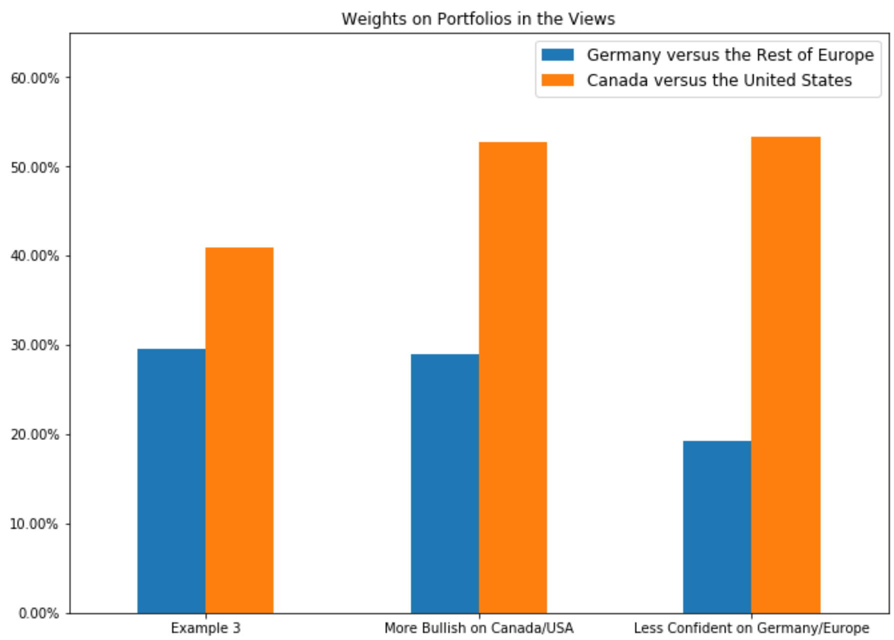
9.3.2 Chart 3B

Country Name	Equilibrium Weights	Optimal Portfolio Weights
AUL	0.021	0.021
CAN	0.027	0.436
FRA	0.057	-0.039
GER	0.05	0.345
JAP	0.111	0.111
UKG	0.119	-0.08
USA	0.615	0.206



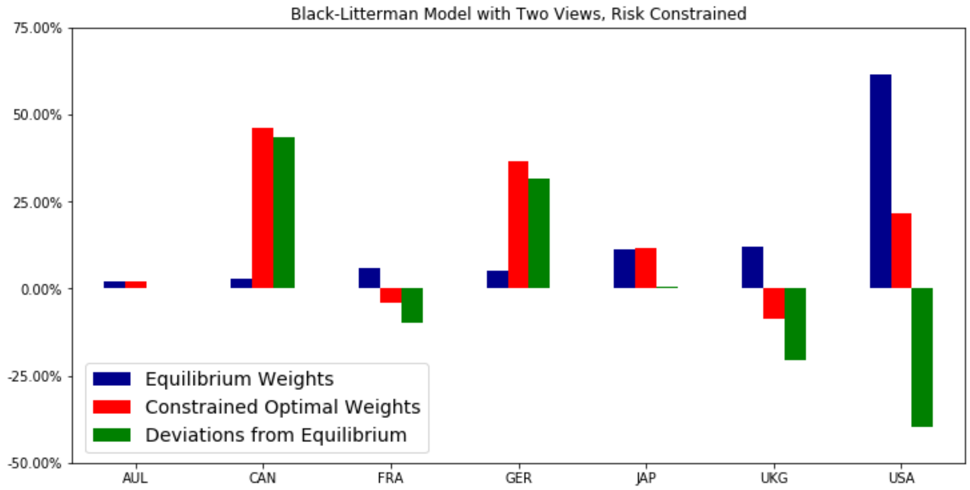
9.4 Chart 4

View	GER vs EUR	CAN vs USA
Example	0.295	0.409
More Bullish on CA/USA	0.290	0.527
Less Confident on GER/EUR	0.193	0.533



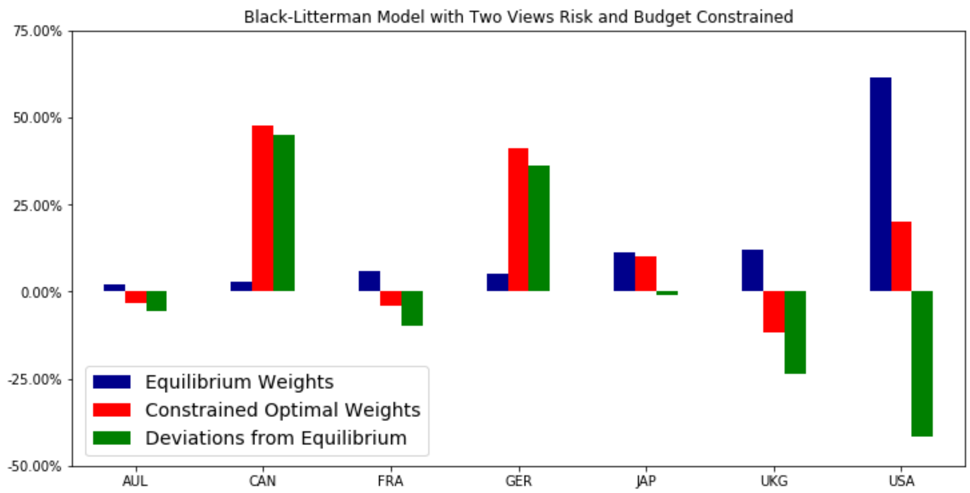
9.5 Chart 5

Country Name	Equilibrium Weights	Constrain Optimal Weight	Deviation
AUL	0.021	0.022	0.001
CAN	0.027	0.462	0.435
FRA	0.057	-0.041	-0.100
GER	0.05	0.366	0.315
JAP	0.111	0.118	0.007
UKG	0.119	-0.086	-0.204
USA	0.615	0.218	-0.397



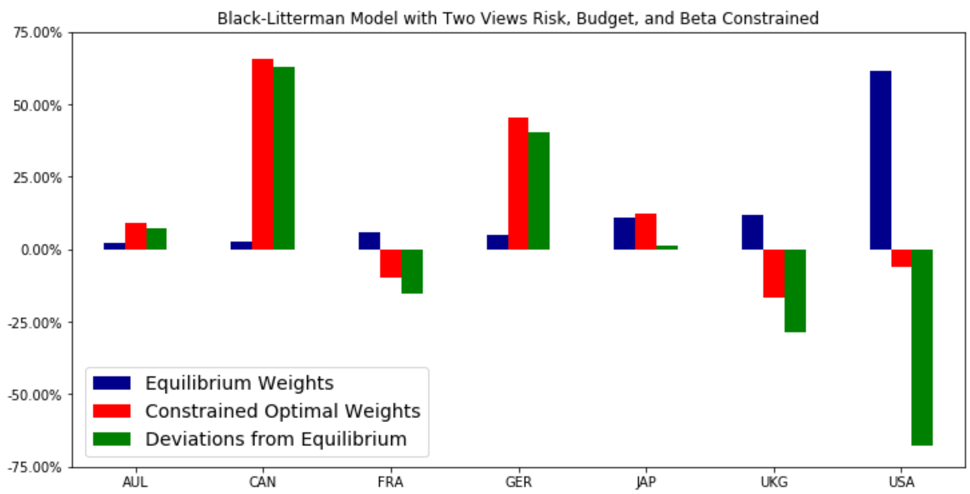
9.6 Chart 6

Country Name	Equilibrium Weights	Constrain Optimal Weight	Deviation
AUL	0.021	-0.033	-0.054
CAN	0.027	0.476	0.449
FRA	0.057	-0.041	-0.098
GER	0.05	0.412	0.363
JAP	0.111	0.102	-0.009
UKG	0.119	-0.117	-0.236
USA	0.615	0.200	-0.415



9.7 Chart 7

Country Name	Equilibrium Weights	Constrain Optimal Weight	Deviation
AUL	0.021	0.092	0.071
CAN	0.027	0.657	0.630
FRA	0.057	-0.100	-0.154
GER	0.050	0.453	0.403
JAP	0.111	0.123	0.012
UKG	0.119	-0.167	-0.286
USA	0.615	-0.061	-0.676



References

[1] Robert Litterman Guangliang He. THE INTUITION BEHIND BLACK-LITTERMAN MODEL PORTFOLIOS. 2002.