DS GA 1002 _ HW4 yp2201@nyu.edu

Q1. (a) Let's assume that Mike grabs $\tilde{x}$ chocolate bars ($0 \le \tilde{x} < 3$)
since all three values have uniform probability, $P_{\tilde{x}}(x) = \frac{1}{3}$, for $x \in \{0,1,2\}$,

and $P_{\tilde{x}}(x) = 0$, otherwise

Laura grabs $\tilde{y}$ Chocolate bars ($0 \le \tilde{y} < 3-\tilde{x}$)

The conditional pmf $P_{\tilde{y}|\tilde{x}}(y|x) = \begin{cases} \frac{1}{3-x}, & 0 \le y \le 2-x \\ 0, & \text{otherwise} \end{cases}$

By the chain rule, joint pmf is as below

$$\underline{P_{\tilde{y},\tilde{x}}(y,x)} = P_{\tilde{y}|\tilde{x}}(y|x) \cdot P_{\tilde{x}}(x) = \boxed{\frac{1}{3} \times \frac{1}{(3-x)}} \text{ for } \begin{cases} x = 0,\dots,2 \\ y = 0,\dots,2-x \end{cases}$$

(b) for the marginal pmf of Laura, sum over Mike's all probabilities.

$$P_{\tilde{y}}(0) = \frac{1}{3}\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{1}\right) = \frac{11}{18}$$

$$P_{\tilde{y}}(1) = \frac{1}{3}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{5}{18}$$

$$P_{\tilde{y}}(2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

c) When $\tilde{y} = 1$, $\tilde{x}$ should be either 0 or 1

So, $$P_{\tilde{x}|\tilde{y}}(0|1) = \frac{P_{\tilde{y},\tilde{x}}(1,0)}{P_{\tilde{y}}(1)} = \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{5}{18}} = \boxed{\frac{2}{5}}$$

$$P_{\tilde{x}|\tilde{y}}(1|1) = \frac{P_{\tilde{y},\tilde{x}}(1,1)}{P_{\tilde{y}}(1)} = \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{5}{18}} = \boxed{\frac{3}{5}}$$

Q2) (a) $\begin{cases} p(\tilde{q}=1) = 0.25 \\ p(\tilde{q}=-1) = 0.75 \end{cases}$ $\begin{cases} \tilde{x}_1 = \tilde{e}_1 \cdot \tilde{q} \\ \tilde{x}_2 = \tilde{e}_2 \cdot \tilde{q} \end{cases}$ $\begin{cases} p(\tilde{e}_1 = 1) = p(\tilde{e}_2 = 1) = 0.8 \\ p(\tilde{e}_1 = -1) = p(\tilde{e}_2 = -1) = 0.2 \end{cases}$

For $\tilde{x}_1 = 1$ and $\tilde{x}_2 = 1$, $\tilde{e}_1 \cdot \tilde{q} = 1$ and $\tilde{e}_2 \cdot \tilde{q} = 1$

$$p(\tilde{x}_1 = 1, \tilde{x}_2 = 1) = p(\tilde{e}_1 \cdot \tilde{q} = 1, \tilde{e}_2 \cdot \tilde{q} = 1)$$

$$\begin{matrix} 1 & 1 & & 1 \\ & & \text{or} & \\ -1 & -1 & & -1 \end{matrix}$$

$$= \underset{p(\tilde{e}_1=1)}{0.8} \times \underset{p(\tilde{e}_2=1)}{0.8} \times \underset{p(\tilde{q}=1)}{0.25} + \underset{p(\tilde{e}_1=-1)\cdot p(\tilde{e}_2=-1)\cdot p(\tilde{q}=-1)}{0.2 \times 0.2 \times 0.75} = 0.19$$

(b) No. While $p(\tilde{x}_1 = 1, \tilde{x}_2 = 1) = 0.19$,

$$p(\tilde{x}_1 = 1) \times p(\tilde{x}_2 = 1) = p(\tilde{e}_1 \cdot \tilde{q} = 1) \times p(\tilde{e}_2 \cdot \tilde{q} = 1)$$

$$= (0.8 \times 0.25 + 0.2 \times 0.75) \times (0.8 \times 0.25 + 0.2 \times 0.75)$$

$$= (0.35)^2 = 0.1225$$

So, $p(\tilde{x}_1 = 1, \tilde{x}_2 = 1) \neq p(\tilde{x}_1 = 1) \times p(\tilde{x}_2 = 1)$

(c) Yes. $p(\tilde{x}_1, \tilde{x}_2 \mid \tilde{q}) = p(\tilde{x}_1 \mid \tilde{q}) \times p(\tilde{x}_2 \mid \tilde{q})$

If $\tilde{x}_1 = 1$, $\tilde{x}_2 = 1$, $\tilde{q} = 1$

$p(\tilde{x}_1 = 1, \tilde{x}_2 = 1 \mid \tilde{q} = 1) = 0.8 \times 0.8$ (since $\tilde{q} = 1$, $\tilde{x}_1 = \tilde{e}_1 \cdot \tilde{q} \neq \boxed{\tilde{e}_1 = 1}$

$p(\tilde{x}_1 = 1 \mid \tilde{q} = 1) = p(\tilde{e}_1 = 1) = 0.8$ $\quad$ $p(\tilde{e}_1) = 0.8$, $p(\tilde{e}_2) = 0.8$

$p(\tilde{x}_2 = 1 \mid \tilde{q} = 1) = p(\tilde{e}_2 = 1) = 0.8$ $\quad$ respectively

$\therefore$ $p(\tilde{x}_1 = 1, \tilde{x}_2 = 1 \mid \tilde{q} = 1) = p(\tilde{x}_1 = 1 \mid \tilde{q} = 1) \times p(\tilde{x}_2 = 1 \mid \tilde{q} = 1)$

$\therefore$ The rest follows by the same argument.

Q3)

$$T = \begin{bmatrix} 0 & 0.1 & 0 \\ 1 & 0 & 1 \\ 0 & 0.9 & 0 \end{bmatrix}$$

T has three eigenvectors: $\lambda_1 = 1$, $\lambda_2 = -1$, $\lambda_3 = 0$

$$q_1 = \begin{pmatrix} \frac{1}{9} \\ \frac{10}{9} \\ 1 \end{pmatrix}, \quad q_2 = \begin{pmatrix} \frac{1}{9} \\ -\frac{10}{9} \\ 1 \end{pmatrix}, \quad q_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Initial state vector is $P_{\tilde{x}}[1] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

By using eigenvector matrix $Q = [q_1 \ q_2 \ q_3]$,

$$Q = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & -1 \\ \frac{10}{9} & -\frac{10}{9} & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad Q^{-1} = \begin{bmatrix} 9/20 & 9/20 & 9/20 \\ 9/20 & -9/20 & 9/20 \\ -9/10 & 0 & 1/10 \end{bmatrix}$$

$$Q^{-1} P_{\tilde{x}}[1] = \begin{bmatrix} 9/20 & 9/20 & 9/20 \\ 9/20 & -9/20 & 9/20 \\ -9/10 & 0 & 1/10 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 9/20 \\ 9/20 \\ -9/10 \end{bmatrix} \begin{matrix} = \alpha[1] \\ = \alpha[2] \\ = \alpha[3] \end{matrix}$$

$$P_{\tilde{x}}[i] = T_{\tilde{x}}^{i-1} \cdot P_{\tilde{x}}[1] = (Q \Lambda Q^{-1})^{i-1} P_{\tilde{x}}[1] = Q \Lambda^{i-1} Q^{-1} P_{\tilde{x}}[1]$$

$$= \sum_{j=1}^{m} \lambda_j^{i-1} \alpha[j] q_j,$$

$(\alpha = Q^{-1} P_{\tilde{x}}[1])$

$$= \underbrace{\lambda_1^{i-1}}_{=1} \alpha[1] q_1 + \underbrace{\lambda_2^{i-1}}_{=-1} \alpha[2] q_2 + \cancel{\underbrace{\lambda_3^{i-1}}_{=0} \alpha[3] q_3}$$

$$= 9/20 \, q_1 + (-1)^{i-1} \cdot 9/20 \, q_2$$

$$= \begin{cases} \text{if } i \text{ is odd}, & 9/20 (q_1 + q_2) = \frac{9}{20} \times \begin{bmatrix} \frac{2}{9} \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0.9 \end{bmatrix} \\ \text{if } i \text{ is even}, & 9/20 (q_1 - q_2) = \frac{9}{20} \times \begin{bmatrix} 0 \\ \frac{20}{9} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{cases}$$

HW4 - Q4

```
# Compute joint pmf of three Bernoulli random variables indicating whether it rains (1) or not (0)
# in Bodega, Coos Bay and Riley
def compute_joint_pmf(station_1,station_2,station_3,data_matrix):
    # creating 2*2*2 array. i.e) counts[0][0][0] = station1 = 0, station2 = 0 , station3 = 0
    counts = np.array([[[0,0],[0,0]],[[0,0],[0,0]]])

    # create data_matrix for each stations
    rv1 = data_matrix[:,station_1]
    rv2 = data_matrix[:,station_2]
    rv3 = data_matrix[:,station_3]

    # loop for every columns and append to each given array index
    for i in range(data_matrix.shape[0]):
        if rv1[i] <= 0 and rv2[i] <= 0 and rv3[i] <= 0:
            counts[0][0][0] += 1
        elif rv1[i] <= 0 and rv2[i] <= 0 and rv3[i] > 0:
            counts[0][0][1] += 1
        elif rv1[i] <= 0 and rv2[i] > 0 and rv3[i] <= 0:
            counts[0][1][0] += 1
        elif rv1[i] <= 0 and rv2[i] > 0 and rv3[i] > 0:
            counts[0][1][1] += 1
        elif rv1[i] > 0 and rv2[i] <= 0 and rv3[i] <= 0:
            counts[1][0][0] += 1
        elif rv1[i] > 0 and rv2[i] <= 0 and rv3[i] > 0:
            counts[1][0][1] += 1
        elif rv1[i] > 0 and rv2[i] > 0 and rv3[i] <= 0:
            counts[1][1][0] += 1
        elif rv1[i] > 0 and rv2[i] > 0 and rv3[i] > 0:
            counts[1][1][1] += 1

    # get probability by dividing total counts
    joint_pmf = counts / data_matrix.shape[0]

    return counts , joint_pmf
```

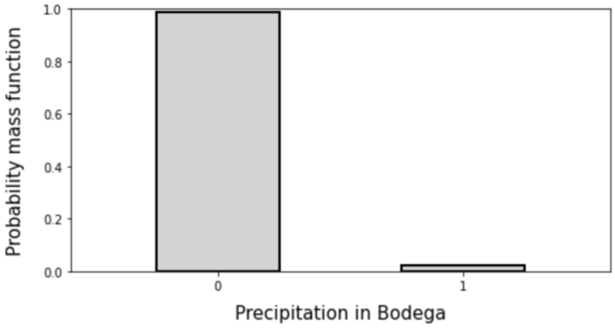

```
    return counts , joint_pmf
```

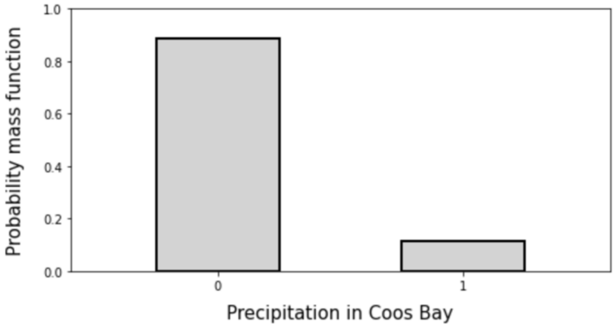

```
counts, joint_pmf = compute_joint_pmf(stations[0],stations[1],stations[2],data_matrix)

print(counts)
print()
print(joint_pmf)
```

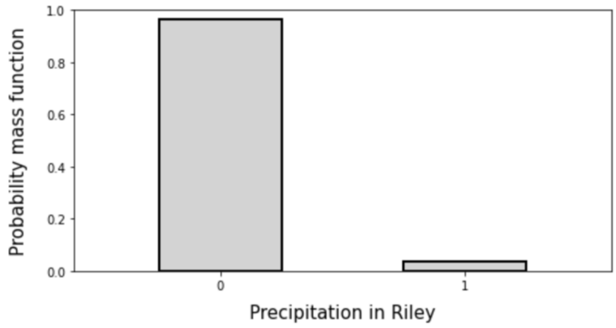
```
[[[7472  186]
  [ 814   89]]

 [[  88   21]
  [  75   15]]]

[[[0.85297 0.02123]
  [0.09292 0.01016]]

 [[0.01005 0.0024 ]
  [0.00856 0.00171]]]
```

```python
# Compute marginal pmf of each of the Bernoulli random variables
def marginal_1_station(joint_pmf):
    # INSERT YOUR CODE HERE

    # for each marginal pmf, add all probabilities fixing certain station by 0 and 1
    marginal_pmf_1 = np.array([joint_pmf[0][0][0] + joint_pmf[0][0][1] + joint_pmf[0][1][0] + joint_pmf[0][0][1],
                               joint_pmf[1][0][0] + joint_pmf[1][0][1] + joint_pmf[1][1][0] + joint_pmf[1][0][1]])

    marginal_pmf_2 = np.array([joint_pmf[0][0][0] + joint_pmf[0][0][1] + joint_pmf[1][0][0] + joint_pmf[1][0][1],
                               joint_pmf[0][1][0] + joint_pmf[0][1][1] + joint_pmf[1][1][0] + joint_pmf[1][1][1]])

    marginal_pmf_3 = np.array([joint_pmf[0][0][0] + joint_pmf[0][1][0] + joint_pmf[1][0][0] + joint_pmf[1][1][0],
                               joint_pmf[0][0][1] + joint_pmf[0][1][1] + joint_pmf[1][0][1] + joint_pmf[1][1][1]])


    return marginal_pmf_1, marginal_pmf_2, marginal_pmf_3


marginal_pmf_1,marginal_pmf_2,marginal_pmf_3 = marginal_1_station(joint_pmf)
print(marginal_pmf_1)
print(marginal_pmf_2)
print(marginal_pmf_3)

vals = [0,1]
ymax = 1.0
xmin = -0.6
xmax = 1.6
plt.figure(figsize=(8,4))
plt.bar(vals,marginal_pmf_1, width = 0.5, color = "lightgray", edgecolor="black",linewidth = 2)
plt.xticks(np.arange(0, 1+1, 1))
plt.xticks(fontsize=font_size_ticks)
plt.yticks(fontsize=font_size_ticks)
plt.ylim([0,ymax])
plt.xlim([xmin,xmax])
plt.ylabel("Probability mass function",fontsize=font_size,labelpad = 15)
plt.xlabel("Precipitation in Bodega",fontsize=font_size,labelpad = 10)
plt.savefig('./precipitation_marginal_pmf_1.pdf',bbox_inches="tight")
```

```
[0.98836 0.0234 ]
[0.88664 0.11336]
[0.9645 0.0355]
```





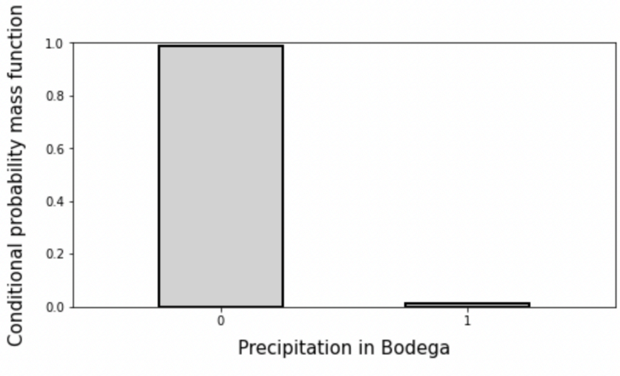


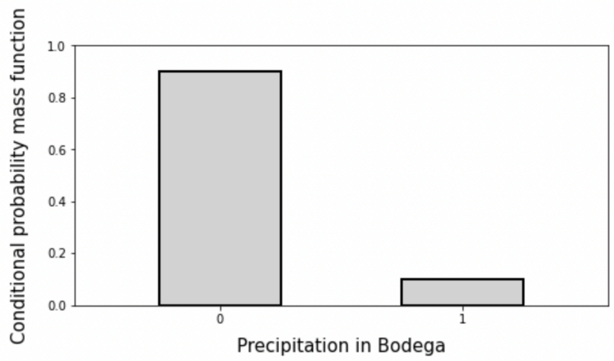
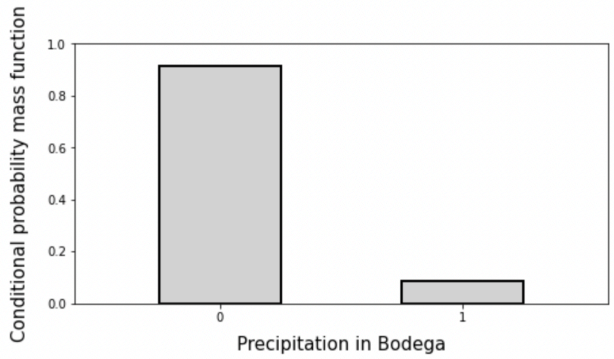
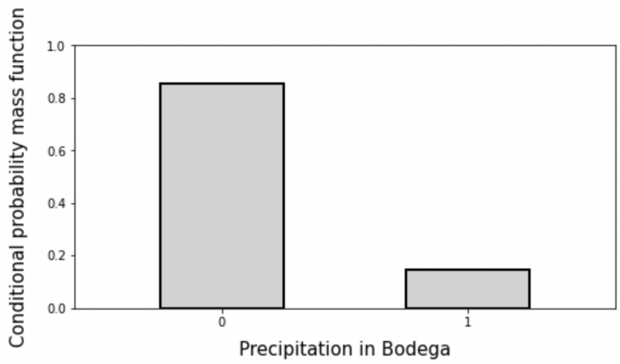
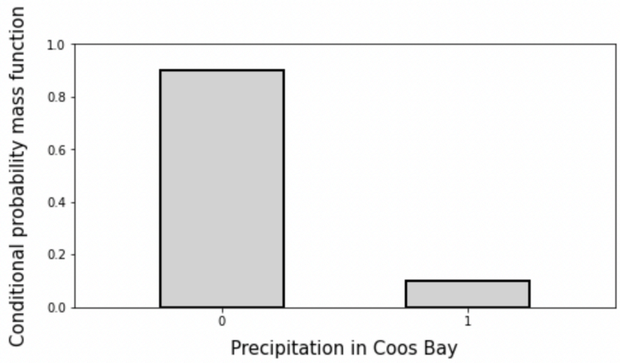

```
# Compute marginal joint pmf of each pair of the Bernoulli random variables
def marginal_2_stations(joint_pmf):
    # INSERT YOUR CODE HERE

    # for certain pmf, think pair as (0,0), (0,1), (1,1), (1,1) and sum excluded station counts for each pair
    marginal_pmf_12 = np.array([[joint_pmf[0][0][0] + joint_pmf[0][0][1], joint_pmf[0][1][0] + joint_pmf[0][1][1]],
                                [joint_pmf[1][0][0] + joint_pmf[1][0][1], joint_pmf[1][1][0] + joint_pmf[1][1][1]]])

    marginal_pmf_13 = np.array([[joint_pmf[0][0][0] + joint_pmf[0][1][0], joint_pmf[0][0][1] + joint_pmf[0][1][1]],
                                [joint_pmf[1][0][0] + joint_pmf[1][0][1], joint_pmf[1][1][0] + joint_pmf[1][1][1]]])

    marginal_pmf_23 = np.array([[joint_pmf[0][0][0] + joint_pmf[1][0][0], joint_pmf[0][0][1] + joint_pmf[1][0][1]],
                                [joint_pmf[0][1][0] + joint_pmf[1][1][0], joint_pmf[0][1][1] + joint_pmf[1][1][1]]])

    return marginal_pmf_12,marginal_pmf_13,marginal_pmf_23

marginal_pmf_12,marginal_pmf_13,marginal_pmf_23 = marginal_2_stations(joint_pmf)

print(marginal_pmf_12)
print()
print(marginal_pmf_13)
print()
print(marginal_pmf_23)
```

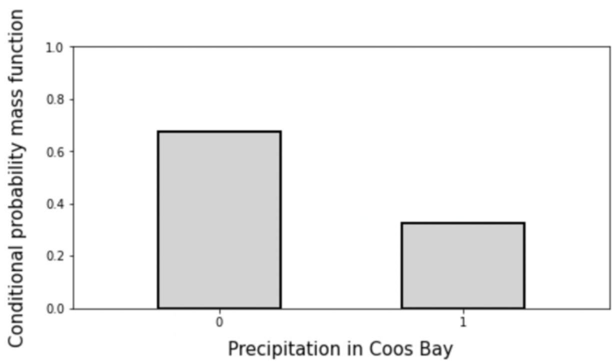
```
[[0.8742  0.10308]
 [0.01244 0.01027]]

[[0.94589 0.03139]
 [0.01244 0.01027]]

[[0.86301 0.02363]
 [0.10148 0.01187]]
```

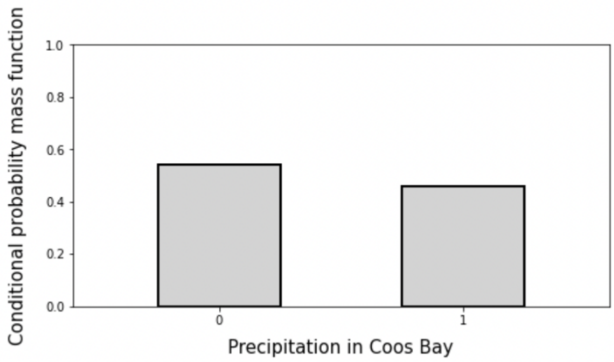
```
# Compute conditional pmf of each of the Bernoulli random variables given the other two
def conditional_1_station_given_2(joint_pmf):
    # INSERT YOUR CODE HERE

    # For example, given 2,3 as (0,0), get station 1 = 0 and station1 = 1 in a list
    # list up for other cases
    cond_1_given_23 = [[[joint_pmf[0][0][0]/(joint_pmf[0][0][0]+joint_pmf[1][0][0]),
                        joint_pmf[1][0][0]/(joint_pmf[0][0][0]+joint_pmf[1][0][0])],
                       [joint_pmf[0][0][1]/(joint_pmf[0][0][1]+joint_pmf[1][0][1]),
                        joint_pmf[1][0][1]/(joint_pmf[0][0][1]+joint_pmf[1][0][1])]],
                      [[joint_pmf[0][1][0]/(joint_pmf[0][1][0]+joint_pmf[1][1][0]),
                        joint_pmf[1][1][0]/(joint_pmf[0][1][0]+joint_pmf[1][1][0])],
                       [joint_pmf[0][1][1]/(joint_pmf[0][1][1]+joint_pmf[1][1][1]),
                        joint_pmf[1][1][1]/(joint_pmf[0][1][1]+joint_pmf[1][1][1])]]]

    cond_2_given_13 = [[[joint_pmf[0][0][0]/(joint_pmf[0][0][0]+joint_pmf[0][1][0]),
                        joint_pmf[0][1][0]/(joint_pmf[0][0][0]+joint_pmf[0][1][0])],
                       [joint_pmf[0][0][1]/(joint_pmf[0][0][1]+joint_pmf[0][1][1]),
                        joint_pmf[0][1][1]/(joint_pmf[0][0][1]+joint_pmf[0][1][1])]],
                      [[joint_pmf[1][0][0]/(joint_pmf[1][0][0]+joint_pmf[1][1][0]),
                        joint_pmf[1][1][0]/(joint_pmf[1][0][0]+joint_pmf[1][1][0])],
                       [joint_pmf[1][0][1]/(joint_pmf[1][0][1]+joint_pmf[1][1][1]),
                        joint_pmf[1][1][1]/(joint_pmf[1][0][1]+joint_pmf[1][1][1])]]]

    cond_3_given_12 = [[[joint_pmf[0][0][0]/(joint_pmf[0][0][0]+joint_pmf[0][0][1]),
                        joint_pmf[0][0][1]/(joint_pmf[0][0][0]+joint_pmf[0][0][1])],
                       [joint_pmf[0][1][0]/(joint_pmf[0][1][0]+joint_pmf[0][1][1]),
                        joint_pmf[0][1][1]/(joint_pmf[0][1][0]+joint_pmf[0][1][1])]],
                      [[joint_pmf[1][0][0]/(joint_pmf[1][0][0]+joint_pmf[1][0][1]),
                        joint_pmf[1][0][1]/(joint_pmf[1][0][0]+joint_pmf[1][0][1])],
                       [joint_pmf[1][1][0]/(joint_pmf[1][1][0]+joint_pmf[1][1][1]),
                        joint_pmf[1][1][1]/(joint_pmf[1][1][0]+joint_pmf[1][1][1])]]]


    return cond_1_given_23, cond_2_given_13, cond_3_given_12


cond_1_given_23, cond_2_given_13, cond_3_given_12 = conditional_1_station_given_2(joint_pmf)
```

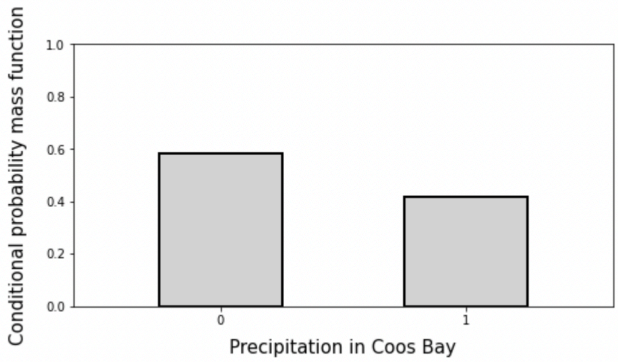
```
[0.9883597883597883, 0.01164021164021164]
[0.8985507246376812, 0.10144927536231885]
[0.9156355455568054, 0.0843644544431946]
[0.8557692307692307, 0.14423076923076922]
[0.9017620082066136, 0.09823799179338642]
[0.6763636363636364, 0.32363636363636367]
[0.539877300613497, 0.4601226993865031]
[0.5833333333333333, 0.4166666666666666]
[0.9757116740663359, 0.024288325933664142]
[0.9014396456256922, 0.09856035437430788]
[0.8073394495412843, 0.1926605504587156]
[0.8333333333333334, 0.16666666666666666]
```

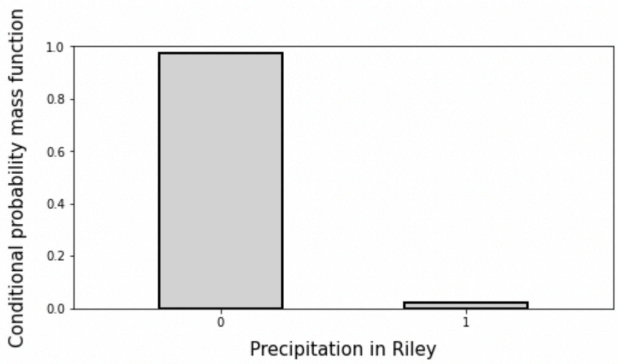


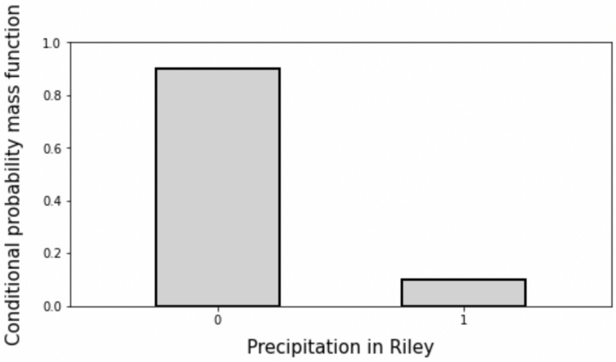

Conditional probability mass function
Precipitation in Bodega

Conditional probability mass function
Precipitation in Bodega

Conditional probability mass function
Precipitation in Bodega

Conditional probability mass function
Precipitation in Coos Bay

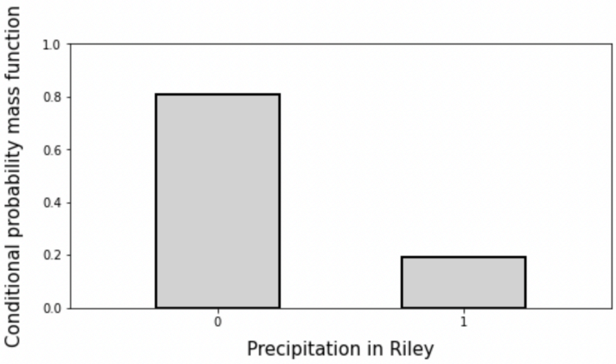

Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Coos Bay

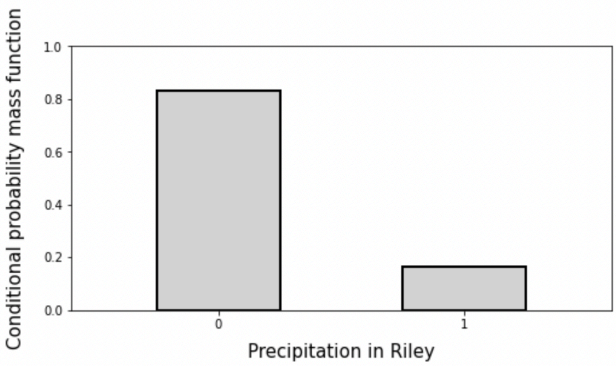

Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Coos Bay


Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Coos Bay


Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Riley


Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Riley


Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Riley


Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Riley

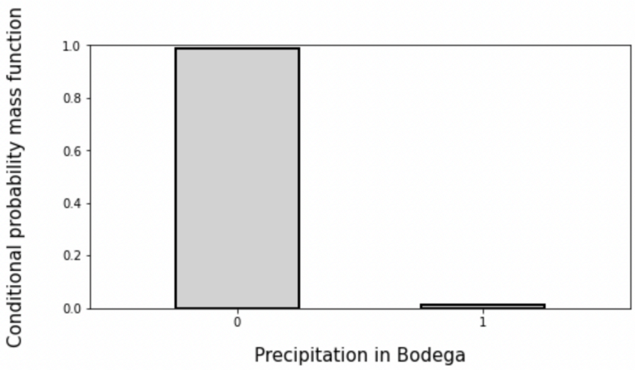
```python
# Compute conditional joint pmf of each pair of the Bernoulli random variables given the other one
def conditional_2_stations_given_1(joint_pmf):
    # INSERT YOUR CODE HERE

    # create sum of each case conditioned by 3,2,1 with condition = 0 or 1
    sum_given_3_0 = joint_pmf[0][0][0] + joint_pmf[0][1][0] + joint_pmf[1][0][0] + joint_pmf[1][1][0]
    sum_given_3_1 = joint_pmf[0][0][1] + joint_pmf[0][1][1] + joint_pmf[1][0][1] + joint_pmf[1][1][1]

    sum_given_2_0 = joint_pmf[0][0][0] + joint_pmf[0][0][1] + joint_pmf[1][0][0] + joint_pmf[1][0][1]
    sum_given_2_1 = joint_pmf[0][1][0] + joint_pmf[0][1][1] + joint_pmf[1][1][0] + joint_pmf[1][1][1]

    sum_given_1_0 = joint_pmf[0][0][0] + joint_pmf[0][0][1] + joint_pmf[0][1][0] + joint_pmf[0][1][1]
    sum_given_1_1 = joint_pmf[1][0][0] + joint_pmf[1][0][1] + joint_pmf[1][1][0] + joint_pmf[1][1][1]


    # for example, given 3 = 0, make a 2X2 matrix with station 1, station 2
    # Also, normailize by given sums that are created on the top
    cond_12_given_3 = np.array([[[joint_pmf[0][0][0]/sum_given_3_0, joint_pmf[0][1][0]/sum_given_3_0],
                                 [joint_pmf[1][0][0]/sum_given_3_0, joint_pmf[1][1][0]/sum_given_3_0]],
                                [[joint_pmf[0][0][1]/sum_given_3_1, joint_pmf[0][1][1]/sum_given_3_1],
                                 [joint_pmf[1][0][1]/sum_given_3_1, joint_pmf[1][0][1]/sum_given_3_1]]])

    cond_13_given_2 = np.array([[[joint_pmf[0][0][0]/sum_given_2_0, joint_pmf[0][0][1]/sum_given_2_0],
                                 [joint_pmf[1][0][0]/sum_given_2_0, joint_pmf[1][0][1]/sum_given_2_0]],
                                [[joint_pmf[0][1][0]/sum_given_2_1, joint_pmf[0][1][1]/sum_given_2_1],
                                 [joint_pmf[1][1][0]/sum_given_2_1, joint_pmf[1][1][1]/sum_given_2_1]]])

    cond_23_given_1 = np.array([[[joint_pmf[0][0][0]/sum_given_1_0, joint_pmf[0][0][1]/sum_given_1_0],
                                 [joint_pmf[0][1][0]/sum_given_1_0, joint_pmf[0][1][1]/sum_given_1_0]],
                                [[joint_pmf[1][0][0]/sum_given_1_1, joint_pmf[1][0][1]/sum_given_1_1],
                                 [joint_pmf[1][1][0]/sum_given_1_1, joint_pmf[1][1][1]/sum_given_1_1]]])


    return cond_12_given_3, cond_13_given_2, cond_23_given_1

cond_12_given_3,cond_13_given_2,cond_23_given_1 = conditional_2_stations_given_1(joint_pmf)
```


```python
    return cond_12_given_3, cond_13_given_2, cond_23_given_1

cond_12_given_3,cond_13_given_2,cond_23_given_1 = conditional_2_stations_given_1(joint_pmf)
```


```python
for ind in range(2):
    print(cond_12_given_3[ind])
print()
for ind in range(2):
    print(cond_13_given_2[ind])
print()
for ind in range(2):
    print(cond_23_given_1[ind])
```

```
[[0.88437 0.09634]
 [0.01042 0.00888]]
[[0.59807 0.28617]
 [0.06752 0.06752]]

[[0.96202 0.02395]
 [0.01133 0.0027 ]]
[[0.81974 0.08963]
 [0.07553 0.01511]]

[[0.8728  0.02173]
 [0.09508 0.0104 ]]
[[0.44221 0.10553]
 [0.37688 0.07538]]
```

```python
# Compute conditional pmf of each Bernoulli random variable given each of the other ones
# (i.e. Bodega just conditioned on Coos Bay, Bodega just conditioned on Riley, etc.)
# Use a dictionary to save the conditional pmfs, for example cond_1["2"] should contain the conditional of the
# first random variable (Bodega)
def conditional_1_station_given_1_station(joint_pmf):
    # INSERT YOUR CODE HERE

    # Initialize result in a dictionary
    cond_1 = {"2": [[0,0],[0,0]], "3":[[0,0],[0,0]]}
    cond_2 = {"1": [[0,0],[0,0]], "3":[[0,0],[0,0]]}
    cond_3 = {"1": [[0,0],[0,0]], "2":[[0,0],[0,0]]}

    # create sum of each cases (to normalize probabilities)
    sum_c1_2_0 = joint_pmf[0][0][0]+joint_pmf[0][0][1]+joint_pmf[1][0][0]+joint_pmf[1][0][1]
    sum_c1_2_1 = joint_pmf[0][1][0]+joint_pmf[0][1][1]+joint_pmf[1][1][0]+joint_pmf[1][1][1]
    sum_c1_3_0 = joint_pmf[0][0][0]+joint_pmf[0][1][0]+joint_pmf[1][0][0]+joint_pmf[1][1][0]
    sum_c1_3_1 = joint_pmf[0][0][1]+joint_pmf[0][1][1]+joint_pmf[1][0][1]+joint_pmf[1][1][1]

    sum_c2_1_0 = joint_pmf[0][0][0]+joint_pmf[0][0][1]+joint_pmf[0][1][0]+joint_pmf[0][1][1]
    sum_c2_1_1 = joint_pmf[1][0][0]+joint_pmf[1][0][1]+joint_pmf[1][1][0]+joint_pmf[1][1][1]
    sum_c2_3_0 = joint_pmf[0][0][0]+joint_pmf[1][0][0]+joint_pmf[0][1][0]+joint_pmf[1][1][0]
    sum_c2_3_1 = joint_pmf[0][0][1]+joint_pmf[1][0][1]+joint_pmf[0][1][1]+joint_pmf[1][1][1]

    sum_c3_1_0 = joint_pmf[0][0][0]+joint_pmf[0][1][0]+joint_pmf[0][0][1]+joint_pmf[0][1][1]
    sum_c3_1_1 = joint_pmf[1][0][0]+joint_pmf[1][1][0]+joint_pmf[1][0][1]+joint_pmf[1][1][1]
    sum_c3_2_0 = joint_pmf[0][0][0]+joint_pmf[1][0][0]+joint_pmf[0][0][1]+joint_pmf[1][0][1]
    sum_c3_2_1 = joint_pmf[0][1][0]+joint_pmf[1][1][0]+joint_pmf[0][1][1]+joint_pmf[1][1][1]


    # for example, this means that for given station2 = 0, care about station1 = 0 or 1 for every station3
    # continue by given station2 = 1, care about station1 = 0 or 1 for every station3
    # continue this logic to every other cases
    cond_1["2"]=[[(joint_pmf[0][0][0]+joint_pmf[0][0][1])/sum_c1_2_0,
                  (joint_pmf[1][0][0]+joint_pmf[1][0][1])/sum_c1_2_0],
                 [(joint_pmf[0][1][0]+joint_pmf[0][1][1])/sum_c1_2_1,
                  (joint_pmf[1][1][0]+joint_pmf[1][1][1])/sum_c1_2_1]]

    cond_1["3"]=[[(joint_pmf[0][0][0]+joint_pmf[0][1][0])/sum_c1_3_0,
                  (joint_pmf[1][0][0]+joint_pmf[1][1][0])/sum_c1_3_0],
                 [(joint_pmf[0][0][1]+joint_pmf[0][1][1])/sum_c1_3_1,
                  (joint_pmf[1][0][1]+joint_pmf[1][1][1])/sum_c1_3_1]]

    cond_2["1"]=[[(joint_pmf[0][0][0]+joint_pmf[0][0][1])/sum_c2_1_0,
                  (joint_pmf[0][1][0]+joint_pmf[0][1][1])/sum_c2_1_0],
                 [(joint_pmf[1][0][0]+joint_pmf[1][0][1])/sum_c2_1_1,
                  (joint_pmf[1][1][0]+joint_pmf[1][1][1])/sum_c2_1_1]]

    cond_2["3"]=[[(joint_pmf[0][0][0]+joint_pmf[1][0][0])/sum_c2_3_0,
                  (joint_pmf[0][1][0]+joint_pmf[1][1][0])/sum_c2_3_0],
                 [(joint_pmf[0][0][1]+joint_pmf[1][0][1])/sum_c2_3_1,
                  (joint_pmf[0][1][1]+joint_pmf[1][1][1])/sum_c2_3_1]]

    cond_3["1"]=[[(joint_pmf[0][0][0]+joint_pmf[0][1][0])/sum_c3_1_0,
                  (joint_pmf[0][0][1]+joint_pmf[0][1][1])/sum_c3_1_0],
                 [(joint_pmf[1][0][0]+joint_pmf[1][1][0])/sum_c3_1_1,
                  (joint_pmf[1][0][1]+joint_pmf[1][1][1])/sum_c3_1_1]]

    cond_3["2"]=[[(joint_pmf[0][0][0]+joint_pmf[1][0][0])/sum_c3_2_0,
                  (joint_pmf[0][0][1]+joint_pmf[1][0][1])/sum_c3_2_0],
                 [(joint_pmf[0][1][0]+joint_pmf[1][1][0])/sum_c3_2_1,
                  (joint_pmf[0][1][1]+joint_pmf[1][1][1])/sum_c3_2_1]]

    return cond_1, cond_2, cond_3


cond_1,cond_2,cond_3 = conditional_1_station_given_1_station(joint_pmf)
```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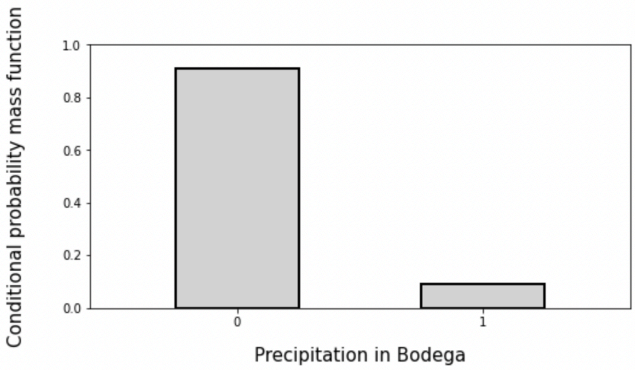
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Bodega


Conditional probability mass function
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0.8
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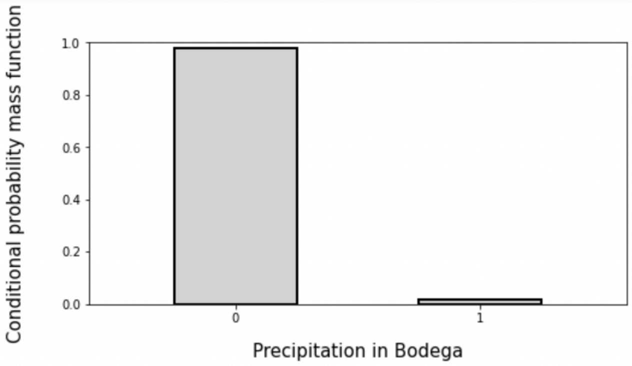
Precipitation in Bodega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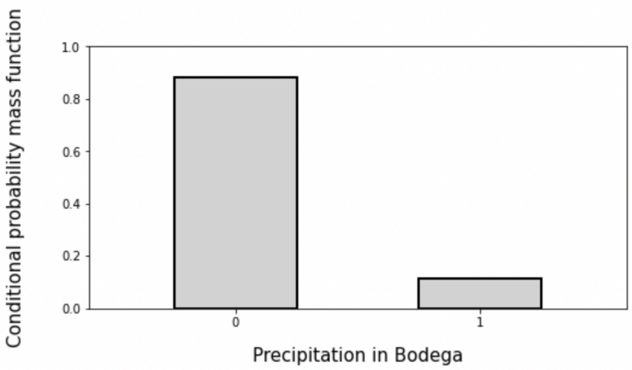
Precipitation in Bodega


Conditional probability mass function
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0.6
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0.2
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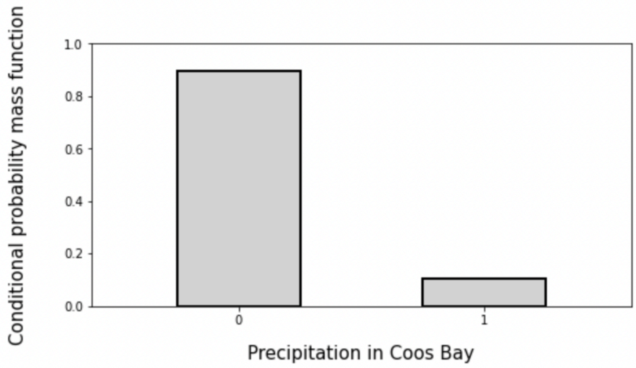
1
Precipitation in Bodega


Conditional probability mass function
1.0
0.8
0.6
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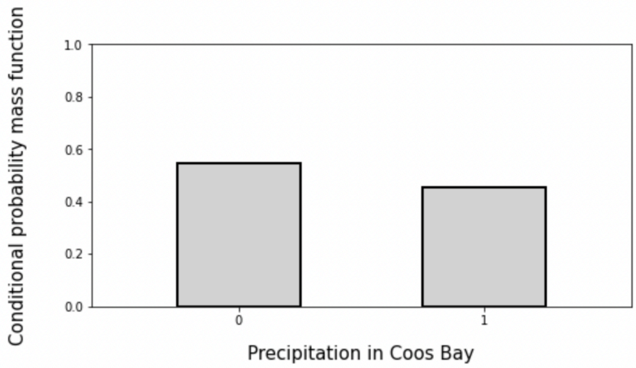
0.2
0.0
0
1
Precipitation in Coos Bay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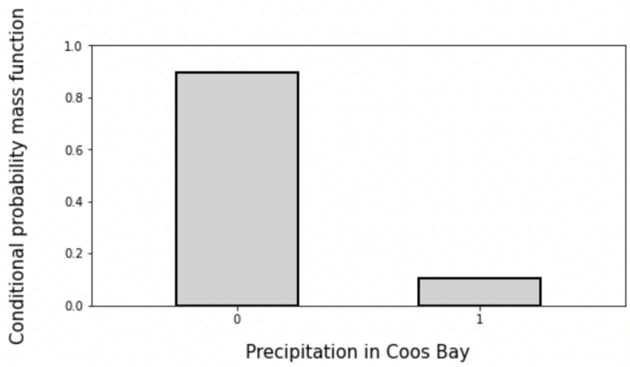
Precipitation in Coos Bay


Conditional probability mass function
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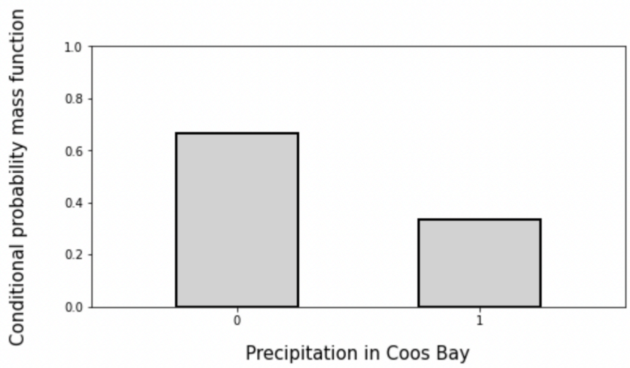
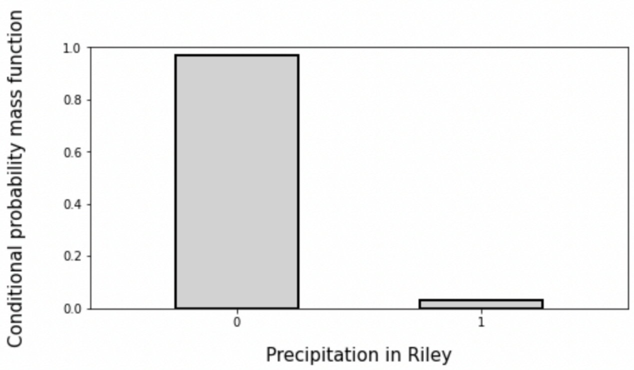
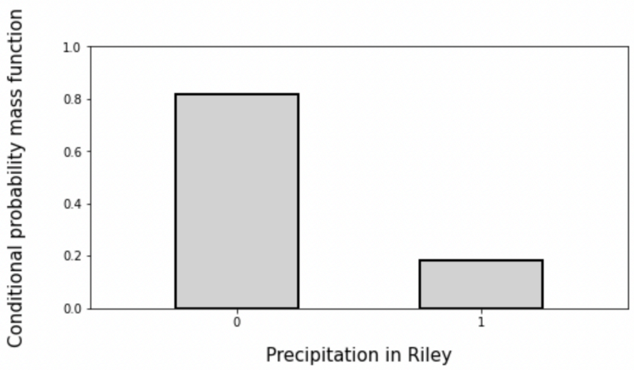
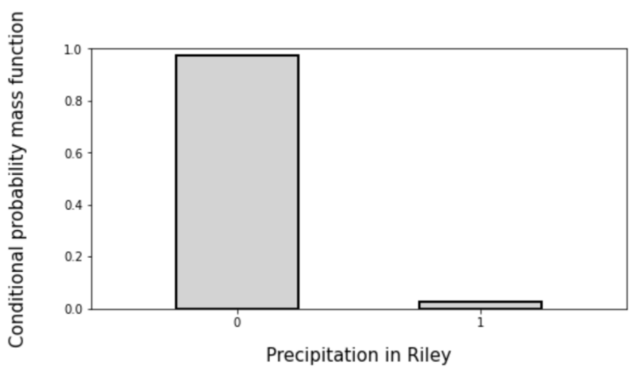
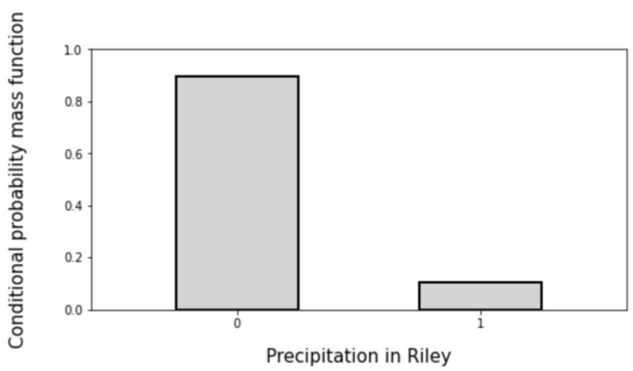
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Coos Bay


Conditional probability mass function
1.0
0.8
0.6
0.4
0.2
0.0
0
1
Precipitation in Coos Bay


Conditional probability mass function
Precipitation in Riley


Conditional probability mass function
Precipitation in Riley


Conditional probability mass function
Precipitation in Riley


Conditional probability mass function
Precipitation in Riley