Forecasting Time Series Homework 9

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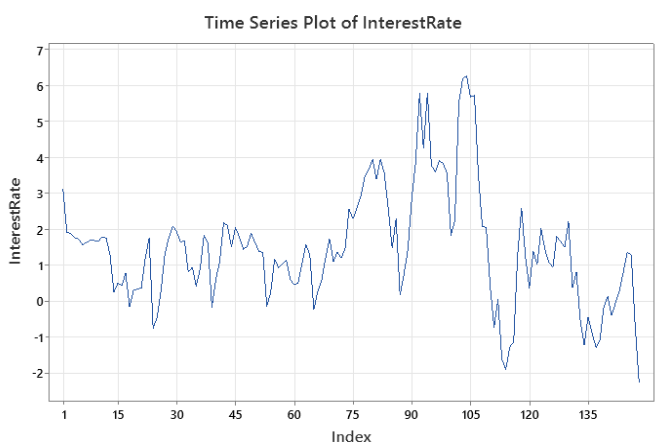
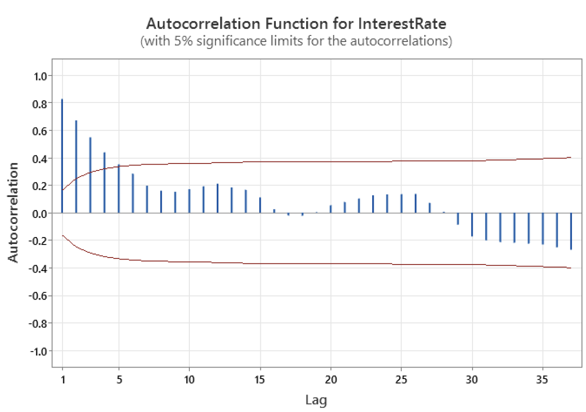
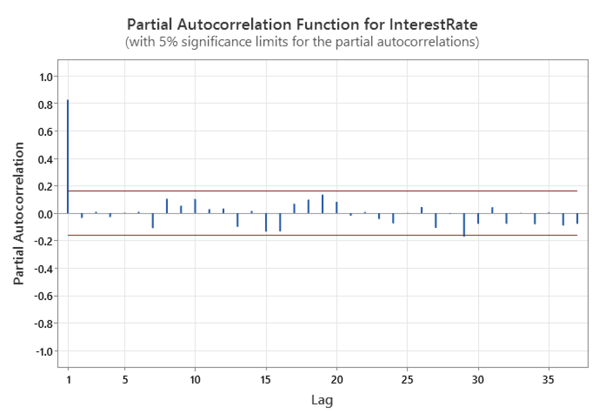
In these problems, we will consider the spread (long term minus short term interest rates) between 20 year UK gilts and 91 day UK Treasury Bills for the period 1952, First Quarter, to 1988 Fourth Quarter. ($n = 148$). You need to create this data set by taking the difference between the contents of the files Long.M and Short.M which are stored in /class/churvich.

1)

Plot the UK interest rate spread. Does the series appear to be stationary? Based on the ACF and the PACF of the raw data, explain why an $AR(1)$ model might be reasonable.







The series doesn't appear to be stationary. The ACF is showing a decreasing pattern, with the correlation at lag 1 being the largest and the correlations at higher lags diminishing as the lag increases. The PACF, on the other hand, shows a sharp cutoff after lag 1, with no significant correlations at higher lags. Therefore, the AR(1) model might be reasonable.

2)

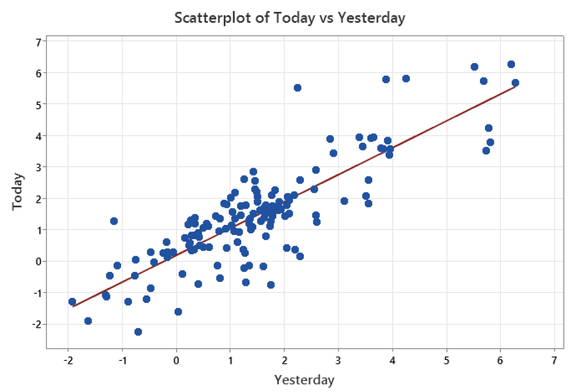
Estimate an $AR(1)$ model by a least-squares regression of $(x_2, \ldots, x_n)$ on $(x_1, \ldots, x_{n-1})$, together with a constant term. Using the standard error from the regression output, and assuming that $\hat{\rho}$ is normally distributed, calculate the p-value for the hypothesis test of $H_0: \rho = 1$ versus $H_1: \rho < 1$, where ρ is the true $AR(1)$ parameter. Does this p-value provide strong evidence against the random walk hypothesis? Is $\hat{\rho}$ significantly less than 1 at level .01?

Regression Equation

Today = 0.1762 + 0.8577 Yesterday

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.1762	0.0982	1.79	0.075	
Yesterday	0.8577	0.0451	19.01	0.000	1.00



Descriptive Statistics

N	Mean	StDev	SE Mean	99% Upper Bound for μ
147	0.0002	1.0043	0.0828	0.1950

μ: population mean of SRES

Test

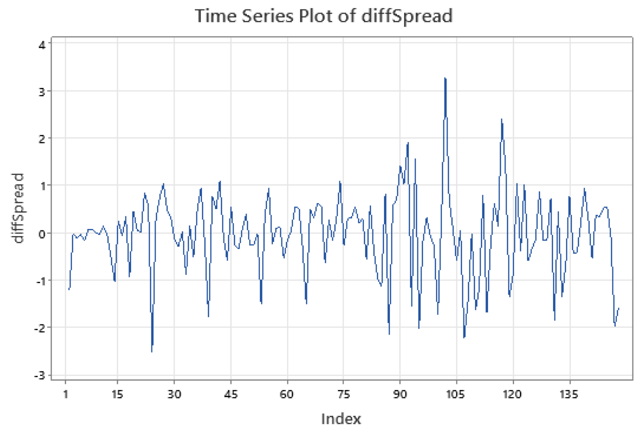
Null hypothesis $H_0: \mu = 1$
Alternative hypothesis $H_1: \mu < 1$

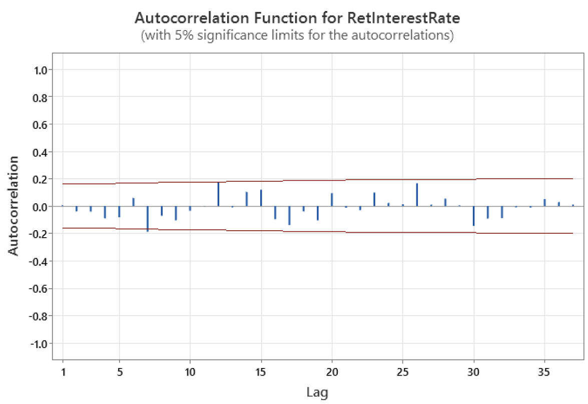
T-Value	P-Value
-12.07	0.000

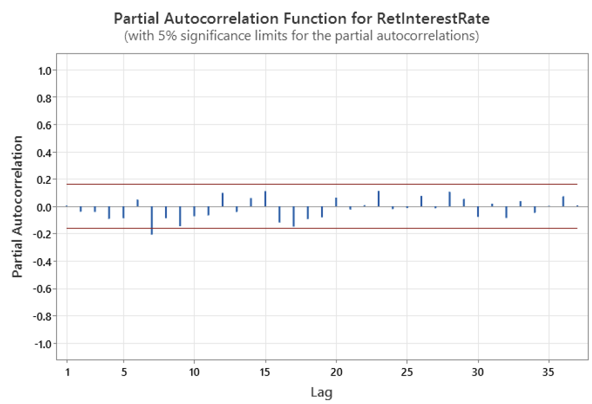
Since the p-value is near 0 for the hypothesis test above, we reject the null hypothesis and conclude that the estimated ρ is significantly less than 1 at level 0.01. This p-value provides strong evidence against the random walk hypothesis, as random walk can be defined as $x_t = \rho x_{t-1} + \varepsilon$, where $\rho = 1$, but in this case, we have ρ which is significantly less than 1 at level 0.01.
Therefore, its AR(1) model is not a random walk, and the p-value provides strong evidence against the random walk hypothesis.

3)

Based on the ACF, PACF, sample mean and sample standard deviation of the differenced data, argue that a random walk without drift might also provide a reasonable model for the UK interest rate spread.







The ACF and PACF of the differenced data do not have any significant cut-off or die-down points. The plots look stationary since they do not explode but only oscillate between some values in which the variance does not change over time. The ACF and PACF plots for the differenced data look like white noise, and therefore stationary, which indicates that a random walk might also provide a reasonable model.

Additionally, we've analyzed sample mean and sample standard deviation to check if random walk without drift might provide a reasonable model for the UK interest rate spread.
The sample mean is -0.0367 and standard deviation of the returns is 0.8915.
The p-value is 0.619, which is greater than alpha = 0.01 (or 0.05), therefore at the significance level of 0.01 (or 0.05), we failed to reject the null hypothesis. There is no strong evidence that the mean of return is significantly different from zero.

Therefore, random walk without drift might provide a reasonable model for the UK interest rate spread.

[95% confidence level]

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
147	-0.0367	0.8915	0.0735	(-0.1820, 0.1087)

μ: population mean of diffSpread

Test

Null hypothesis	H_0: μ = 0
Alternative hypothesis	H_1: μ ≠ 0

T-Value	P-Value
-0.50	0.619

[99% confidence level]

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
147	-0.0367	0.8915	0.0735	(-0.2286, 0.1552)

μ: population mean of diffSpread

Test

Null hypothesis	H_0: μ = 0
Alternative hypothesis	H_1: μ ≠ 0

T-Value	P-Value
-0.50	0.619

4)

Perform the Dickey-Fuller test (τ_μ) of driftless random walk versus stationary $AR(1)$. Compute the approximate p-value for the $AR(1)$ hypothesis, based on the Dickey-Fuller table. Is $\hat{\rho}$ significantly less than 1 at level .01? Accordingly, based on a hypothesis test at level .01, decide whether the UK interest rates are driftless random walk or stationary $AR(1)$. This conclusion is different from the one you arrived at in Problem 1 (if you have not made a mistake). What is the reason for the difference? Which conclusion is more justifiable, on statistical grounds? If you used a significance level of .05, what would you conclude from the Dickey-Fuller test?

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Table 8.5.2. Empirical cumulative distribution of $\hat{\tau}$ for $\rho = 1$

Sample Size n	Probability of a Smaller Value: 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
				$\hat{\tau}$				
25	−2.66	−2.26	−1.95	−1.60	0.92	1.33	1.70	2.16
50	−2.62	−2.25	−1.95	−1.61	0.91	1.31	1.66	2.08
100	−2.60	−2.24	−1.95	−1.61	0.90	1.29	1.64	2.03
250	−2.58	−2.23	−1.95	−1.62	0.89	1.29	1.63	2.01
500	−2.58	−2.23	−1.95	−1.62	0.89	1.28	1.62	2.00
∞	−2.58	−2.23	−1.95	−1.62	0.89	1.28	1.62	2.00
				$\hat{\tau}_\mu$				
25	−3.75	−3.33	−3.00	−2.63	−0.37	0.00	0.34	0.72
50	−3.58	−3.22	−2.93	−2.60	−0.40	−0.03	0.29	0.66
100	−3.51	−3.17	−2.89	−2.58	−0.42	−0.05	0.26	0.63
250	−3.46	−3.14	−2.88	−2.57	−0.42	−0.06	0.24	0.62
500	−3.44	−3.13	−2.87	−2.57	−0.43	−0.07	0.24	0.61
∞	−3.43	−3.12	−2.86	−2.57	−0.44	−0.07	0.23	0.60
				$\hat{\tau}_\tau$				
25	−4.38	−3.95	−3.60	−3.24	−1.14	−0.80	−0.50	−0.15
50	−4.15	−3.80	−3.50	−3.18	−1.19	−0.87	−0.58	−0.24
100	−4.04	−3.73	−3.45	−3.15	−1.22	−0.90	−0.62	−0.28
250	−3.99	−3.69	−3.43	−3.13	−1.23	−0.92	−0.64	−0.31
500	−3.98	−3.68	−3.42	−3.13	−1.24	−0.93	−0.65	−0.32
∞	−3.96	−3.66	−3.41	−3.12	−1.25	−0.94	−0.66	−0.33

This table was constructed by David A. Dickey using the Monte Carlo method. Details are given in Dickey (1975). Standard errors of the estimates vary, but most are less than 0.02.

To extend the results for the first order process with $\rho = 1$ to the pth order autoregressive process, we consider the time series

Regression Equation

Today = 0.1762 + 0.8577 Yesterday

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.1762	0.0982	1.79	0.075	
Yesterday	0.8577	0.0451	19.01	0.000	1.00

We start from using an ordinary regression model, and get Coefficient (ρ) of 0.8577 and SE Coefficient of 0.0451. Also, we get the critical value (p-value) of -3.51 from the Dicky-Fuller.

To identify if ρ is significantly less than 1 at level 0.01, we calculate $\tau_{\mu} = \frac{0.8577 - 1}{0.0451} = -3.15521064302$ and compare the result with the Dicky-Fuller tabled value. As n=148, $\tau_{\mu} = -3.155 > -3.51$, and therefore we fail to reject the null hypothesis test. So, ρ is not significantly less than 1, and by this, UK interest rates are a driftless random walk, which is different from the answer from Problem 1.

The reason for the difference is that, Dicky-Fuller table provides a more strict table to compare τ_{μ}. For example, if we use a standard normal table, the value to be compared would be around -2.59, and we can say that $\tau_{\mu} = -3.155 < -2.59$, and ρ is significantly less than 1.

On statistical grounds, the Dicky-Fuller test is more reasonable, as it provides a more strict standard to the analysis. Also, actual data distributions may not follow the assumption of normality. If we use the standard normal, then it might have too many incorrect rejections of null hypothesis.

Finally, if we used 0.05-standard, the value to be compared would be around -2.89, so $\tau_{\mu} = -3.155 < -2.89$. Therefore, we reject the null and conclude that ρ is significantly less than 1. In other words, at 0.05-standard, we conclude that UK interest rates are not driftless random walk but stationary AR(1).