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1. Given $\tilde{a}$ a geometric random variable with parameter α,
geometric distribution is : $P_{\tilde{a}}(a) = (1-\alpha)^{a-1}\alpha^1$
if we assume a_1 = $\tilde{a}$ equals a for $a = 1,2,3,\dots$,
and a_2 = $\tilde{a}$ equals a for $a > 5$,

the probability should be $P_{\tilde{a}}(a_1) / P_{\tilde{a}}(a_2) = \dfrac{P_{\tilde{a}}(a_1 \cap a_2)}{P_{\tilde{a}}(a_2)}$

if) $a = 1,2,3,4,5$ then $P_{\tilde{a}}(a_1 \cap a_2) = 0$,
Since there will be no intersection of value a.

if) $a > 5$, then $P_{\tilde{a}}(a_1 \cap a_2) = P_{\tilde{a}}(a_1)$
Since every a_1 will be an intersection of a_2

$P_{\tilde{a}}(a_1) = (1-\alpha)^{(a_1-1)} \cdot \alpha^1$.

$$
\begin{aligned}
P_{\tilde{a}}(a_2) = P_{\tilde{a}}(a>5) &= 1 - \left(P_{\tilde{a}}(1) + P_{\tilde{a}}(2) + P_{\tilde{a}}(3) + P_{\tilde{a}}(4) + P_{\tilde{a}}(5)\right) \\
&= 1 - (\alpha + (1-\alpha)\cdot\alpha + (1-\alpha)^2\alpha + (1-\alpha)^3\alpha + (1-\alpha)^4\alpha) \\
&= (1 - \alpha - (1-\alpha)\alpha - (1-\alpha)^2\alpha - (1-\alpha)^3\alpha - (1-\alpha)^4\alpha) \\
&= (1-\alpha)(1 - \alpha - (1-\alpha)\alpha - (1-\alpha)^2\alpha - (1-\alpha)^3\alpha) \\
&= (1-\alpha)^2(1 - \alpha - (1-\alpha)\alpha - (1-\alpha)^2\alpha) \\
&= (1-\alpha)^3(1 - \alpha - (1-\alpha)\alpha) \\
&= (1-\alpha)^4(1-\alpha) \\
&= (1-\alpha)^5
\end{aligned}
$$

$\therefore$ if $a \le 5$, $P_{\tilde{a}}(a_1)/P_{\tilde{a}}(a_2) = \dfrac{P_{\tilde{a}}(a_1 \cap a_2)}{P_{\tilde{a}}(a_2)} = 0$

$a > 5$, $P_{\tilde{a}}(a_1)/P_{\tilde{a}}(a_2) = \dfrac{P_{\tilde{a}}(a_1 \cap a_2)}{P_{\tilde{a}}(a_2)} = \dfrac{P_{\tilde{a}}(a_1)}{P_{\tilde{a}}(a_2)} = \dfrac{(1-\alpha)^{(a_1-1)} \cdot \alpha^1}{(1-\alpha)^5} = \boxed{(1-\alpha)^{(a_1-6)} \cdot \alpha^1}$

2. (a) $P(\theta, a) = \frac{10!}{4!4!2!}\theta^4 a^2 (1-\theta-\alpha)^4$

$$\log P(\theta, a) = \log\frac{10!}{4!4!2!} + 4\log(\theta) + 2\log(a) + 4\log(1-\theta-a)$$

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(b) By taking the derivative of both θ and a, we can calulate θ, a.

~~Let~~ 1) $\frac{d}{d\theta}\log P(\theta, a) = 0 + \frac{4}{\theta} + 0 + \frac{4\times(-1)}{(1-\theta-a)} = 0$

$\Rightarrow \frac{4}{\theta} = \frac{4}{(1-\theta-a)}, \quad \theta = (1-\theta-a) \quad \therefore \theta = \frac{1-a}{2}$

2) $\frac{d}{da}\log P(\theta, a) = 0 + 0 + \frac{2}{a} + \frac{4(-1)}{(1-\theta-a)} = 0, \quad 2a = 1-\theta-a \quad \therefore \theta = 1-3a$

Since $\theta = \frac{1-a}{2} = 1-3a$, $1-a = 2-6a$, $5a = 1$, $\boxed{a = \frac{1}{5}}$, $\boxed{\theta = \frac{1-\frac{1}{5}}{2} = \frac{2}{5}}$

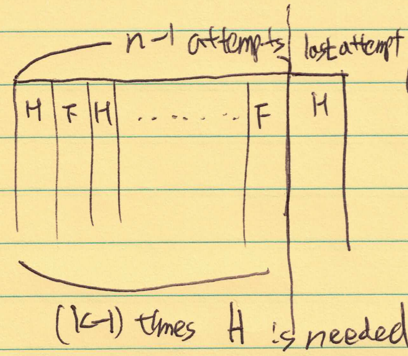
(c) $P(\text{Gorry wins}) = \frac{4}{10} = 2/5$

$P(\text{Anish wins}) = \frac{2}{10} = 1/5$

$P(\text{draw}) = \frac{4}{10} = 2/5.$

from 10 chess games.
This result is same as (b),
which makes this nonparametric model
same as parametric model

3. If a player needs total n attempts and hits a certain number k times, last attempt should be hitting a certain number.



(H : Hits a certain number)

F : fails to hit a certain number)

So, for $(n-1)$ attempts, we have $(k-1)$ Hits and $(n-1)-(k-1)$ none hits in binomial distribution.

Assuming the probability of success in each attempt as θ, and number of required attempts k, pmf should be as below

pmf) $\underline{{}_{n-1}C_{k-1} \cdot (\theta)^{k-1} \cdot (1-\theta)^{(n-1)-(k-1)}} \cdot \theta$

$p(\text{last attempt} = \theta)$

$$\boxed{= {}_{n-1}C_{k-1} (\theta)^{k} \cdot (1-\theta)^{n-k}}$$