Forecasting Time Series Homework 6

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The data sets chaos1 and chaos2 (available on the course website) were generated with $n = 50$ by iterating the "tent map",

$$f(x) = \begin{cases} x/.6 & \text{if } 0 \le x \le .6 \\ (1-x)/.4 & \text{if } .6 < x \le 1 \end{cases} .$$

We used $x_0 = .5$ for chaos1 and $x_0 = .501$ for chaos2.

1) Check that $x_1 = f(x_0)$ where $\{x_t\}$ is the series of observations on chaos1, and f is the function defined above.

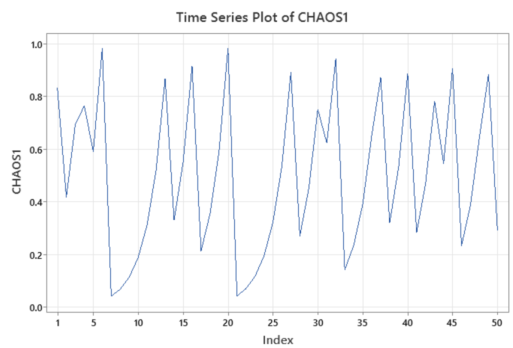
1)

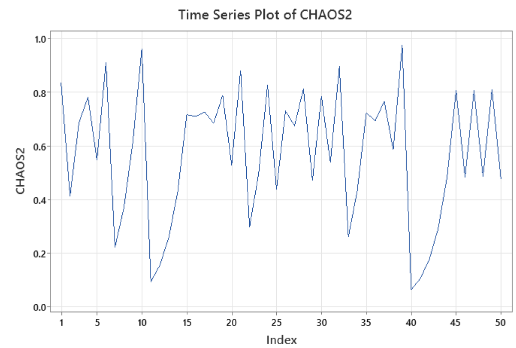
$x_1 = 0.8333$

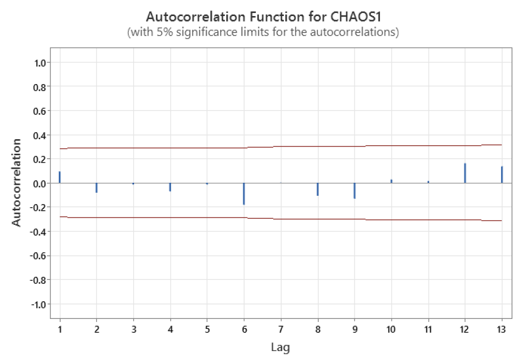
$f(x_0) = f(0.5) = 0.5 / 0.6 = 0.8333$

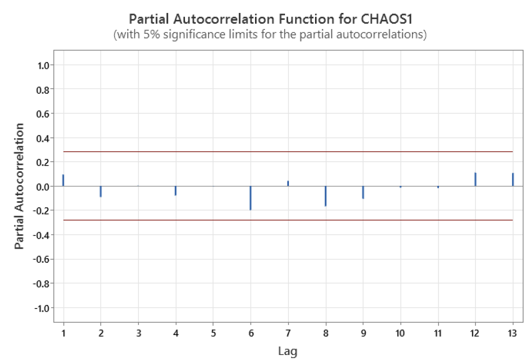
$\therefore x_1 = f(x_0)$

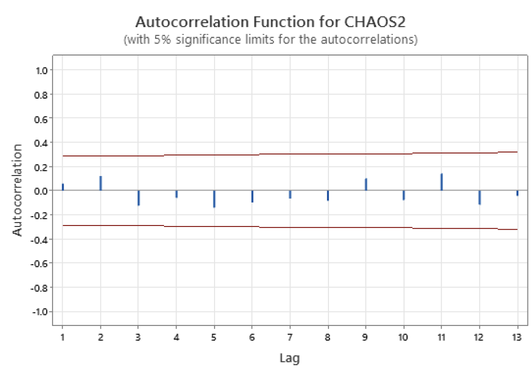
2) Plot chaos1 and chaos2, in separate plots. Do the series look random? Are they in fact random? Do the series look stationary?

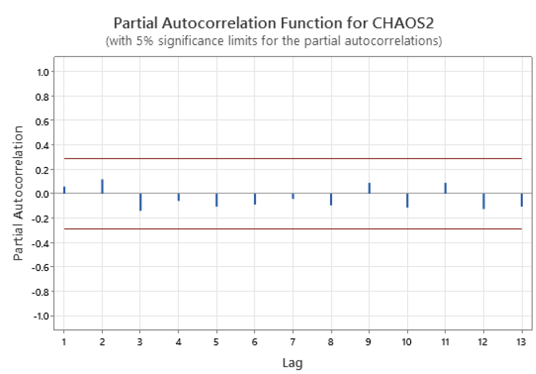










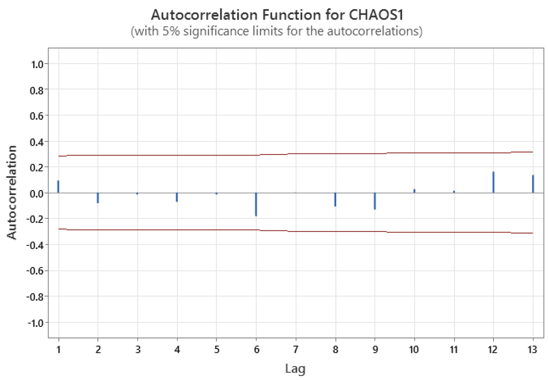


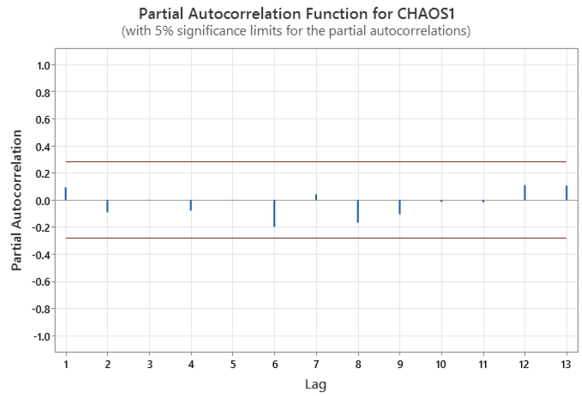
Both time series plots look random since they do not have any repeated periodic patterns.

In fact, the plots are not random because the series is deterministic calculated from the given function, so we can predict x_{t+1} from x_t using the defined function, f.

The plots look stationary since they do not explode but only oscillate between some values in which the variance does not change over time. To see more precisely, we checked ACF and PACF of both CHAOS1 and CHAOS2. As you can see, ACF and PACF are not statistically significant at any lag points. Therefore, we can say that CHAOS1 and CHAOS2 are both white noise, and therefore stationary.

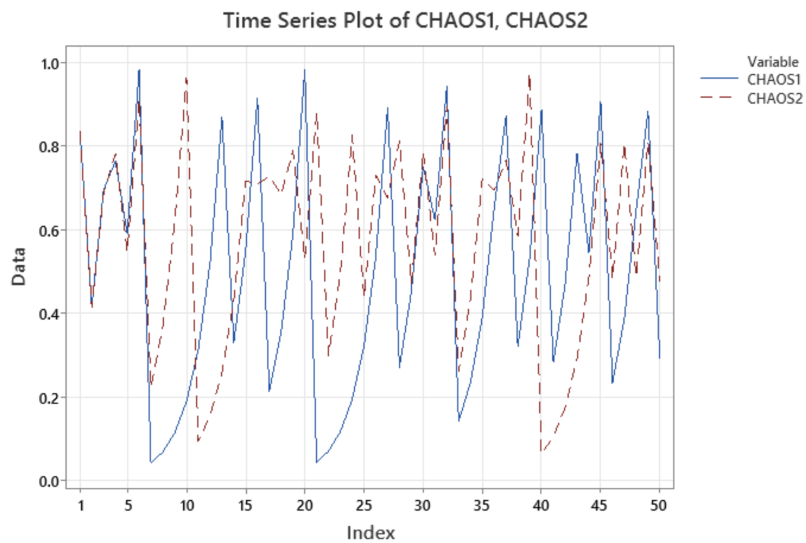
3) Plot the ACF and PACF for chaos1. Based on these, suggest an ARMA model. Would this model provide the best possible forecasts?





The ACF and PACF do not have any significant cut-off or die-down points, the CHAOS 1 seems to be white noise. Therefore, we can suggest a model of ARMA(0, 0). Since the series is deterministic which means the best forecast would be granted by the given function, the ARMA(0, 0) model does not provide the best possible forecasts.

4) Plot both chaos1 and chaos2 on the same plot. Do the paths look similar? Should they look similar when t is close to 1? Why? What should happen if chaos1 and chaos2 happen to get very close together at some later time? Use the plot to help justify your answer.

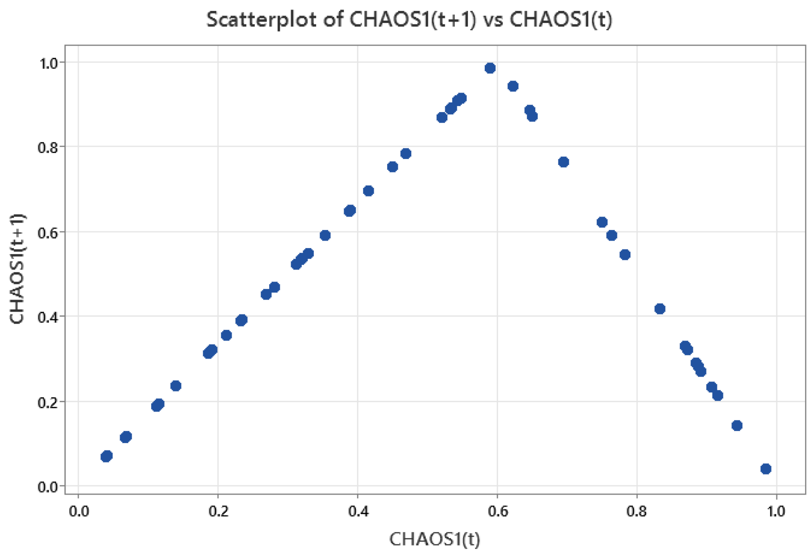


The paths for CHAOS 1 and CHAOS 2 look similar in the beginning but get deviated as t increases. However, both plots still oscillate between 0.0 and 1.0 with the similar variance.

When t is close to 1, which is the beginning of the series, their plots look very similar. (When $1 \leq t \leq 5$, plots almost overlap each other). They should look similar when t is close to 1, because they have initial points that are close in value ($x_0 = .5$ in CHAOS1, $x_0 = .501$ in CHAOS2)

Even though CHAOS1 is very close to CHAOS2 in the beginning, the plots become different at later times as shown in the plot above. If CHAOS1 and CHAOS2 happen to get very close together, even closer than the current, then the region that the plots overlap might get extended but at some later time, the plots get different eventually, no matter how close the beginning points are.
Also, even if the process is not random and f is known, even a small uncertainty in x can cause the future predicted value to become less and less predictable because error in x_t will be magnified with each iteration of f, meaning that the future value will eventually get dominated by the error. This phenomenon is called sensitive dependence on initial conditions (or the butterfly effect), and is a hallmark of chaos.

5) Plot $x_2, \ldots, x_{50}$ versus $x_1, \ldots, x_{49}$, where $\{x_t\}$ is the series of observations on chaos1. Does this reveal the map (in other words, the function f) which generated the data? Do you see why this f is called the tent map? Does this plot help us to see that $\{x_t\}$ is not an $AR(1)$ series? How?



The scatter plot above reveals the map as defined by the function *f* above, which is used to generate the data.

Based on the scatter plot, it looks like a tent, so this *f* is called a tent map.

This plot helps us to see that $\{x_t\}$ is not an AR(1) series since the plot does not show a single linear trend. It shows a positive linear line where $0 \leq t \leq 0.6$ and changes to a negative linear line where $0.6 \leq t \leq 1$.