Forecasting Time Series Homework 8

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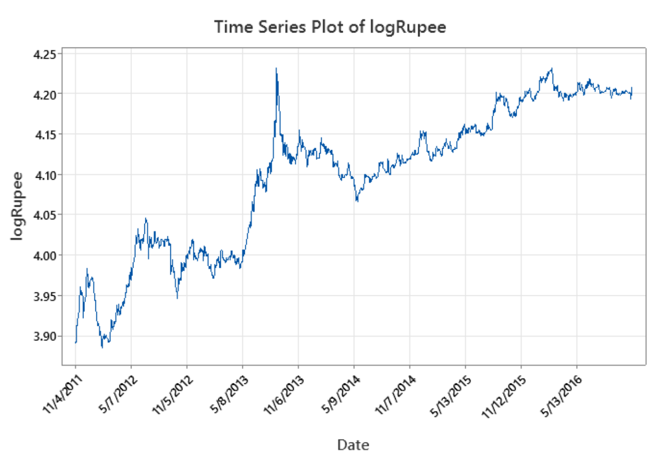
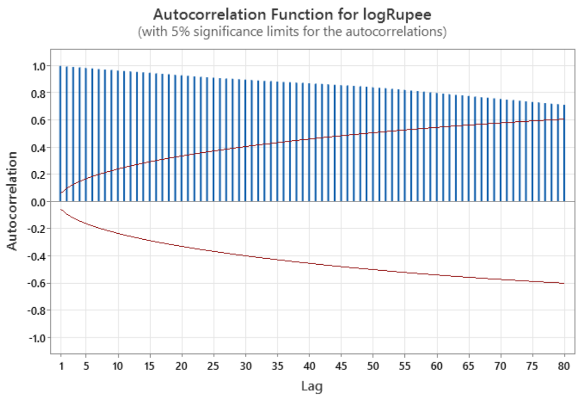
FORECASTING HOMEWORK 8

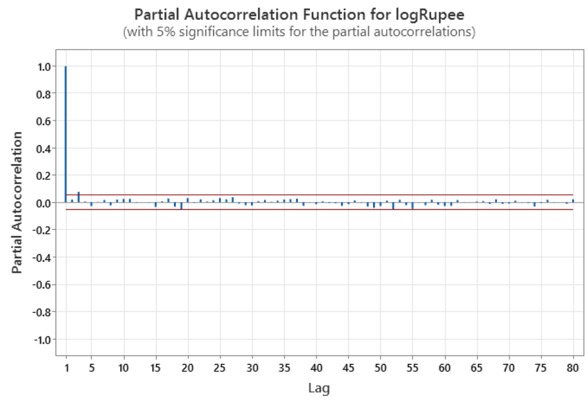
In these problems, we will consider Rupee, the exchange rate for the Indian Rupee to 1 U.S. Dollar. The data is daily from November 4th, 2011 to November 10th, 2016. (n=1259). We will work with the logs of the exchange rates. We will be fitting ARIMA-ARCH models to the data. To do this we will use R. You can use R on apps.stern.nyu.edu or download it from http://cran.r-project.org/. See the handout on using R (Windows and Mac versions), which has some basic facts about R and also details on transferring files between R and Minitab. Examples of fitting ARCH models in R are given in the handout on Estimation and Automatic Selection of ARCH Models.

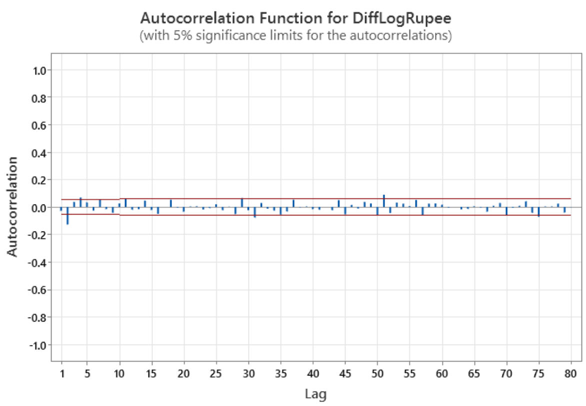
1)

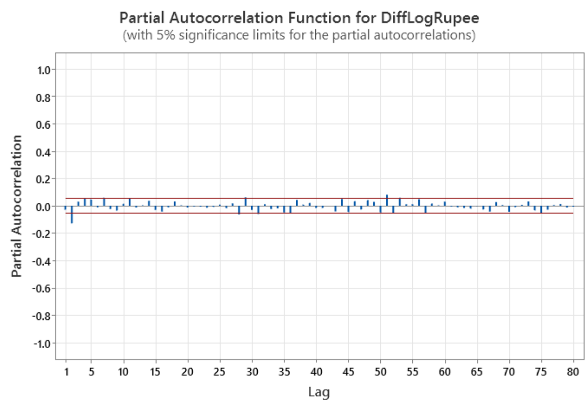
Plot the logs of Rupee. Based on this plot, and the ACF and PACF of the logs and differenced logs, does the series appear to be stationary? Can you identify an $ARIMA(p\ ,d\ ,q)$ model from these plots?











Based on the log Rupee time series plot, the series of log Rupee is not stationary. Based on ACF and PACF of log Rupee, we can see that PACF is showing a significant cut off after lag 1, and ACF of log Rupee dies down. Also, ACF and PACF of difference of log Rupee do not show any statistically significant lags. Therefore, we can identify ARIMA (1,1,0).

2)

Using AIC_C, select an $ARIMA(p, 1, q)$ (without constant) with $0 \le p \le 2$, $0 \le q \le 2$. Write the complete form of the fitted model. Save the residuals and fitted values for the model you selected, using Storage → Residuals, Fits. The residuals will be stored in RESI1 and the fitted values will be stored in FITS1. (Note that FITS1 starts with one missing value, while at time t it represents $f_{t-1,1}$, the one-step forecast for the log exchange rate at time t made from time $t-1$). Also, get Minitab to compute the (ARIMA) one step ahead forecast and 95% forecast interval.

Based on AICc scores, we've selected ARIMA (2,1,2) without constant as our best model. The complete form of the fitted model is as below:

Ans) $x_t = -0.133x_{t-1} - 0.491x_{t-2} + \varepsilon_t + 0.107\varepsilon_{t-1} + 0.366\varepsilon_{t-2}$, *where* x_t *= diff(log Rupee)*

Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
AR 1	-0.133	0.162	-0.82	0.412
AR 2	-0.491	0.161	-3.05	0.002
MA 1	-0.107	0.173	-0.62	0.537
MA 2	-0.366	0.173	-2.12	0.034

Differencing: 1 Regular
Number of observations after differencing: 1258

Model Selection

Model (d = 1)	LogLikelihood	AICc	AIC	BIC
p = 2, q = 2*	4750.69	-9491.34	-9491.38	-9465.70
p = 2, q = 0	4748.58	-9491.14	-9491.16	-9475.75
p = 2, q = 1	4748.93	-9489.83	-9489.87	-9469.32
p = 0, q = 2	4747.17	-9488.31	-9488.33	-9472.92
p = 1, q = 2	4748.05	-9488.07	-9488.10	-9467.55
p = 1, q = 1	4740.38	-9474.74	-9474.76	-9459.34
p = 0, q = 0	4737.95	-9473.90	-9473.90	-9468.77
p = 0, q = 1	4738.52	-9473.04	-9473.05	-9462.77
p = 1, q = 0	4738.38	-9472.75	-9472.76	-9462.49

** Best model with minimum AICc. Output for the best model follows.*

Forecasts from Time Period 1259

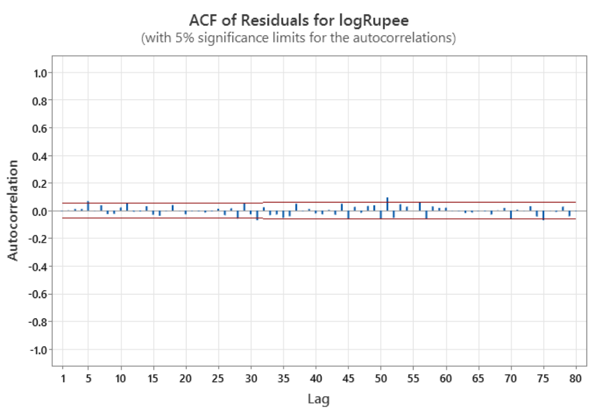
Time Period	Forecast	SE Forecast	95% Limits Lower	95% Limits Upper	Actual
1260	4.20725	0.0055510	4.19637	4.21813	

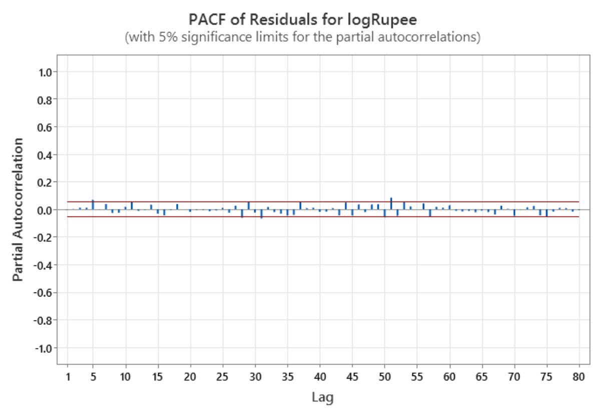
3)

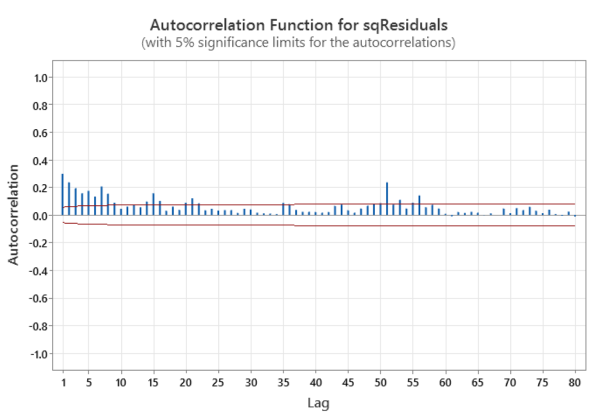
Plot the residuals, as well as ACF and PACF of both the residuals and the squared residuals. Use these plots to argue that the residuals, although approximately uncorrelated, are not independent; instead, they show evidence of conditional heteroscedasticity.

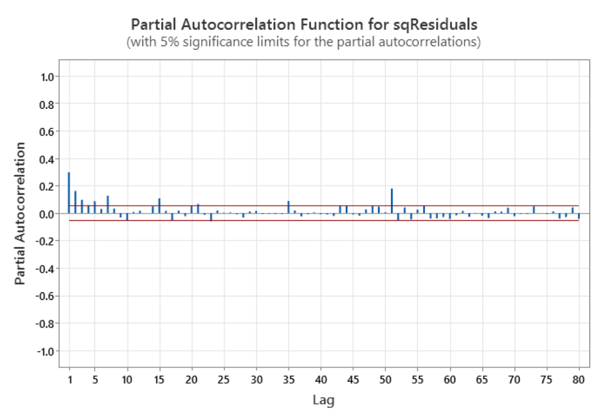
ACF and PACF of Residuals for log Rupee don't have statistically significant lags, and seems like a white noise. However, ACF and PACF of the squared residuals shows multiple lags showing statistically significant.

When we just look at the log Rupee, then they seem uncorrelated (as they seem like a white noise). However, when just squaring the residuals, they show some statistically significant points, showing evidence that they are not actually independent. It means that the variances are correlated, and this can be an evidence of conditional heteroscedasticity.









We next need to save the residuals (RESI1) to an external file, to read into R. To do this, use File → New → Worksheet in Minitab, then select the column of residuals in the old worksheet and copy and paste into the new worksheet. Save the new worksheet using File → Save Worksheet As. Save as type: Text (ANSI). I will use RES.TXT for the output file name. If you will be using R from apps.stern, save the file in your H: directory. If you will be using a version of R that is installed on your computer, save the file in a local directory (see the document on using R for detailed instructions). Next, minimize the Minitab window, and open R.

To read the data set into R use the command >scan() or >file.choose() as described in the usingR document. Download and load the tseries package that will be used to fit ARCH models, as described in the usingR document. You can now fit your arch models, as described in class and in the handout on Estimation and Automatic Selection of ARCH models.

4)

Using R on the residuals from the ARIMA model, find the log likelihood values and AIC_C values for $ARCH(q)$ models where q ranges from 0 to 10. You will need to calculate the log likelihood for the $ARCH(0)$ model by hand. See the handout on Estimation and Automatic Selection of ARCH models.

Next, consider a $GARCH(1,1)$ model. If the residuals from Minitab are stored in an R data set x, then the R command is >model=garch(x,c(1,1)). Evaluate AIC_C for the $GARCH(1,1)$ model, using q=2 in the formula for AIC_C. If the $GARCH(1,1)$ is preferred by AIC_C, use it as your selected model. Comment on the statistical significance of the parameter values of your selected model, as given by the summary(model) command. Write the complete form of the ARCH or GARCH model you have selected. Hand in the R output for the selected model, that is, the results of both summary(model) and logLik(model), but only for the one model that was selected by AIC_C. Also evaluate the unconditional (marginal) variance of the shocks in this model.

For ARCH(0) log likelihood manual calculation, result is as follows,

```
> -0.5 * (length(x)-1) * (1 + log(2 * pi * mean(x^2)))
[1] 4746.964
```

Also, after we calculate the log likelihood of ARCH (q) by R, we calculated AICc by the below formula. The result is as follows,

$$AIC_C = -2\log\text{likelihood} + 2(q+1)\frac{N}{N-q-2}$$

q	logLik	N	AICc
0	4746.964	1259	-9491.9248
1	4801.309	1259	-9598.6084
2	4829.003	1259	-9651.9869
3	4832.305	1259	-9656.5781
4	4835.804	1259	-9661.5601
5	4858.062	1259	-9704.0569
6	4857.211	1259	-9700.3325
7	4869.956	1259	-9723.7968
8	4866.369	1259	-9714.5939
9	4863.337	1259	-9706.4977
10	4858.37	1259	-9694.5283

GARCH(1,1) result is as follows, and we get a log likelihood of 4903.779. By running the above formula for AICc and setting q=2, we get AICc = -9801.5389.
We also found that GARCH(1,1) is preferred by AICc, as it resulted in the lowest AICc, and will use this model for our further analysis.

We can see that all parameter values show statistical significance, showing 4.1e-05 for a0, and less than 2e-16 for both a1 and b1.

Complete form of the model is as follows:

$$h_t = 1.488e\text{-}07 + 3.923e\text{-}02(\varepsilon_{t-1})^2 + 9.555e\text{-}01 h_{t-1},\ \textit{where } h_t = \textit{conditional variance}$$

```
GARCH(1,1)

Residuals:
     Min       1Q   Median       3Q      Max 
-7.84118 -0.48502  0.03399  0.61348  5.82063 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 1.488e-07   3.627e-08    4.102  4.1e-05 ***
a1 3.923e-02   3.714e-03   10.561  < 2e-16 ***
b1 9.555e-01   2.910e-03  328.301  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1218, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 2.2352, df = 1, p-value = 0.1349

> logLik(model)
'log Lik.' 4903.779 (df=3)
```

[Reference: ARCH(0) ~ ARCH(10)]

```
Call:
garch(x = x, order = c(0, 1))

Model:
GARCH(0,1)

Residuals:
    Min      1Q  Median      3Q     Max
-7.6481 -0.4329  0.0268  0.5282  4.9910

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 2.342e-05   5.884e-07   39.797  < 2e-16 ***
a1 2.290e-01   3.299e-02    6.943 3.85e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1970.8, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.80811, df = 1, p-value = 0.3687

> logLik(model)
'log Lik.' 4801.309 (df=2)
```

```
Call:
garch(x = x, order = c(0, 2))

Model:
GARCH(0,2)

Residuals:
     Min       1Q   Median       3Q      Max
-8.37710 -0.44057  0.02678  0.54443  5.55170

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.971e-05   5.104e-07   38.622  < 2e-16 ***
a1 1.486e-01   3.196e-02    4.649 3.34e-06 ***
a2 1.873e-01   2.715e-02    6.899 5.24e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 2533.8, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.27322, df = 1, p-value = 0.6012

> logLik(model)
'log Lik.' 4829.003 (df=3)
```

```
garch(x = x, order = c(0, 3))

Model:
GARCH(0,3)

Residuals:
     Min       1Q   Median       3Q      Max
-8.37729 -0.44541  0.02524  0.54479  5.55417

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.785e-05   6.044e-07   29.529  < 2e-16 ***
a1 1.202e-01   3.251e-02    3.696 0.000219 ***
a2 1.783e-01   2.803e-02    6.362 1.99e-10 ***
a3 1.089e-01   2.529e-02    4.308 1.65e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 2536.4, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.14264, df = 1, p-value = 0.7057

> logLik(model)
'log Lik.' 4832.305 (df=4)
```

```
Model:
GARCH(0,4)

Residuals:
     Min       1Q   Median       3Q      Max
-8.63685 -0.45706  0.02729  0.54436  5.51902

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.631e-05   5.810e-07   28.076  < 2e-16 ***
a1 1.201e-01   3.333e-02    3.602 0.000316 ***
a2 1.469e-01   2.662e-02    5.521 3.38e-08 ***
a3 1.033e-01   2.401e-02    4.302 1.69e-05 ***
a4 9.115e-02   2.350e-02    3.879 0.000105 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 2893.3, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.1528, df = 1, p-value = 0.6959

> logLik(model)
'log Lik.' 4835.804 (df=5)
```

```
Model:
GARCH(0,5)

Residuals:
     Min       1Q   Median       3Q      Max
-8.06382 -0.46694  0.02895  0.56249  5.47468

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.196e-05   8.311e-07   14.393  < 2e-16 ***
a1 1.211e-01   3.307e-02    3.660 0.000252 ***
a2 1.405e-01   2.484e-02    5.656 1.55e-08 ***
a3 1.070e-01   2.232e-02    4.793 1.64e-06 ***
a4 9.619e-02   2.168e-02    4.438 9.09e-06 ***
a5 1.863e-01   2.143e-02    8.692  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1821.5, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.055437, df = 1, p-value = 0.8139

> logLik(model)
'log Lik.' 4858.062 (df=6)
```

```
Model:
GARCH(0,6)

Residuals:
     Min       1Q   Median       3Q      Max
-8.08305 -0.47387  0.02895  0.56098  5.67560

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.141e-05   8.163e-07   13.973  < 2e-16 ***
a1 1.214e-01   3.223e-02    3.767 0.000165 ***
a2 1.254e-01   2.452e-02    5.114 3.16e-07 ***
a3 8.914e-02   2.029e-02    4.394 1.11e-05 ***
a4 8.760e-02   2.338e-02    3.746 0.000180 ***
a5 1.826e-01   2.124e-02    8.598  < 2e-16 ***
a6 6.109e-02   2.482e-02    2.461 0.013848 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1870.3, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.066995, df = 1, p-value = 0.7958

> logLik(model)
'log Lik.' 4857.211 (df=7)
```

```
garch(x = x, order = c(0, 7))

Model:
GARCH(0,7)

Residuals:
     Min       1Q   Median       3Q      Max
-8.13640 -0.47589  0.02982  0.58185  5.80243

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.122e-05   7.844e-07   14.298  < 2e-16 ***
a1 1.215e-01   3.368e-02    3.608 0.000309 ***
a2 7.576e-02   2.684e-02    2.823 0.004759 **
a3 9.576e-03   2.092e-02    0.458 0.647157
a4 1.026e-01   2.417e-02    4.245 2.19e-05 ***
a5 1.273e-01   2.342e-02    5.436 5.45e-08 ***
a6 2.654e-02   2.186e-02    1.214 0.224758
a7 1.778e-01   3.640e-02    4.884 1.04e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1749.9, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.17088, df = 1, p-value = 0.6793

> logLik(model)
'log Lik.' 4869.956 (df=8)
```

```
Model:
GARCH(0,8)

Residuals:
    Min      1Q  Median      3Q     Max
-8.1442 -0.4783  0.0297  0.5795  5.8058

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 1.118e-05   7.857e-07   14.234  < 2e-16 ***
a1 1.196e-01   3.429e-02    3.489 0.000485 ***
a2 7.470e-02   2.728e-02    2.739 0.006172 **
a3 9.990e-03   2.095e-02    0.477 0.633446
a4 1.015e-01   2.424e-02    4.189 2.80e-05 ***
a5 1.271e-01   2.342e-02    5.427 5.75e-08 ***
a6 2.673e-02   2.185e-02    1.223 0.221216
a7 1.774e-01   3.636e-02    4.878 1.07e-06 ***
a8 4.392e-03   1.681e-02    0.261 0.793942
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1762.3, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.15799, df = 1, p-value = 0.691

> logLik(model)
'log Lik.' 4866.369 (df=9)
```

```
Model:
GARCH(0,9)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.1945 -0.4764  0.0299  0.5755  5.8044 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 1.095e-05   7.721e-07   14.182  < 2e-16 ***
a1 1.170e-01   3.399e-02    3.442 0.000577 ***
a2 6.820e-02   2.726e-02    2.502 0.012356 *  
a3 4.955e-03   2.099e-02    0.236 0.813405    
a4 1.012e-01   2.413e-02    4.195 2.72e-05 ***
a5 1.244e-01   2.349e-02    5.296 1.19e-07 ***
a6 2.736e-02   2.209e-02    1.238 0.215537    
a7 1.804e-01   3.686e-02    4.894 9.86e-07 ***
a8 4.873e-03   1.665e-02    0.293 0.769816    
a9 2.190e-02   1.989e-02    1.101 0.270913    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals 
X-squared = 1833.8, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.12074, df = 1, p-value = 0.7282

> logLik(model)
'log Lik.' 4863.337 (df=10)
```

```
GARCH(0,10)

Residuals:
     Min       1Q   Median       3Q      Max 
-8.51797 -0.47984  0.02892  0.56147  5.84057 

Coefficient(s):
     Estimate  Std. Error  t value Pr(>|t|)    
a0  1.139e-05   7.703e-07   14.781  < 2e-16 ***
a1  7.932e-02   2.899e-02    2.736 0.006214 ** 
a2  7.032e-02   2.539e-02    2.770 0.005613 ** 
a3  3.503e-02   2.235e-02    1.567 0.117032    
a4  7.927e-02   2.566e-02    3.090 0.002003 ** 
a5  9.189e-02   1.967e-02    4.671    3e-06 ***
a6  5.043e-02   2.755e-02    1.831 0.067152 .  
a7  1.087e-01   2.927e-02    3.714 0.000204 ***
a8  3.215e-02   2.300e-02    1.398 0.162140    
a9  4.138e-02   2.625e-02    1.576 0.114967    
a10 1.516e-13   1.862e-02    0.000 1.000000    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals 
X-squared = 2206.9, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.00033918, df = 1, p-value = 0.9853

> logLik(model)
'log Lik.' 4858.337 (df=11)
```

5)

Using the Minitab output from problem 2, and the R output from your selected model in problem 4, construct a 95% one step ahead forecast interval for the log exchange rate, based on your ARIMA-ARCH model. (If you decided to use a $GARCH(1,1)$ model, you will need to first get the conditional variances from R. See Problem 6.) Compare this to the interval based on the ARIMA only model from problem 2. Also compute the 5th percentile of the conditional distribution of the next period's log exchange rate.

95% one step ahead forecast interval for the log exchange rate based on our ARIMA-ARCH model.

From problem 2, we have below forecast with 95% forecast interval.

Forecasts from Time Period 1259

			95% Limits		
Time Period	Forecast	SE Forecast	Lower	Upper	Actual
1260	4.20725	0.0055510	4.19637	4.21813	

Using our ARIMA-ARCH model, we first use conditional variances from R. Using the result from problem 6, we get conditional variances h_t.
Using h_{1259} = 0.00000873205733200221 and ε_{1259} = *0.0100313,*
we can get h_{1260} = 0.0000124398771939668 based on the formula

$$h_t = 1.488e\text{-}07 + 3.923e\text{-}02(\varepsilon_{t-1})^2 + 9.555e\text{-}01h_{t-1}, \text{ where } h_t = \text{conditional variance}$$

Using h_{1260} = 0.0000124398771939668, best one-step forecast $f_{1259,1}$ = 4.20725, and the formula,

$$f_{t,1} \pm 1.96\sqrt{h_{t+1}}.$$

we can get 95% intervals of (4.20033703882, 4.21416296118).

Comparing the above results with ARIMA forecast interval (4.19637, 4.21813),
Our ARIMA-ARCH model shows more narrow intervals.

Also, the 5th percentile of the conditional distribution of the next period's log exchange rate is
$f_{1259,1} - 1.645\sqrt{h_{1260}}$ = 4.20144805044112

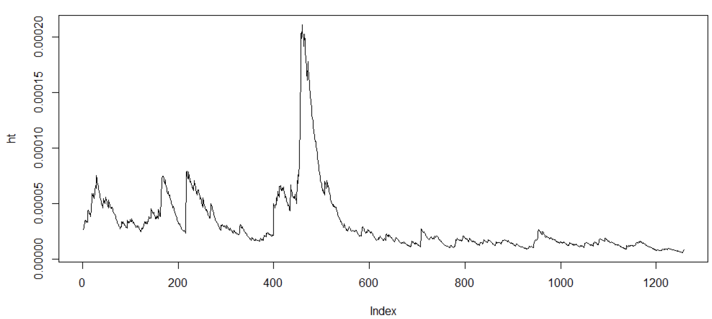
6)

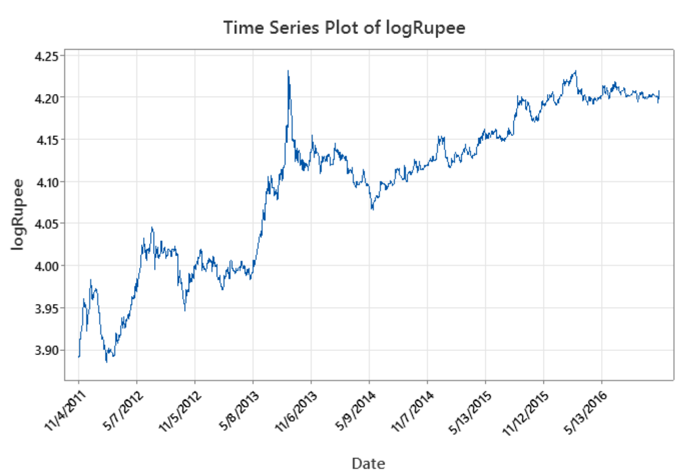
Plot the conditional variances, ht, for your fitted ARCH model from problem 4. (See instructions below). Use this plot to locate bursts of high volatility. Do these highly volatile periods agree with those found from examination of the time series plot of the log exchange rates themselves?

To save the conditional variances and read them into Minitab, proceed as follows. First, re-fit your selected ARCH model and store it in the variable "model". Compute and store the conditional variances with the command >ht=model$fit[,1]^2.

Next, write the ht dataset to a file named htfile.txt using the write() or file.choose() command as described in the usingR document. Minimize R and re-enter Minitab. Read htfile.txt into a new Minitab worksheet, using File → Open. The resulting column (ht) should have a length of 1259. So, for example, the last value is h_{1259}, the conditional variance for time 1259 (which can be computed based on information available at time 1258). Copy and paste this column into the original Minitab worksheet.

By comparing highly volatile periods, which occur around the index of 475 or September 2013, the two highly volatile periods agree with those found from examination of the time series plot of the log exchange rates.





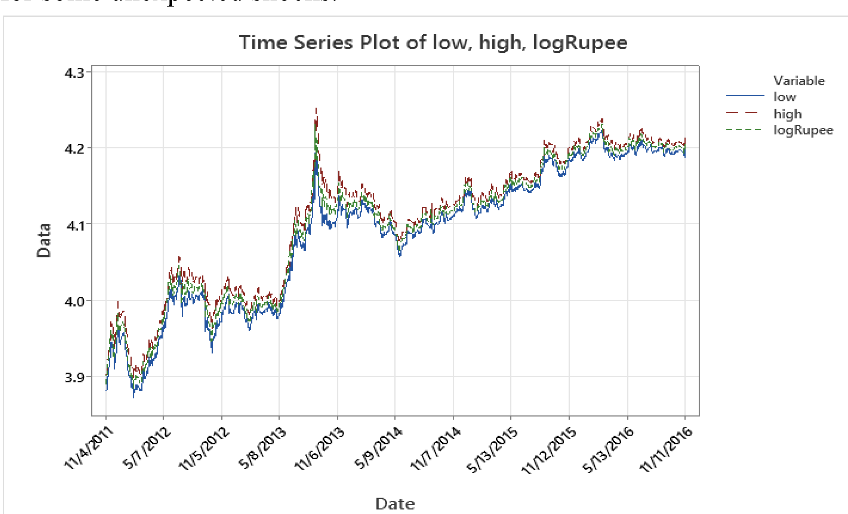
7)

Make a time series plot which simultaneously shows the log exchange rates, together with the ARIMA-ARCH one-step-ahead 95% forecast intervals based on information available the previous day. (See instructions below). Using the plot, together with the numerical values in your Minitab worksheet, comment on the accuracy and practical usefulness of the forecast intervals. Keep in mind that the performance may be somewhat better here than in an actual forecasting context, since the ARIMA-ARCH parameters are estimated from the entire data set, not just the observations up to the time at which the forecast is to be constructed.

To compute the forecast intervals in Minitab, proceed as follows. First, get "low" and "high", the lower and upper endpoints of the 95% forecast intervals, using Calc → Calculator → Store result in variable: low , Expression : FITS1-1.96*sqrt(ht) → OK, and similarly for high. For a given t, the interval between low and high represents a one-step-ahead 95% forecast interval for the log exchange rate at time t based on information which was available at time $t-1$.

To plot the intervals along with the log exchange rates in Minitab, use Graph → Time Series Plot → Multiple. In Data View, click the box for "Connect Line" but un-check the box for "Symbols".

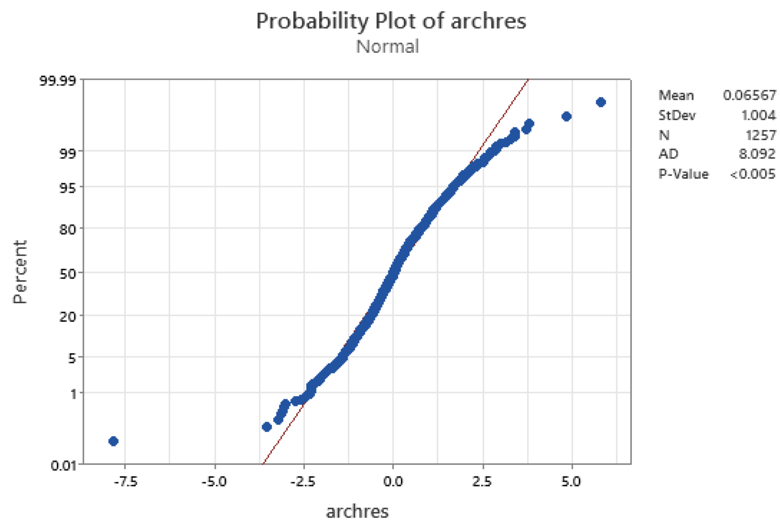
Our model's accuracy seems better, showing a narrower interval compared to ARIMA only model. It is expected as our model's parameters are estimated from the entire data set with the conditional variances that keep updated over the entire course of period. It is useful, as the actual forecasting performance may be better except for some unexpected shocks.



8)

Compute the residuals from your ARIMA-ARCH model, that is, $e_t = \varepsilon_t / \sqrt{h_t}$. If the ARIMA-ARCH model is adequate, these residuals should be normally distributed with mean zero and variance 1. To compute these residuals in Minitab, use Calc → Calculator → Store result in variable: archres, Expression: RESI1/sqrt(ht). Make a normal probability plot of archres, using Stat → Basic Statistics → Normality Test. Does the model seem to have adequately described the leptokurtosis ("long-tailedness") in the data?

The model seems to have long-tail, which seems to fail the normality test. Therefore, it adequately describes the leptokurtosis in the data. We've also checked the p-value, and the value is < 0.005. This also gives evidence to reject the null hypothesis of normality test, indicating a non-normal distribution.



9)

From the formula for the prediction intervals, it follows that the 95% prediction interval constructed yesterday fails to cover today's log exchange rate whenever today's residual exceeds 1.96 in absolute value. Use Calculator to count up how many failures there were, using sum(abs(archres)>1.96). What percentage of the time did the intervals fail? (Keep in mind that there are not 1259 data values in archres).

Based on the calculation, there are 72 failures, and therefore for a total 1,257 data values in archres, the percentage is 5.72792362768 %.