	Simple LR	Multiple LR (full model)	Centered Form Regression	Reduced Model	General Linear
Formula	$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$	$y_i = \beta_0 j_n + \beta_1 x_i + \cdots + \beta_k x_k + \epsilon_i$ $= (X_1, X_2)\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} + \epsilon$ $= X_1\beta_1 + X_2\beta_2 + \epsilon$	$y_i = \beta_0 j_n + \beta_1 x_{i1} + \cdots + \beta_k x_i k + \epsilon_i$ $= \alpha + \beta_1(x_{i1} - \bar{x}_1) + \cdots$ $+\beta_k(x_{k1} - \bar{x}_k) + \epsilon_i$ $y = (j, X_c)\begin{pmatrix}\alpha\\ \beta_1\end{pmatrix} + \epsilon$, where $X_c = \left(I - \frac{1}{n}J\right)X_1$	$y = X_1\beta_1^* + \epsilon^*$, $var(\epsilon^*) = \sigma^2 I$	
Least-Squares Estimator β	$\beta_0 = \bar{y} - \hat{\beta}_1\bar{x}$ $\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$	$\hat{\beta} = (X'X)^{-1}X'y$	$\hat{\alpha} = \bar{y}$ $\hat{\beta}_1 = (X_c'X_c)^{-1}X_c'y = S_{xx}^{-1}S_{yx}$ $\hat{\beta}_0 = \hat{\alpha} - \hat{\beta}_1\overline{x_1} - \cdots - \hat{\beta}_k\overline{x_k}$ $= \bar{y} - \hat{\beta}_1'\bar{\mathrm{x}} = \bar{y} - S_{yx}'S_{xx}^{-1}\bar{x}$ $S_{xx} = \frac{X_c'X_c}{n-1} = \widehat{cov(x,x)}$, $S_{yx} = \frac{X_c'y}{n-1} = \widehat{cov(x,y)}$	$\hat{\beta}_1^* = (X_1'X_1)^{-1}X_1'y$	$\hat{\beta}_c$ $= \hat{\beta} - (\mathrm{X'X})^{-1}\mathrm{C'}[\mathrm{C}(\mathrm{X'X})^{-1}\mathrm{C'}]^{-1}\mathrm{C}\beta$
Estimator for σ²	$s_2 = SSE/(n-2)$	$s_2 = SSE/(n-k-1)$ $\sigma^2 = \frac{1}{n}(\mathrm{y} - \mathrm{X}\hat{\beta})'(\mathrm{y} - \mathrm{X}\hat{\beta})$			
Mean, Variance	$E(\hat{\beta}_0) = \beta_0, E(\hat{\beta}_1) = \beta_1$ $var(\hat{\beta}_0) = \sigma^2\left[\frac{1}{n} + \frac{\bar{x}}{\sum(x_i - \bar{x})^2}\right]$ $var(\hat{\beta}_1) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2}$	$E(\hat{\beta}) = \beta$ $E(s^2) = \sigma^2$ $cov(\hat{\beta}) = var(\hat{\beta})$ $= \sigma^2(X'X)^{-1}$ $\widehat{cov}(\hat{\beta}) = s^2(X'X)^{-1}$		$E(\hat{\beta}_1^*) = \beta_1 + A\beta_2$, where $A = (X'X)^{-1}X_1'X_2$ $E(\hat{\beta}_1^*) = \beta *_1$ $cov(\hat{\beta}_1^*) = \sigma^2(X_1'X_1)^{-1}$	$E(\hat{\beta}_c)$ $= \beta - (\mathrm{X'X})^{-1}\mathrm{C'}[\mathrm{C}(\mathrm{X'X})^{-1}\mathrm{C'}]^{-1}\mathrm{C}\beta$ $cov(\hat{\beta}_c)$ $= \sigma^2(\mathrm{X'X})^{-1}$ $- \sigma^2(\mathrm{X'X})^{-1}\mathrm{C'}[\mathrm{C}(\mathrm{X'X})^{-1}\mathrm{C'}]^{-1}\mathrm{C}(\mathrm{X'X})^{-1}$
Hypothesis H_0: $\beta_1 = 0$		If true, $F = \frac{SSR/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$, where $SSR/\sigma^2 \sim \chi^2(k, \lambda_1)$, $SSE/\sigma^2 \sim \chi^2(n-k-1)$ If not, $F \sim F(k, n-k-1, \lambda_1), \lambda_1 = \hat{\beta}_1'\mathrm{X_c'X_c}\hat{\beta}_1/2\sigma^2$			
Hypothesis H_0: $\beta_2 = 0$	-	If true, $F = \frac{SS(\beta_2 \mid \beta_1)/h}{SSE/(n-k-1)} = \frac{(R^2 - R_r^2)/h}{(1-R^2)/(n-k-1)} \sim F(h, n-k-1)$, where $SS(\beta_2 \mid \beta_1)/\sigma^2 \sim \chi^2(n-k-$ $1), SSE/\sigma^2 \sim \chi^2(h, \lambda_1), R_r^2 = \frac{\widehat{\beta*}_1'X'y - n\bar{y}^2}{y'y - n\bar{y}^2}, R^2 = \frac{SSR}{SST}$ If not, $F \sim F(h, n-k-1, \lambda_1), , \lambda_1 = \Vert(P_{C(X)} - P_{C(X_1)})\mu\Vert^2 = \Vert\mu - \mu_0\Vert^2/2\sigma^2 = \beta'_2[X_2'X_2 -$ $X_2'X_1(X_1'X_1)^{-1}X_1'X_2]\beta_2/2\sigma^2$			$E\left(\frac{SSE}{SST}\right) = \frac{n-k-1}{n-1}$ $E(R_a^2) = 0$ $E(R^2) = 1 - \frac{n-k-1}{n-1} = \frac{k}{n-1}$
Hypothesis H_0: $C\beta = 0$		If true, $F = \frac{SSH/q}{SSE/(n-k-1)} \sim F(q, n-k-1)$, where $SSH/\sigma^2 \sim \chi^2(q, \lambda)$, $SSE/\sigma^2 \sim \chi^2(n-k-1)$ If not, $F \sim F(q, n-k-1, \lambda), where\ \lambda = (\mathrm{C}\hat{\beta})'[C(X'X)^{-1}C']C\hat{\beta}/2\sigma^2$			
Hypothesis H_0: $C\beta = t$		If true, $F = \frac{SSH/q}{SSE/(n-k-1)} \sim F(q, n-k-1)$, where $SSH/\sigma^2 \sim \chi^2(q, \lambda)$, $SSE/\sigma^2 \sim \chi^2(n-k-1)$ If not, $F \sim F(q, n-k-1, \lambda), where\ \lambda = (\mathrm{C}\beta - t)'[C(X'X)^{-1}C'](\mathrm{C}\beta - t)/2\sigma^2$			
Hypothesis H_0: $a'\beta = 0$		If true, $F = \frac{SSH/q}{SSE/(n-k-1)} = \frac{(a'\hat{\beta})^2}{s^2 a'(X'X)^{-1}a} \sim F(1, n-k-1)$ or $t = \frac{a'\hat{\beta}}{s\sqrt{a'(X'X)^{-1}a}} \sim t(n-k-1)$			
Hypothesis H_0: $\beta_j = 0$	for j=1, $t = \frac{\hat{\beta}_1}{s/\sqrt{\sum(x_i - \bar{x})^2}}$	If true, $F = \frac{\hat{\beta}_j^2}{s^2 g_{jj}} \sim F(1, n-k-1)\ or\ t_i = \frac{\hat{\beta}_j}{s\sqrt{g_{jj}}} \sim t(n-k-1), where\ g_{jj} = diag((X'X)^{-1})$			
CI	$\hat{\beta}_1 \pm t_{\alpha/2, n-2}\frac{s}{\sum(x_i - \bar{x})^2}$	$\hat{\beta}_j \pm t_{1-\alpha/2, n-k-1}s\sqrt{g_{jj}}, s = \sqrt{SSE/(n-k-1)}$ (for β_j) $a'\hat{\beta} \pm t_{1-\alpha/2, n-k-1}s\sqrt{a'(X'X)^{-1}a}$ (for $a'\beta$) $x_0'\hat{\beta} \pm t_{1-\alpha/2, n-k-1}s\sqrt{x_0'(X'X)^{-1}x_0}$ (for E(y)) $x_0'\hat{\beta} \pm t_{1-\alpha/2, n-k-1}s\sqrt{1 + x_0'(X'X)^{-1}x_0}$ (for prediction)			

- $\boldsymbol{SST} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \left\|(I - P_{j_n}y)y\right\|^2 = \|(y - \bar{y}j_n)\|^2 = y'y - n\bar{y}^2 = \left(\hat{\beta}'X'y - n\bar{y}^2\right) + (y'y - n\bar{y}^2) = y'\left(I - \frac{1}{n}J\right)y = y'(I-H)y + y'(H-H_1)y + y'\left(H_1 - \frac{1}{n}J\right)y = y'H_c y + y'\left(I - \frac{1}{n}J - H_c\right)y = SSR + SSE$, where $y'(I-H)y \sim \chi^2(n-k-1), y'(H-H_1)y \sim \chi^2(h, \lambda_1), \lambda_1 = \beta'_2\left[X'_2X_2 - X'_2X_1\left(X'_1X_1\right)^{-1}X'_1X_2\right]\beta_2/2\sigma^2$

- $\mathbf{SSR} = \hat{\beta}'_1X'_cX_c\hat{\beta}_1 = \hat{\beta}'X'y - n\bar{y}^2 = \left(P_{C(X)}y\right)'y - y'P'_{\mathcal{L}(j_n)}P_{\mathcal{L}(j_n)}y = \hat{\beta}'_1X'_cy = y'X_c(X'_cX_c)^{-1}X'_cy = \left\|P_{X_c}y\right\|^2 = y'H_cy$

- $\boldsymbol{SSE} = \varepsilon'\varepsilon = \sum_{i=1}^{n}\left(y_i - x'_i\hat{\beta}\right) = \left(y - X\hat{\beta}\right)'\left(y - X\hat{\beta}\right) = y'\left[I - P_{C(X)}\right]y = y'[I - X(X'X)^{-1}X']y = y'y - \hat{\beta}'X'y = \sum_{i=1}^{n}(y_i - \bar{y})^2 - \hat{\beta}'_1X'_cy = \sum_{i=1}^{n}(y_i - \bar{y})^2 - \left\|P_{X_c}y\right\|^2 = y'\left(I - \frac{1}{n}J - H_c\right)y = \|(I-H)y\|^2 = \left\|P_{C(X)^\perp}y\right\|^2$

- $\boldsymbol{SS}(\boldsymbol{\beta_2}|\boldsymbol{\beta_1}) = SSR(full) - SSR(reduced) = SSE(reduced) - SSE(full) = \left\|\hat{y} - \hat{y}_0\right\|^2 = \left\|y - \hat{y}_0\right\|^2 - \|y - \hat{y}\|^2$

  $= y'(H - H_1)y = \widehat{\beta}'X'y - \hat{\beta}_1^{*'}X'y = \hat{\beta}'_1X'_cX_c\hat{\beta}_1 - \hat{\beta}_1^{*'}X'_cX_c\hat{\beta}_1$

- $\boldsymbol{SSH} = y'\left(P_{C(X)} - P_{C(X_1)}\right)y = \left\|\left(P_{C(X)} - P_{C(X_1)}\right)y\right\|^2 = \left(C\hat{\beta}\right)'[C(X'X)^{-1}C']^{-1}C\hat{\beta} = \left(C\hat{\beta} - t\right)'[C(X'X)^{-1}C']^{-1}\left(C\hat{\beta} - t\right)$

- $\boldsymbol{R^2} = \frac{SSR}{SST}$

- $\boldsymbol{R_a^2} = \frac{(n-1)R^2 - k}{n-k-1}$

$$< y \sim N_n(X\beta, \sigma^2 I) >$$

$H_0: \beta_1 = 0$ (ANOVA table)

variation	df	Sum of Squares	Mean Squares	E(Mean Squares)
β_1	k	$SSR = \hat{\beta}_1' X_c' y = \hat{\beta}' X' y - n\bar{y}^2$	SSR/k	$\sigma^2 + \frac{1}{k} \hat{\beta}_1' X_c' X_c \hat{\beta}_1$
Error	n-k-1	$SSE = y'y - \hat{\beta}' X' y$	SSE/(n-k-1)	σ^2
Total	n-1	$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$		

$H_0: \beta_2 = 0$ (ANOVA table)

variation	df	Sum of Squares	Mean Squares	E(Mean Squares)
β_2 *adjusted for* β_1	h	$S(\beta_2 \mid \beta_1) = \hat{\beta}' X' y - \hat{\beta}_1^{*\prime} X' y$ $= y'\left(P_{C(X)} - P_{C(X_1)}\right) y$	$SS(\beta_2 \mid \beta_1)/h$	$\sigma^2 + \frac{1}{h} \beta'_2 \left[X_2' X_2 - X_2' X_1 \left(X_1' X_1 \right)^{-1} X_1' X_2 \right] \beta_2$ $= \sigma^2 + \frac{1}{h} \lvert\lvert \mu - \mu_0 \rvert\rvert$
Error	n-k-1	$SSE = y'y - \hat{\beta}' X' y$ $= y'(I - P_{C(X)}) y$	SSE/(n-k-1)	σ^2
Total	n-1	$SST = y'y - n\bar{y}^2$		

$H_0: C\beta = 0$ (ANOVA table), $C\hat{\beta} \sim N_q(C\beta, \sigma^2 C(X'X)^{-1} C')$

variation	df	Sum of Squares	Mean Squares	E(Mean Squares)
General Linear	q	$SSH = \hat{\beta}' X' y - \hat{\beta}_c' X' y = \left(C\hat{\beta} \right)' [C(X'X)^{-1} C']^{-1} \left(C\hat{\beta} \right)$	SSH/q	$\sigma^2 + \frac{1}{q} (C\beta)' [C(X'X)^{-1} C']^{-1} (C\beta)$
Error	n-k-1	$SSE = y'[I - X(X'X)^{-1} X'] y$	SSE/(n-k-1)	σ^2
Total	n-1	$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$		

$H_0: C\beta = t$ (ANOVA table), $C\hat{\beta} - t \sim N_q(C\beta - t, \sigma^2 C(X'X)^{-1} C')$

variation	df	Sum of Squares	Mean Squares	E(Mean Squares)
General Linear	q	$SSH = \left(C\hat{\beta} - t \right)' [C(X'X)^{-1} C']^{-1} \left(C\hat{\beta} - t \right)$	SSH/q	$\sigma^2 + \frac{1}{q} (C\beta - t)' [C(X'X)^{-1} C']^{-1} (C\beta - t)$
Error	n-k-1	$SSE = y'[I - X(X'X)^{-1} X'] y$	SSE/(n-k-1)	σ^2
Total	n-1	$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$		