

# Teaching Transformers to Solve Combinatorial Problems through Efficient Trial & Error

Panagiotis Giannoulis<sup>1</sup>

Yorgos Pantis<sup>2,3</sup>

Christos Tzamos<sup>2,3</sup>

<sup>1</sup>National Technical University of Athens, Greece

<sup>2</sup>National and Kapodistrian University of Athens, Greece

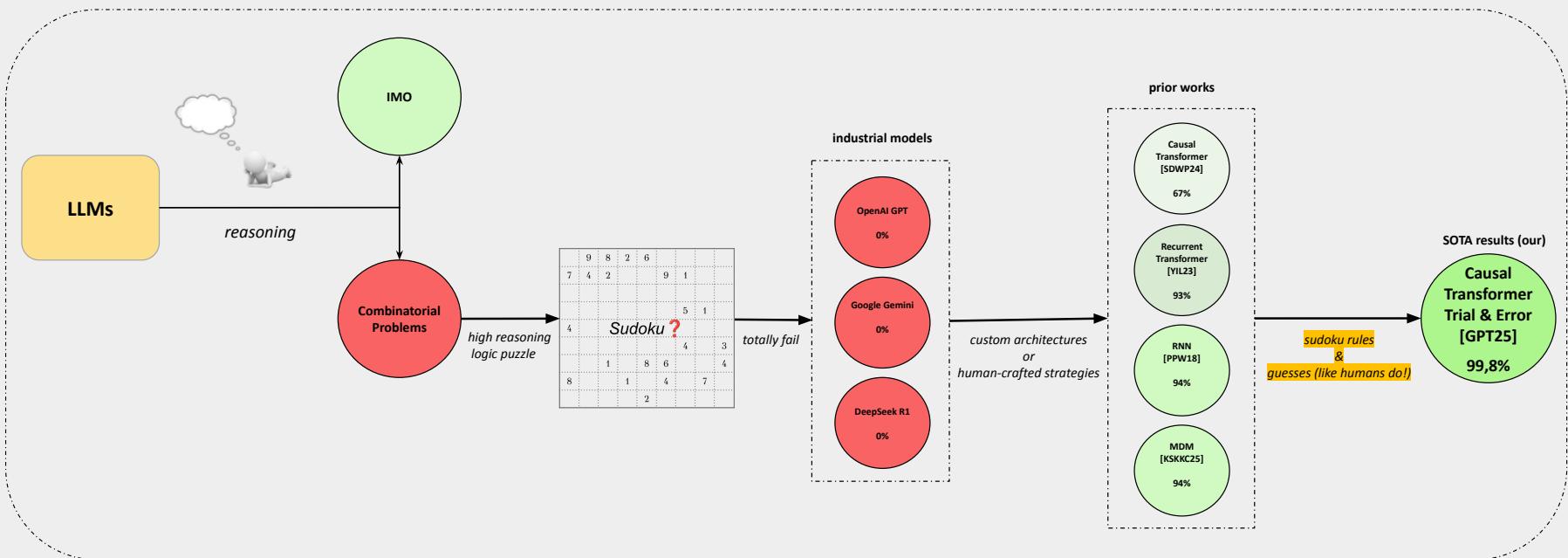
<sup>3</sup>Archimedes, Athena Research Center, Greece

*Authors listed alphabetically; equal contribution*



# Motivation

LLMs have shown surprisingly strong results in mathematical tasks, but...



# Key Contributions

- A generalized framework for NP-class problems
- No custom architecture: valina-decoder only GPT-2 with 42M parameters
- Training transcripts solely based on Sudoku rules (DFS method) and guesses
  - An approach that can solve Sudoku which we do not know yet deterministic strategies to solve them
  - Close related approach to human nature!
- Minimization of guesses via a novel loss inspired by Min-Sum Set Cover problem
- SOTA results; accuracy 99.8% on randomly generated Sudoku
- SudokuPy: a Python library for generating random Sudoku puzzles

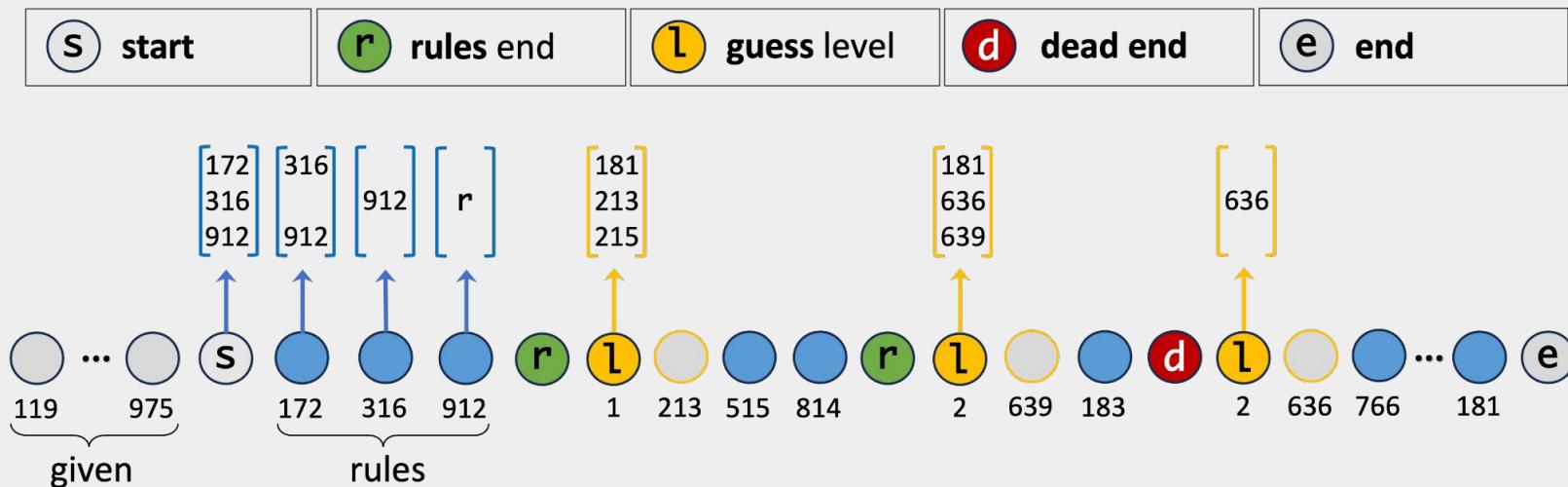
# (1/2) Imitation Learning

*Sudoku rules + informed multiple guesses*

- Encoding: Each move encoded as a 3-digit number rcv ( $111 \rightarrow 999$ ); row\_column\_value
- Multiple Targets: Combinatorial puzzles allow multiple valid next moves
  - Instead of a single deterministic label, we support multiple next-token predictions  
Cross Entropy loss:  $-\log p_i$  → Multiple target loss:  $-\sum \log p_i$   
over all valid next tokens
- Results: Accuracy 98,9%; SOTA compared to other prior works  
Accuracy 99,1% in 1-3 SAT problem (NP-complete problem)

# (2/2) Imitation Learning

*Sudoku rules + informed multiple guesses*



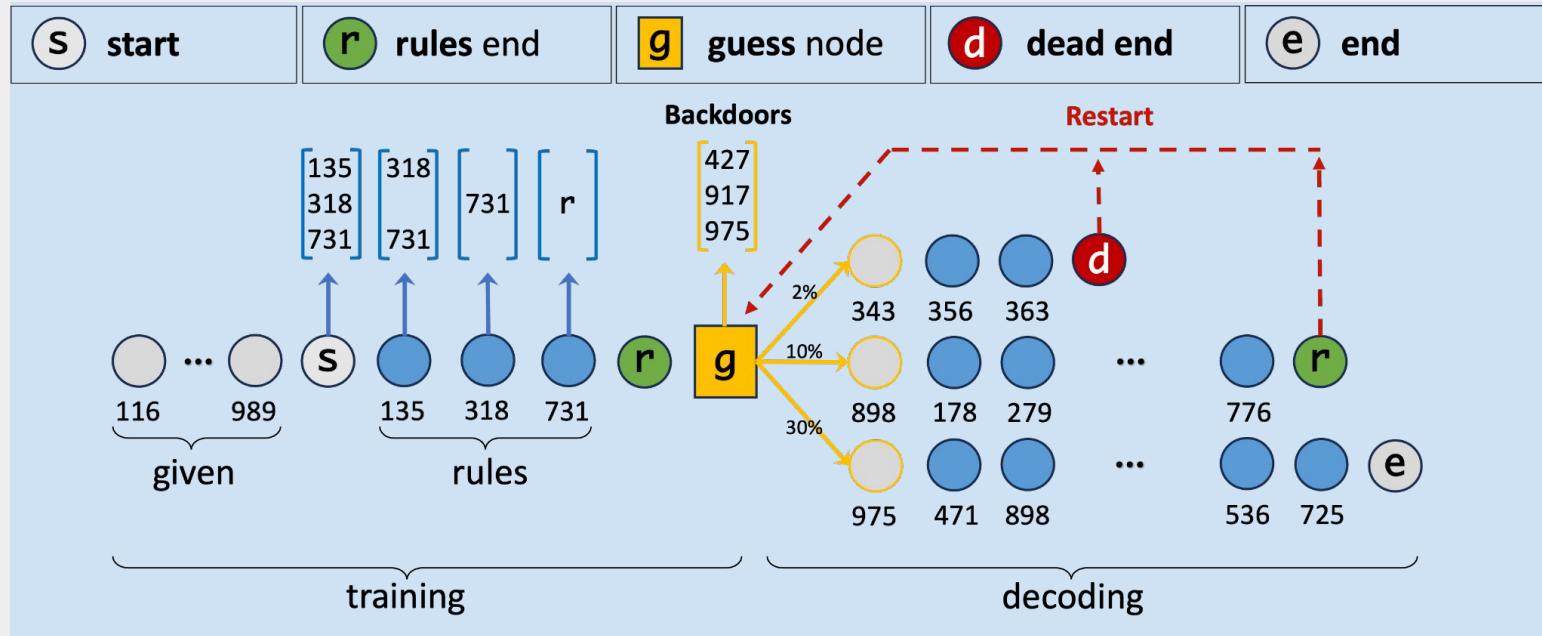
# (1/4) Beyond Imitation Learning

*Sudoku rules + a single guess*

- Encoding & Multiple Targets: Same as Imitation Learning
- Guesses: A single guess (backdoor; once identified, applying simple rules leads to the full solution)
- Loss Function: We still support multiple next-token predictions and a new loss formulation for guesses  
Multiple target loss:  $-\log p_i$  & Guesses loss:  $1/\sum p_i$  over all valid guesses (1)
- Results: Accuracy 99.8%; new SOTA, outperforming even our previous best results
- Insights: This approach shows empirically that 99.8% of Sudokus can be solved by using only one guess (backdoor)

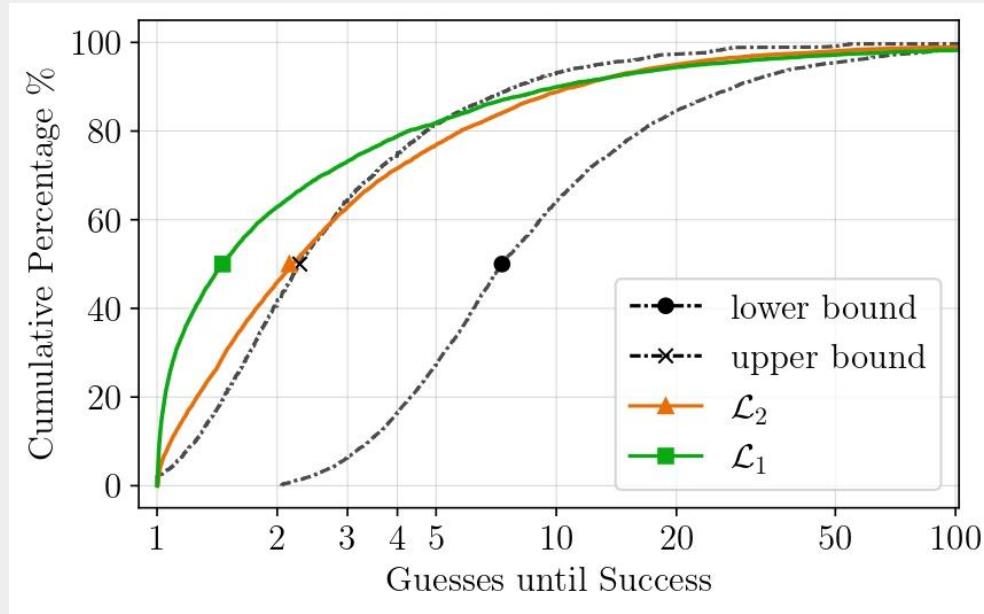
# (2/4) Beyond Imitation Learning

*Sudoku rules + a single guess*



# (3/4) Beyond Imitation Learning

*Sudoku rules + a single guess*



Remark: For half of all Sudokus, the backdoor guess can be found in about 1,5 guesses

# (4/4) Beyond Imitation Learning

*Mathematical insights*

- Assumptions: depth-1 of search and non adaptive policy
- Challenge:
  - You face  $n$  possible choices, but only a hidden subset  $S$  is valid
  - Subset  $S$  is drawn from a known distribution  $D$
  - Each test costs 1 time unit and once it is made you only learn if it is valid
- Goal: Find a policy  $\pi$  that minimizes the expected time to discover a valid choice

**Theorem.** For any distribution  $\mathcal{D}$  over sets  $S \subseteq [n]$ , it holds that for any permutation  $\tau$ :

$$\min_{\pi \in \Delta(n)} \mathbb{E}_{S \sim \mathcal{D}} \left[ \frac{1}{\sum_{i \in S} \pi_i} \right] \leq H_n \cdot \mathbb{E}_{S \sim \mathcal{D}} \left[ \arg \min_{i=1}^n \{\tau_i \in S\} \right]$$

where  $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \Theta(\log n)$  is the  $n$ -th harmonic number.

Remark: Loss function (1) yields solutions with a bounded approximation to the optimal policy, whereas treating the problem as a multi-class classification task (e.g., weighted Cross-Entropy Loss) leads to much worse approximations

# References

- R. Palm, U. Paquet, O. Winther. Recurrent relational networks. NeurIPS'18.
- Z. Yang, A. Ishay, J. Lee. Learning to solve constraint satisfaction problems with recurrent transformer. ICLR'23.
- K. Shah, N. Dikkala, X.Wang, R.Panigrahy. Causal language modeling can elicit search and reasoning capabilities on logic puzzles. NeurIPS'24.
- J.Kim, K. Shah, V. Kontonis, S. Kakade, S. Chen. Train for the worst, plan for the best: Understanding token ordering in masked diffusions. ICML'25.

# Questions & Answers

For questions, feel free to reach us via email

or

visit us in San Diego, USA!

For more details, read our full paper:



Thank you for your time!