Question 4

Prove, mathematically, that both roots are negative.

See the equations in Question 5 and let $a = 1$ and $c = 1$, then we can see:
$x_1 = (-b + sqrt(b * b - 4))/2$
$x_2 = (-b - sqrt(b * b - 4))/2$
when b in $(1e1, 1e2...1e15)$, and $b >= 10$
so we can get:
$b * b > (b * b - 4)$
$-b + sqrt(b * b - 4) < 0$ and $-b - sqrt(b * b - 4) < 0$
Thus, x_1, x_2 are both negative.

Question 5

What is $x_1 x_2$ in terms of a, b, c?

$x_1 = (-b + sqrt(b * b - 4 * a * c))/(2 * a)$
$x_2 = (-b - sqrt(b * b - 4 * a * c))/(2 * a)$

Question 6

Look at `roots.txt`. What do you notice about one of the roots?

1. The precision of of the x_1 are smaller than 17.
2. The x_1 becomes 0.000000 after $b >= 9$, but accroding to Qestion 4, the value of it should be negative.

Question 7

Look at the formula for the quadratic equation for the solution x_1. For fixed a and c, how do the magnitudes of terms in the numerator compare as b gets large?

1. The precision of x_1 gets smaller as b gets larger.
2. Mathematically, as b gets larger, x_1 becomes bigger and x_2 becomes smaller. In the case of $a = 1c = 1$, x_1 should become bigger but not bigger than 0.

Question 8

Given your analysis in 8, discuss what you think is happening in the finite precision calculation of x_1?

1. Since x_1 is the double type in the c file, so it's width is 64 bits and it only contains 17 precision digits. When the number of decimal exceeds 17 decimal digits, machine memory overflows. During the calculating process, $b - sqrt(b^2 - 4ac)$ is a very small number after $b >= 1e9$ and the percision is reduced dramatically. Thus, we can't get the specific the decimal digits even before 16.
2. Also after the percision exceeding 17 digits, machine can not operate it. After $b > 1e8$, x_1 becomes zero.