Introduction to Financial Engineering

Option Pricing

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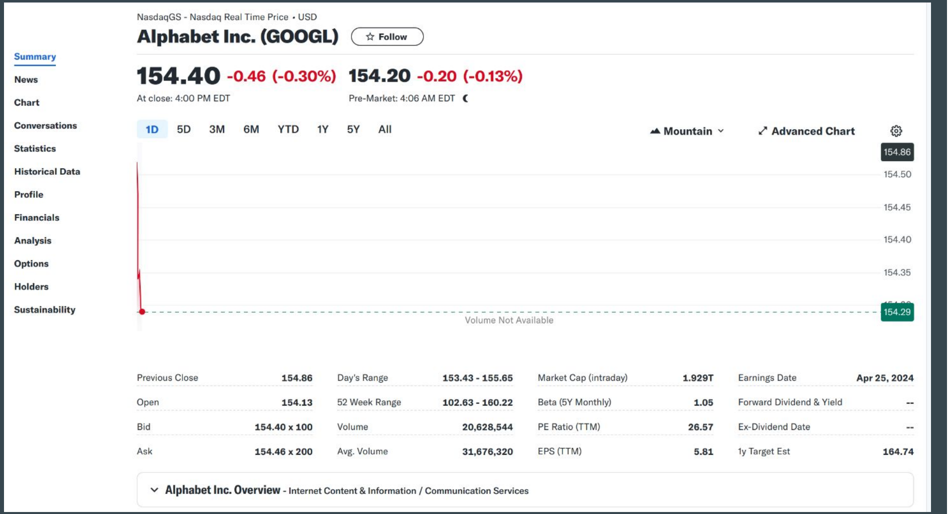
Stock Selection

Alphabet Inc Class A (GOOGL)

The closing price dataset over the last 1 year for the chosen stock was taken from Yahoo Finance.

Stock's History
(Yahoo Finance)



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NasdaqGS - Nasdaq Real Time Price • USD

Alphabet Inc. (GOOGL) ☆ Follow

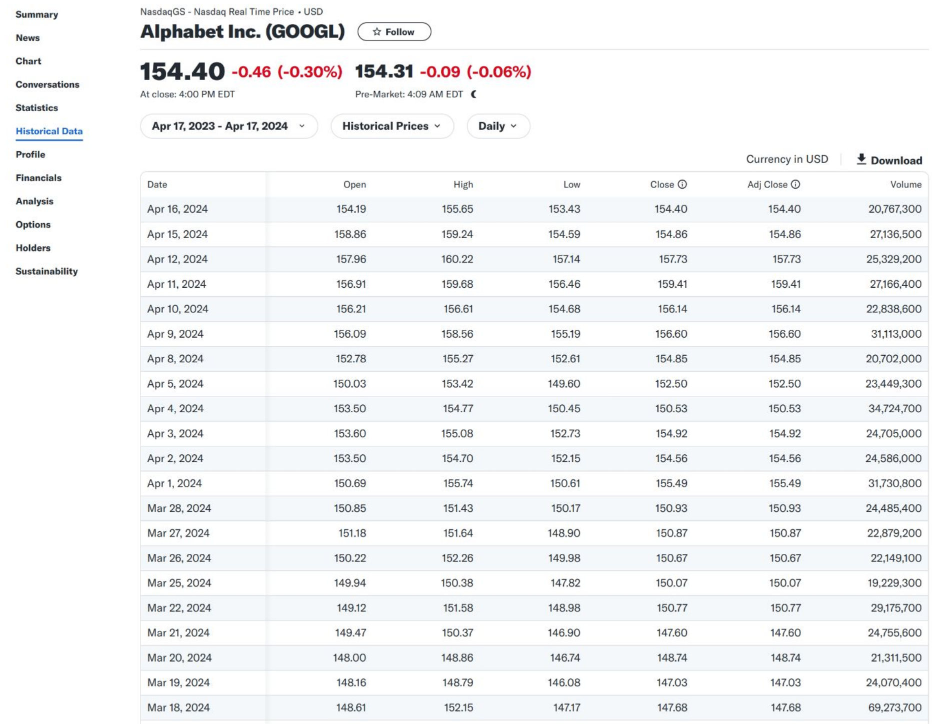
154.40 -0.46 (-0.30%) **154.20** -0.20 (-0.13%)

At close: 4:00 PM EDT Pre-Market: 4:06 AM EDT

1D 5D 3M 6M YTD 1Y 5Y All Mountain Advanced Chart

Previous Close	154.86	Day's Range	153.43 - 155.65	Market Cap (intraday)	1.929T	Earnings Date	Apr 25, 2024
Open	154.13	52 Week Range	102.63 - 160.22	Beta (5Y Monthly)	1.05	Forward Dividend & Yield	--
Bid	154.40 x 100	Volume	20,628,544	PE Ratio (TTM)	26.57	Ex-Dividend Date	--
Ask	154.46 x 200	Avg. Volume	31,676,320	EPS (TTM)	5.81	1y Target Est	164.74

Alphabet Inc. Overview - Internet Content & Information / Communication Services



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NasdaqGS - Nasdaq Real Time Price • USD

Alphabet Inc. (GOOGL) ☆ Follow

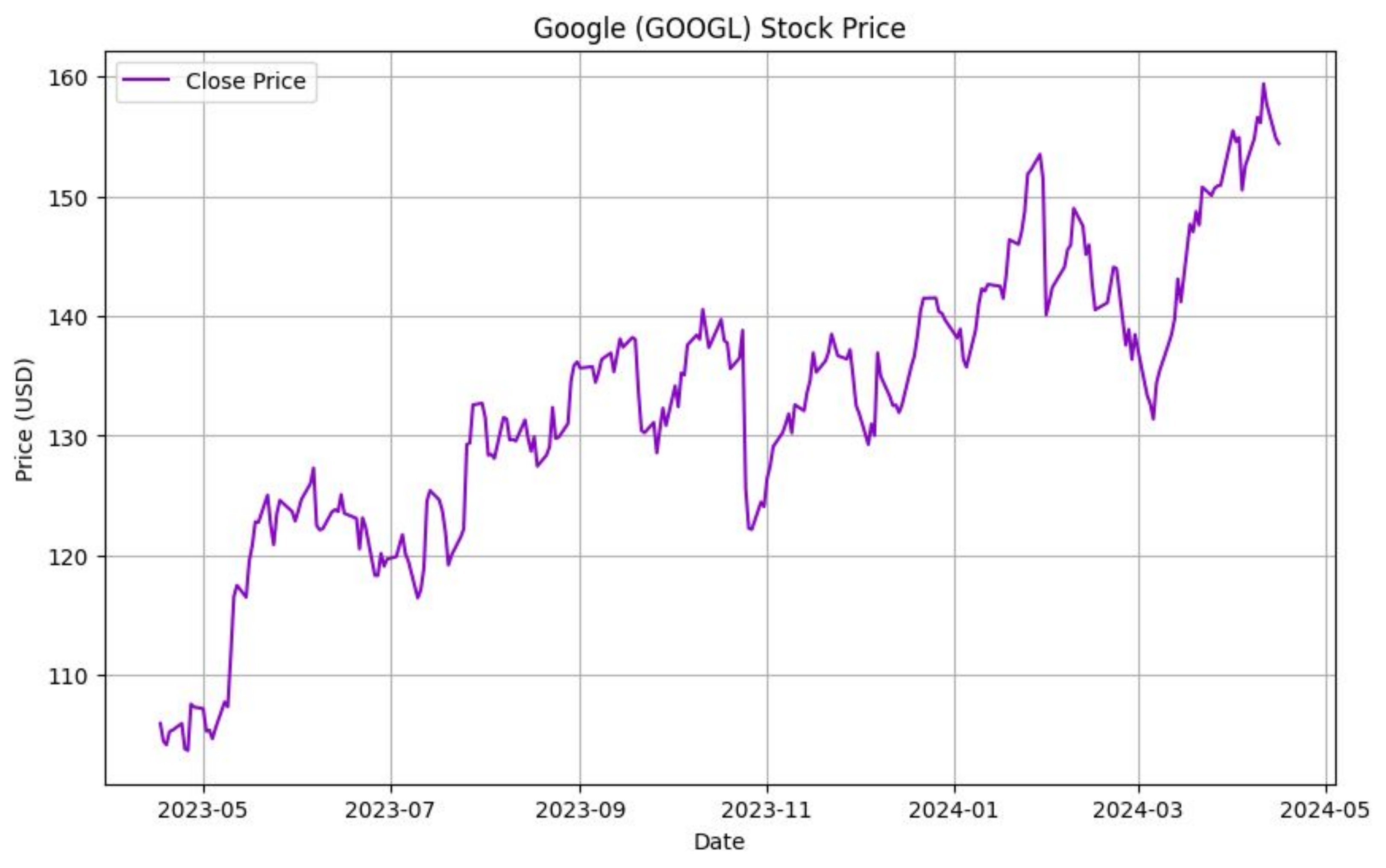
154.40 -0.46 (-0.30%) **154.31** -0.09 (-0.06%)

At close: 4:00 PM EDT Pre-Market: 4:09 AM EDT

Apr 17, 2023 - Apr 17, 2024 | Historical Prices | Daily

Currency in USD Download

Date	Open	High	Low	Close	Adj Close	Volume
Apr 16, 2024	154.19	155.65	153.43	154.40	154.40	20,767,300
Apr 15, 2024	158.86	159.24	154.59	154.86	154.86	27,136,500
Apr 12, 2024	157.96	160.22	157.14	157.73	157.73	25,329,200
Apr 11, 2024	156.91	159.68	156.46	159.41	159.41	27,166,400
Apr 10, 2024	156.21	156.61	154.68	156.14	156.14	22,838,600
Apr 9, 2024	156.09	158.56	155.19	156.60	156.60	31,113,000
Apr 8, 2024	152.78	155.27	152.61	154.85	154.85	20,702,000
Apr 5, 2024	150.03	153.42	149.60	152.50	152.50	23,449,300
Apr 4, 2024	153.50	154.77	150.45	150.53	150.53	34,724,700
Apr 3, 2024	153.60	155.08	152.73	154.92	154.92	24,705,000
Apr 2, 2024	153.50	154.70	152.15	154.56	154.56	24,586,000
Apr 1, 2024	150.69	155.74	150.61	155.49	155.49	31,730,800
Mar 28, 2024	150.85	151.43	150.17	150.93	150.93	24,485,400
Mar 27, 2024	151.18	151.64	148.90	150.87	150.87	22,879,200
Mar 26, 2024	150.22	152.26	149.98	150.67	150.67	22,149,100
Mar 25, 2024	149.94	150.38	147.82	150.07	150.07	19,229,300
Mar 22, 2024	149.12	151.58	148.98	150.77	150.77	29,175,700
Mar 21, 2024	149.47	150.37	146.90	147.60	147.60	24,755,600
Mar 20, 2024	148.00	148.86	146.74	148.74	148.74	21,311,500
Mar 19, 2024	148.16	148.79	146.08	147.03	147.03	24,070,400
Mar 18, 2024	148.61	152.15	147.17	147.68	147.68	69,273,700



Google (GOOGL) Stock Price
Close Price
160
150
140
130
120
110
Price (USD)
2023-05
2023-07
2023-09
2023-11
2024-01
2024-03
2024-05
Date

Annual Volatility ≈ 0.27

$$volatility_{annual} = \sigma_{returns} \times \sqrt{(252)}$$

10 Years U.S. Treasury Rate

Treasury Yields

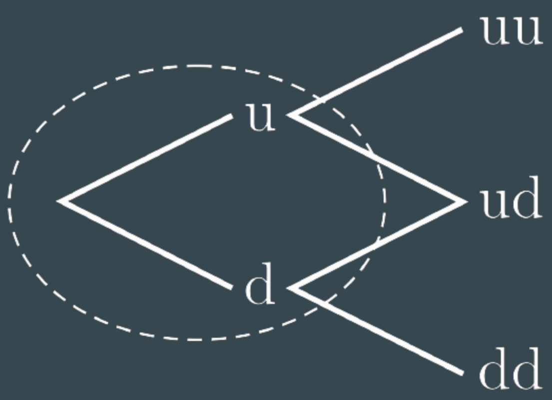
NAME	COUPON	PRICE	YIELD	1 MONTH	1 YEAR	TIME (EDT)
GB3:GOV **3 Month**	0.00	5.18	5.33%	-3	+31	4:22 AM
GB6:GOV **6 Month**	0.00	5.15	5.36%	+4	+32	1:44 AM
GB12:GOV **12 Month**	0.00	4.91	5.17%	+12	+36	4:08 AM
GT2:GOV **2 Year**	4.50	99.15	4.96%	+24	+77	4:22 AM
GT5:GOV **5 Year**	4.13	97.59	4.67%	+35	+97	4:21 AM
GT10:GOV **10 Year**	4.00	94.92	4.52%	+34	+105	4:21 AM
GT30:GOV **30 Year**	4.25	92.02	4.75%	+32	+94	4:22 AM

Rate of Interest
4.52%

Source: *https://www.bloomberg.com/markets/rates-bonds/government-bonds/us*

Binomial Model

- It values options using an iterative approach utilizing multiple periods to value options.
- With the model, there are two possible outcomes with each iteration — **a move up** or **a move down** that follow a binomial tree.



Binomial Model

$$\tilde{\boldsymbol{p}} = (e^{\mathrm{r}\Delta \mathrm{t}} - d) / (u - d)$$

$$\boldsymbol{\Delta t} = T / n$$

$$\boldsymbol{u} = e^{\sqrt{(\sigma \Delta t)}}$$

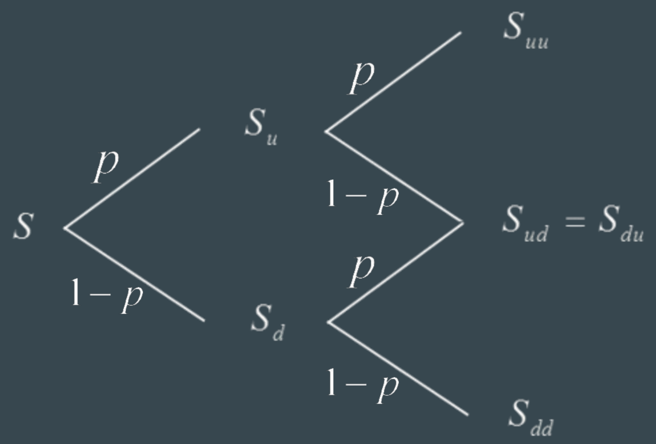
$$\boldsymbol{d} = e^{-\sqrt{(\sigma \Delta t)}}$$

$\boldsymbol{T}$: time of maturity (in years)
$\boldsymbol{r}$: risk-free interest rate
$\boldsymbol{\sigma}$: annual volatility of the underlying asset
$\boldsymbol{n}$: number of time steps

Binomial Model

$$\begin{cases} S^{\mathrm{u}} = S(0)(u) \\ S^{\mathrm{d}} = S(0)(d) \end{cases}$$

Call Option	$(S(T) - K)^{+}$
Put Option	$(K - S(T))^{+}$



Binomial Model

Call Option Pricing Formula

$$C = \sum_{n=0}^{N} \binom{N}{n} p^n (1-p)^{N-n} \quad (S \quad u^n \cdot d^{N-n} - K)^{+} \cdot e^{-rT}$$

Put Option Pricing Formula

$$P = \sum_{n=0}^{N} \binom{N}{n} p^n (1-p)^{N-n} \quad (K - S \cdot u^n \quad d^{N-n})^{+} \quad e^{-rT}$$

Put-Call Parity

This relation holds between the prices of European call and put options, both with exercise price X and exercise time T, for a stock that pays no dividends.

$$C^{\mathrm{E}} - P^{\mathrm{E}} = S(0) - X\mathrm{e}^{-rT}$$

Call Option Price Evaluation (Binomial Model)

K	T = 1 month	T = 5 months	T = 9 months
85.97	20.3232	22.3536	25.147
88.97	17.3345	20.1237	22.9427
91.97	14.3458	17.8937	20.7383
94.97	11.3719	15.6637	18.534
97.97	9.1205	13.4337	16.3297
100.97	6.869	11.2037	14.1254
103.97	4.6176	8.9737	11.921
106.97	2.8643	7.2343	10.1998
109.97	2.1072	6.476	9.4449
112.97	1.35	5.7177	8.69
115.97	0.5929	4.9594	7.935
118.97	0	4.2011	7.1801
121.97	0	3.4428	6.4252
124.97	0	2.6845	5.6703

Put Option Price Evaluation (Binomial Model)

K	T = 1 month	T = 5 months	T = 9 months
85.97	0	0.7497	2.2814
88.97	0	1.4637	2.9771
91.97	0	2.1778	3.6728
94.97	0.0148	2.8918	4.3685
97.97	0.7521	3.6059	5.0642
100.97	1.4894	4.3199	5.7598
103.97	2.2267	5.034	6.4555
106.97	3.4621	6.2386	7.6343
109.97	5.6937	8.4243	9.7794
112.97	7.9253	10.61	11.9245
115.97	10.1569	12.7958	14.0696
118.97	12.5527	14.9815	16.2146
121.97	15.5414	17.1672	18.3597
124.97	18.5302	19.3529	20.5048

Black Scholes Model

It is a differential equation widely used to price options contracts.

The model requires five input variables: the strike price of an option, the current stock price, the time to expiration, the risk-free rate, and the volatility.

Black Scholes Model

$$C = N(d_1)S \ - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln\frac{S}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

where,

- $\boldsymbol{C}$: call option price
- $\boldsymbol{N}$: CDF of normal distribution
- $\boldsymbol{S}$: current asset price
- $\boldsymbol{K}$: strike price
- $\boldsymbol{r}$: risk-free interest rate
- $\boldsymbol{T}$: time of maturity
- $\boldsymbol{\sigma}$: volatility of asset

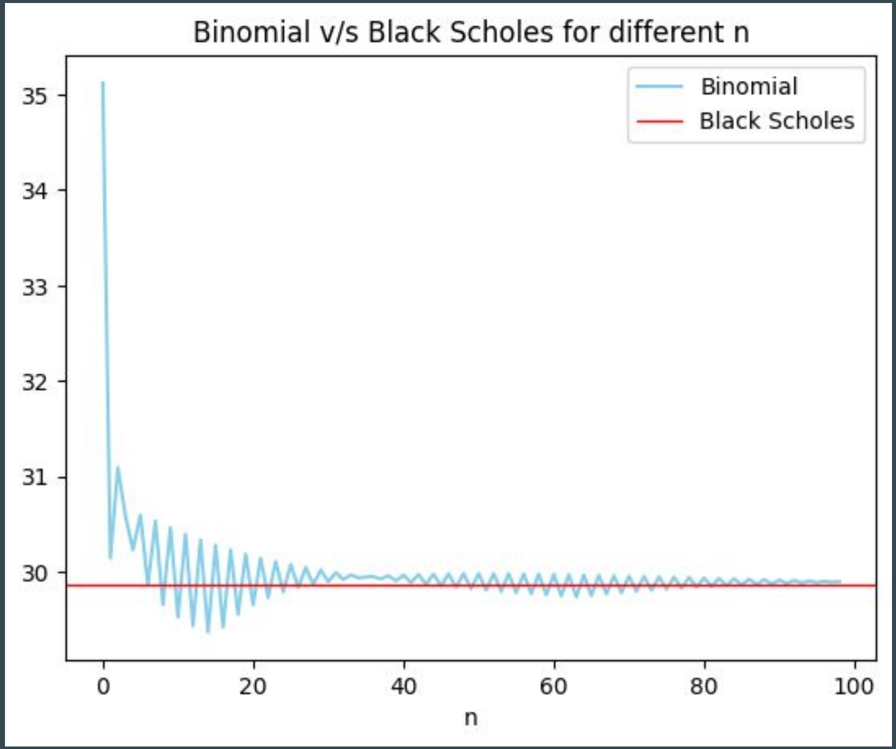
Call Option Price Evaluation (Black Scholes Model)

K	T = 1 month	T = 5 months	T = 9 months
85.97	20.3303	22.3395	24.5531
88.97	17.3628	19.782	22.228
91.97	14.4384	17.3589	20.0261
94.97	11.6105	15.0907	17.956
97.97	8.9617	12.9945	16.0239
100.97	6.5917	11.0825	14.2334
103.97	4.5912	9.3616	12.5857
106.97	3.0133	7.833	11.0797
109.97	1.8575	6.4929	9.7123
112.97	1.0736	5.333	8.4786
115.97	0.5815	4.3415	7.3724
118.97	0.2954	3.5039	6.3861
121.97	0.1409	2.8045	5.5117
124.97	0.0633	2.2268	4.7405

Put Option Price Evaluation (Black Scholes Model)

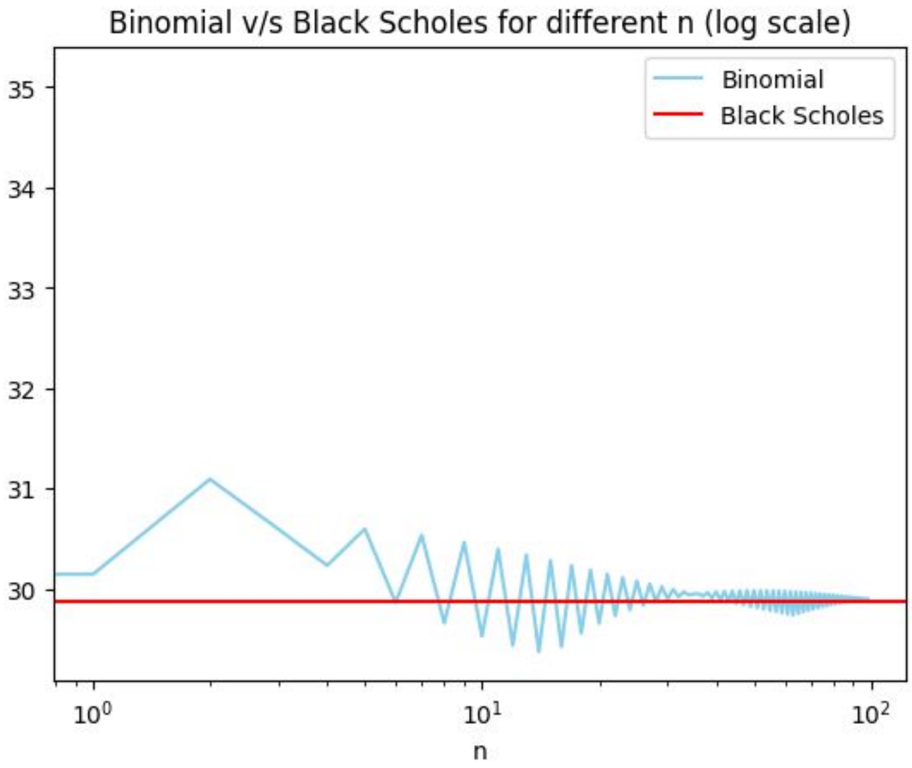
K	T = 1 month	T = 5 months	T = 9 months
85.97	0.0071	0.7355	1.6876
88.97	0.0283	1.1221	2.2624
91.97	0.0926	1.643	2.9606
94.97	0.2535	2.3189	3.7905
97.97	0.5934	3.1667	4.7584
100.97	1.2121	4.1987	5.8678
103.97	2.2003	5.4218	7.1201
106.97	3.6112	6.8372	8.5142
109.97	5.4441	8.4412	10.0468
112.97	7.6489	10.2253	11.7132
115.97	10.1455	12.1778	13.5069
118.97	12.8481	14.2843	15.4206
121.97	15.6824	16.5289	17.4462
124.97	18.5934	18.8952	19.5751

↑sing number of steps (Binomial Model)



for:

$$S = 200, K = 220 \ \& \ T = 2 \text{ years}$$



Increasing the number of steps in Binomial Model

As the number of steps in the binomial tree increases, the model converges towards the Black-Scholes model, which is a continuous-time model for option pricing.

This highlights the flexibility of the Binomial Option Pricing Model, allowing for a more accurate valuation as the number of steps approaches infinity

Comparison with **Actual Market** Data

Actual Market Data (Yahoo Finance)

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154.40 -0.46 (-0.30%) **154.31** -0.09 (-0.06%)

At close: 4:00 PM EDT Pre-Market: 4:09 AM EDT

Apr 19, 2024 | All Strike Prices | List | All Options

Calls

In The Money

Contract Name	Last Trade Date (EDT)	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
GOOGL240419C00005000	4/16/2024 6:34 PM	5	149.80	0.00	0.00	0.00	0.00%	1	0	0.00%
GOOGL240419C00035000	2/27/2024 7:16 PM	35	103.96	113.75	118.45	0.00	0.00%	-	8	0.00%
GOOGL240419C00045000	4/15/2024 1:56 PM	45	113.44	0.00	0.00	0.00	0.00%	1	0	0.00%
GOOGL240419C00050000	4/4/2024 6:52 PM	50	102.50	0.00	0.00	0.00	0.00%	20	0	0.00%
GOOGL240419C00055000	3/4/2024 8:51 PM	55	78.69	99.05	100.50	0.00	0.00%	5	5	564.84%
GOOGL240419C00060000	3/4/2024 8:48 PM	60	73.89	93.85	95.50	0.00	0.00%	5	13	496.48%
GOOGL240419C00065000	11/17/2023 4:27 PM	65	70.70	68.30	69.35	0.00	0.00%	1	0	0.00%
GOOGL240419C00070000	4/15/2024 3:43 PM	70	88.50	0.00	0.00	0.00	0.00%	5	0	0.00%
GOOGL240419C00075000	3/7/2024 5:25 PM	75	59.73	77.40	77.95	0.00	0.00%	1	3	0.00%
GOOGL240419C00080000	4/8/2024 5:29 PM	80	74.55	0.00	0.00	0.00	0.00%	2	0	0.00%
GOOGL240419C00085000	4/16/2024 7:53 PM	85	69.74	0.00	0.00	0.00	0.00%	2	0	0.00%
GOOGL240419C00090000	3/26/2024 5:45 PM	90	62.13	0.00	0.00	0.00	0.00%	1	0	0.00%
GOOGL240419C00095000	4/15/2024 1:30 PM	95	63.37	0.00	0.00	0.00	0.00%	1	0	0.00%
GOOGL240419C00100000	4/15/2024 5:46 PM	100	56.01	0.00	0.00	0.00	0.00%	22	0	0.00%
GOOGL240419C00105000	4/15/2024 3:47 PM	105	53.50	0.00	0.00	0.00	0.00%	5	0	0.00%
GOOGL240419C00110000	4/16/2024 3:51 PM	110	45.00	0.00	0.00	0.00	0.00%	1	0	0.00%
GOOGL240419C00115000	4/16/2024 6:28 PM	115	39.72	0.00	0.00	0.00	0.00%	1	0	0.00%

Option Price Data

Expiry Date: **April 19th 2024**

Last trade dates lying in the last 1 year: between **28th March 2023** & **1st April 2024**.

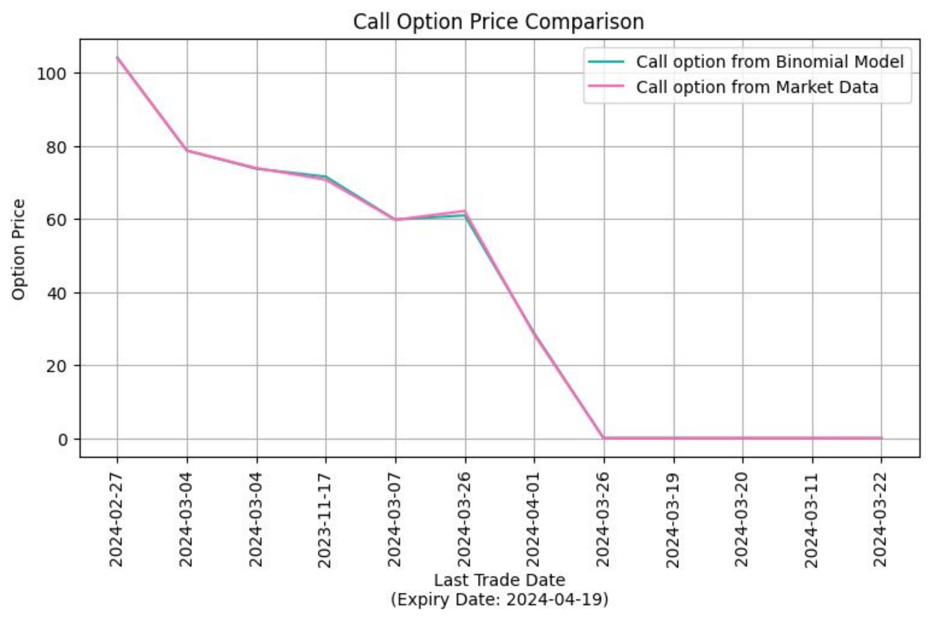
Calculations

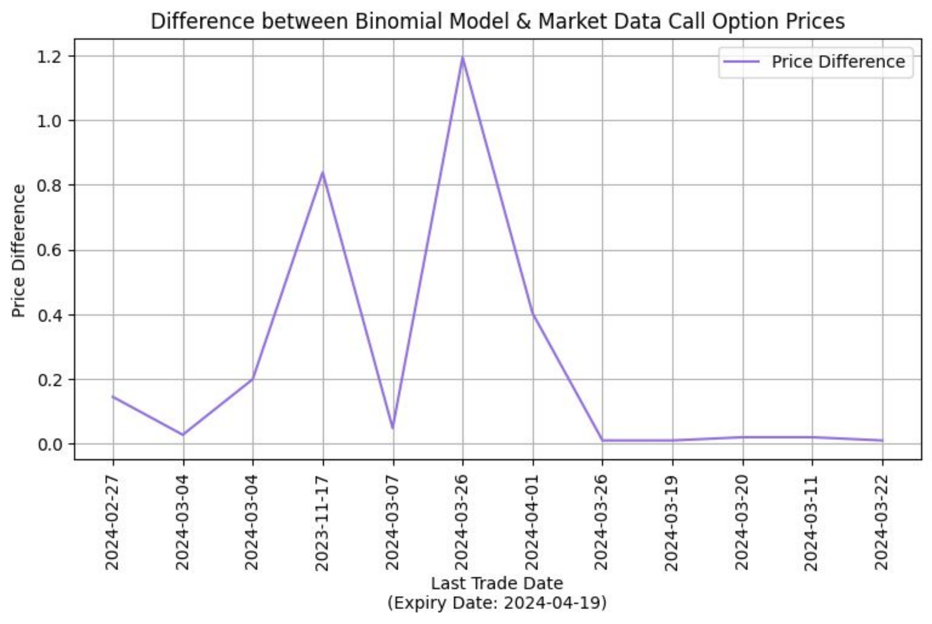
Initial Stock Price (S)	Calculated using the closing price for the corresponding date in the overall stock data for the given last trade date.
Maturity Period (T)	(Expiry Date - Last Trade Date) / 365.
Strike Price (K)	Corresponding to the given option.
Time Steps (n)	Number of days in the maturity period.

Call Option Prices

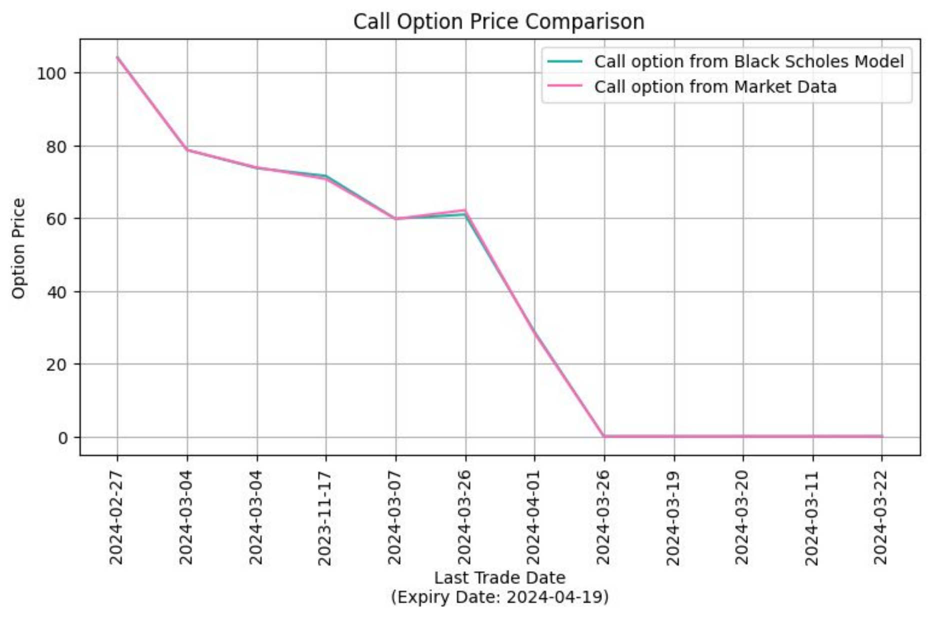
Comparisons

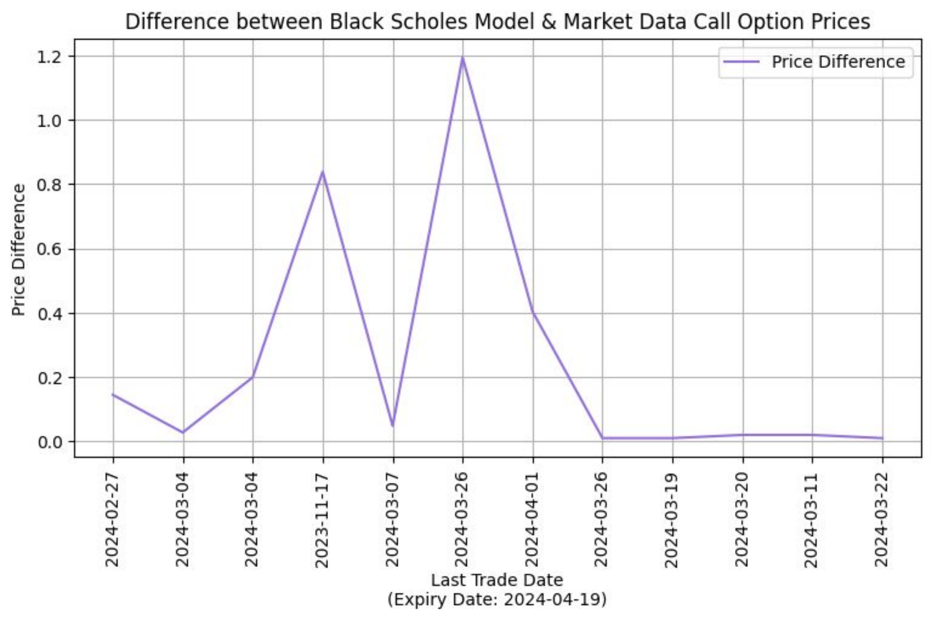
Call Option Prices Comparison (Binomial Model)

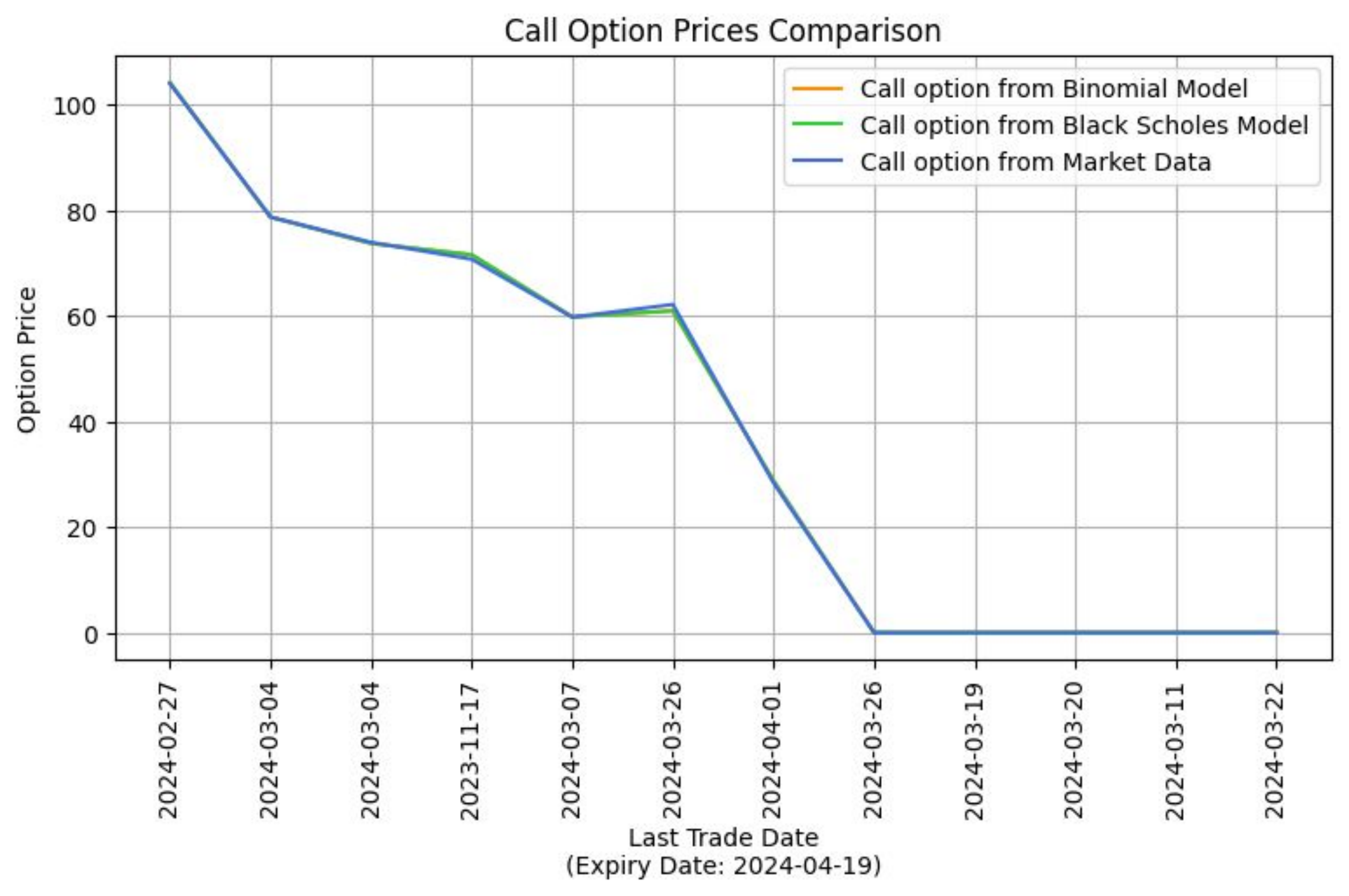




Call Option Prices Comparison (Black Scholes Model)





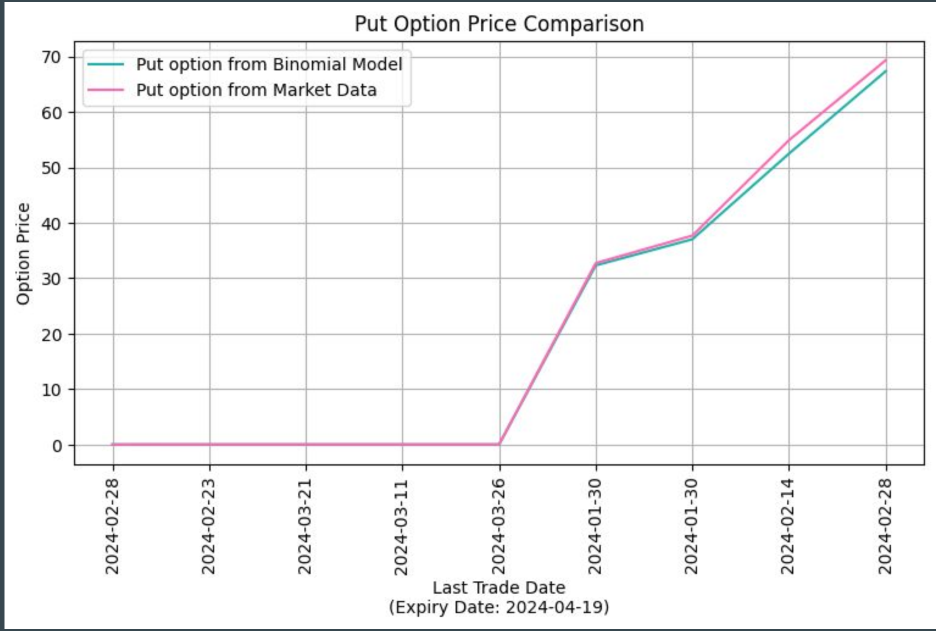


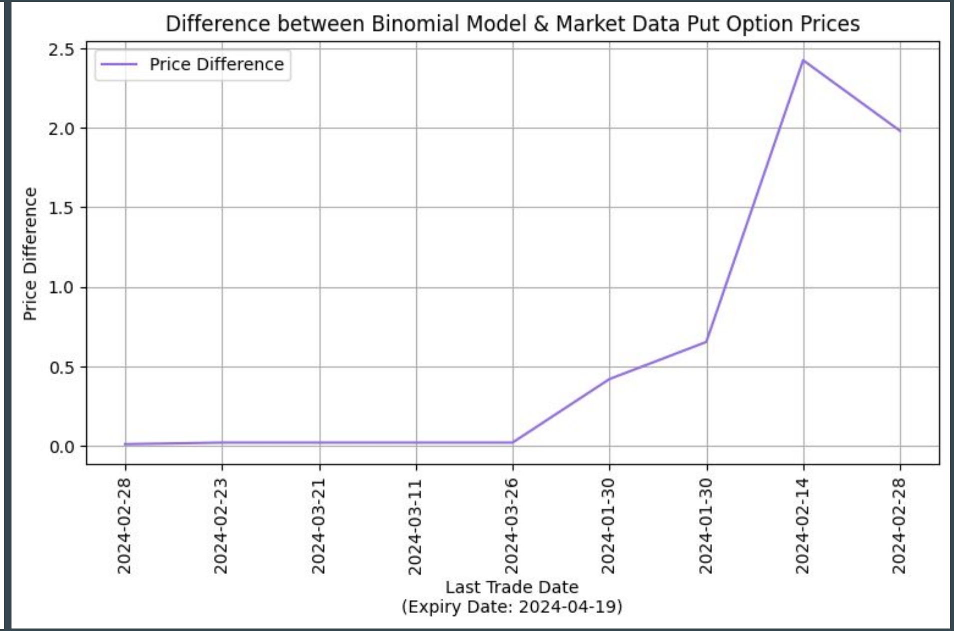
Call Option Prices Comparison
Call option from Binomial Model
Call option from Black Scholes Model
Call option from Market Data
Option Price
100
80
60
40
20
0
2024-02-27
2024-03-04
2024-03-04
2023-11-17
2024-03-07
2024-03-26
2024-04-01
2024-03-26
2024-03-19
2024-03-20
2024-03-11
2024-03-22
Last Trade Date
(Expiry Date: 2024-04-19)

Put Option Prices

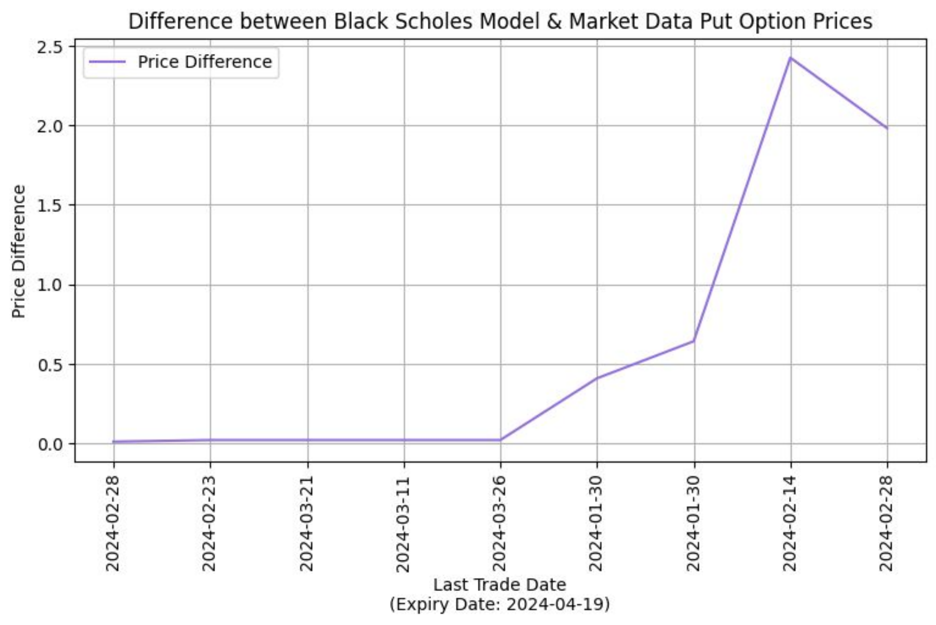
Comparisons

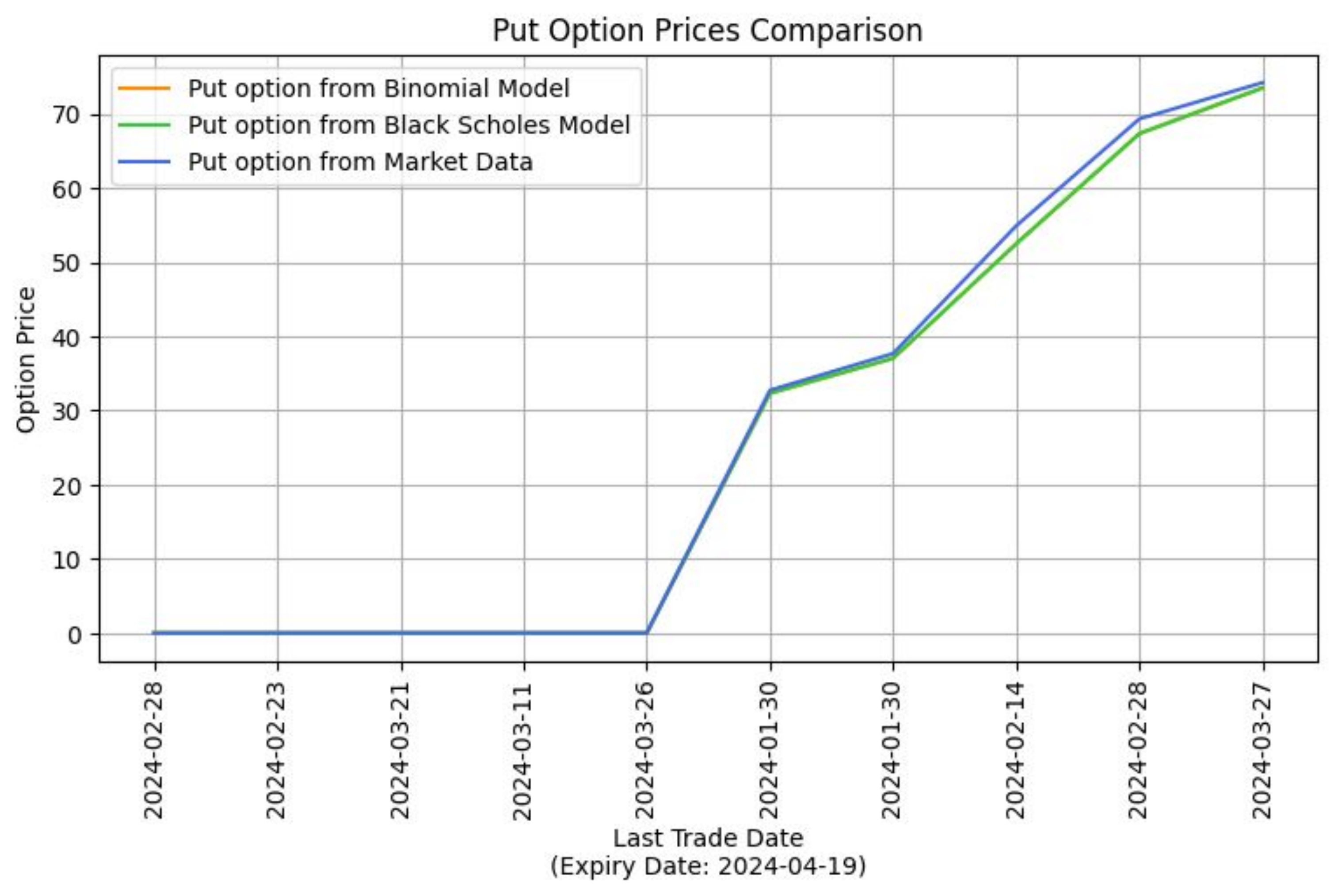
Put Option Prices Comparison (Binomial Model)





Put Option Prices Comparison (Black Scholes Model)




Put Option Prices Comparison
Put option from Binomial Model
Put option from Black Scholes Model
Put option from Market Data
Option Price
0
10
20
30
40
50
60
70
2024-02-28
2024-02-23
2024-03-21
2024-03-11
2024-03-26
2024-01-30
2024-01-30
2024-02-14
2024-02-28
2024-03-27
Last Trade Date
(Expiry Date: 2024-04-19)

Delta Neutral Portfolio

Delta Neutral Portfolio

It is a portfolio strategy that uses multiple positions to balance positive and negative deltas so the overall delta of the assets totals zero.

A delta-neutral portfolio evens out the response to market movements for a certain range to bring the net change of the position to zero.

Delta Calculation

For the European call option price in the Black–Scholes model given by $C^E(S)$, the delta is given by:

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}S} C^{\mathrm{E}}(S) = N(d_1)}$$

Portfolio Calculation

$$Call\ Price = S_0N(d_1) - Ke^{-rT}N(d_2)$$

$$\frac{\delta c}{\delta S_0} = \frac{\delta}{\delta S_0}[S_0N(d_1) - Ke^{-rT}N(d_2)]$$

$$\frac{\delta c}{\delta S_0} = (1)N(d_1) - 0$$

$$\frac{\delta c}{\delta S_0} = N(d_1)$$

$$Put\ Price = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$

$$\frac{\delta p}{\delta S_0} = \frac{\delta}{\delta S_0}[Ke^{-rT}N(-d_2) - S_0N(-d_1)]$$

$$\frac{\delta p}{\delta S_0} = 0 - (1)N(-d_1)$$

$$\frac{\delta p}{\delta S_0} = -N(-d_1) = -(1 - N(d_1))$$

$$\frac{\delta p}{\delta S_0} = -1 + N(d_1)$$

$$\frac{\delta p}{\delta S_0} = -N(d_1)$$

Portfolio (Number of Options) : (Batch Size X Delta) / (1 - Delta)

Portfolio (Call Options)

Strike Price (K)	Maturity Period (T in days)	Risk-Free Rate (r)	Volatility (σ)	Call Delta	Portfolio (# put options)
35	52	0.0452	0.26969	0.998072	5176.001406
55	46	0.0452	0.26969	0.994385	1770.995286
60	46	0.0452	0.26969	0.993575	1546.419732
65	154	0.0452	0.26969	0.999964	280974.6344
75	43	0.0452	0.26969	0.989639	955.202046
90	24	0.0452	0.26969	0.969375	316.525459
127	18	0.0452	0.26969	0.927862	128.623379
215	24	0.0452	0.26969	0.887352	78.77188
220	31	0.0452	0.26969	0.921548	117.466668
225	30	0.0452	0.26969	0.915644	108.545862
230	39	0.0452	0.26969	0.943408	166.703815
240	28	0.0452	0.26969	0.898779	88.793906

Portfolio (Put Options)

Strike Price (K)	Maturity Period (T in days)	Risk-Free Rate (r)	Volatility (σ)	Put Delta	Portfolio (# call options)
60	51	0.0452	0.26969	-0.004852	0.048754
65	56	0.0452	0.26969	-0.003939	0.039542
70	29	0.0452	0.26969	-0.016081	0.163443
75	39	0.0452	0.26969	-0.012244	0.123955
90	24	0.0452	0.26969	-0.030625	0.31593
185	80	0.0452	0.26969	-0.004368	0.043871
190	80	0.0452	0.26969	-0.004512	0.045321
200	65	0.0452	0.26969	-0.010911	0.110312
205	51	0.0452	0.26969	-0.025691	0.263686
225	23	0.0452	0.26969	-0.12684	1.452658

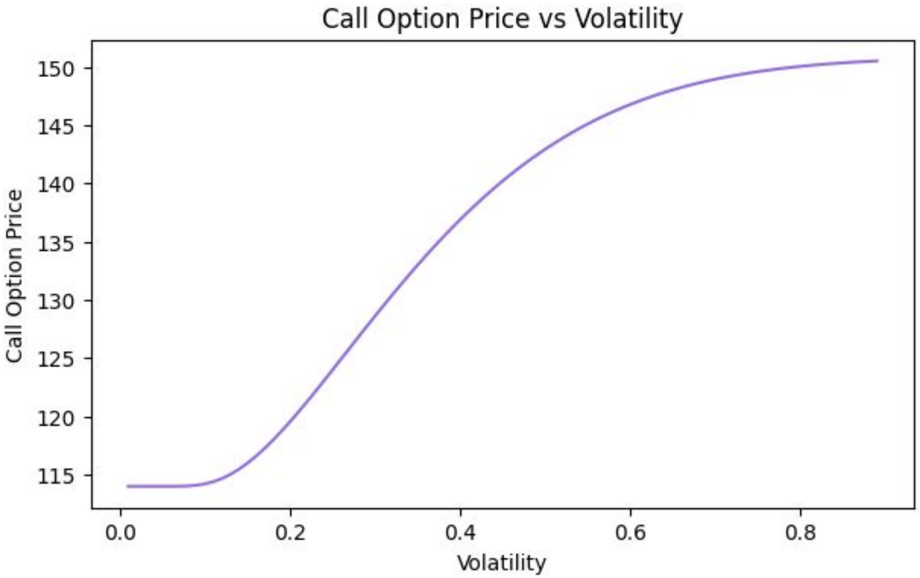
Implied Volatility

Implied Volatility

It is derived from the Black-Scholes formula, and using it can provide significant benefits to investors.

It is an estimate of the future variability for the asset underlying the options contract.

from the actual market data:



Implied Volatility

The objective is to find the volatility level so that the Black-Scholes price is equal to the observed option price.

$$Call_{BS}(\sigma_{implied}) = P$$

We find the root of the function Black-Scholes price minus observed price.

$$f(x) = Call_{BS}(x) - P$$

Newton-Raphson Method

It is a simple algorithm to find the root of a function.

x_0 is the initial guess.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Then, the function is estimated by its tangent line, and the new estimate is the x-intercept of this tangent line.

Newton-Raphson Method

Then a tolerance level is fixed and the algorithm is iterated until the difference between two consecutive estimations is below this level.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\text{until } \left|x_{n+1} - x_n\right| \leq \varepsilon$$

Newton-Raphson Method

The Newton-Raphson method is preferred over brute force methods for calculating implied volatility because it is significantly faster and more accurate.

The method iteratively refines the guess for the implied volatility by using the derivative of the function

Implied Volatility Calculation

The root of the (square of the difference between the price predicted by the Black Scholes Formula & the actual market price) gives the implied volatility.

f (.) = (predicted option price - actual market price)^2

Implied Volatility = root of f .

Implied Volatility (Call Options)

Strike Price (K)	Call Option Price (C)	Time to Maturity (T in days)	Initial Price (S0)	Risk-Free Rate (r)	Implied Volatility (%)
35	103.96	52	138.880005	0.0452	0
55	78.69	46	133.350006	0.0452	92.549613
60	73.89	46	133.350006	0.0452	106.416472
65	70.7	154	135.309998	0.0452	0
75	59.73	43	134.380005	0.0452	0
90	62.13	24	150.669998	0.0452	135.459682
127	28.37	18	155.490005	0.0452	0
215	0.01	24	150.669998	0.0452	47.049792
220	0.01	31	147.029999	0.0452	46.229409
225	0.02	30	148.740005	0.0452	51.195372
230	0.02	39	137.669998	0.0452	54.646775
240	0.01	28	150.770004	0.0452	55.438197

Implied Volatility (Put Options)

Strike Price (K)	Put Option Price (P)	Time to Maturity (T in days)	Initial Price (S0)	Risk-Free Rate (r)	Implied Volatility (%)
65	0.02	56	143.960007	0.0452	73.036732
70	0.02	29	147.600006	0.0452	94.986516
75	0.02	39	137.669998	0.0452	68.321246
90	0.02	24	150.669998	0.0452	73.945618
185	32.73	80	151.460007	0.0452	30.530998
190	37.68	80	151.460007	0.0452	33.555262
200	54.9	65	145.940002	0.0452	60.743598
205	69.31	51	136.380005	0.0452	79.217592
225	74.2	23	150.869995	0.0452	91.311712

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THANK YOU!