Introduction to Financial Engineering

Markowitz Portfolio Optimization

Team Members

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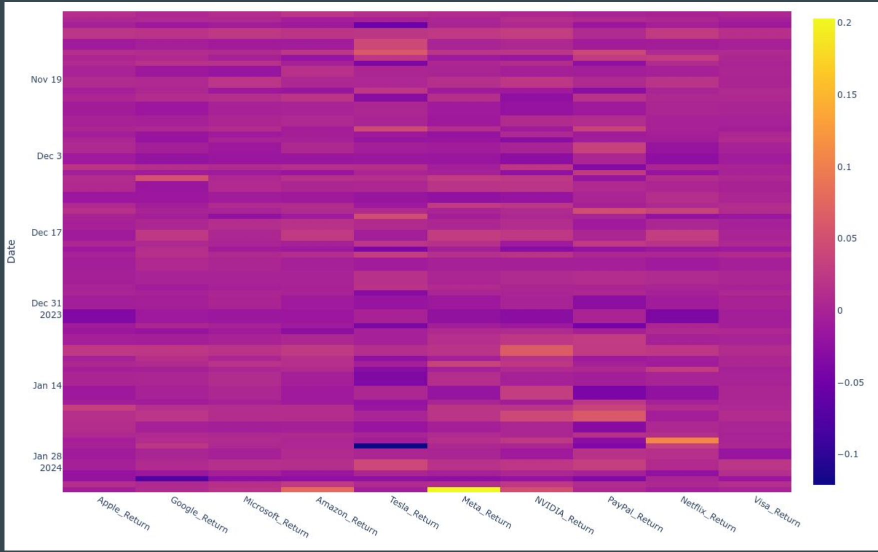
Asset Selection

The closing price data (over the past 3 months) of the following 10 selected risky assets was taken from **Yahoo Finance**.

Apple Inc. **(AAPL)**	Alphabet Inc. **(GOOGL)**	Microsoft Corporation **(MSFT)**	Amazon.com, Inc. **(AMZN)**	Tesla, Inc. **(TSLA)**
Meta Platforms, Inc. **(META)**	NVIDIA Corporation **(NVDA)**	PayPal Holdings, Inc. **(PYPL)**	Netflix, Inc. **(NFLX)**	Visa Inc. **(V)**

Return Calculation



Visualization showing the returns for each company (asset)

Percentage returns for each asset on each date were calculated by determining the percentage change in the values across a series.

Return Calculation

Date	Apple_Return	Google_Return	Microsoft_Return	Amazon_Return	Tesla_Return	Meta_Return	NVIDIA_Return	PayPal_Return	Netflix_Return	Visa_Return
2024-01-22	0.012163	-0.002664	-0.005418	-0.003605	-0.015976	-0.004355	0.002740	-0.031601	0.005715	0.001107
2024-01-23	0.006653	0.007192	0.006028	0.008011	0.001628	0.008958	0.003671	0.016316	0.013341	0.000221
2024-01-24	-0.003484	0.011289	0.009175	0.005448	-0.006264	0.014278	0.024869	-0.027169	0.107032	0.001438
2024-01-25	-0.001697	0.021318	0.005738	0.005610	-0.121253	0.006348	0.004156	-0.036655	0.031439	0.003534
2024-01-26	-0.009013	0.002107	-0.002322	0.008685	0.003395	0.002442	-0.009510	0.017625	0.014982	-0.017131
2024-01-29	-0.003586	0.008673	0.014334	0.013449	0.041910	0.017456	0.023496	0.032049	0.009414	0.021348
2024-01-30	-0.019246	-0.013354	-0.002758	-0.014015	0.003457	-0.002394	0.004947	-0.001255	-0.022473	0.012753
2024-01-31	-0.019358	-0.075003	-0.026946	-0.023899	-0.022444	-0.024796	-0.019865	-0.036589	0.002239	-0.014036
2024-02-01	0.013341	0.007566	0.015594	0.026289	0.008383	0.011893	0.024380	0.010921	0.006027	0.013869
2024-02-02	-0.005405	0.008643	0.018426	0.078666	-0.005030	0.203176	0.049709	0.006449	-0.005057	0.000469

Dataframe showing the returns for each company (asset)

Return Calculation

Visualization showing company (asset) returns over time

Risk Measure Calculation

Apple	0.011458
Google	0.016344
Microsoft	0.010810
Amazon	0.016321
Tesla	0.027877
Meta	0.029283
NVIDIA	0.019487
PayPal	0.023020
Netflix	0.019923
Visa	0.007228

Each asset's risk was determined using its percentage returns' standard deviation.

MARKOWITZ MEAN-VARIANCE OPTIMIZATION

Markowitz Mean-Variance Optimization

The Markowitz Mean-Variance Optimization Model is a mathematical framework first introduced by the economist **Harry Markowitz** in 1952.

It is based on the idea that investors are **highly averse to risk** and will only accept more risk if compensated by **higher expected returns**.

Initialization

Initially, the mean (μ) returns and the covariance (Σ) matrix were calculated based on the complete asset return dataset.

The covariance matrix served as the "Risk Model" in this case.

Optimization Problem Formulation

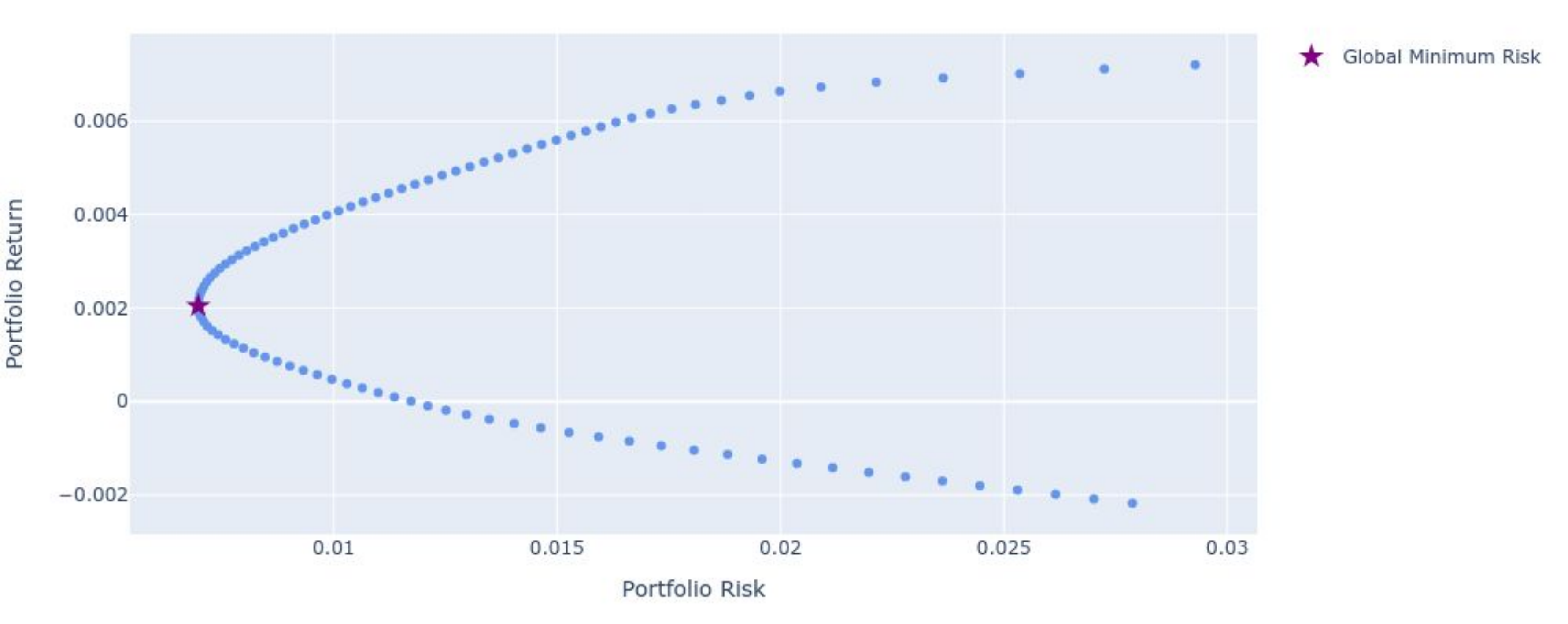
Minimize the portfolio risk with the constraint expressing a lower bound on the portfolio return:

$$
\begin{array}{lrcl}
\text{minimize} & \mathbf{x}^\top \boldsymbol{\Sigma} \mathbf{x} & & \\
\text{subject to} & \boldsymbol{\mu}^\top \mathbf{x} & \geq & r_{\min}, \\
 & \mathbf{1}^\top \mathbf{x} & = & 1.
\end{array}
$$

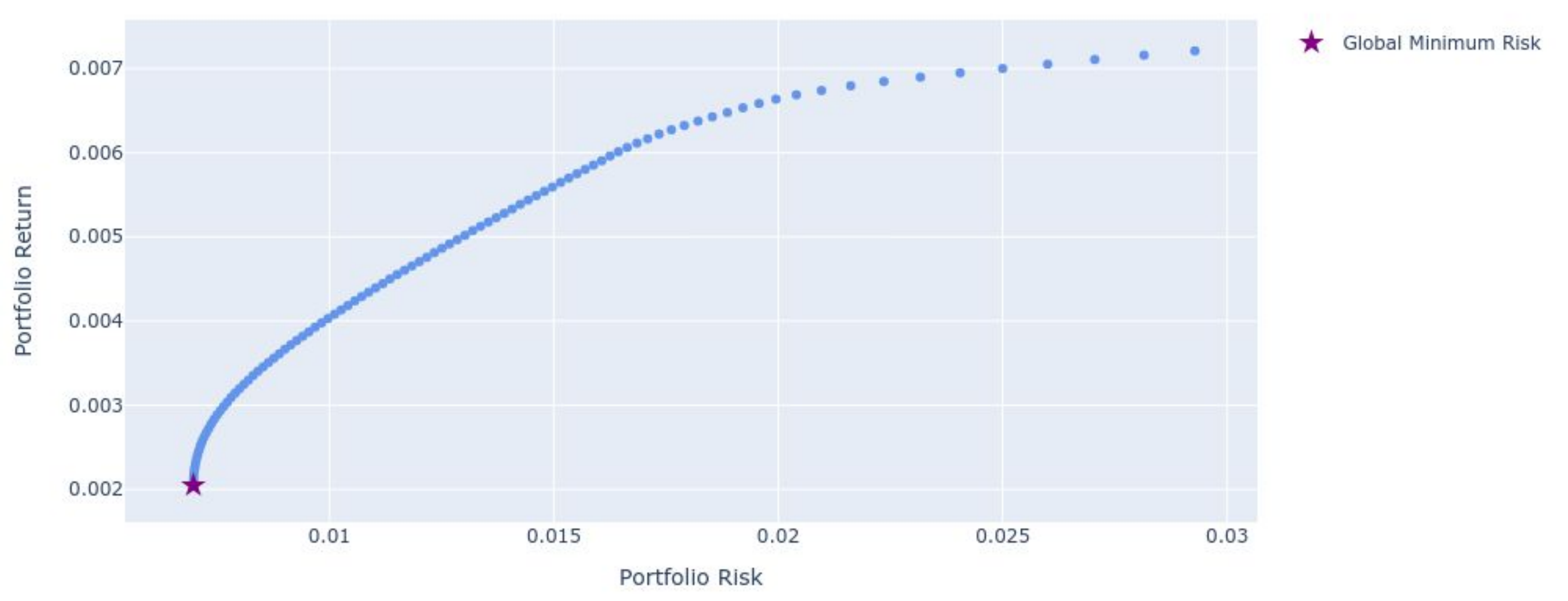
Here $r_{\min}$ is the target expected return that the investor wishes to reach.

It is a quadratic optimization (QO) problem.

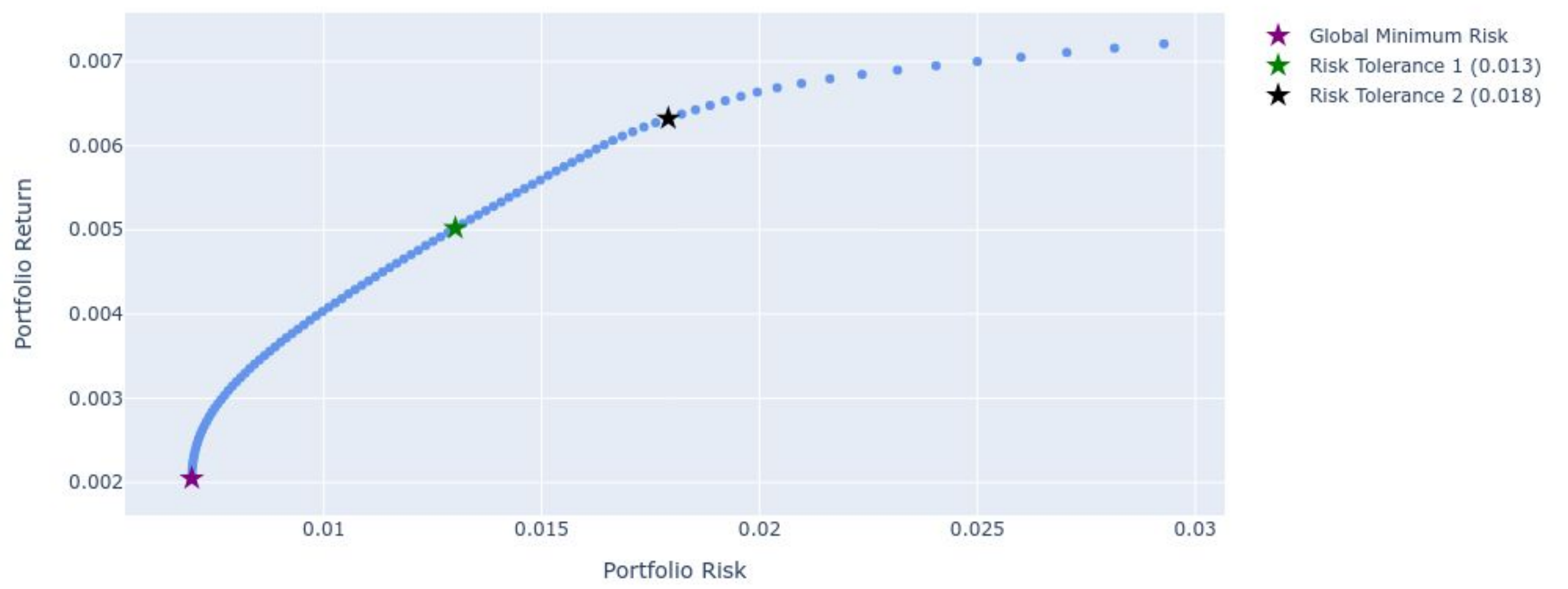
Efficient Frontier



Efficient Frontier (only the upper half)



Selecting 2 points on the efficient frontier (representing different risk tolerance levels)



Selecting 2 points on the efficient frontier (representing different risk tolerance levels)

Asset	Weights (risk tolerance: 0.013)	Weights (risk tolerance: 0.018)
Apple	0.000	0.000
Google	0.000	0.000
Microsoft	0.000	0.000
Amazon	0.000	0.000
Tesla	0.000	0.000
Meta	0.147	0.253
NVIDIA	0.376	0.610
PayPal	0.001	0.000
Netflix	0.225	0.137
Visa	0.252	0.000

Trade-Off Between Risk & Return

(in portfolio choices)

Factors influencing the trade-off

- **Investor's Risk Tolerance**
  Different investors have varying levels of risk tolerance based on their individual financial goals and psychological comfort with uncertainty.

- **Time Horizon**
  The time an investor has to achieve their financial objectives influences the appropriate level of risk.
  Longer time horizons may allow for a higher tolerance of risk.

Calculating Risk-Return

- **Alpha Ratio**
  Measures the excess return of an investment compared to a benchmark.

- **Beta Ratio**
  Indicates the correlation of a stock's returns to the overall market.

- **Sharpe Ratio**
  Evaluates the risk-adjusted return of an investment.

Risk-Reward Ratio

It is calculated by dividing the expected return on a trade by the capital at risk.

Higher ratios suggest a potentially more favorable risk-reward profile.

While the trade off implies a positive relationship, it doesn't guarantee better returns with higher risks.

Balancing risk and return is crucial for building a portfolio that aligns with an investor's goals and risk tolerance.

Markowitz Optimization Limitations

Normal Distribution Assumption

It assumes normality in returns, leading to underestimation of risk during extreme events and difficulty in distinguishing between upside and downside moves.

Mitigation
Exploring alternative risk measures and non-Gaussian models.

Static Inputs and Time-Varying Market Conditions

The framework ignores dynamic changes in variances and correlations over time, particularly during market turbulence, resulting in potential underestimation of joint negative returns.

Mitigation
Continuously applying optimization algorithms and models that capture changing market dynamics.

Estimation Error and Uncertainty

It is highly sensitive to estimation errors and uncertainties in expected returns, often leading to inefficient portfolios.

Mitigation
Applying weight constraints, conducting sensitivity analysis, and using advanced models like Black-Litterman.

Single-Period Framework and Multi-Period Objectives

The model assumes a single-period decision-making process, overlooking investors' multi-period objectives.

Mitigation
Overcome with AI-driven asset managers using advanced multi-period mean-variance models.

Markowitz Optimization Real-World Applications

Portfolio Construction

It optimizes returns and manages risk by constructing diversified portfolios through the combination of multiple assets, enabling investors to achieve higher returns without unacceptable risk levels.

Risk Management

It assesses risk and return characteristics within a portfolio, facilitating effective risk management.
For instance, adding government bonds to a stock portfolio can reduce overall variance due to their negative correlation with stocks.

Use in ETFs (Exchange-Traded Funds)

It allows investors to build efficient and diversified portfolios using the accessibility and variety offered by these funds.

Efficient Frontier

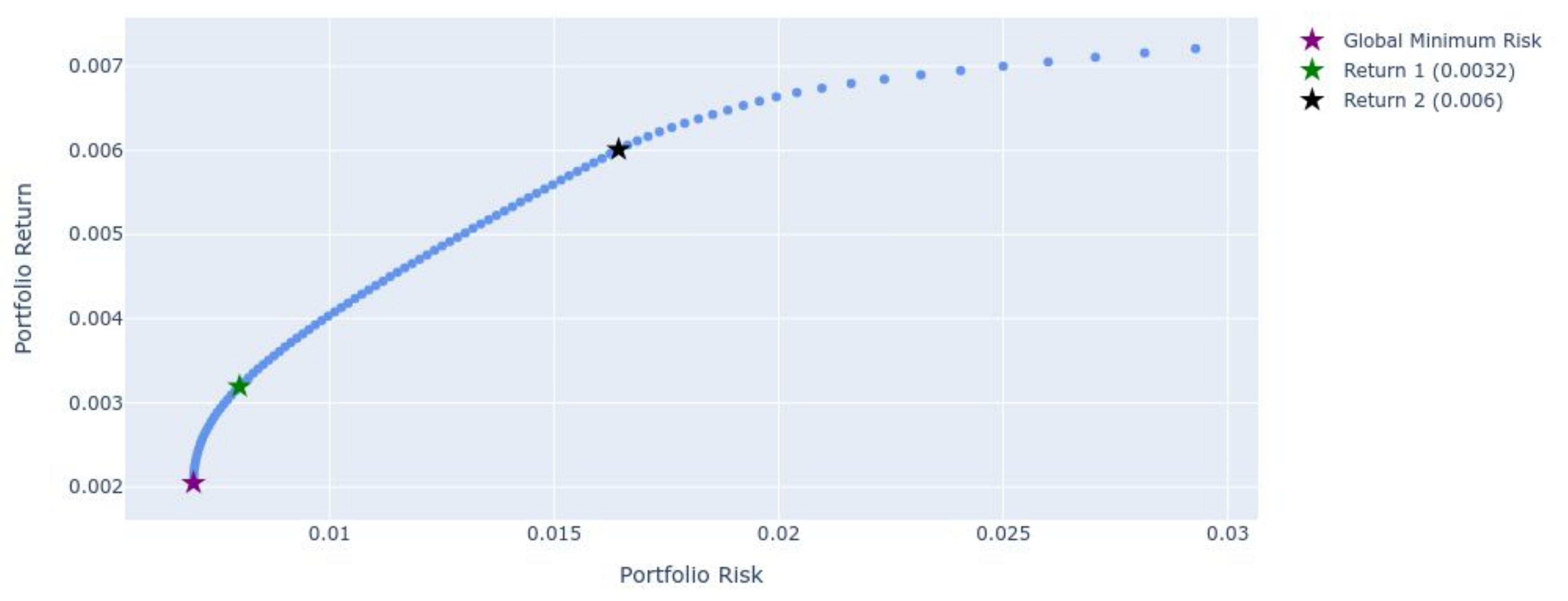
It guides investors to the optimal combination of investments for the highest return at the lowest risk.
This is particularly valuable for those seeking a balanced approach to risk and return in their portfolios.

Additional Work

In addition to the prior section, **weights were calculated for all points on the efficient frontier**, corresponding to various risk tolerance levels.

We also added **a feature to compute the global minimum risk for a given return** using the Markowitz Efficient Frontier.

Selecting 2 points on the efficient frontier (representing different returns)



Selecting 2 points on the efficient frontier (representing different returns)

Asset	Weights (return: 0.0032)	Weights (return: 0.0060)
Apple	0.000	0.000
Google	0.000	0.000
Microsoft	0.000	0.000
Amazon	0.000	0.000
Tesla	0.000	0.000
Meta	0.074	0.187
NVIDIA	0.086	0.534
PayPal	0.007	0.000
Netflix	0.117	0.279
Visa	0.717	0.000

Additional Work

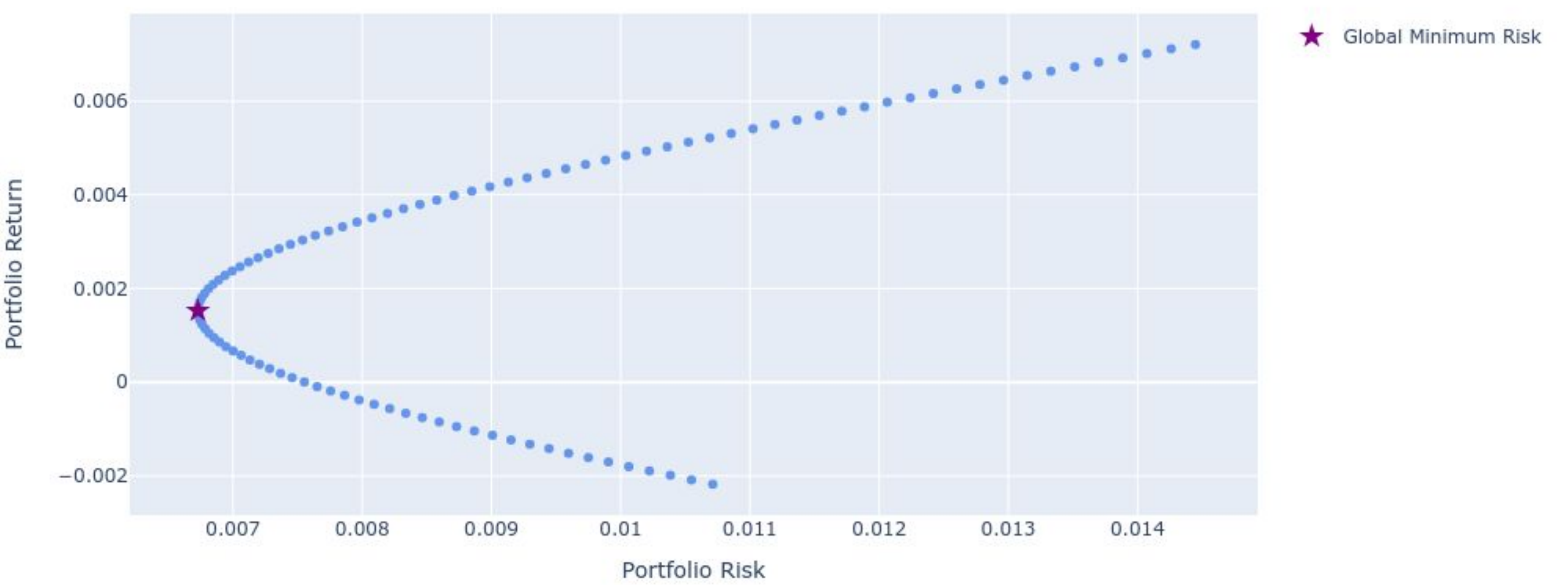
In the previous formulation of the Markowitz Optimization Problem, only positive weights were considered which excluded the case for short-selling **(allows negative weights)**.

Short-Selling

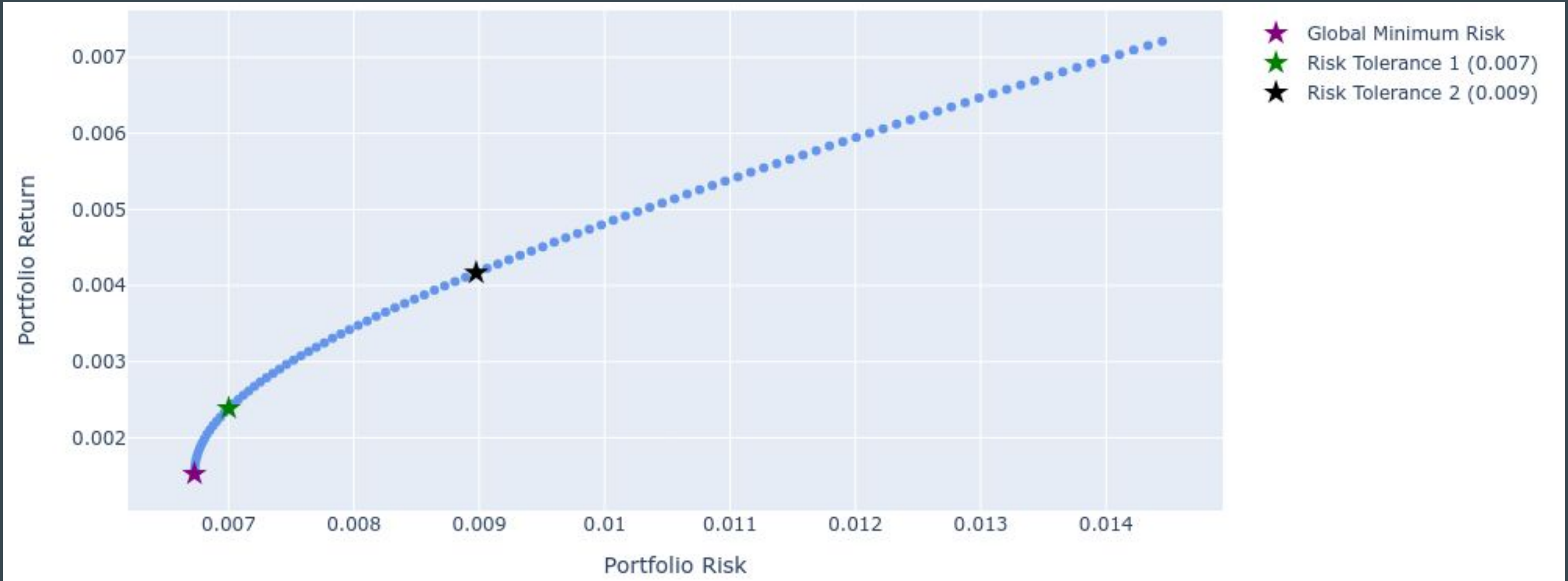
Short-selling is an investment strategy that involves borrowing and selling an asset with the expectation that its price will decline.

The investor then repurchases the asset at a lower price, returning it to the lender and pocketing the difference.

Short-Selling: Markowitz Efficient Frontier



Short-Selling: Selecting 2 random points on the frontier



Short-Selling: Weights assigned to maximize the return

Asset	Weights (risk tolerance: 0.007)	Weights (risk tolerance: 0.009)
Apple	0.049	-0.173
Google	-0.043	-0.083
Microsoft	0.095	0.022
Amazon	0.015	-0.026
Tesla	-0.005	-0.086
Meta	0.037	0.071
NVIDIA	-0.039	0.153
PayPal	0.003	0.075
Netflix	0.066	0.131
Visa	0.821	0.916

THANK YOU!