Introduction to Financial Engineering

MARKOWITZ PORTFOLIO OPTIMIZATION

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PROBLEM STATEMENT

The project aims to optimize portfolios comprising 10 selected risky assets by applying Markowitz's mean-variance optimization. The process involves choosing assets, collecting closing prices over the past 3 months, calculating returns, and constructing the efficient frontier. Two points on the efficient frontier, representing varying risk tolerance levels, are selected. For each point, the corresponding weights for assets are determined to construct portfolios maximizing expected return for given risk levels. The deliverables include optimized portfolios and insights into risk-return trade-offs. The methodology and prediction accuracy are reported, showcasing the effectiveness of the mean-variance optimization approach.

ASSET SELECTION

The closing price data (over the past 3 months) of 10 selected risky assets was taken from **Yahoo Finance**, as listed below:

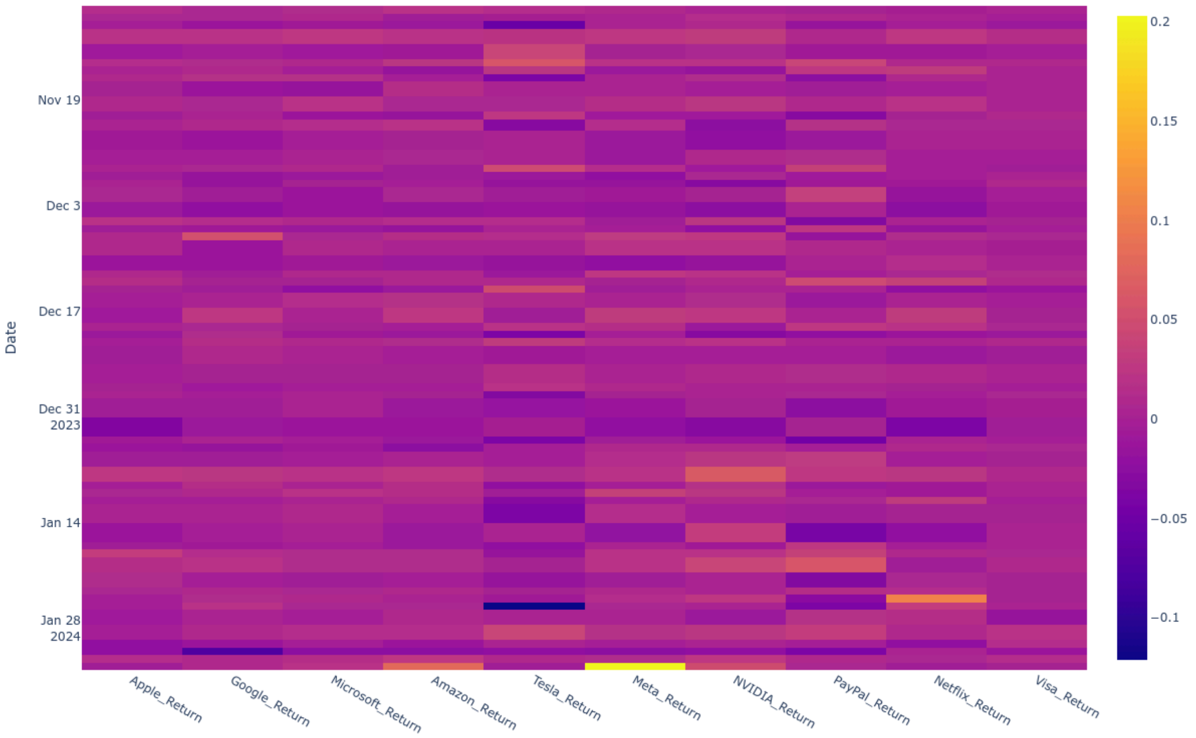
- Apple Inc. (AAPL)
- Alphabet Inc. (GOOGL)
- Microsoft Corporation (MSFT)
- Amazon.com, Inc. (AMZN)
- Tesla, Inc. (TSLA)
- Meta Platforms, Inc. (META)
- NVIDIA Corporation (NVDA)
- PayPal Holdings, Inc. (PYPL)
- Netflix, Inc. (NFLX)
- Visa Inc. (V)

RETURN & RISK MEASURE CALCULATION

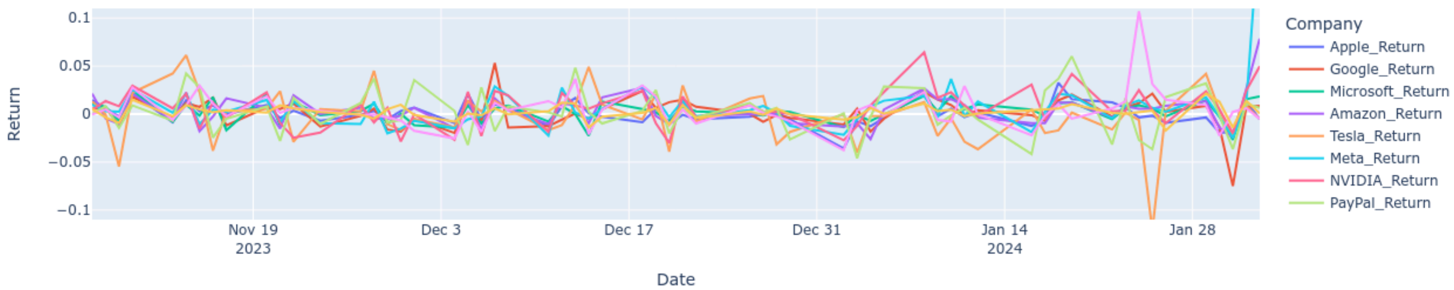
The percentage returns were calculated for each asset for each date by calculating the percentage change in the values through a series.

Date	Apple_Return	Google_Return	Microsoft_Return	Amazon_Return	Tesla_Return	Meta_Return	NVIDIA_Return	PayPal_Return	Netflix_Return	Visa_Return
2024-01-22	0.012163	-0.002664	-0.005418	-0.003605	-0.015976	-0.004355	0.002740	-0.031601	0.005715	0.001107
2024-01-23	0.006653	0.007192	0.006028	0.008011	0.001628	0.008958	0.003671	0.016316	0.013341	0.000221
2024-01-24	-0.003484	0.011289	0.009175	0.005448	-0.006264	0.014278	0.024869	-0.027169	0.107032	0.001438
2024-01-25	-0.001697	0.021318	0.005738	0.005610	-0.121253	0.006348	0.004156	-0.036655	0.031439	0.003534
2024-01-26	-0.009013	0.002107	-0.002322	0.008685	0.003395	0.002442	-0.009510	0.017625	0.014982	-0.017131
2024-01-29	-0.003586	0.008673	0.014334	0.013449	0.041910	0.017456	0.023496	0.032049	0.009414	0.021348
2024-01-30	-0.019246	-0.013354	-0.002758	-0.014015	0.003457	-0.002394	0.004947	-0.001255	-0.022473	0.012753
2024-01-31	-0.019358	-0.075003	-0.026946	-0.023899	-0.022444	-0.024796	-0.019865	-0.036589	0.002239	-0.014036
2024-02-01	0.013341	0.007566	0.015594	0.026289	0.008383	0.011893	0.024380	0.010921	0.006027	0.013869
2024-02-02	-0.005405	0.008643	0.018426	0.078666	-0.005030	0.203176	0.049709	0.006449	-0.005057	0.000469

Dataframe showing the returns for each company (asset)



Visualization showing the returns for each company (asset)



Visualization showing company (asset) returns over time

The risk for each asset was calculated using the standard deviation of percentage returns for each asset, as show below:

- `Apple      0.011458`
- `Google     0.016344`
- `Microsoft  0.010810`
- `Amazon     0.016321`
- `Tesla      0.027877`
- `Meta       0.029283`
- `NVIDIA     0.019487`
- `PayPal     0.023020`
- `Netflix    0.019923`
- `Visa       0.007228`

MARKOWITZ MEAN-VARIANCE OPTIMIZATION

The Markowitz Mean-Variance Optimization Model is a mathematical framework first introduced by the economist Harry Markowitz in 1952. It is based on the idea that investors are highly averse to risk and will only accept more risk if compensated by higher expected returns.

The steps involved in the portfolio optimization are as follows:

- **Initialization**
  Initially, the mean ($\boldsymbol{\mu}$) returns and the covariance ($\boldsymbol{\Sigma}$) matrix were calculated based on the complete asset return dataset. The covariance matrix served as the "Risk Model" in this case.

- **Optimization Problem Formulation**
  Minimize the portfolio risk with the constraint expressing a lower bound on the portfolio return:

$$\begin{array}{lrcl} \text{minimize} & \mathbf{x}^\mathsf{T}\boldsymbol{\Sigma}\mathbf{x} & & \\ \text{subject to} & \boldsymbol{\mu}^\mathsf{T}\mathbf{x} & \geq & r_{\min}, \\ & \mathbf{1}^\mathsf{T}\mathbf{x} & = & 1. \end{array}$$

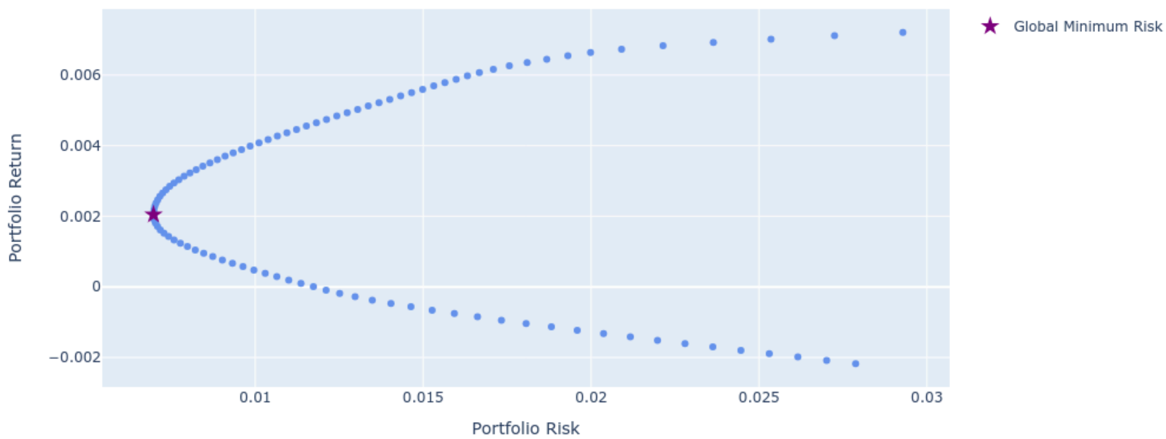
  Here $r_{\min}$ is the target expected return that the investor wishes to reach. In this problem, the objective function is convex quadratic, and all the constraints are linear. Thus, it is a quadratic optimization (QO) problem.
  The above optimization problem was solved using the python library CYXPY.

Note: *Only positive weights were considered in the above formulation (short-selling case was not allowed).*

- **Efficient Frontier Plotting**
  Efficient Frontier is the line that indicates the combination of investments that will provide the highest level of return for the lowest level of risk. Portfolios to the right face higher risk for the expected return, while those below the frontier provide a lower return for the risk undertaken.

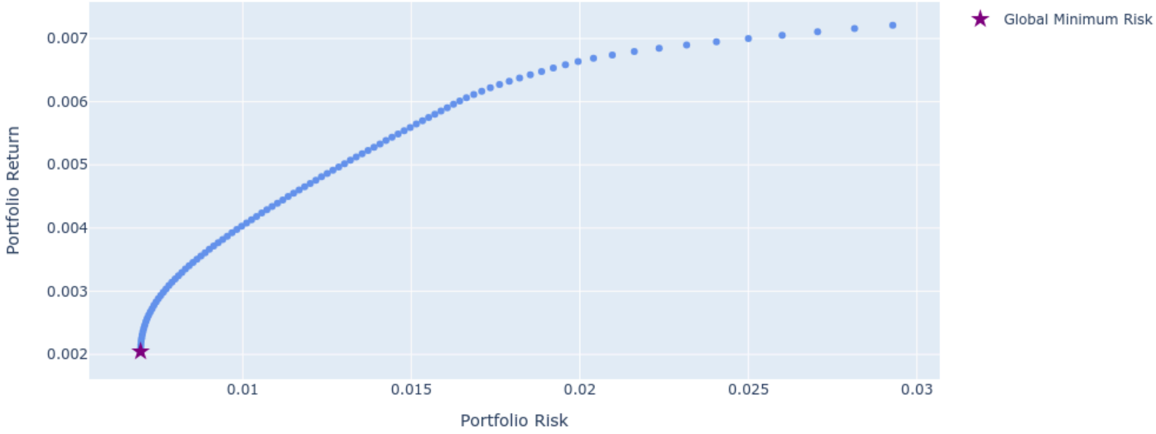
  The efficient frontier was plotted by varying the target return levels. For each target return, a new optimization problem was solved with constraints on the expected return. The resulting portfolio risk and return pairs were plotted, forming the efficient frontier.

  The portfolio with the minimum risk on the efficient frontier was identified and marked on the plot.



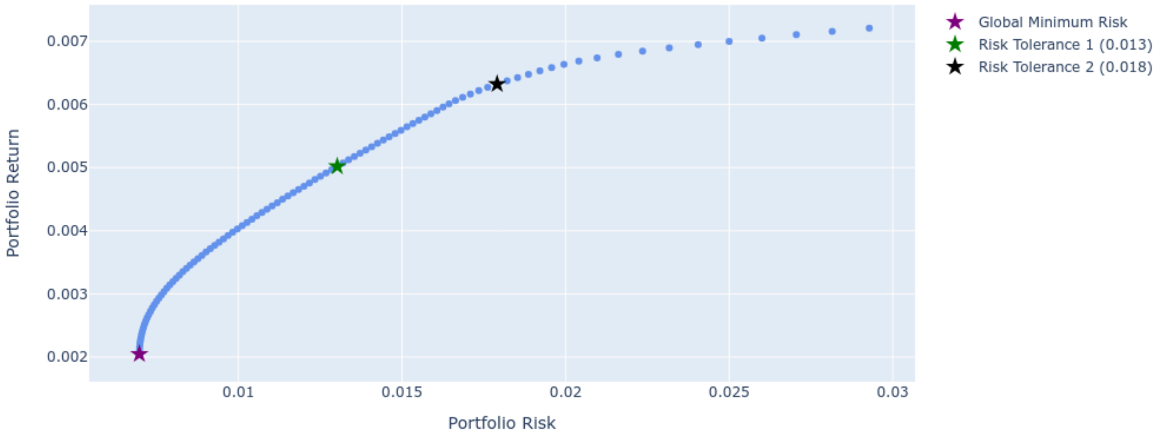
Plot showing the Markowitz Efficient Frontier

Furthermore, the lower half of the efficient frontier (below the minimum risk point) was excluded, and only the upper half was plotted. This decision was made because the lower half represented points with higher risk and lower return, which are not practical to consider when investing.



Visualization showing the Markowitz Efficient Frontier (upper half)

- **Selecting 2 points on the efficient frontier representing different risk tolerance levels**
  Two random points on the frontier were selected with risk tolerance levels 0.013 and 0.018 respectively.



Visualization showing the two selected points on the Markowitz Efficient Frontier

The corresponding weights for each asset to construct a portfolio that maximize the expected return for the selected levels of risk are shown below:

Asset	Weights (risk tolerance: 0.013)	Weights (risk tolerance: 0.018)
Apple	0.000	0.000
Google	0.000	0.000
Microsoft	0.000	0.000
Amazon	0.000	0.000
Tesla	0.000	0.000
Meta	0.147	0.253
NVIDIA	0.376	0.610
PayPal	0.001	0.000
Netflix	0.225	0.137
Visa	0.252	0.000

TRADE-OFF BETWEEN RISK & RETURN (in portfolio choices)

Risk-return tradeoff states that the potential return rises with an increase in risk. Individuals associate low levels of uncertainty with low potential returns, and high levels of uncertainty or risk with high potential returns.

Factors influencing the trade-off include: influencing factors include an investor's risk tolerance and time horizon, and diversification is a strategy to manage risk at both individual and portfolio levels.

Calculating risk-return:

- **Alpha Ratio:** Measures the excess return of an investment compared to a benchmark.
- **Beta Ratio:** Indicates the correlation of a stock's returns to the overall market.
- **Sharpe Ratio:** Evaluates the risk-adjusted return of an investment.

Risk-Reward Ratio:

- Calculated by dividing the expected return on a trade by the capital at risk.
- Higher ratios suggest a potentially more favorable risk-reward profile.

While the trade off implies a positive relationship, it doesn't guarantee better returns with higher risks. Investors must consider their financial situation, goals, and risk tolerance. Balancing risk and return is essential for constructing a portfolio aligned with an investor's objectives and risk tolerance.

LIMITATIONS OF MARKOWITZ OPTIMIZATION

- **Normal Distribution Assumption**
  - It assumes normality in returns, leading to underestimation of risk during extreme events and difficulty in distinguishing between upside and downside moves.
  - **Mitigation:** Incorporating alternative risk measures like semi-variance and skewness, and considering non-Gaussian models or stress-testing to account for extreme events.

- **Static Inputs and Time-Varying Market Conditions**
  - The framework ignores dynamic changes in variances and correlations over time, particularly during market turbulence, resulting in potential underestimation of joint negative returns.
  - **Mitigation:** Continuous application of optimization algorithms, such as Markowitz, can adapt to changing market conditions. Implementing models that capture time-varying volatilities and correlations enhances accuracy.

- **Estimation Error and Uncertainty**
  - It is highly sensitive to estimation errors and uncertainties in expected returns, often leading to inefficient portfolios.
  - **Mitigation:** Applying reasonable weight constraints, conducting sensitivity analysis, and utilizing advanced models like the Black-Litterman framework and regularly updating input parameters to reflect changing market conditions.

- **Single-Period Framework and Multi-Period Objectives**
  - The model assumes a single-period decision-making process, overlooking investors' multi-period objectives.
  - **Mitigation:** Employing AI-driven asset managers with computationally advanced multi-period mean-variance models and utilizing machine learning to dynamically adjust portfolios over different investment stages, aligning with diverse time horizons and objectives.

MARKOWITZ OPTIMIZATION REAL-WORLD APPLICATIONS

- **Portfolio Construction**
  It facilitates the construction of diversified portfolios, optimizing returns while managing risk. Through the combination of multiple assets, investors can achieve higher returns without exposing themselves to unacceptable levels of risk.

- **Risk Management**
  It assesses the risk and return characteristics of individual investments within a portfolio, allowing effective risk management. For example, incorporating government bonds into a stock portfolio can reduce overall portfolio variance due to their typically negative correlation with stocks.

- **Use in ETFs**
  Its relevance has been enhanced by the rise of Exchange-Traded Funds (ETFs). Investors can construct efficient and diversified portfolios using the accessibility and variety offered by ETFs.

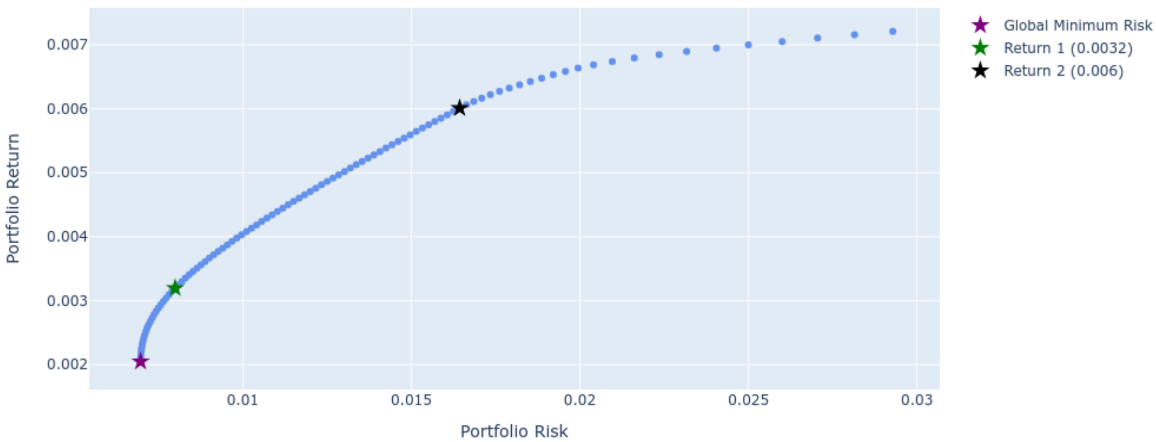
- **Efficient Frontier**
  It introduces the concept of the efficient frontier, indicating the optimal combination of investments for the highest return at the lowest risk. This is particularly valuable for investors seeking a balanced approach to risk and return in their portfolios.

ADDITIONAL WORK

Calculating the global minimum risk for a desired return value)

In addition to the previous section, apart from the two points mentioned above, corresponding weights were calculated for all the points on the efficient frontier, representing different risk tolerance levels.

Also, a feature to calculate the global minimum risk for a desired return value was implemented, making use of the Markowitz Efficient Frontier, illustrated with an example below:



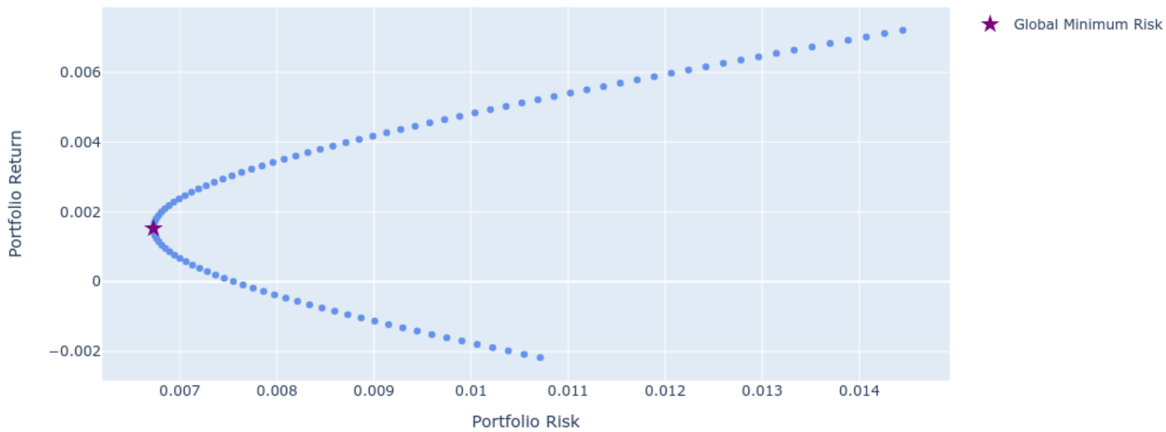
Visualization showing the two selected points on the Markowitz Efficient Frontier

The corresponding weight values for the above example points are shown below:

Asset	Weights (return: 0.0032)	Weights (return: 0.0060)
Apple	0.000	0.000
Google	0.000	0.000
Microsoft	0.000	0.000
Amazon	0.000	0.000
Tesla	0.000	0.000
Meta	0.074	0.187
NVIDIA	0.086	0.534
PayPal	0.007	0.000
Netflix	0.117	0.279
Visa	0.717	0.000

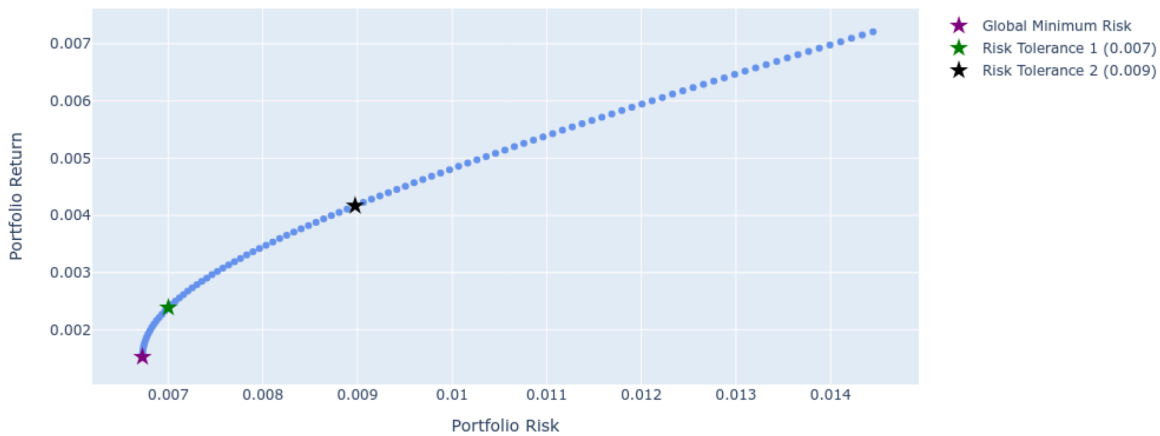
Allowing short-selling

In the previous formulation of the Markowitz Optimization Problem, only positive weights were considered which excluded the case for short-selling (which allows negative weights).
Short-selling is an investment strategy that involves borrowing and selling an asset with the expectation that its price will decline. The investor then repurchases the asset at a lower price, returning it to the lender and pocketing the difference.
The efficient frontier for the short-selling case was plotted, as shown below:



Plot showing the Markowitz Efficient Frontier (short-selling allowed)

Two random points on the frontier were selected with risk tolerance levels 0.007 and 0.009 respectively.



Visualization showing the two selected points on the Markowitz Efficient Frontier (short-selling allowed)

The corresponding weights for each asset to construct a portfolio that maximize the expected return for the selected levels of risk are shown below:

Asset	Weights (risk tolerance: 0.007)	Weights (risk tolerance: 0.009)
Apple	0.049	-0.173
Google	-0.043	-0.083
Microsoft	0.095	0.022
Amazon	0.015	-0.026
Tesla	-0.005	-0.086
Meta	0.037	0.071
NVIDIA	-0.039	0.153
PayPal	0.003	0.075
Netflix	0.066	0.131
Visa	0.821	0.916

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