Chapter 10

Properties of Stock Options

Practice Questions

Consolidate

Problem 10.9
What is a lower bound for the price of a six-month call option on a non-dividend-paying stock when the stock price is $80, the strike price is $75 and the risk-free interest rate is 10% per annum?

The lower bound is

$$80 - 75e^{-0.1\times0.5} = \$8.66$$

Problem 10.10
What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is $58, the strike price is $65 and the risk-free interest rate is 5% per annum?

The lower bound is

$$65e^{-0.05\times2/12} - 58 = \$6.46$$

Problem 10.12
A one-month European put option on a non-dividend-paying stock is currently selling for $2.50. The stock price is $47, the strike price is $50 and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur?

In this case the present value of the strike price is $50e^{-0.06\times1/12} = 49.75$. Because

$$2.5 < 49.75 - 47.00$$

the condition in equation (10.5) is violated. An arbitrageur should borrow $49.50 at 6% for one month, buy the stock and buy the put option. This generates a profit in all circumstances.

If the stock price is above $50 in one month, the option expires worthless, but the stock can be sold for at least $50. A sum of $50 received in one month has a present value of $49.75 today. The strategy therefore generates profit with a present value of at least $0.25.

If the stock price is below $50 in one month, the put option is exercised and the stock owned is sold for exactly $50 (or $49.75 in present value terms). The trading strategy therefore generates a profit of exactly $0.25 in present value terms.

Problem 10.13

Give an intuitive explanation of why the early exercise of an American put becomes more attractive as the risk-free rate increases and volatility decreases.

The early exercise of an American put is attractive when the interest earned on the strike price is greater than the insurance element lost. When interest rates increase, the value of the interest earned on the strike price increases making early exercise more attractive. When volatility decreases, the insurance element is less valuable. Again this makes early exercise more attractive.

Problem 10.14

The price of a European call that expires in six months and has a strike price of $30 is $2. The underlying stock price is $29. The term structure is flat, with all risk-free interest rates being 10%. What is the price of a European put option that expires in six months and has a strike price of $30?

Using the notation in the chapter, put-call parity [equation (10.10)] gives

$$c + Ke^{-rT} = p + S_0$$

or

$$p = c + Ke^{-rT} - S_0$$

In this case

$$p = 2 + 30e^{-0.1\times 6/12} - 29 =$$

In other words, the put price is $1.537.