Date : 7 mai Place : 5

Epreuve de : MAP

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Note et appréciations : 20

## Exercice 1 :

$F_1(p) = \dfrac{3}{p(1-e^{-4p})} \Rightarrow f_1(t) = 3\sum_{n=0}^{\infty} \gamma(t-4n)$

$TL^{-1}\left[\dfrac{1}{p}\right] = \gamma(t)$

$F_2(p) = \dfrac{e^{-p}}{p(1-e^{-4p})} \Rightarrow f_2(t) = \sum_{n=0}^{\infty} \gamma(t-1-4n)$

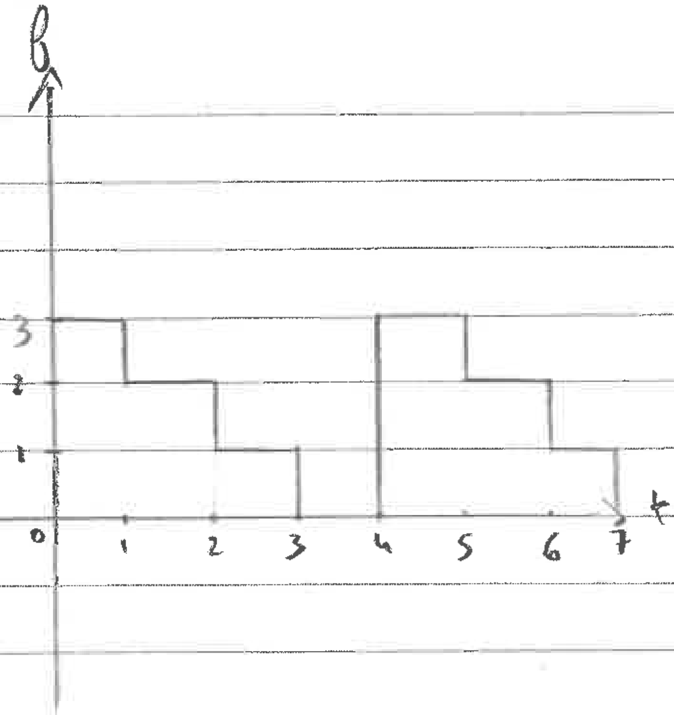
$F_3(p) = \dfrac{e^{-2p}}{p(1-e^{-4p})} \Rightarrow f_3(t) = \sum_{n=0}^{\infty} \gamma(t-2-4n)$

$F_4(p) = \dfrac{e^{-3p}}{p(1-e^{-4p})} \Rightarrow f_4(t) = \sum_{n=0}^{\infty} \gamma(t-3-4n)$

$F(p) = F_1(p) - F_2(p) - F_3(p) - F_4(p)$

$TL^{-1}[F(p)] = 3\sum_{n=0}^{\infty} \gamma(t-4n) - \sum_{n=0}^{\infty} \gamma(t-1-4n) - \sum_{n=0}^{\infty} \gamma(t-2-4n) - \sum_{n=0}^{\infty} \gamma(t-3-4n)$

$f(t) = \sum_{n=0}^{\infty} \left(3\gamma(t-4n) - \gamma(t-1-4n) - \gamma(t-2-4n) - \gamma(t-3-4n)\right)$




## Exercice 2 :

$$y(t) \xrightarrow{TL} Y(p)$$
$$y'(t) \xrightarrow{TL} pY(p) \cancel{- y(0)}$$
$$y''(t) \xrightarrow{TL} p^2 Y(p) \cancel{- p y(0)} \cancel{- y'(0)}$$
$$(1+e^{-t})\gamma(t) \xrightarrow{TL} \frac{1}{p} + \frac{1}{p+1}$$

Soit $p^2 Y(p) + 2pY(p) + Y(p) = \frac{1}{p} + \frac{1}{p+1}$

$p^2+2p+1 = (p+1)(p+1)$

$$\Leftrightarrow Y(p)(p^2+2p+1) = \frac{1}{p} + \frac{1}{p+1}$$
$$\Leftrightarrow Y(p) = \left(\frac{1}{p} + \frac{1}{p+1}\right)\left(\frac{1}{(p+1)^2}\right)$$
$$= \frac{1}{p(p+1)^2} + \frac{1}{(p+1)^3}$$

$$Y_1(p) = \frac{A}{p} + \frac{B}{p+1} + \frac{C}{(p+1)^2}$$

$\lim_{p \to 0} p Y_1(p) = A = 1$

$\lim_{p \to -1} (p+1)^2 Y_1(p) = C = -1$

$\lim_{p \to \infty} p Y_1(p) = A + B = 0 \Rightarrow B = -1$

et $TL^{-1}\left[\frac{1}{(p+1)^3}\right] = t^2 e^{-t} \gamma(t)$

$$Y_1(p) = \frac{1}{p} - \frac{1}{p+1} - \frac{1}{(p+1)^2}$$

soit $y(t) = TL^{-1}[Y(p)] = \gamma(t)[1 - e^{-t} - te^{-t} + t^2 e^{-t}]$

## Exercice 3 :

$g(t)$ est impaire donc $a_n = 0$

$$b_n = \frac{4}{T_0} \times \int_0^{\frac{T_0}{2}} \sin\left(n \frac{2\pi}{T_0} t\right) g(t)\, dt$$

$$= \frac{4}{T_0} \times \int_0^{\frac{T_0}{2}} 1 \sin\left(\frac{n 2\pi t}{T_0}\right) dt$$

$$= \frac{4}{T_0} \times \left[ \frac{-\cos\left(n \frac{2\pi}{T_0} t\right)}{n \frac{2\pi}{T_0}} \right]_0^{\frac{T_0}{2}}$$

$$= \frac{4}{T_0} \times \left[ \frac{-\cos\left(n \frac{2\pi}{T_0} \frac{T_0}{2}\right)}{n \frac{2\pi}{T_0}} + \frac{1}{n \frac{2\pi}{T_0}} \right]$$

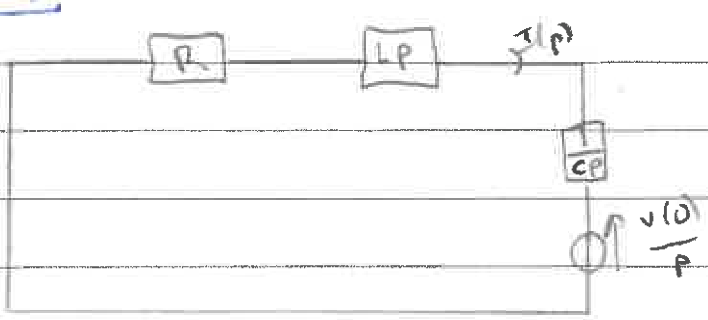
$$b_n = \frac{2}{n\pi} \left( -\cos(n\pi) + 1 \right)$$

Si $b_n$ est pair : $b_n = 0$, si impair : $b_n = \frac{4}{n\pi}$

soit $g_p(t) = \sum_{n=0}^{\infty} b_n \sin\left(n \frac{2\pi}{T_0} t\right)$ $\qquad b_{2k+1} = \frac{4}{(2k+1)\pi}$

$$g_p(t) = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{(2k+1)} \sin\left((2k+1) \frac{2\pi}{T_0} t\right)$$

## Exercice 4 :



a) $RI(p) - LpI(p) - \frac{1}{Cp}I(p) + \frac{v(0)}{p} = 0$

$\Leftrightarrow I(p)\left(R - Lp - \frac{1}{Cp}\right) = -\frac{v(0)}{p}$

$\Leftrightarrow I(p) = \frac{v(0)}{p} \cdot \frac{1}{-R + Lp + \frac{1}{Cp}}$

soit $I(p) = \frac{V(0) \times C}{-RCp + LCp^2 + 1} = \frac{V(0)}{L} \cdot \frac{1}{p^2 - \frac{R}{L}p + \frac{1}{LC}}$ V

$= 1000 \cdot \frac{1}{p^2 - 5000p + 6\,060\,606}$

$\Delta = 25\,000\,000 - 4 \times 6\,060\,606 = 757\,576$ et $\sqrt{\Delta} = 870$

soit $\alpha_1 = \frac{5000 - 870}{2} = 2065$ et $\alpha_2 = 2935$

d'où :

$I(p) = 1000 \cdot \left(\frac{1}{(p-2065)(p-2935)}\right)$

$I_1(p) = \frac{A}{p-2065} + \frac{B}{p-2935}$ soit $\lim_{p \to 2065} (p-2065)I_1(p) = A = -\frac{1}{870}$

et $\lim_{p \to 2935} (p-2935)I_1(p) = B = \frac{1}{870}$

soit $I(p) = \frac{1000}{870}\left(-\frac{1}{p-2065} + \frac{1}{p-2935}\right)$

V d'où $i(t) = TL^{-1}[I(p)] = \frac{1000}{870}\left(-e^{2065t} + e^{2935t}\right)\gamma(t)$

b) On a $V(p) = \frac{v(0)}{p} + \frac{I(p)}{Cp} = \frac{5}{p} + 34831\left(-\frac{1}{p(p-2065)} + \frac{1}{p(p-2935)}\right)$

$V(p)$ étant la tension aux bornes du condensateur

$V_1(p) = \frac{1}{p(p-2065)} = \frac{A}{p} + \frac{B}{p-2065}$ | $V_2(p) = -\frac{1}{2935}\frac{1}{p} + \frac{1}{2935}\frac{1}{p-2935}$

soit $\lim_{p \to 0} pV_1(p) = A = -\frac{1}{2065}$

et $\lim_{p \to 2065} (p-2065)V_1(p) = B = \frac{1}{2065}$

$V_1(p) = -\frac{1}{2065}\frac{1}{p} + \frac{1}{2065}\frac{1}{p-2065}$

V

Soit $V(p) = \frac{5}{p} + 34831\left(-\frac{1}{2065}\left(\frac{1}{p} + \frac{1}{p-2065}\right) + \frac{1}{2935}\left(-\frac{1}{p} + \frac{1}{p-2935}\right)\right)$

D'où $v(t) = \gamma(t)\left(5 + 34831\left(-\frac{1}{2065}\left(-1 + e^{2065t}\right) + \frac{1}{2935}\left(-1 - e^{2935t}\right)\right)\right)$