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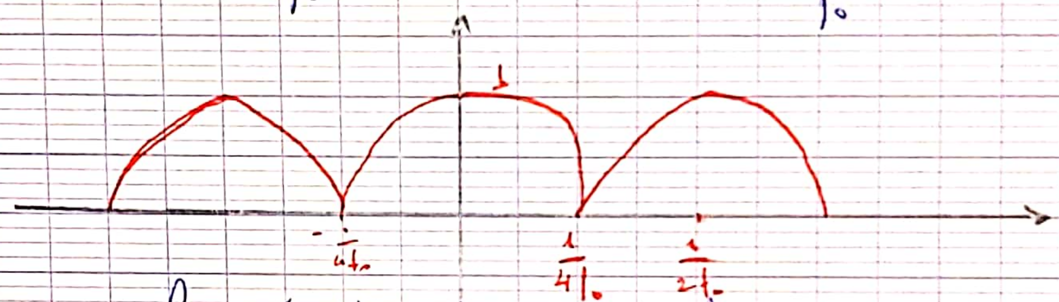
Devoir individuel 2020

1) $= \cos(2\pi f_0 t)$

du cos $\quad 2\pi f_0 T = 2\pi \iff T = \dfrac{1}{f_0}$



la période de cette fonction

$$T = \frac{1}{4f_0} + \frac{1}{4f_0} = \frac{1}{2f_0}$$

2) On a le signal est paire,
donc $\forall n \in \mathbb{N}^+ \quad b_n = 0$

$$a_0 = 2\times 2 \times f_0 \times \int_0^{\frac{1}{4f_0}} \cos(2\pi f_0 t)\, dt$$

$$= 4 f_0 \left[\frac{\sin(2\pi f_0 t)}{2\pi f_0} \right]_0^{\frac{1}{4f_0}} = \frac{4 f_0}{2\pi f_0} \overbrace{\sin\left(\frac{\pi}{2}\right)}^{2} - 0$$

$$= \frac{2}{\pi}$$

3) $$a_n = 2 \times 4 f_0 \times \int_0^{\frac{1}{4f_0}} \cos(2\pi f_0 t) \times \cos(4\pi f_0 t n)\, dt$$

$$= 8 f_0 \int_0^{1/4f_0} \frac{\cos(2\pi f_0 t(1+2n)) + \cos(2\pi f_0 t(1-2n))}{2}\, dt$$

$$= 4 f_0 \left[\frac{\sin(2\pi f_0 t(1-2n))}{2\pi f_0 (1-2n)} + \frac{\sin(2\pi f_0 t(1+2n))}{2\pi f_0 (1+2n)} \right]_0^{1/4f_0}$$

$$= 2 f_0 \times \frac{\sin\left(\frac{\pi}{2}(1-2n)\right)}{\pi f_0 (1-2n)} + \frac{\sin\left(\frac{\pi}{2}(1+2n)\right)}{\pi f_0 (1+2n)}$$

$$v(t) = \frac{2}{\pi} + \frac{2}{f_0} \sum_{n=1}^{n=+\infty} \cos(n\omega_0 t) \left[\frac{\sin\left(\frac{\pi}{2}(1-2n)\right)}{f_0 \pi (1-2n)} + \frac{\sin\left(\frac{\pi}{2}(1+2n)\right)}{f_0 \pi (1+2n)} \right]$$

$$v(t) = \frac{2}{\pi} + \frac{2}{f_0} \sum_{n=1}^{+\infty} \cos(n\omega_0 t) \left[\frac{\sin\left(\frac{\pi}{2}(1-2n)\right)}{f_0 \pi (1-2n)} + \frac{\sin\left(\frac{\pi}{2}(1+2n)\right)}{f_0 \pi (1+2n)} \right]$$

3) $|x(t)| = |a(t)| \cdot \overbrace{v_2(t)}^{\text{entre } [-B;B]} = -a(t) \cdot \overbrace{v_2(t)}^{\text{entre } [-B;B]}$

$\xrightarrow{T.F} |X(f)| = -A(f) * V_2(f)$

avec $v(t)$ p.s $v_2(t)$ est la restriction de $v(t)$ entre $[-B; B]$

3) $|x(t)| = |a(t)| \times v(t) = -a(t) \times v(t)$

$\xrightarrow{T.F} -A(f) * V(f)$

avec $V(f) = \sum_{n=-\infty}^{+\infty} C_n \, \delta(f - n f_0)$

$= \frac{a_0}{2} \delta(f) + \sum_{n=1}^{n=+\infty} \frac{a_n}{2} \delta(f - n f_0)$

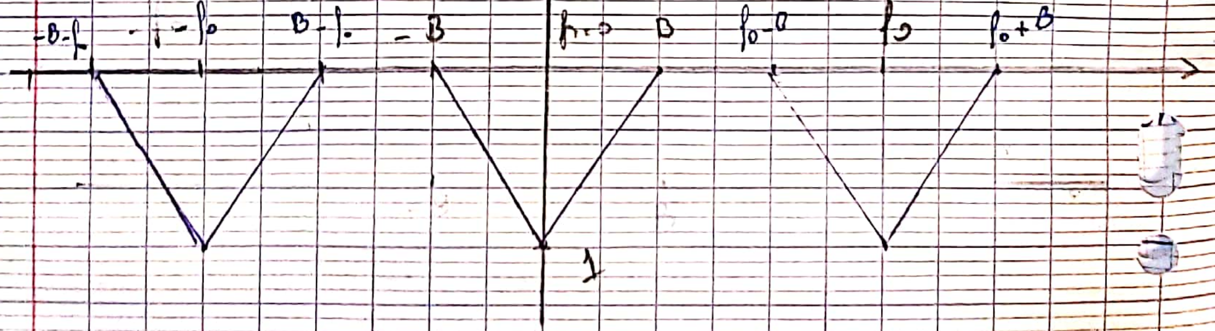
on peut aussi dire que

$= \sum_{n=0}^{+\infty} \frac{a_n}{2} \delta(f - n f_0)$

TF $X(f) = -A(f) * \sum_{n=0} \frac{a_n}{2} \delta(f - n f_0)$

4)

$V(f) = -\sum_{n=0}^{+\infty} \frac{a_n}{2} A(f - n f_0)$



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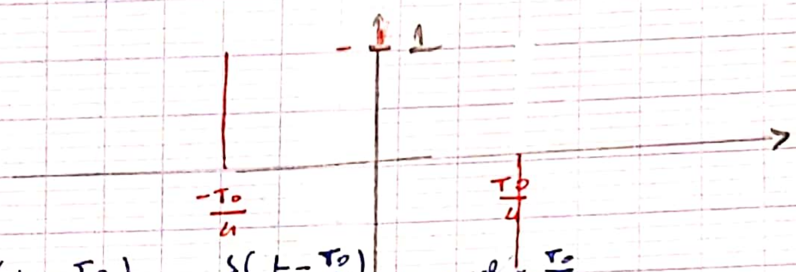
7)



$$\frac{dw(t)}{dt} = \delta\left(t+\frac{T_0}{4}\right) - \delta\left(t-\frac{T_0}{4}\right)$$

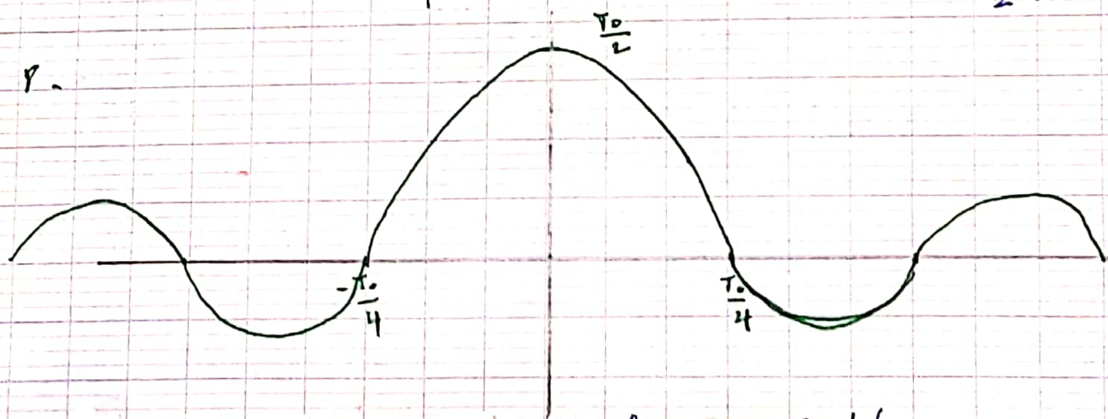
$$\xrightarrow{TF} e^{+j2\pi\frac{T_0}{4}f} - e^{-j2\pi f\frac{T_0}{4}}$$

$$= e^{j\frac{\pi}{2}T_0 f} - e^{-j\frac{\pi}{2}T_0 f}$$

$$w(t) \xrightarrow{TF} W(f) = \frac{e^{j\frac{\pi}{2}T_0 f} + e^{-j\frac{\pi}{2}T_0 f}}{j2\pi f}$$

$$= \frac{1}{\pi f}\cdot \sin\left(\frac{\pi}{2}f\,T_0\right) = \frac{T_0}{2}\times\frac{\sin\left(f\,T_0\frac{\pi}{2}\right)}{\frac{\pi}{2}f\,T_0}$$

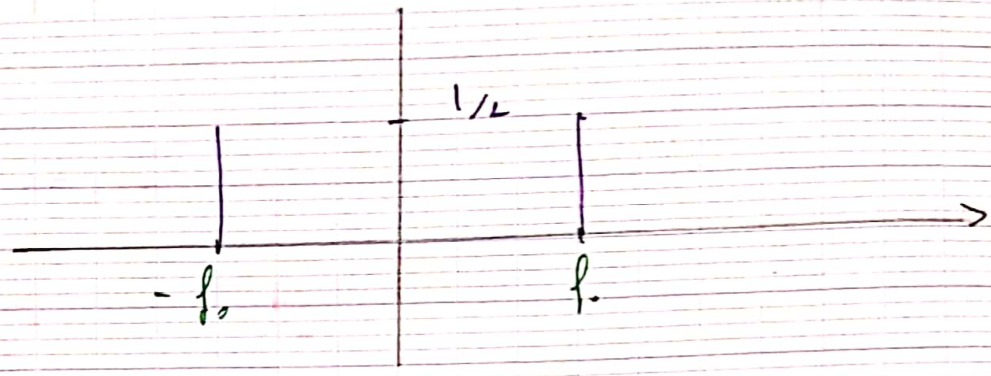
P.



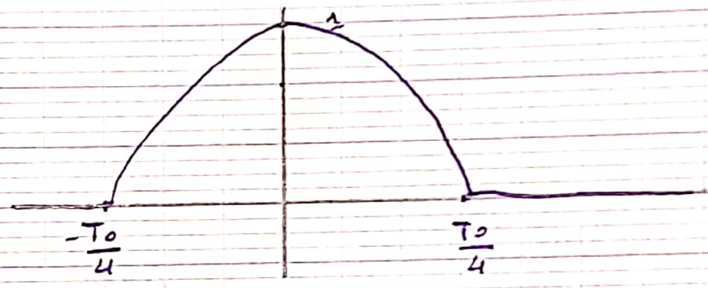
8. $g(t) = \cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$

$$\xrightarrow{T.F} G(f) = \frac{1}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right)$$

10)



11) $f(t) = w(t) \cdot g(t)$



12)

$$f(t) \xrightarrow{TF} F(f) = W(f) * G(f)$$

$$= \frac{1}{2} \frac{1}{f\pi} \sin\left(\frac{\pi}{2} f\, T_0\right) * \left(\delta(f-f_0) + \delta(f+f_0)\right)$$

$$= \frac{1}{2\pi}\left[\frac{\sin\left(\frac{\pi}{2}T_0(f-f_0)\right)}{f-f_0} + \frac{\sin\left(\frac{\pi}{2}T_0(f+f_0)\right)}{f+f_0}\right]$$

13) $\cancel{V(f) = \frac{1}{T_0}}$

$$V(f) = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} F(nf_0)\, \delta(f-nf_0)$$

$$= \frac{1}{2T_0\pi} \sum_{n=-\infty}^{+\infty} \left(\frac{\sin\left(\frac{\pi}{2}(2nf_0-f_0)T_0\right)}{2nf_0-f_0} + \frac{\sin\left(\frac{\pi}{2}(2nf_0-f_0)T_0\right)}{2nf_0+f_0}\right)\delta(f-n$$

$$= \frac{1}{2T_0\pi} \sum_{n=-\infty}^{+\infty} \frac{\sin\left(\frac{\pi}{2}f_0(2n-1)T_0\right)}{f_0(2n-1)} + \frac{\sin\left(\frac{\pi}{2}f_0(2n-1)T_0\right)}{f_0(2n+1)}$$

14) On a $F(f) = \sum_{n=-\infty}^{+\infty} C_n \, \delta(f - n f_0)$

Donc $C_n = \left(\frac{\sin(\frac{\pi}{2}(2n f_0 - f_0) T_0)}{2n f_0 - f_0} + \frac{\sin(\frac{\pi}{2}(2n f_0 + f_0) T_0)}{2n f_0 + f_0} \right) \frac{1}{2 T_0 \pi}$

et puisque $b_n = 0$

et $C_n = \frac{a_n - j b_n}{2} \Rightarrow a_n = 2 C_n$

Scanné avec CamScanner