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Price and Quantity Competition among Heterogeneous Suppliers with Two-Part Pricing: Applications to Clubs, Local Public Goods, Networks, and Growth Controls

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Abstract

Models of club good, local public good, and growth control appear to have theoretical structures distinct from usual oligopoly models. This article shows however that they are special cases of a generalized oligopoly model that incorporates the possibility of two-part pricing and externalities between consumers (either congestion or network externalities). Our generalized two-part pricing model not only serves as a synthesis of a wide range of models but also allows us to obtain several new results on equilibrium prices. Another advantage of our model is that it can be considered as a reduced form of more complicated models that have spatial structures. This allows us to easily generalize the analysis to the case where firms are heterogeneous and the number of firms is arbitrary.

1. Introduction

Models of club good, local public good, and growth control appear to have theoretical structures quite distinct from standard oligopoly models. This article shows however that they are special cases of a generalized oligopoly model that incorporates the possibility of two-part pricing and externalities between consumers (either congestion or network externalities). We thus offer a synthesis of many different types of models, i.e., a differentiated product model with two-part pricing in Calem and Spulber (1984), a shared facility model of Scotchmer (1985b), club good models of Buchanan (1965), McGuire (1974), Berglas (1976), and Scotchmer (1985a), local public good models of Stiglitz (1977), Wooders (1978), Kanemoto (1980), Wildasin (1980), Brueckner (1983), and Scotchmer (1986), and growth control models of Epple et al. (1988), Brueckner (1990), Engle et al. (1992), Helsley and Strange (1995), Brueckner (1995), Sakashita (1995), and Brueckner and Lai (1996).

Our generalized two-part pricing model not only serves as a synthesis of a wide range of models but also allows us to obtain several new results on equilibrium prices. For example, if neither congestion nor network externality exists on the consumer side, the access fee as well as unit prices is efficient in a Bertrand-type price competition case, but it is not efficient in a Cournot-type quantity competition case. We also show that the access fee is *lower* than the marginal social cost of an additional subscriber in the Bertrand case if network externalities exist.

Another advantage of our model is that it can be interpreted as a reduced form of more complicated models that have spatial structures. This facilitates the extension to the case where firms are heterogeneous and the number of firms is arbitrary.

As Oi (1971) has shown, two-part pricing eliminates monopolistic price distortion when consumers are homogeneous. Because a monopolist can use a lump-sum access fee to capture the consumer's surplus, he/she does not have to raise the unit price. In an oligopoly setting access fees distort distribution of customers among different suppliers. This distortion is of a second-order magnitude however and Oi's result carries over to oligopoly models, as pointed out by Scotchmer (1985b). This feature of two-part pricing distinguishes our model from the standard oligopoly model.

With two-part pricing, potential choice variables for a firm are two types of prices, the access fee and unit prices, and two types of quantity variables, the number of customers and quantities of outputs. One can define an equilibrium concept using a combination of these four types of choice variables. In this paper we restrict our attention to the case where a firm is a price taker concerning the access fee.[2] Concerning unit prices and quantities of outputs, we consider both price taking and quantity taking cases.

In both equilibria, no distortion arises for the unit prices and capacity investment, but the access fee does not in general equal the marginal social cost of an additional subscriber. As the number of firms increases, the access fee approaches the marginal social cost. Even with a finite number of firms, however, a special case exists where the access fee is not distorted, i.e., a Bertrand equilibrium (a Nash equilibrium in prices) with neither congestion nor network externality on the consumption side.

All the results in this paper follow from profit maximization of a firm and do not require other equilibrium conditions. This means that these results hold even if other

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[2] It is not difficult to analyze quantity taking behavior concerning the number of customers, but, in our model where all consumers are homogeneous and the total number of consumers is fixed, a firm is faced with a fixed number of customers if those of the rest of the firms are fixed. In order to obtain a non-trivial equilibrium we have to allow for endogenous population size or to assume that some communities fix the access fees rather than quantities. Helsley and Strange (1995) adopts the latter approach assuming that one of the communities is passive, i.e., fixes the access fee at zero.

firms are not maximizing profits. It is worth emphasizing that many of the qualitative results on equilibrium prices can be obtained without a full characterization of Nash equilibria.

The organization of this paper is as follows. Section 2 formulates an oligopoly model of two-part pricing. Section 3 examines price taking behavior concerning unit prices, and Section 4 turns to the quantity taking case. Section 5 concludes the paper.

2. Model

Consider firms that supply a vector of goods and services (denoted *X*), charging a lump-sum access fee (denoted *f*) and a vector of unit prices (denoted *p*). The goods and services *X* may be club goods such as golf courses and tennis courts, local public services such as parks and roads, and network services such as telecommunication and electricity.

Suppliers of *X* may differ in their cost structures. The cost function of the *j*-th firm is $C^j = C^j(X^j, n^j, k^j)$, where X^j is the output vector of the firm, n^j is the number of subscribers, k^j is the capacity of the firm (or a capital input), and the cost function satisfies $C_X^j > 0$, $C_n^j \geq 0$, and $C_k^j > 0$. The assumption of $C_n^j \geq 0$ reflects the possibility that an increase in the number of subscribers is costly by itself. This is common in network industries where connecting a new user to a network requires additional facilities. For clubs and local public goods, an increase in membership may cause congestion even if the total consumption X^j is the same. For example, a pool which attracts 1,000 swimmers per day each of whom swims only for ten minutes would require more frequent cleaning than a pool with 100 swimmers per day each of whom swims for a hundred minutes.

The profits of firm j are $\Pi^j = f^j n^j + p^j X^j - C^j(X^j, n^j, k^j)$. We omit superscript j when this does not cause confusion.

We assume that all consumers have the same twice-differentiable and quasi-concave utility function, $U(z, x, X, n, k)$, where z is the consumption of the composite consumer good which represents all goods other than goods X; x is a consumption vector of goods X; and X, n, and k are respectively the output vector, the number of subscribers, and the capacity of the particular supplier that the consumer subscribes to. We assume that the first derivatives of the utility function satisfy $U_z > 0$, $U_x > 0$, and $U_k \geq 0$.

Goods X may involve congestion on the consumption side as well as the production side. Both the total consumption X and the number of subscribers n can cause congestion. For example, it is uncomfortable to swim if there are too many swimmers in a pool, in which case we have $U_X \leq 0$. An increase in the membership of a swimming club would cause congestion even if the total swim hours X were the same, because a swimmer who enters a pool disrupts the flow of swimmers. In this case, inequality $U_n \leq 0$ holds.

Note that in our model the capacity constraint can be 'soft.' The total consumption X can be increased without increasing 'capacity' k if consumers are willing to tolerate congestion. Our model however allows a strict capacity constraint as a limiting case. If for example the total consumption X can never exceed capacity k, then U_X is minus infinity at $X = k$.

Network externalities analyzed by Artle and Averous (1973), Littlechild (1975), Orens and Smith (1981), Rohlfs (1974), and Squire (1973) can be considered as “negative” congestion. In the case of telecommunication, a new subscriber gives external benefits to other subscribers because they now have the opportunity to call one more subscriber. We have $U_n \geq 0$ in this case.

Even in the presence of network externalities, more than one firm can coexist in equilibrium if congestion on the production side is strong enough to offset network externalities on the consumption side. This paper focuses on such a case.

A consumer may purchase X from more than one supplier, but nobody will do so in equilibrium because he/she can reduce the payment of access fees by trading with only one firm. From this and the homogeneous consumer assumption, we have $X = nx$ in equilibrium.

The budget constraint for a consumer is $y = z + f + px$, where y, f, and p are the consumer's income, an access fee, and a vector of unit prices of goods X respectively, and the composite consumer good is taken to be the numeraire. Define the expenditure function

$$E(p, X, n, k, u) = \min_{\{z,x\}}\{z + px\colon\ U(z, x, X, n, k) \geq u\}.$$

Note that this expenditure function does not include the access fee. In equilibrium the budget constraint satisfies $y = f + E(p, X, n, k, u)$. The partial derivatives of the expenditure function are

$$E_X(p, X, n, k, u) = -U_X/U_z\,,$$

$$E_k(p, X, n, k, u) = -U_k/U_z < 0\,,$$

$$E_n(p, X, n, k, u) = -U_n/U_z\,,$$

and

$$x(p, X, n, k, u) = E_p(p, X, n, k, u).$$

The last equation yields a recursive relationship,

$$X(p, k, n, u) = nx(p, X(p, k, n, u), n, k, u)\,,$$

which defines a demand function that a firm is faced with. This reduced-form demand function satisfies

$$X_p = nx_p/(1 - nx_X)$$

$$X_n = (x + nx_n)/(1 - nx_X)$$

$$X_u = nx_u/(1 - nx_X).$$

We assume that $nx_X < 1$, $x + nx_n > 0$, and $x_u > 0$. The first two inequalities exclude perverse cases. If the first inequality does not hold, congestion externality is so strong that the demand curve that the firm is faced with is upward sloping in the unit price. The second inequality excludes the case where an increase in the number of subscribers reduces the total demand for *X*. The last inequality is equivalent to normality of goods *X*. Under these assumptions, we have $X_p < 0$, $X_n > 0$, and $X_u > 0$.

We assume that the total number of consumers is *N* and fixed. Because a consumer trades with one firm only, the population constraint,

$$\sum_{j=1}^{J} n^j = N, \tag{2.1}$$

must hold, where *J* is the number of firms. In equilibrium all consumers obtain the same utility level (denoted by *u*) which yields

$$E^j(p^j, X^j(p^j, k^j, n^j, u), n^j, k^j, u) = y - f^j, \qquad j = 1, 2, \ldots, J. \tag{2.2}$$

Models of private goods, club goods, local public goods, and growth controls are special cases of our model.

(a) Private Goods

Calem and Spulber (1984) analyzed two-part pricing in an oligopoly model of private goods. This model assumes a utility function $U(z, x)$ and a cost function $C(X)$. With two-part pricing the budget constraint for a consumer is $y = z + f + px$.

(b) Club Goods:

(i) A fixed use intensity model

Club models of the simplest type assume that consumption of X is fixed exogenously. This fixed use intensity model is a special case of our model with utility function $U(z,n,k)$ and cost function $C(k)$. In this case the unit price is unnecessary and a firm charges only the access fee (membership fee). The budget constraint is then $y = z + f$.

(ii) Variable use intensity and shared facility models

The variable use intensity model in Berglas (1976) is more complicated and assumes a utility function $U(z,x,X,k)$ and a cost function $C(k, X)$.

The shared facility model of Scotchmer (1985b) assumes congestion only on the consumption side: the utility function is $U(z,x,X)$ and the cost of the facility is fixed. Suppliers adopt two-part pricing and the budget constraint for a consumer is $y = z + f + px$.

(c) Local Public Goods:

The local public good model of McGuire (1974), Wildasin (1980), Brueckner (1983), and Scotchmer (1986) assumes that a consumer must purchase residential land to consume local public goods. If we interpret x and k as land and local public goods respectively, the local public good model is a special case of our model with utility function $U(z,x,k)$ and cost function,

$$C(k, X) = \begin{cases} C(k) & \text{if } X \le H \\ \infty & \text{if } X > H \end{cases}$$

where H is the total available land in a jurisdiction.

Except in Scotchmer (1986), the head tax (or the access fee) is assumed impossible and local public goods are financed by a (100%) tax on land rent. The budget constraint for a consumer is then $y = z + px$; and the 'profit' of a local government is the total land rent minus the cost of the public good, $pH - C(k)$. If the head tax is available, the budget constraint becomes $y = z + f + px$. The 'profit' of a local government in this case is $pH + fn - C(k)$.

(d) Growth Controls

Models of growth controls typically have a spatial dimension and the city government restricts the physical size of the city. Examples of these models are in Epple et al. (1988), Brueckner (1990), Engle et al. (1992), Helsley and Strange (1995), Brueckner (1995), Sakashita (1995), and Brueckner and Lai (1996). Here we use the framework of Helsley and Strange (1995).

They assume that each community occupies a linear strip of land with one unit of land at each distance and that each household consumes one unit of land. Everybody commutes to the Central Business District (CBD) located at the left edge of the strip. Under these assumptions the population of a community, denoted *n*, coincides with the length of the residential zone. Commuting costs per unit distance is *t*. The utility function of a household is $u = \tilde{z} + a(n)$, where $\tilde{z}$ is the composite consumer good and $a(n)$ represents the amenity level that depends on the population size. We use the notation $\tilde{z}$ because the conversion of this spatial model into our non-spatial framework requires a change of variable as will be seen later. Helsley and Strange fully characterize Nash equilibria assuming that the amenity function has a linear form, $a(n) = -an$. The opportunity costs of the residential land are zero so that if there is no growth control, the residential land rent at the edge of the city is also zero. Growth controls raise this boundary land rent to a positive level.

All residents in a community receive the same utility level in equilibrium. This allows us to convert the spatial model into our non-spatial framework by focusing on a resident at the edge of the city. Because the length of the residential zone is *n*, the commuting costs for this resident are *tn*. Consider a growth control that raises the land rent at the edge from zero to *f*. The budget constraint for the resident is then $y = \tilde{z} + tn + f$. Now, we redefine the consumer good as $z = \tilde{z} + tn$. The budget constraint is then

$$y = z + f$$

and the utility function is

$$U(z, x, X, n, k) = z - an - tn .$$

The per-capita cost of public service provision is a constant *c*. The community developer receives the land rent in the residential zone that equals $fn + \frac{t}{2}n^2$. The profit of a community developer is then

$$\pi = (f - c)n + \frac{t}{2}n^2 .$$

In our non-spatial framework, the cost function of public services that is compatible with this profit function is

$$C(n, X, k) = cn - \frac{t}{2}n^2 .$$

Thus, the non-spatial counterpart of the growth control model has congestion on the consumer side represented by the last term in the utility function and an offsetting scale economy on the production side represented by the last term in the cost function. These properties will play a crucial role in creating inefficiencies of growth controls.

In the growth control model of Helsley and Strange (1995), the levels of local public services *X* are exogenous and their prices *p* do not exist. Their price control game assumes that a community takes the access fees (rather than unit prices) of other communities as given, whereas in their population control game populations of other communities are taken as given. This paper does not consider the latter game but it is not difficult to do so.

3. Price Competition

This section examines price competition where a firm maximizes its profit, taking other firms' prices as given. A Nash equilibrium in prices (or a Bertrand equilibrium) is obtained when all firms simultaneously engage in this type of behavior. We do not, however, attempt a full characterization of a Nash equilibrium. Instead, we derive a formula for profit maximizing prices given the price-taking behavior of a firm. Our results therefore hold even when other firms are not maximizing profits.

With two-part pricing, a firm chooses a price schedule that consists of the access fee f and unit prices p. In this section we assume that it takes both of them as given. The quantity competition that we examine in the next section assumes that a firm takes the access fee f and quantities X as given. The price competition applies when firms set price schedules and accept whatever demand the price schedules generate. The quantity competition would be relevant when other firms are faced with strict capacity constraints so that the firm believes that others cannot increase supply even if it raises its price schedule.

In the local public good model, which equilibrium concept is relevant depends on the ownership structure of land. If private individuals own land and land rent is determined by the market, land rent (which is one of the unit prices in our model) is not a direct choice variable for a local government. In such a case a Cournot assumption is more appropriate. If the local government owns the land and sets the land rent, then a

Bertrand assumption may be applicable although there is no a priori reason to exclude the Cournot equilibrium even in this case.

In addition to these two variables, a firm can choose capacity k. If the capacity constraint is strict in the sense that k is an upper bound for the firm's production X, then it must adjust capacity k to meet whatever demand its price schedule generates. In such a case capacity k cannot be an independent choice variable. In our model with congestion, however, the firm can increase production X without expanding capacity k although the firm and consumers must incur congestion costs. In sum, each firm chooses the access fee f, the unit prices p, and capacity k, taking other firms' choices of these variables as given.

If prices and capacities of all firms are given, equilibrium conditions (2.1) and (2.2) yield the number of subscribers and the utility level,

$$n^j = n^j(\mathbf{f},\mathbf{p},\mathbf{k}), \quad j = 1,\ldots,J, \tag{3.1}$$

$$u = u(\mathbf{f},\mathbf{p},\mathbf{k}), \tag{3.2}$$

where $\mathbf{f} = (f^1,\ldots,f^J)$, $\mathbf{p} = (p^1,\ldots,p^J)$, $\mathbf{k} = (k^1,\ldots,k^J)$. Demand for firm j is then

$$X^j(p^j,k^j,n^j(\mathbf{f},\mathbf{p},\mathbf{k}),u(\mathbf{f},\mathbf{p},\mathbf{k})), \quad j = 1,\ldots,J, \tag{3.3}$$

where $X^j(p^j,k^j,n^j,u)$ is the reduced form demand function derived in the preceding section.

Let us first consider profit maximization of firm 1 that takes other firms' policies, $(f^2,\ldots,f^J)$, $(p^2,\ldots,p^J)$, $(k^2,\ldots,k^J)$, as given. Suppressing other firms' policies, we can rewrite (3.1) and (3.2) as

$$n^1 = n^1(f^1,p^1,k^1) \tag{3.4}$$

$$u = u(f^1,p^1,k^1). \tag{3.5}$$

The profit of firm 1 can then be written as a function of its choice variables (f^1,p^1,k^1).

We can write this function as

$$\begin{aligned}\Pi(f,n,k) = {} & f\,n(f,p,k) + pX(p,k,n(f,p,k),u(f,p,k)) \\ & - C(n(f,p,k), X(p,k,n(f,p,k),u(f,p,k)),k)\end{aligned}, \tag{3.6}$$

where we suppress superscript 1 when this does not cause confusion. Maximization of this profit function with respect to (f,p,k) yields the following first order conditions.

$$f - C_n = -\frac{1}{n_f}\{n + (p - C_X)(X_n n_f + X_u u_f)\} \tag{3.7}$$

$$p - C_X = -\frac{X + (f - C_n)n_p}{X_p + X_u u_p + X_n n_p} \tag{3.8}$$

$$(f - C_n)n_k + (p - C_X)(X_n n_k + X_k + X_u u_k) = C_k. \tag{3.9}$$

If the unit price equaled the marginal cost of production (i.e., $p = C_X$), then the usual monopoly pricing formula would hold for the access fee *f*:

$$\mathrm{MC_n} = \mathrm{MR_n} = f(1 + \frac{n}{n_f f}) \leq f.$$

If the access fee equaled the marginal cost of a subscriber (i.e., $f = C_n$), then the similar monopoly pricing formula would be obtained for the unit prices *p*:

$$\mathrm{MC_X} = \mathrm{MR_X} = p(1 + \frac{X/p}{X_p + X_u u_p + X_n n_p}) \leq p.$$

In a monopoly model, Oi (1971) showed that the access fee is more efficient than the unit price in capturing the monopoly rent because it is a non-distortionary lump-sum charge. As noted by Scotchmer (1985b), this result extends to an oligopoly model. We show that distortion in unit prices does not occur also in our model. It is noteworthy that this is true even when access fees distort the distribution of customers between different suppliers.

Let us first evaluate the derivatives of $n^1(f^1, p^1, k^1)$ and $u(f^1, p^1, k^1)$.

LEMMA 1. Partial derivatives of $n^1(f^1, p^1, k^1)$ and $u(f^1, p^1, k^1)$ satisfy

$$n_f^1 = -\frac{1}{e_n^1}\frac{\sum_{j=2}^{J}(e_u^j / e_n^j)}{\sum_{j=1}^{J}(e_u^j / e_n^j)}; \qquad u_f = -\frac{1}{e_n^1}\frac{1}{\sum_{j=1}^{J}(e_u^j / e_n^j)}$$

$$n_p^1 = (x^1 + E_X^1 X_p^1)n_f^1; \qquad u_p = (x^1 + E_X^1 X_p^1)u_f$$

$$n_k^1 = (E_k^1 + E_X^1 X_k^1)n_f^1; \qquad u_k = (E_k^1 + E_X^1 X_k^1)u_f$$

where

$$e_n^j = E_X^j X_n^j + E_n^j \text{ and } e_u^j = E_X^j X_u^j + E_u^j \quad \text{for} \quad j = 1, ..., J.$$

PROOF:

The derivatives of $n^1(f^1, p^1, k^1)$ and $u(f^1, p^1, k^1)$ follow from

$$\begin{bmatrix} \quad \mathsf{W} \quad \end{bmatrix}\begin{bmatrix} dn^1 \\ \vdots \\ dn^J \\ du \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}df^1 - \begin{bmatrix} x^1 + E_X^1 X_p^1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}dp^1 - \begin{bmatrix} E_k^1 + E_X^1 E_k^1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}dk^1 \qquad (3.10)$$

where

$$\mathsf{W} = \begin{bmatrix} e_n^1 & 0 & \cdots & 0 & e_u^1 \\ 0 & e_n^2 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & e_n^J & e_u^J \\ 1 & \ldots & 1 & 1 & 0 \end{bmatrix}.$$

Define

$$\omega = |\mathsf{W}| = -\left[\prod_{j=1}^{J} e_n^j\right]\sum_{j=1}^{J}\frac{e_u^j}{e_n^j}$$

$$\phi = \begin{vmatrix} 1 & 0 & \cdots & \cdots & 0 & e_u^1 \\ 0 & e_n^2 & 0 & \cdots & 0 & e_u^2 \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & e_n^J & e_u^J \\ 0 & 1 & \cdots & 1 & 1 & 0 \end{vmatrix}$$

and

$$\psi = \begin{vmatrix} e_n^1 & 0 & \cdots & 0 & 1 \\ 0 & e_n^2 & \ddots & \vdots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & e_n^J & \vdots \\ 1 & \ldots & 1 & 1 & 0 \end{vmatrix} = -\prod_{j=2}^{J} e_n^j .$$

Then, by Cramer's rule, we obtain

$$n_f^1 = -\frac{\phi}{\omega} \quad \text{and} \quad u_f = -\frac{\psi}{\omega} ,$$

which yields the first two equalities in the lemma. The rest of the lemma is obvious from (3.10). **Q.E.D.**

According to the following proposition, distortion occurs only in the access fee, and the unit prices and capacity investment are not distorted.

PROPOSITION 1. If a firm maximizes its profit, taking the access fees, unit prices, and capacities of other firms as given, then its unit prices equal the marginal costs of producing goods *X* plus the marginal congestion costs,

$$p^j = C_X^j - n^j \frac{U_X^j}{U_z^j}, \quad j = 1, \cdots, J ,$$

and its capacity investment is carried out until the marginal benefit of capacity expansion equals the marginal cost,

$$n^j \frac{U_k^j}{U_z^j} = C_k^j, \quad j = 1, \cdots, J .$$

The access fee diverges from the marginal social cost of a new subscriber:

$$f^j - C_n^j + n^j U_n^j / U_z^j = n^j \frac{1}{U_z^j \sum_{i \neq j} (e_u^i / e_n^i)}, \qquad j = 1, \cdots, J ,$$

where

$$e_n^i = -\frac{U_n^i}{U_z^i} - \frac{U_X^i}{U_z^i} \frac{x^i + n^i x_n^i}{1 - n^i x_X^i},$$

$$e_u^i = \frac{1}{U_z^i} - \frac{U_X^i}{U_z^i}\frac{n^i x_u^i}{1-n^i x_X^i}.$$

PROOF:

Let us restrict our attention to $j=1$ and suppress the superscript. The same result holds for other firms. From Lemma 1, we have $n_p = (x + E_X X_p)n_f$, $u_p = (x + E_X X_p)u_f$, $n_k = (E_k + E_X X_k)n_f$, and $u_k = (E_k + E_X X_k)u_f$, which yields

$$\Pi_p = (x + E_X X_p)\Pi_f + (p - C_X - nE_X)X_p,$$

and

$$\Pi_k = (E_k + E_X X_k)\Pi_f + (p - C_X - nE_X)X_k - (nE_k + C_k).$$

Combining these relationships with the first order conditions for profit maximization (i.e., $\Pi_f = \Pi_p = \Pi_k = 0$) shows that, if $X_p \neq 0$, then $p = C_X + nE_X$ and $-nE_k = C_k$.

Next, from Lemma 1 and

$$\Pi_f = n + \{(f - C_n) + (p - C_X)X_n\}n_f + (p - C_X)X_u u_f = 0,$$

we obtain

$$\begin{aligned} f &= C_n - nE_X X_n - \frac{n}{n_f}(1 + E_X X_u u_f) \\ &= C_n + nE_n + n\frac{1}{\sum_{j=2}^{J}(1/e_n^j)(e_u^j/E_u^1)} \end{aligned}$$

Q.E.D.

A surprising result is obtained in a special case where neither congestion nor network externality exists on the consumer side. In such a case, $E_X^j = E_n^j = e_n^j = 0$, which implies that no distortion occurs for the access fee.[3]

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3 Note that the absence of congestion on the consumer side does not necessarily imply natural

COROLLARY 1. If neither congestion nor network externality exists on the consumer side (i.e., $U_X = U_n = 0$), then the access fee equals the social marginal cost of an additional subscriber, i.e., $f^j = C_n^j$ for any *j*.

This corresponds to the well-known result that in a Bertrand model the price equals the marginal cost. Because we assumed that a firm takes other firms' access fees as given, it is faced with a horizontal demand curve if neither congestion nor network externality exists on the consumption side.

Congestion on the consumer side makes the demand curve downward sloping. The reason is that, because a reduction in the number of customers eases congestion, the firm can charge a higher access fee. With a downward sloping demand curve, the profit maximizing level of access fee exceeds the marginal social cost of an additional subscriber.

The network externality has the opposite effect of making the demand curve upward sloping. If $U_n > 0$ and $U_X = 0$, then the above proposition shows that the access fee is lower than the marginal social cost of a subscriber. Note that the second order condition for profit maximization is satisfied even in this case if congestion on the production side is strong enough.

COROLLARY 2. If $U_X \le 0$ and $U_n \le 0$, then the access fee is higher than or equal to the marginal social cost of an additional subscriber:

$$f^j \ge C_n^j - n^j U_n^j \big/ U_z^j, \quad j = 1, \cdots, J.$$

If $U_n > 0$ and $U_X = 0$, then the access fee is lower than the marginal social cost.

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monopoly because congestion on the production side tends to favor smaller firm size.

If $U_X > 0$ and $U_n > 0$, then it is not clear whether or not the access fee is lower than the social marginal cost because e_u may become negative.

Although the access fee is in general distorted, the total number of subscribers is fixed at N and will not be distorted. Distortions in real resource allocation are therefore limited to the distribution of subscribers among firms.

The formula for the access fee becomes simpler in a symmetric equilibrium where all firms have the same cost structure and charge the same price.

COROLLARY 3. In a symmetric equilibrium, we have

$$f - C_n + \frac{nU_n}{U_z} = \frac{1}{J-1} n(E_X X_n + E_n) \frac{E_u}{E_X X_u + E_u}.$$

This corollary implies that, as the number of firms becomes larger, the access fee approaches the social marginal cost of a subscriber. The distortion in access fee is proportional to $1/(J-1)$. For example, the distortion becomes a half as the number of firms increases from 2 to 3. In our model the first best allocation is attained in the symmetric case. Because all consumers purchase goods X and the number of consumers is fixed, the access fees are equivalent to non-distortionary lump-sum taxes so long as all firms charge equal fees.

Equilibrium prices in the examples are as follows.

(a) Private Goods: $p = C_X$ and $f = 0$.

In the case of private goods, the access fee is zero because no congestion exists on the consumption side and $C_n = 0$. Because the unit price equals the marginal cost, increasing returns to scale at the margin are not compatible with an equilibrium with nonnegative profits.

(b) Club Goods

(i) A Fixed Use Intensity Model: $p = 0$ and $f = \frac{J}{J-1}(-nU_n/U_z)$.

Because the consumption of the club good, *x*, is fixed exogenously in this case, we can safely assume that the unit price is zero. The membership fee (the access fee) is positive because an increase in membership causes congestion. When the number of clubs is two, the access fee is twice as high as the congestion cost of an additional club member. The ratio between the access fee and the marginal congestion cost quickly approaches one as the number of clubs increases.

(ii) Variable Use Intensity and Shared Facility Models:

In both cases the access fee is

$$f = \frac{1}{J-1} \frac{x}{1-nx_X} p \frac{E_u}{E_X X_u + E_u}.$$

The unit price is

$$p = C_X - n\frac{U_X}{U_z}$$

in a variable use intensity model and

$$p = -n\frac{U_X}{U_z}$$

in a shared facility model.

In both cases the unit price equals the marginal congestion cost of *X*. Because an additional user of a facility does not cause any increase in social costs so long as the total consumption *X* is the same, the efficient level of the access fee is zero. With a finite number of firms, the access fee is positive, but it approaches zero as the number increases.

If the utility function is quasi-linear as in Scotchmer (1985b), then $X_u = 0$ and

$x_X = 0$, which yields $f = \frac{1}{J-1} xp$ in the shared facility model. This coincides with Scotchmer's result. In this case the access fee happens to equal the revenue from unit prices, i.e., $f = px$, when the number of firms is two.

(c) Local Public Goods: $p = U_X / U_z$ and $f = 0$.

A local government owns all the land in its jurisdiction and sets the head tax and land rent to maximize its fiscal surplus, i.e., the sum of the head tax and land rent revenues minus the cost of local public goods. In equilibrium, land rent is equal to the marginal rate of substitution between land and the consumer good. The supply of a local public good is optimal even if the number of local governments is finite.

In the local public good model, neither congestion nor network externality exists on the consumption side. Furthermore, an increase in membership does not cause congestion on the production side, either. The local public good model therefore satisfies $U_X = U_n = C_n = 0$ and the head tax (or the access fee) is always zero in equilibrium. This means that the head tax is not necessary to attain the efficient supply of local public goods. This result depends crucially on our Bertrand assumption. We shall see below that in a Nash equilibrium in quantities the head tax is positive.

(d) Growth Controls

Section 4 of Helsley and Strange (1995) contains the analysis of price competition where two active communities engage in price controls and a passive community sets the price equal to zero. In our model, this corresponds to the case where the active communities choose the access fee *f* optimally, taking other communities' access fees fixed. Substituting the utility function $U(z, x, X, n, k) = z - an - tn$ and the cost function $C(n, X, k) = cn - \frac{t}{2} n^2$ into Proposition 1 yields

$$f = c + \frac{(3a+t)n}{2}$$

for an active community, where n is the population of the community. This shows that a community's choice of growth control is always inefficient because the 'price' for entry exceeds the per capita cost of local public services, i.e., $f > c$.

Note that the inefficiency result holds even if the population size does not affect the amenity level. The reason is that an increase in population size results in higher commuting costs. Although the utility function in the original spatial model does not include commuting costs, the utility function in the non-spatial counterpart embodies a change in commuting costs induced by an increase in population.

Helsley and Strange obtain the full characterization of the Nash equilibrium and show, in our notations, that

$$n = \frac{2[(a+t)N - c]}{9a+7t}$$

$$f = \frac{(a+t)[6c + (3a+t)N]}{9a+7t}.$$

This is consistent with our result.

4. Quantity Competition

In this section we consider a Cournot case where a firm takes the access fees (f^j's) and outputs (X^j's) of other firms as given. This case is relevant if it takes time for a firm to change the access fee and outputs compared with the number of customers and unit prices. As noted in the preceding section, a typical example would be a local public good model where private individuals own land and the local government levies taxes on land rent.

Consider profit maximization of firm 1 that takes other firms' policies,

$(f^2,\dots,f^J)$, $(X^2,\dots,X^J)$, $(k^2,\dots,k^J)$, as given. Solving $X^j = X(p^j,k^j,n^j,u)$ for p^j, we obtain $p^j = p^j(X^j,k^j,n^j,u)$. Using this relationship, we can rewrite the market clearing condition (2.2) as

$$E^j(p^j(X^j,k^j,n^j,u), X^j,n^j,k^j,u) = w - f^j, \quad j = 1,2,\dots,J. \tag{4.1}$$

Then, in the same way as in the preceding section, we can write the number of subscribers and their utility level as functions of firm 1's choice variables:

$$n^1 = n^1(f^1, X^1, k^1) \tag{4.2}$$

$$u = u(f^1, X^1, k^1). \tag{4.3}$$

Firm 1 maximizes

$$\Pi^1 = f^1 n^1 + p^1(X^1,k^1,n^1,u)X^1 - C^1(n^1, X^1, k^1) \tag{4.4}$$

with respect to (f^1, X^1, k^1) subject to (4.2) and (4.3). We suppress superscript 1 when obvious.

The next lemma obtains the derivatives of (4.2) and (4.3).

LEMMA 2. Partial derivatives of $n^1(f^1, X^1, k^1)$ and $u(f^1, X^1, k^1)$ satisfy

$$n_f^1 = -\frac{1}{e_n^1}\frac{\sum_{j=2}^{J}(e_u^j/e_n^j)}{\sum_{j=1}^{J}(e_u^j/e_n^j)}; \quad u_f = -\frac{1}{e_n^1}\frac{1}{\sum_{j=1}^{J}(e_u^j/e_n^j)}$$

$$n_X^1 = (xp_X + E_X)n_f; \qquad u_X = (xp_X + E_X)u_f$$

$$n_k^1 = (E_k + xp_k)n_f; \qquad u_k = (E_k + xp_k)u_f.$$

PROOF:

If we replace elements of Ω in the preceding section by

$$e_n^j = x^j p_n^j + E_n^j = -x^j\frac{X_n^j}{X_p^j} + E_n^j,$$

$$e_u^j = x^j p_u^j + E_u^j = -x^j \frac{X_u^j}{X_p^j} + E_u^j \geq 0,$$

then we obtain

$$\begin{bmatrix} & \mathrm{W} & \end{bmatrix} \begin{bmatrix} dn^1 \\ \vdots \\ dn^J \\ du \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} df^1 - \begin{bmatrix} x^1 p_X^1 + E_X^1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} dX^1 - \begin{bmatrix} E_k^1 + x^1 p_k^1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} dk^1.$$

The lemma then immediately follows by applying the same argument as in Lemma 1.

Q.E.D.

Equilibrium prices can be characterized in the same way as in the preceding section.

PROPOSITION 2. If a firm maximizes its profit, taking the access fees, output levels, and capacities of other firms as given, then unit prices and capacity investment satisfy the same conditions as in the price competition case. The condition for the access fee is also the same as in Proposition 1 if e_n^j and e_u^j are modified as

$$e_n^j \equiv -\frac{U_n^j}{U_z^j} - x^j \frac{X_n^j}{X_p^j} \text{ and } e_u^j \equiv \frac{1}{U_z^j} - x^j \frac{X_u^j}{X_p^j}.$$

PROOF:

From Lemma 2,

$$\Pi_X = (xp_X + E_X)\Pi_f + (p - C_X - nE_X)$$

$$\Pi_k = (xp_k + E_X)\Pi_f - (nE_k + C_k).$$

Hence, the first order conditions for profit maximization, $\Pi_f = \Pi_X = \Pi_k = 0$, yield

$$p = C_X + nE_X$$

and

$$-nE_k = C_k.$$

Now, from

$$\Pi_f = n + (f - C_n + Xp_n)n_f + Xp_u u_f = 0,$$

we get

$$f = C_n + nE_n + n^1 \frac{1}{\sum_{j=2}^{J} (1/e_n^j)(e_u^j / E_u^1)}.$$

Q.E.D.

Thus, no distortion arises in unit prices and capacity investment also in the quantity competition case. The access fee is distorted as in the Bertrand equilibrium but the precise formulae are different.

An important difference from the previous case is that distortion in the access fee does not vanish even when neither congestion nor network externality exists on the consumer side. Even if $E_X^j = E_n^j = 0$, we have $e_n^j = -\left(x^j\right)^2 \Big/ \left(n^j x_p^j\right) > 0$, which yields the following result.

COROLLARY 1. Even if neither congestion nor network externality exists on the consumption side, the access fee exceeds the social marginal cost of an additional subscriber: $f^j > C_n^j$.

If neither congestion nor network externality exists on the consumer side, a firm in a Bertrand model perceives that a slight rise in its access fee (with unit prices fixed) induces a mass exodus of its customers because it takes other firms' unit prices and access fee as given. In a Cournot model however it takes other firms' output levels as fixed and believes that their unit prices will be adjusted to meet the capacity constraint. In such a case a small rise in the access fee induces only a small reduction in the number of customers.

Corollary 1 implies that the access fee can be higher than the marginal social cost of an additional subscriber even in the case of network externality.

COROLLARY 2. Even if $U_n > 0$ and $U_X = 0$, the access fee can exceed the marginal social cost of an additional subscriber.

In a symmetric equilibrium we obtain the following corollary.

COROLLARY 3. In a symmetric equilibrium we have

$$f = C_n + nE_n + \frac{1}{J-1}\left[-X\left(X_n/X_p\right) + nE_n\right]\frac{E_u}{-x\left(X_u/X_p\right) + E_u}.$$

As in the Bertrand case, the distortion in the access fee will vanish as the number of firms approaches infinity. Even if the access fee is distorted, however, a symmetric equilibrium is first best efficient, since the total number of consumers is fixed.

Equilibrium prices in our examples are as follows. We do not have the growth control example here because local public services X are exogenous.

(a) Private Goods: $p = C_X$ and $f = -\frac{1}{J-1}\frac{(x)^2}{x_p}\frac{E_u}{-x(X_u/X_p) + E_u}$.

Unlike in the Bertrand case, the access fee is positive except in the limit as the number of firms approaches infinity.

(b) Club Goods

(i) A Fixed Use Intensity Model: $p = 0$ and $f = \frac{J}{J-1}(-nU_n/U_z)$.

Equilibrium prices are the same as in the Bertrand equilibrium.

(ii) Variable Use Intensity and Shared Facility Models:

In both cases the unit price is the same as in the Bertrand equilibrium. The access fee,

$$f = -\frac{1}{J-1}\frac{(x)^2}{x_p}\frac{E_u}{-x(X_u/X_p)+E_u},$$

is however different from the Bertrand equilibrium.

(c) Local Public Goods: $p = U_x/U_z$ and $f = -\frac{1}{J-1}\frac{(x)^2}{x_p}\frac{E_u}{-x(X_u/X_p)+E_u}$.

In a Bertrand equilibrium in the preceding section, the head tax (the access fee) is zero in the local public good model. In a Cournot equilibrium the head tax is positive. An implication of this result is that if the head tax is restricted to zero, the unit price and/or capacity choice will be distorted in a Nash equilibrium in quantities.[4] If the utility function is quasi-linear, then $X_u = 0$ and we obtain the same result as in Scotchmer (1986):

$$f = -\frac{1}{J-1}\frac{(x)^2}{x_p}.$$

In the limit as the number of firms approaches infinity, the access fee becomes zero. There is an extensive literature on this case, e.g., Brueckner (1983), Kanemoto (1980), and Henderson (1985). An important issue that is treated in this literature is whether a property tax on housing serves as a congestion tax. Consider an extension of the (pure) local public good model to allow for congestion, i.e., the cost function is now $C(k,n,X)$. Then, the first best allocation requires the access fee (or the head tax) as a congestion tax. Hamilton (1975) shows that the property tax on housing consumption serves as a congestion fee. His result however relies on zoning regulation that correct

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[4] Scotchmer (1986) obtained this result in a local public good model.

distortion in housing consumption caused by the property tax. Hoyt (1991), Krelove (1993), and Wilson (1997) examine whether the property tax can serve as a congestion fee even when there are no zoning restrictions on zoning. This issue may be analyzed in our framework by assuming that the output vector consists of two components, land and housing. We do not spell out the results here because they coincide with Wilson's.

5. Concluding Remarks

With two-part pricing, many of the results in the oligopoly theory must be modified. The most important modification is that, with homogeneous consumers, unit prices and capacity investment are not distorted even when a firm has monopoly power. The access fee is distorted, but the distortion disappears as the number of firms increases.

Scotchmer (1985b) obtained these results in a Bertrand equilibrium of a simple shared facility model. This paper extends the results to Bertrand and Cournot equilibria of a fairly general model with heterogeneous suppliers. Our model includes private goods, club goods, local public goods, shared facilities, and growth controls as special cases. This extension clarifies relationships among these models and between the two types of equilibria. Furthermore, we obtain interesting new results. For example, if there is neither congestion nor network externality on the consumption side, a Bertrand equilibrium involves no distortion even in the access fee. The local public good model is an important example of this case. Another interesting result is that network externalities on the consumption side tend to make the access fee *lower* than the social marginal cost of an additional subscriber.

We assumed that the profits of the firms are given to absentee share holders. If the consumers own the shares of the firms, then repercussions through distribution of profit income are introduced. The analysis of such a case is somewhat more complicated, but similar (though more complicated) formulas are obtained. All the qualitative results remain the same.

The assumption of homogeneous consumers is crucial to our results. With heterogeneous consumers, the analysis becomes much more complicated because the price structure serves an additional role of a self-selection device. Much of the literature on nonlinear pricing in a monopoly model focused on this aspect, and extending their results to oligopolistic competition is a fruitful direction of future research.[5]

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[5] See Brown and Sibley (1986) for an excellent textbook treatment of nonlinear pricing with

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