summary statistics

2018/09/11

In this tutorial, you get familiarised with the basic ideas of time series data, using Japanese financial data from Jan 1975 to Dec 1995. Dataset contains 10-year bond return (end of month), 3-month Tokyo Interbank Offerd Rate (TIBOR) and Collaterised overnight call rate (montly average).

First, I import the libraries and dataset, and then compute the spread, which is the difference of 10-year bond return and 3-month TIBOR.

```
library(magrittr)
library(tidyverse)
library(xts)
```

```
dat<- read.csv("~/Documents/GitHub/Applied_TimeSeries_Analysis/dat/spreadCall.csv",
               header=F, skip=2, stringsAsFactors = F) %>%
  set_colnames(c('date','tenYearBond', 'TIBOR', 'call')) %>%
  mutate(spread=tenYearBond-TIBOR)
datXts <- xts(dat[,c(4,5)], order.by = as.yearmon(dat[,1], format="%Y/%m"))
first(datXts, "3 months")
```

```
##           call spread
##  1 1975 12.674  -2.37
##  2 1975 13.000  -4.48
##  3 1975 12.920  -3.93
```

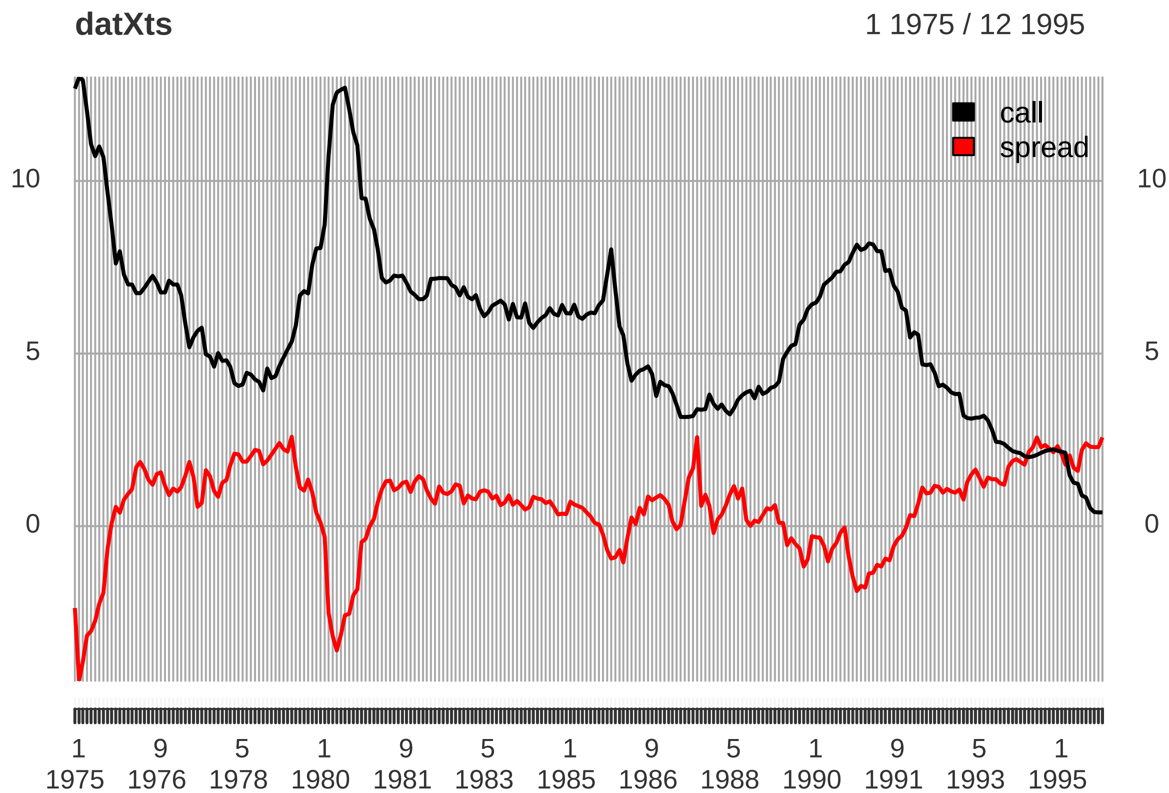
```
last(datXts, "3 months")
```

```
##         call spread
## 10 1995 0.41   2.29
## 11 1995 0.40   2.29
## 12 1995 0.40   2.57
```

1. Basics of time series data

First thing to do is to vidualise the data.

```
plot.xts(datXts, legend.loc = 7)
```



The sample means of `call` and `spread` are

```
colMeans(datXts)
```

```
##      call    spread
## 5.7216383 0.5613095
```

Next, variance covariance[correlation] matrix gives a bries idea of the data series.
To get the correlation matrix, we divide variance covariance matrix by the standard diviation matrix, which I call `sdMat` (Varinaces on diagonal and cross product of standard diviations on off-diagonal elements).

```
div<-scale(datXts, scale=F) # diviation from mean
vcov<-t(div)%*%div/(nrow(div)-1)
vcov
```

```
##             call    spread
## call    6.281399 -2.463163
## spread -2.463163  1.663924
```

```
sdCall<- sd(datXts$call)
sdSpread<- sd(datXts$spread)
sdMat<- matrix(c(sdCall^2, sdCall*sdSpread, sdCall*sdSpread, sdSpread^2),2)
sdMat
```

```
##          [,1]     [,2]
## [1,] 6.281399 3.232920
## [2,] 3.232920 1.663924
```

```
corrMat<- vcov/sdMat
corrMat
```

```
##              call     spread
## call    1.0000000 -0.7619003
```

```
## spread -0.7619003  1.0000000
```

In addition to mean, variance and covariance, it is important to report autocovariance since time series data is characterised by its time-dependent order.
auto-variance covariance matrix is shown as follows where k represents the lag length.

$$\begin{bmatrix} cov(spread_t, spread_{t-k}) & cov(spread_t, call_{t-k}) \\ cov(call_t, spread_{t-k}) & cov(call_t, call_{t-k}) \end{bmatrix}$$

As examples, I consider k=1 and k=12.

```
autoCov_k1<- t(div[2:nrow(datXts),]) %*% div[1:(nrow(datXts)-1),]/(nrow(div)-2) # k=1
autoCov_k12<- t(div[13:nrow(datXts),]) %*% div[1:(nrow(datXts)-12),]/(nrow(div)-13) # k=12

autoCov_k1
```

```
##              call    spread
## call    6.078412 -2.365019
## spread -2.369980  1.560101
```

```
autoCov_k12
```

```
##               call     spread
## call    2.8124375 -0.6140454
## spread -0.2104592  0.1605558
```

```
autoCov_k1/sdMat
```

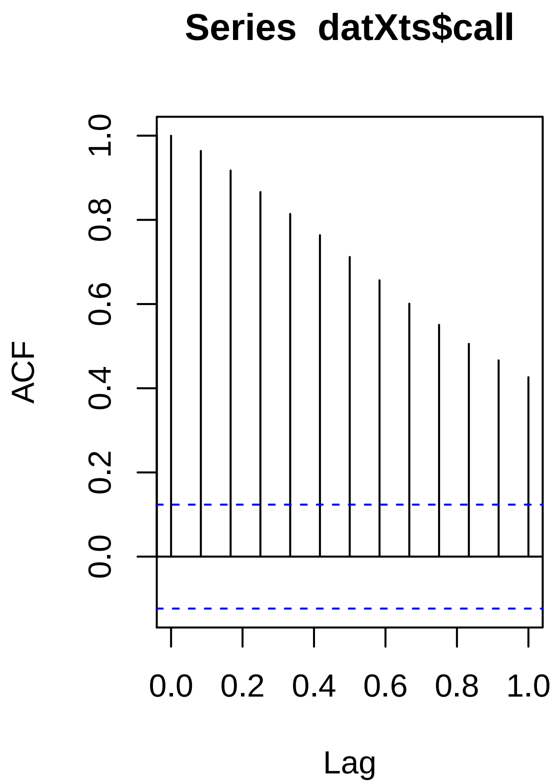
```
##               call     spread
## call    0.9676844 -0.7315429
## spread -0.7330772  0.9376037
```

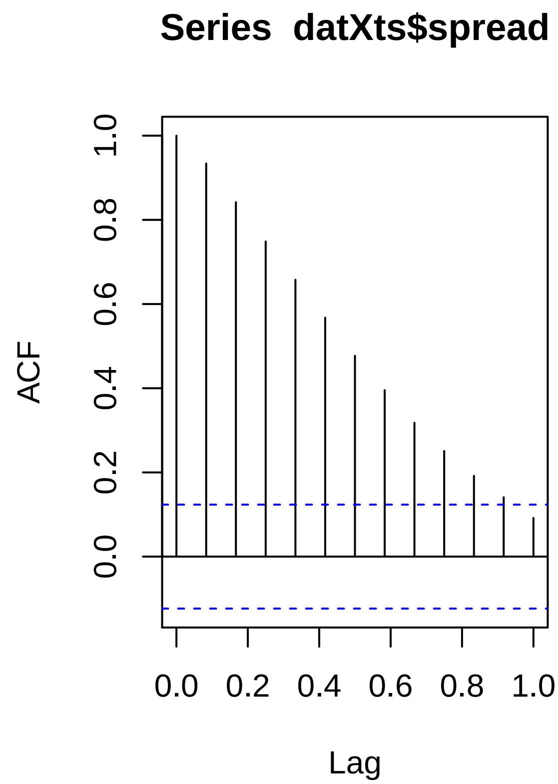
```
autoCov_k12/sdMat
```

```
##                call      spread
## call    0.44774062 -0.18993526
## spread -0.06509881  0.09649227
```

The autocorrelation plot (correlogram) for `call` and `spread` are

```
par(mfrow=c(1,2))
acf(datXts$call,lag.max = 12)
acf(datXts$spread,lag.max = 12)
```





Blue dotted line shows the critical value for the autocorrelation test. Under the null hypothesis of no autocorrelation, the test statistics (auto correlaiton function) follows standard normal distribution divided by the square root of the number of observations, i.e. $1.96/\sqrt{252} = 0.123$. As we see from the correlogram and the correlation matrix, the null hypothesis of no autocorrelation is rejected for all the observations except 12-lag spread. Note however, that this test is only for a specic lag (imagin t-test).
Ljung-Box test offers a statistical way to examine if any of a group of autocorrelations of a time series data is different from zero. Null hypothesis therefore is that the autocorrelations for up to 12 lags are all zero ($\rho_1 = \rho_2 = .. = \rho_{12} = 0$). We test for lag length of one, six and twelve (k=1, 6 and 12).

```
# Ljung-Box test
for (i in c(1,6,12)){
  print(Box.test(datXts$spread, lag=i, type='L'))
}
```

```
##
##  Box-Ljung test
##
## data:  datXts$spread
## X-squared = 222.4, df = 1, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  datXts$spread
## X-squared = 802.52, df = 6, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  datXts$spread
```

```
## X-squared = 903.72, df = 12, p-value < 2.2e-16
```

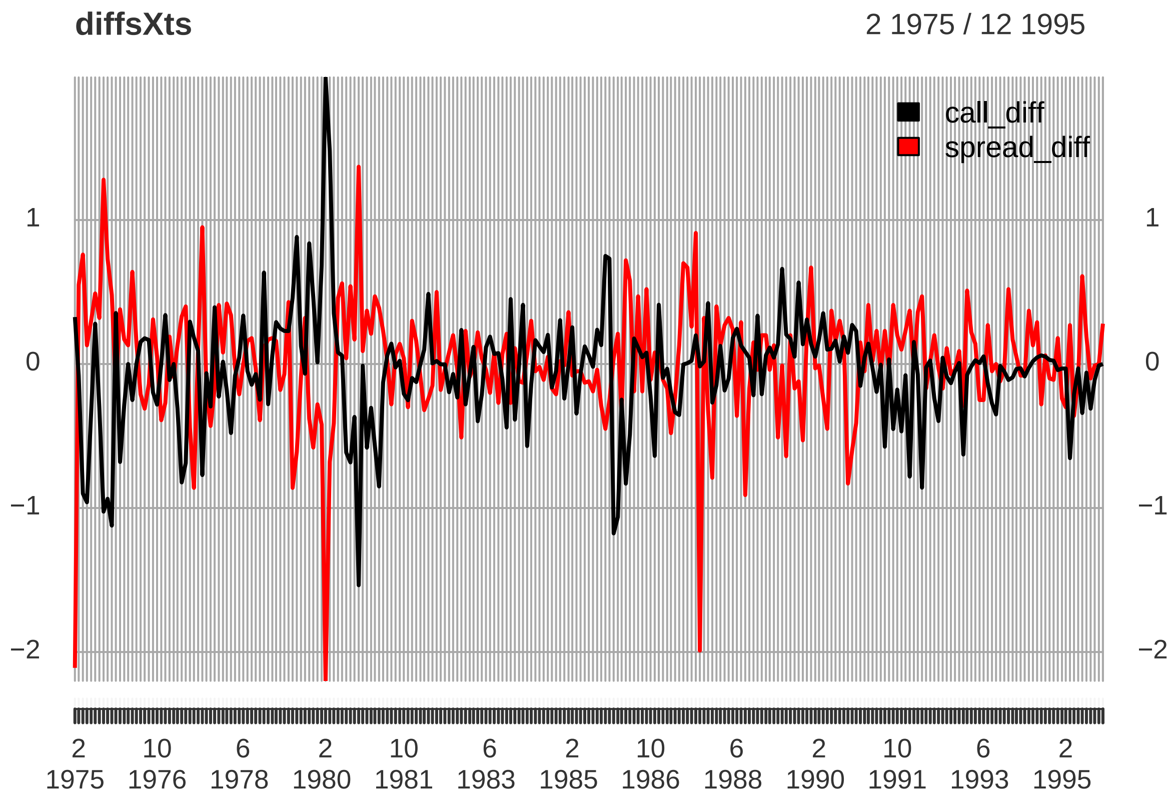
```
for (i in c(1,6,12)){
  print(Box.test(datXts$call, lag=i, type='L'))
}
```

```
##
##  Box-Ljung test
##
## data:  datXts$call
## X-squared = 236.9, df = 1, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  datXts$call
## X-squared = 1099.3, df = 6, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  datXts$call
## X-squared = 1560.4, df = 12, p-value < 2.2e-16
```

The result rejects the null hypothesis at all lags inspected, indicating `call` and `spread` indeed have autocorrelation.

Finally, I repeat the same steps, but this time for series of first order difference, which I call `call_diff` and `spread_diff`, respectively.

```
diffsXts<- diff.xts(datXts) %>%
  set_colnames(c("call_diff","spread_diff")) %>%
  na.omit
plot.xts(diffsXts,legend.loc = 7)
```




```
first(diffsXts, "3 months")
```

```
##          call_diff spread_diff
##  2 1975     0.326       -2.11
##  3 1975    -0.080        0.55
##  4 1975    -0.900        0.76
```

```
last(diffsXts,"3 months")
```

```
##          call_diff spread_diff
## 10 1995     -0.11       -0.01
## 11 1995     -0.01        0.00
## 12 1995      0.00        0.28
```

```
colMeans(diffsXts)
```

```
##   call_diff spread_diff
## -0.04890040  0.01968127
```

Calculate the variance covariance matrix of `spread_diff`,`call_diff`

```
div_diff<-scale(diffsXts, scale=F) # diviation from mean
vcov_diff<-t(div_diff)%*%div_diff/(nrow(div_diff)-1)
vcov_diff
```

```
##                call_diff spread_diff
## call_diff     0.14720440 -0.08578896
## spread_diff -0.08578896  0.17005830
```

```
sdCall_diff<- sd(diffsXts$call_diff)
sdSpread_diff<- sd(diffsXts$spread_diff)
```

```
sdMat_diff<- matrix(c(sdCall_diff^2, sdCall_diff*sdSpread_diff,
                 sdCall_diff*sdSpread_diff, sdSpread_diff^2),2)
sdMat_diff
```

```
##              [,1]      [,2]
## [1,] 0.1472044 0.1582192
## [2,] 0.1582192 0.1700583
```

```
corrMat_diff<- vcov_diff/sdMat_diff
corrMat_diff
```

```
##                call_diff spread_diff
## call_diff    1.0000000  -0.5422157
## spread_diff -0.5422157   1.0000000
```

Autocovariance and autocorrelation are;

```
autoCov_diff_k1<- t(div_diff[2:nrow(diffsXts),]) %*% div_diff[1:(nrow(diffsXts)-1),]/(nrow(div_diff)-2)
autoCov_diff_k12<- t(div_diff[13:nrow(diffsXts),]) %*% div_diff[1:(nrow(diffsXts)-12),]/(nrow(div_diff)

autoCov_diff_k1
```

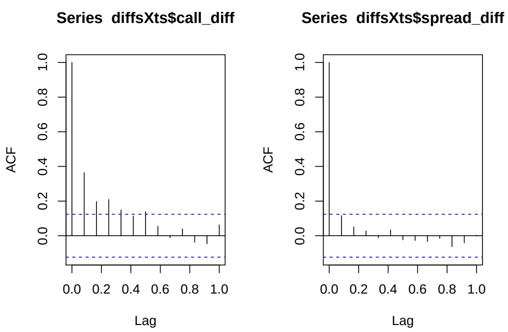
```
##                 call_diff spread_diff
## call_diff    0.05385098 -0.04883770
## spread_diff -0.04089622  0.01980912
```

```
autoCov_diff_k12
```

```
##                 call_diff  spread_diff
## call_diff   0.009660003 0.0032060610
## spread_diff 0.003012920 0.0005562539
```

Correlograms are;

```
par(mfrow=c(1,2))
acf(diffsXts$call_diff,lag.max = 12)
acf(diffsXts$spread_diff,lag.max = 12)
```



Critical value for autocorrelation test (dotted line) is $1.96/\sqrt{251} = 0.124$. `Call_diff` shows the sign of autocorrelation, but smaller than the level series. On the other hand, `spread_diff` doesn't have significant autocorrealtion at any lags inspected.

Again, I conduct Ljung-Box test to examine the autocorrealtion up to 12 lags.

```
for (i in c(1,6,12)){
  print(Box.test(diffsXts$call_diff, lag=i, type='L'))
}
```

```
##
```

```
##  Box-Ljung test
##
## data:  diffsXts$call_diff
## X-squared = 33.722, df = 1, p-value = 6.356e-09
##
##
##  Box-Ljung test
##
## data:  diffsXts$call_diff
## X-squared = 68.776, df = 6, p-value = 7.285e-13
##
##
##  Box-Ljung test
##
## data:  diffsXts$call_diff
## X-squared = 71.956, df = 12, p-value = 1.377e-10
```

```
for (i in c(1,6,12)){
  print(Box.test(diffsXts$spread_diff, lag=i, type='L'))
}
```

```
##
##  Box-Ljung test
##
## data:  diffsXts$spread_diff
## X-squared = 3.4191, df = 1, p-value = 0.06445
##
##
##  Box-Ljung test
##
## data:  diffsXts$spread_diff
## X-squared = 4.7598, df = 6, p-value = 0.575
##
##
##  Box-Ljung test
##
## data:  diffsXts$spread_diff
## X-squared = 6.7867, df = 12, p-value = 0.8714
```

`spread_diff` fails to reject the null hypothesis of no-autocorrelation at lag 1, 6 and 12, while `call_diff` rejects the null at each lag inspected.