MAT 5030: Homework 1

- Assigned on September 14 (Wed), 2016.
- **Due at 4:30pm on September 21 (Wed)**, 2016.
- Submit **by e-mail at seikibunpu@gmail.com**
- The e-mail title will be "M5030 HW1 - Your Name".
- Use the following format.
  - R-code, output, and brief explanations in one TXT or MS-Word file.
  - Graphics in one MS-Word or PDF file.
  - In case of MS-Word, you can put both in one file.
- Truncate the output if it exceeds about 20 lines in the R console.
- A late submission will reduce your score as follows. E.g, if you submit 19 hours late and your work gets 9 points out of 10, your score will be $9 \times 0.8 = 7.2$ out of 10.

Delayed by (in days)	0-0.5	0.5-1	1-1.5	$\cdots$	4-4.5	4.5-
Multiplier	0.9	0.8	0.7	$\cdots$	0.1	0

1. Let

$$f(x) = \sqrt{\frac{x^3 + 3x^2 + 1}{x^4 + 5x^3 + 7x + 9}} \quad (x \geq 0)$$

(a) Draw a line graph of $(x, f(x))$ for $0 \leq x \leq 10$ with increments of 0.01.

(b) Find numerically the maximum value of $f(x)$ and the maximizer x (report x to the second decimal place. For instance, $x = 1.23$).

2. Type in R:

```
set.seed(1);
X <- rnorm(1000)
```

Then the object X will be a vector of 1,000 standard normal random numbers.

(a) Calculate a standard deviation of the 1,000 numbers.

(b) Find the 100-th smallest number out of the 1,000, that is, approximately the 10-th percentile of the data.

(c) Is the result in (b) roughly consistent with a normal probability table?

See `http://www.stat.ufl.edu/~athienit/Tables/Ztable.pdf`, and explain briefly (no R-coding necessary).

3. Suppose

$$A = \begin{bmatrix} 0.979 & 0.144 \\ 0.147 & -0.999 \end{bmatrix}$$

(a) Calculate A^2 (a matrix product).

(b) Calculate A^8, A^{32} and A^{1024} (Hint: $8 = 2^3,\ 32 = 2^5,\ 1024 = 2^{10}$).

(c) Calculate A^{1000} (Hint: $1000 = 1024 - 32 + 8$).

(d) Obtain the eigenvalues and eigenvectors of A.

(e) Calculate A^{1000} by using the result in (d). (Hint: Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be the (column) eigenvectors of A, and λ_1 and λ_2 be the corresponding eigenvalues. Then,

$$A^n(\mathbf{x}_1, \mathbf{x}_2) = (\lambda_1^n \mathbf{x}_1, \lambda_2^n \mathbf{x}_2). \quad)$$