MAT 5030 Chapter 1: Basics

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Download **R** language from CRAN (Comprehensive R Archive Network) and install it to your computer.

CRAN Website:

```
http://cran.r-project.org/
```

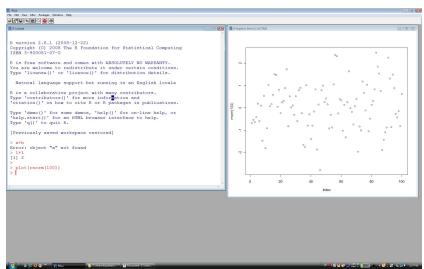
R is available for Windows, Mac OS X and Linux. Some references:

An Introduction to R:

```
http://cran.r-project.org/doc/manuals/R-intro.pdf
```

- "Introductory Statistics with R" (P. Dalgaard)
 - Available online at the WSU library website.
- "The R book" (J.J. Faraway)
 - ► Chapter 1 (and a part of 2) are free with Amazon Kindle.

A screen shot of R:

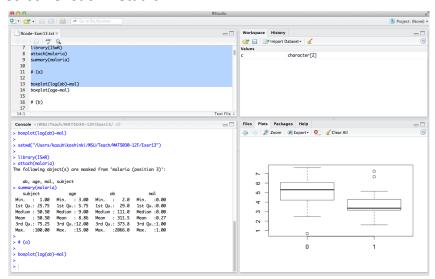


R Studio:

The R Studio is a free integrated development environment for R. It does not change the essentials of R, but it makes easier to edit codes, saves images and manage objects.

Download and install R Studio to your computer from http://www.rstudio.com/ if you want.

A screen shot of R Studio:



Basics

```
> 1+1 # addition
[1] 2
> 4/3-1/3 # subtraction
[1] 1
> 8*31 # multiplication
[1] 248
> 20/11 # division
[1] 1.818182
```

Basics (cont'd)

```
> 2^3 # power
[1] 8
> sqrt(2) # square root
[1] 1.414214
> 2^(1/2) # another way for square root
[1] 1.414214
> abs(-5.5) # absolute value of -5.5
[1] 5.5
```

Calculations: Scalar – Rounding

```
> round(2.98765) # nearest integer
[1] 3
> round(2.12345)
[1] 2
> round(2.12345, 2) # rounds to x.xx
[1] 2.12
> floor(8.81) # round-down
[1] 8
> ceiling(8.31) # round-up
[1] 9
```

Log & exponential

```
> exp(4) # e^4 (exponential)
Γ17 54.59815
> log(10) # logarithm of 10 (base = e)
Γ17 2.302585
> log10(100) # logarithm of 100 (base = 10)
[1] 2
> log(exp(3)) # log and exp are inverse each other.
[1] 3
> \exp(\log(3))
[1] 3
> (\exp(0.01) - 1)/0.01 # an example of using parentheses
Γ17 1.005017
```

Triangle functions

```
> pi
[1] 3.141593
> sin(pi/2)
[1] 1
> cos(pi/4)
[1] 0.7071068
> tan(pi/4)
[1] 1
```

Triangle functions (cont'd)

```
> asin(1/2) # arcsine
[1] 0.5235988

> acos(1/2) # arccos
[1] 1.047198

> atan(1) # arctan
[1] 0.7853982

> cosh(3) # hyperbolic cosine (= (exp(3)+exp(-3))/2)
[1] 10.06766
```

Gamma function

Since R does not have a function for factorial (e.g., 3!), the Gamma function is useful for combinatorics.

Recall that the Gamma function is defined by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \tag{1}$$

and satisfies

$$\Gamma(n) = (n-1)! \tag{2}$$

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Gamma function (cont'd)

```
> gamma(4) # 'gamma function of 4' = (4-1)! = 3!
[1] 6
> factorial(3) # gamma(4) = factorial(3)
[1] 6
> 3! # "!" is not defined in R
Error: unexpected '!' in "3!"
> gamma(7) /(gamma(5) * gamma(3)) # = 6!/(4!2!) = 6 Combination 4
[1] 15
```

[1] 4

Assignments

```
> x < -2 # we define x := 2
> x # show the value of x
[1] 2
> x = 2 \# another way to define x := 2
> y < -3 \# we define y := 3
> x + y
[1] 5
> 2 * x
```

Calculations: Vector - Basics

- > Weight <- c(3.1, 3.9, 4.3, 2.5, 3.6) # 5-dimensional vector
- > Weight # show "Weight"
- [1] 3.1 3.9 4.3 2.5 3.6
- > Height <- c(0.50, 0.54, 0.53, 0.47, 0.51) # 5-dim
- > BMI <- Weight/Height^2 # calculation by element (5-dim)</pre>

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Calculations: Vector – Elements

```
> BMI
[1] 12.40000 13.37449 15.30794 11.31734 13.84083
> BMI[2] # 2nd element (scalar)
[1] 13.37449
> BMI[2:4] # 2nd-4th elements (3-dim)
[1] 13.37449 15.30794 11.31734
> BMI[c(1,5)] # 1st and 5th elements (2-dim)
[1] 12.40000 13.84083
```

Calculations: Vector - Sum & Mean

```
> Weight
[1] 3.1 3.9 4.3 2.5 3.6
> sum(Weight) # summation
[1] 17.4
> length(Weight) # the length of vector
[1] 5
> sum(Weight)/length(Weight) # the mean of Weight
[1] 3.48
> mean(Weight) # mean
[1] 3.48
```

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Calculations: Vector – Variance & Standard deviation

Recall that the sample variance s_x^2 of the values: x_1, x_2, \dots, x_n is defined by

$$s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$
 (3)

where \bar{x} is the sample mean (average).

The sample standard deviation s_x is the square root of the sample variance. Note that s_x has the same unit as x_1, \dots, x_n , while s_x^2 does not. s_x is a measure of dispersion.

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Variance & Standard deviation (Cont'd)

```
> var(Weight) # sample variance of weight
[1] 0.492
> 1/4 * (sum((Weight -mean(Weight))^2)) # another way for variance
[1] 0.492
> sd(Weight) # sample standard deviation
[1] 0.7014271
```

Sequences

```
> 5:10 # integers from 5 to 10
[1] 5 6 7 8 9 10
> 4:-3
[1] 4 3 2 1 0 -1 -2 -3
> seq(0,10, length =6) # arithmetic sequence
[1] 0 2 4 6 8 10
> seq(0,12, by = 3) # arithmetic sequence
[1] 0 3 6 9 12
> rep(1:3, length = 10) # repeat c(1,2,3) to make sequence with length = 10
 [1] 1 2 3 1 2 3 1 2 3 1
> rep(1:3, times = 3) # repeat c(1,2,3) 3 times
[1] 1 2 3 1 2 3 1 2 3
```

Max, min, quantiles

```
> A <- 1:100 # integers from 1 to 100 (100-dim vector)</pre>
> max(A) # maximum (scalar)
[1] 100
> min(A) # minimum
[1] 1
> median(A) # median
[1] 50.5
> quantile(A, 0.25) # 25 percentile
 25%
25.75
> quantile(A, 0.75) # 75 percentile
 75%
75.25
> fivenum(A) # (min, 25%, median, 75%, max), 5-dim vector
[1] 1.0 25.5 50.5 75.5 100.0
```

Note:

Some discrepancies exist above for quantiles, since there are numerous definitions for quantiles. The 25 percentile is at least between 25 and 26. Type "help(quantile)" in R to see 9 definitions.

How to find commands?

- Google "R + key words".
- See documents on the Internet and books.
- Use search in R:

```
help.search("correlation test")
```

How to see the details of commands?

Use R-manual:

```
help(read.table)
    # help for the command: "read.table"
?read.table
    # same as above
```

Data manipulation 1

```
> A <- c(20, 11, 9, 2, 12)
> sort(A) # sort by increasing order
[1] 2 9 11 12 20
> sort(A, decreasing = TRUE) # sort by decreasing order
[1] 20 12 11 9 2
> rev(A) # reverse the order of A
[1] 12 2 9 11 20
> rank(A) # e.g., "20" is the 5th smallest observation in A
[1] 5 3 2 1 4
> order(A) # e.g., 4-th element of A is the smallest
[1] 4 3 2 5 1
> A[order(A)[2]] # 2nd smallest number in A
[1] 9
```

Data manipulation 2

```
> diff(A) # 1st difference of A: c(11-20, 9-11, 2-9, 12-2)
[1] -9 -2 -7 10
> range(A) # same as "c(min(A), max(A))"
[1] 2 20
> prod(A) # product of all elements: 20 * 11 * 9 * 2 * 12
[1] 47520
```

Some calculations

```
> Mid1 < -c(10.50.70.90.30) # midterm 1 scores for 5 students
> Mid2 <- c(20,80,100,60,40) # midterm 2 scores for the 5 students
> pmax(Mid1, Mid2) # best scores for each student
[1] 20 80 100 90 40
> pmin(Mid1, Mid2) # worst scores for each student
[1] 10 50 70 60 30
> c(Mid1, Mid2) # combine vectors
 [1] 10 50 70 90 30 20 80 100 60 40
> cor(Mid1. Mid2) # correlation between the two
Γ1] 0.7
```

Correlation Coefficient:

Recall that the (Pearson's) sample correlation coefficient r for observations $(x_1, y_1), \dots, (x_n, y_n)$ is defined by:

$$r = \frac{s_{xy}}{s_x s_y} \tag{4}$$

where

$$s_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

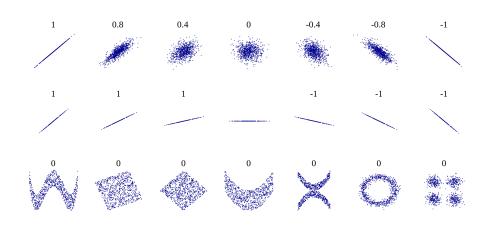
$$\bar{x} = \text{mean of } x_i \text{'s}$$

$$\bar{y} = \text{mean of } y_i \text{'s}$$

Correlation coefficient

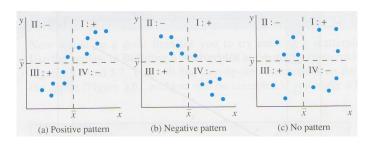
r represents the strength of linear relationship between x and y.

- -1 ≤ r ≤ 1.
- r > 0 means x and y have positive linear relationship.
- r < 0 means x and y have negative linear relationship.
- r = 0 means x and y does not have **linear** relationship.



The signs of $(x_i - \bar{x})(y_i - \bar{y})$:

By definition, the sum of $(x_i - \bar{x})(y_i - \bar{y})$ (over all i's) determines the sign of r. One can see that positive $(x_i - \bar{x})(y_i - \bar{y})$'s correspond to a positive relationship.



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Recalculate \mathbf{r} from the definition.

```
x <- Mid1; y <- Mid2; # ";" can divide the line
> Sxy <- 1/4 * sum((x - mean(x))*(y-mean(y)))
> r <- Sxy/(sd(x)*sd(y))
> r
[1] 0.7
```

We got the same answer as "cor(Mid1,Mid2)".

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Statistical Procedures

Paired t-test

(Note: we will study t-test later in Chapter 5.) Want to test whether or not one exam has significantly higher mean than the other.

Statistical Procedures

Our null hypothesis is that the two means are the same. The output says that:

- The t-statistic follows t-distribution with degrees of freedom 4 under the null.
- The t-statistic is -0.9129.
- The p-value is 0.4130.

Since p - value > 0.05, the null hypothesis is not rejected. There are no evidence that the two means are significantly different.

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Character vectors

Generating character vectors

```
> BabyName <- c("Andy", "Brian", "Cathy") # 3 by 1 vector
> BabyName
[1] "Andy" "Brian" "Cathy"
> BabyName[2]
[1] "Brian"
```

Factor Vectors

Statistical analysis often deals with categorical variables such as gender, country of origin and ratings (e.g., "excellent", "good" or "poor"). R can recognize these as positive integers.

```
> G <- c("male", "female", "male", "female", "unknown") # character vector</pre>
> G
[1] "male" "female" "male" "female" "unknown"
> GF <- as.factor(G) # "factor" vector</pre>
> GF
[1] male female male female unknown
Levels: female male unknown
> GN <- as.numeric(GF) # as a numeric vector</pre>
> GN
Γ11 2 1 2 1 3
> G2 <- as.character(GN) # as a character vector</pre>
> G2
[1] "2" "1" "2" "1" "3"
```

Matrices

Most statistical inferences (such as regression analysis and analysis of variance) are better described when we use matrix representation. Statistical languages such as R/S-Plus, Matlab, Gauss and Ox specialize in matrix calculation.

Generating matrices

Generating matrices 2

Checking matrices

```
> nrow(A) # the number of rows
[1] 2
>
> ncol(A) # the number of columns
[1] 3
> dim(A) # dimension
[1] 2 3
>
> row(A) # row number for each element
    [,1] [,2] [,3]
[1,] 1 1 1
[2,] 2 2 2
> col(A) # column number for each element
    [,1] [,2] [,3]
[1,] 1 2 3
[2,] 1 2
```

Extracting Elements

```
> A[2,2] \# (2,2)-th element of A
[1] 4
> A[1.] # 1st row as a 3-dimensional vector
\lceil 1 \rceil 1 3 5
> A[,2] # 2nd column as a 2-dimensional vector
Γ17 3 4
> as.matrix(A[,2]) # 2nd column as a matrix
     [,1]
[1,] 3
[2,] 4
> A[1,c(1,3)] # 1st row and 1st and 3rd column
[1] 1 5
```

Vectors to a matrix

```
> W < -c(3.1, 2.9, 3.8)
> H <- c(50, 46, 53)
> cbind(W,H) # bind columns
       W
         Н
[1,] 3.1 50
[2,] 2.9 46
[3,1] 3.8 53
>
> rbind(W,H) # bind rows
  [,1] [,2] [,3]
W 3.1 2.9 3.8
H 50.0 46.0 53.0
```

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Calculation

```
> A <- rbind(c(1,2), c(3,4))
> A
    [,1] [,2]
[1,]
    1
[2,] 3
            4
> B < - rbind(c(3,0), c(0,2))
> B
    [,1] [,2]
[1,]
       3
[2,]
       0
> A + B # addition
    [,1] [,2]
[1,]
       4
[2,]
       3
> A - B # subtraction
    [,1] [,2]
[1,] -2
       3
[2,]
```

Calculation

```
> A * B # product element by element
     [,1] [,2]
[1,] 3
[2,] 0 8
> A %*% B # matrix product
     \lceil ,1 \rceil \lceil ,2 \rceil
[1,] 3 4
[2,] 9 8
> solve(A) # inverse of A
     \lceil , 1 \rceil \lceil , 2 \rceil
[1,] -2.0 1.0
[2,] 1.5 -0.5
> A %*% solve(A) # A times A inverse = I (some rounding errors)
     [,1]
                   Γ.21
[1,] 1 1.110223e-16
Γ2.1
        0 1.000000e+00
```

Vector and matrix

```
> A
     \lceil , 1 \rceil \lceil , 2 \rceil
[1,] 1 3
[2,] 2 4
> X < -c(1,2)
> A %*% X  # X is recognized as a column vector (2 by 1)
     [,1]
[1,] 7
[2,] 10
> X %*% A  # X is recognized as a row vector (1 by 2)
     [,1] [,2]
\lceil 1, \rceil 5 11
```

Diagonal and transposed matrices

```
> diag(2) # 2 by 2 identity matrix
    [,1] [,2]
[1,] 1 0
[2,] 0
> diag(3:2) # 2 by 2 diagonal matrix
    [,1] [,2]
[1,] 3 0
[2,] 0 2
> t(A) # transposed A
    \lceil ,1 \rceil \lceil ,2 \rceil
[1,] 1 3
[2,] 2 4
```

Determinant, eigenvectors

Verifying eigenvectors

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Row and column names

Array

Generating arrays

A matrix has two indices; row and column. R can also produce arrays with more than two indices.

```
> A <- array(1:24, dim=c(3,4,2))
> A
, , 1
    [.1] [.2] [.3] [.4]
[1,]
                     10
[2,] 2
                 8 11
[3,]
                 9
                     12
, , 2
    [,1] [,2] [,3] [,4]
[1,]
      13
           16
                19
                     22
[2,] 14
                     23
           17
                20
```

18 21

24

15

[3,]

Array

Transposed Array

```
> aperm(A, perm = c(2,1,3)) # interchange 1st and 2nd indices
, , 1
    [,1] [,2] [,3]
[1,]
       1
           2
[2,] 4 5
                6
[3,] 7 8
                9
[4,] 10
           11
               12
, , 2
    [,1] [,2] [,3]
\lceil 1, \rceil 13 14
             15
[2,] 16 17 18
[3,] 19
          20 21
```

23

24

[4,] 22

We sometimes want to combine multiple numbers, vectors and matrices into an object. It is called a "**list**" in R.

```
> Instructor <- c("Kaz")</pre>
> \text{Time} < - \text{rbind}(c(1250, 1345), c(1250, 1345), c(1250, 1345))
> rownames(Time) <- c("Mon","Wed","Fri")</pre>
> colnames(Time) <- c("Start","End")</pre>
> M5030 <- list(Inst=Instructor, Time=Time)</pre>
> M5030
$Inst
[1] "Kaz"
$Time
    Start End
Mon 1250 1345
Wed 1250 1345
Fri 1250 1345
```

Elements of a list

```
> M5030$Inst # "Inst" of M5030
[1] "Kaz"
M5030[[1]] # 1st object of M5030
[1] "Kaz"
> M5030[[2]][3,] # 3rd row of the 2nd object
       End
Start
1250 1345
> M5030[[2]][3,] <- c(1200, 1800) # change 3rd row of 2nd object
> M5030[[2]]
   Start End
Mon 1250 1345
Wed 1250 1345
Fri 1200 1800
```

Example: t-test

Statistical procedures often output a list.

```
> TT <- t.test(Mid1-Mid2) # paired t-test (see the earlier slide)</pre>
> TT[[1]]
-0.912871
> TT[[2]]
df
 4
> TT[[3]]
Γ17 0.4129544
> TT[[4]]
[1] -40.41443 20.41443
attr(,"conf.level")
[1] 0.95
```

If you just type "TT", the format of the results is a little different (see the earlier slide).

The elements of a list may have names. You can see more details by typing "help(t.test)".

Data Frame

Data frame is a set of equal-sized vectors, for example, gender, weight and height of 4 patients.

```
> Gender <- c("male","female","male","female") # gender: factor
> Height <- c(174,160, 182, 166) # height in cm: numeric
> Weight <- c(62, 59, 95, 75) # weight in kg: numeric
> D<- data.frame(Gender, Height, Weight)
> D
    Gender Height Weight
1 male 174 62
2 female 160 59
3 male 182 95
4 female 166 75
```

Data frame allows vectors to have different types (e.g., numeric, character, factor), while matrix requires all elements to have the same type.

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Data Frame

You can use similar commands for data frames as for matrices.

```
> D$Gender
[1] male female male female
Levels: female male
> D[2,2]
[1] 160
```

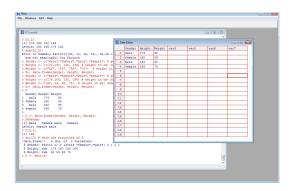
A useful command to see a data frame at a glance.

```
> str(D) # show the structure of D
'data.frame': 4 obs. of 3 variables:
$ Gender: Factor w/ 2 levels "female", "male": 2 1 2 1
$ Height: num 174 160 182 166
$ Weight: num 62 59 95 75
```

Data Frame

Spreadsheet

You can edit the dataframe on a spreadsheet by "D <- edit(D)" ("D" is the name of the data frame).



The "apply" function (and its analogues) calculate statistics such as mean and standard deviation for each row or column.

apply

```
> A < -c(7,10, 8, 9); B < -c(7, 8, 9,10); C < -c(12, 8, 9, 7);
> D <- data.frame(A,B,C)</pre>
> D
  ABC
1 7 7 12
2 10 8 8
3 8 9 9
4 9 10 7
>
> apply(D, 1, sum) # sum for each row
Γ17 26 26 26 26
> apply(D, 2, mean) # mean for each column
  A B C
8.5 8.5 9.0
```

lapply, sapply

```
> E < -2.5; F < -c(0.7,0.8,0.9); G < -c(0.1,0.2,0.3,0.4,0.5);
> L1 <- list(E=E,F=F,G=G)
> 1.1
$E
[1] 2.5
$F
[1] 0.7 0.8 0.0 9.0
$G
[1] 0.1 0.2 0.3 0.4 0.5
> sapply(L1, mean) # mean of each object as a vector
    E
      F
2.500 2.625 0.300
```

```
> lapply(L1, mean) # mean of each object as a list
$E
[1] 2.5

$F
[1] 2.625
$G
[1] 0.3
```

tapply

The "tapply" calculates group means by factor.

Logical Operations

Equality, inequality

```
> X < -5: Y < -6
> X == 5 \# X = 5?
[1] TRUE
> X > 5 \# X > 5?
[1] FALSE
> c(X,Y) > c(6, 5) \# X > 6?, Y > 5?
[1] FALSE TRUE
> X != 4 # "X is not equal to 4" is true?
[1] TRUE
> X >= 4 # X is larger than or equal to 4?
[1] TRUE
> X <= 4 # X is smaller than or equal to 4?
[1] FALSE
```

Logical Operations

AND, OR, XOR and NOT

```
> (X == 5) & (Y == 6) # Is "X = 5 and Y = 6" true?
[1] TRUE
> (X == 5) | (Y == 5) # Is "X = 5 or Y = 5" true?
[1] TRUE
> xor(X == 5, Y == 6) # Exactly one of the two conditions is true?
[1] FALSE
> !(1 == 0) # "Not 1 = 0" is true?
[1] TRUE
```

Note: "&&" and "||" behave similar to "&" and "|" respectively, but only "&" and "|" can apply for vectors.

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Conditioning

Recall the data frame "D" with Gender, Height and Weight.

```
> Cond <- D$Gender == "female" # condition: female
> Cond
[1] FALSE TRUE FALSE TRUE
```

- > D[Cond,] # extract rows for female
 Gender Height Weight
 2 female 160 59
- 4 female 166 75

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Missing Values

Missing values are left as NA.

```
> Weight <- c(70, 80, 55, NA, 99)
> Weight
Γ17 70 80 55 NA 99
> mean(Weight) # mean of all observations
Γ17 NA
> mean(Weight, na.rm=TRUE) # mean, ignoring NAs
[1] 76
> is.na(Weight) # logical operation to look for NAs.
[1] FALSE FALSE FALSE TRUE FALSE
> Weight[!is.na(Weight)] # Exclude NAs from the data
[1] 70 80 55 99
> Weight == NA # This does not work!
[1] NA NA NA NA NA
```

Missing Values

```
> Gender <- c("M","F","M","F", NA)
> Gender
[1] "M" "F" "M" "F" NA
> Gender == "M"
[1] TRUE FALSE TRUE FALSE NA
```

X < -c(1.5.3.8.2.10.4.9)

R generates scatter plots, bar charts, histograms etc., in a different window. You can save the figure as jpg, eps, pdf or some other formats. To save the figure, choose "File > Save as" in the menu bar when the figure window is active.

```
Y <- c(2,6,4,7,1,9,3,10)

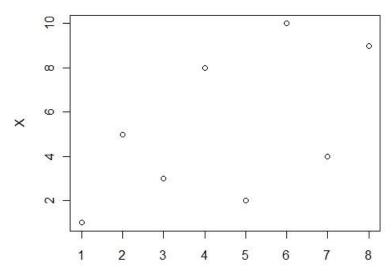
plot(X) # plot (horizontal: index, vertical: X)

plot(X, Y) # scatter plot (horizontal: X, vertical:Y)

RN <- rnorm(100) # 100 standard normal random variables
hist(RN) # histogram of RN
```

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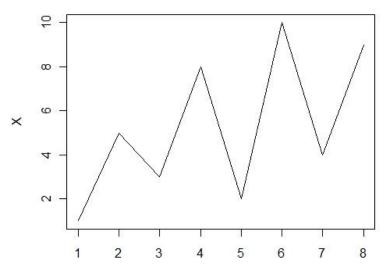
plot(X), plot against index



₹ 1000

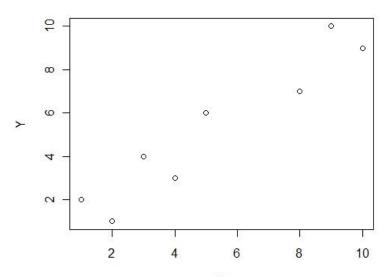
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plot(X, type="l"), plot against index (line plot)



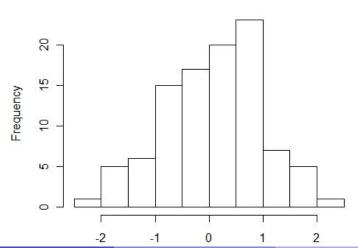
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plot(X,Y), scatter plot



hist(RN), histogram

Histogram of rnorm(100)



Shinki (Wayne State U)

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Note:

See "help(plot)" and "help(par)" (or search documents on the Internet) for more graphic options.

Customizing figures in R is not very convenient, while technically you can customize almost everything. You may consider to use a different software such as MS-Excel for complicated figures.