MAT 5030: Homework 1 Solutions

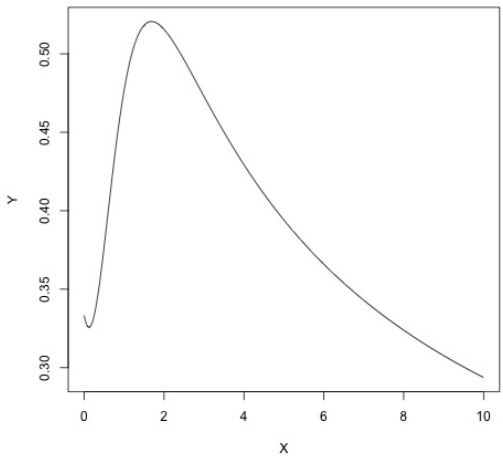
Problem 1

```
setwd("/Users/kazuhikoshinki/WSU/Teach/MAT5030-16F/HW1")


# (a)
> X <- 0.01 * (0:1000) # 0 <= X <= 1 with increments 0.01
> Y <- sqrt((X^3 + 3*X^2 + 1)/(X^4+5*X^3+7*X+9))


> jpeg("Fig1a.jpg")
> plot(X,Y,type="l")
> dev.off()
quartz
     2
dev.off() # end of jpeg file


# (b)
> order(Y)[1001] # show where is the largest observation in Y
[1] 170
> Y[170]
[1] 0.5205625
# maximizer x=1.69 (170th observation of X)
# maximum = f(1.69) = 0.5205625
```



Problem 2

```
# (a)
> set.seed(1) # set a seed for random numbers
> X <- rnorm(1000) # 1000 standard normal random numbers
> sd(X)
[1] 1.034916

# (b)
> sort(X)[100] # 100th smallest nubmer in X
[1] -1.34413

# (c)
# the 10th percentile of standard normal distribution
# is about -1.28 since Pr(X < -1.28) = 0.1003.
# -1.34413 in (b) is close to -1.28.
```

Problem 3

```
# (a)(b)
> A <- rbind(c(0.979,0.144),c(0.147,-0.999))
> A2 <- A %*% A # A^2
> A4 <- A2 %*% A2
> A8 <- A4 %*% A4 # A^8
> A16 <- A8 %*% A8
> A32 <- A16 %*% A16 # A^{32}
> A64 <- A32 %*% A32
> A128 <- A64 %*% A64
> A256 <- A128 %*% A128
> A512 <- A256 %*% A256
> A1024 <- A512 %*% A512 # A^{1024}

> A2
          [,1]      [,2]
[1,]  0.979609 -0.002880
[2,] -0.002940  1.019169
> A8
```

```
            [,1]       [,2]
[1,]  0.92094709 -0.0115035
[2,] -0.01174315  1.0789604
> A32
            [,1]        [,2]
[1,]  0.72011695 -0.04630104
[2,] -0.04726564  1.35611313
> A1024
           [,1]      [,2]
[1,]    98.7977 -1336.554
[2,] -1364.3989 18457.853

# (c)
> A1024 %*% solve(A32) %*% A8 # A^1000 = A^1024 (A^32)^{-1} A^8
            [,1]      [,2]
[1,]    78.47005 -1061.558
[2,] -1083.67339 14660.143

# (d)
> eigen(A)
$values
[1] -1.0096444  0.9896444

$vectors
            [,1]       [,2]
[1,] -0.07222204 0.99727908
[2,]  0.99738858 0.07371857

# (e)
>lam <- eigen(A)$values
> X  <- eigen(A)$vectors
> X %*% diag(lam^1000) %*% solve(X)
># A^1000 = X diag(lam[1]^1000, lam[2]^1000) X^{-1}
            [,1]      [,2]
[1,]    78.47005 -1061.558
[2,] -1083.67339 14660.143
```