# Møller Scattering

This notebook calculates and writes the t-channel amplitude of Møller scattering.  Strictly speaking, the Møller cross section should consider both the t- and u- channels.  However, as we are only considering small momentum transfers in our simulation, we can assume that one channel is always much greater than the other (section 3.1).  Taking advantage of the indistinguishability of the outgoing electrons, the simplest way make use of this approximation is to calculate only the t-channel and then multiply the resulting amplitude by two.

## Set directory

The notebook will write the t-channel amplitude expression into a text file in the directory that contains this Mathematica notebook.

```
In[∘]:= SetDirectory[NotebookDirectory[]];
```

## Definitions from Bjorken and Drell

Load FeynCalc to access the complexConjugate command.

```
In[∘]:= << FeynCalc`
```

```
FeynCalc 9.3.0 (stable version). For help, use the
  documentation center, check out the wiki or write to the mailing list.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

 • V. Shtabovenko, R. Mertig and F. Orellana,
     P3H–20–002, TTP19–020, TUM–EFT 130/19, arXiv:2001.04407

 • V. Shtabovenko, R. Mertig and F. Orellana,
     Comput. Phys. Commun., 207, 432–444, 2016, arXiv:1601.01167

 • R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.
```

Gamma matrices in the Dirac basis.

```
In[∘]:= G0 = DiagonalMatrix[{1, 1, -1, -1}];
        G1 = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}};
        G2 = {{0, 0, 0, -I}, {0, 0, I, 0}, {0, I, 0, 0}, {-I, 0, 0, 0}};
        G3 = {{0, 0, 1, 0}, {0, 0, 0, -1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};
```

Electron spinors.  The states have been multiplied by Sqrt[2m] to match the normalization used by Peskin and Schroeder.

In[•]:=

$$\texttt{u1[p\_] := Sqrt[p[[1]] + m]} \left\{1,\ 0,\ \frac{\texttt{p[[4]]}}{\texttt{p[[1]] + m}},\ \frac{\texttt{p[[2]] + I p[[3]]}}{\texttt{p[[1]] + m}}\right\}$$

$$\texttt{u2[p\_] := Sqrt[p[[1]] + m]} \left\{0,\ 1,\ \frac{\texttt{p[[2]] - I p[[3]]}}{\texttt{p[[1]] + m}},\ \frac{\texttt{-p[[4]]}}{\texttt{p[[1]] + m}}\right\}$$

“Barred” electron spinors.

```
In[•]:= u1b[p_] := ComplexConjugate[u1[p]].G0
        u2b[p_] := ComplexConjugate[u2[p]].G0
```

Components of each momentum.  The numbers 1 and 2 label the incoming electrons and 3 and 4 the outgoing electrons. p1 is the momentum of the beam electron, so it has no x or y components.

```
In[•]:= p1 = {E1, p1x, p1y, p1z};
        p2 = {E2, p2x, p2y, p2z};
        p3 = {E3, p3x, p3y, p3z};
        p4 = {E4, p4x, p4y, p4z};
```

# Contract spinor indices

Contract spinor indices by sandwiching gamma matrices between electron spinors.

```
In[•]:= u1g0u1[p_, q_] := u1b[p].G0.u1[q] // Simplify
        u1g1u1[p_, q_] := u1b[p].G1.u1[q] // Simplify
        u1g2u1[p_, q_] := u1b[p].G2.u1[q] // Simplify
        u1g3u1[p_, q_] := u1b[p].G3.u1[q] // Simplify

        u2g0u2[p_, q_] := u2b[p].G0.u2[q] // Simplify
        u2g1u2[p_, q_] := u2b[p].G1.u2[q] // Simplify
        u2g2u2[p_, q_] := u2b[p].G2.u2[q] // Simplify
        u2g3u2[p_, q_] := u2b[p].G3.u2[q] // Simplify

        u1g0u2[p_, q_] := u1b[p].G0.u2[q] // Simplify
        u1g1u2[p_, q_] := u1b[p].G1.u2[q] // Simplify
        u1g2u2[p_, q_] := u1b[p].G2.u2[q] // Simplify
        u1g3u2[p_, q_] := u1b[p].G3.u2[q] // Simplify

        u2g0u1[p_, q_] := u2b[p].G0.u1[q] // Simplify
        u2g1u1[p_, q_] := u2b[p].G1.u1[q] // Simplify
        u2g2u1[p_, q_] := u2b[p].G2.u1[q] // Simplify
        u2g3u1[p_, q_] := u2b[p].G3.u1[q] // Simplify
```

# Contract vector indices

Sum over all possible incoming and outgoing spin combinations.

```
In[∘]:= S0[p_, q_] := (u1g0u1[p, q] + u2g0u2[p, q] + u1g0u2[p, q] + u2g0u1[p, q]) // Simplify
        S1[p_, q_] := (u1g1u1[p, q] + u2g1u2[p, q] + u1g1u2[p, q] + u2g1u1[p, q]) // Simplify
        S2[p_, q_] := (u1g2u1[p, q] + u2g2u2[p, q] + u1g2u2[p, q] + u2g2u1[p, q]) // Simplify
        S3[p_, q_] := (u1g3u1[p, q] + u2g3u2[p, q] + u1g3u2[p, q] + u2g3u1[p, q]) // Simplify
```

Contract vector indices using the Minkowski metric. Divide by 2 to account for the beam and material ground state being unpolarized, i.e., superpositions of equal parts spin up and down. Write the resulting expression to a text file “M.txt.” In the final python code, the t-channel amplitude is obtained by dividing the resulting expression by t = (p1 - p4)^2 and multiplying by the the electron charge squared, e^2.

```
In[∘]:= (S0[p4, p1] * S0[p3, p2] - S1[p4, p1] * S1[p3, p2] -
            S2[p4, p1] * S2[p3, p2] - S3[p4, p1] * S3[p3, p2]) / 2 // FullSimplify
        (*Export[
         "M.txt",
         %]*)
```

Out[∘]=

$$\begin{aligned}&\frac{1}{\sqrt{\mathrm{E1}+m}\,\sqrt{\mathrm{E2}+m}\,\sqrt{\mathrm{E3}+m}\,\sqrt{\mathrm{E4}+m}}\\
&2\,(((\mathrm{E1}+m)(\mathrm{E4}+m)+\mathrm{p1x}\,\mathrm{p4x}+(\mathrm{p1y}+i\,\mathrm{p1z})(\mathrm{p4y}-i\,\mathrm{p4z}))((\mathrm{E2}+m)(\mathrm{E3}+m)+\mathrm{p2x}\,\mathrm{p3x}+(\mathrm{p2y}+i\,\mathrm{p2z})(\mathrm{p3y}-i\,\mathrm{p3z}))+\\
&\quad(\mathrm{p4x}\,(\mathrm{E1}+m)+\mathrm{p1x}\,(\mathrm{E4}+m))\,(-(\mathrm{p3x}\,(\mathrm{E2}+m)+\mathrm{p2x}\,(\mathrm{E3}+m)))-\\
&\quad(-i\,\mathrm{p4z}\,(\mathrm{E1}+m)+\mathrm{E1}\,\mathrm{p4y}+\mathrm{E4}\,(\mathrm{p1y}+i\,\mathrm{p1z})+m\,\mathrm{p1y}+i\,m\,\mathrm{p1z}+m\,\mathrm{p4y})\\
&\qquad(-i\,\mathrm{p3z}\,(\mathrm{E2}+m)+\mathrm{E2}\,\mathrm{p3y}+\mathrm{E3}\,(\mathrm{p2y}+i\,\mathrm{p2z})+m\,\mathrm{p2y}+i\,m\,\mathrm{p2z}+m\,\mathrm{p3y})-\\
&\quad(\mathrm{E1}\,(\mathrm{p4z}+i\,\mathrm{p4y})+\mathrm{E4}\,(\mathrm{p1z}-i\,\mathrm{p1y})+m\,(-i\,\mathrm{p1y}+\mathrm{p1z}+i\,\mathrm{p4y}+\mathrm{p4z}))\\
&\qquad(\mathrm{E2}\,(\mathrm{p3z}+i\,\mathrm{p3y})+\mathrm{E3}\,(\mathrm{p2z}-i\,\mathrm{p2y})+m\,(-i\,\mathrm{p2y}+\mathrm{p2z}+i\,\mathrm{p3y}+\mathrm{p3z})))\end{aligned}$$