Write a program to estimate the value of the *percolation threshold* via Monte Carlo simulation.

Install our Java programming environment (optional). Install our custom IntelliJ programming environment by following these step-by-step instructions for your operating system [Mac OS X · Windows · Linux].

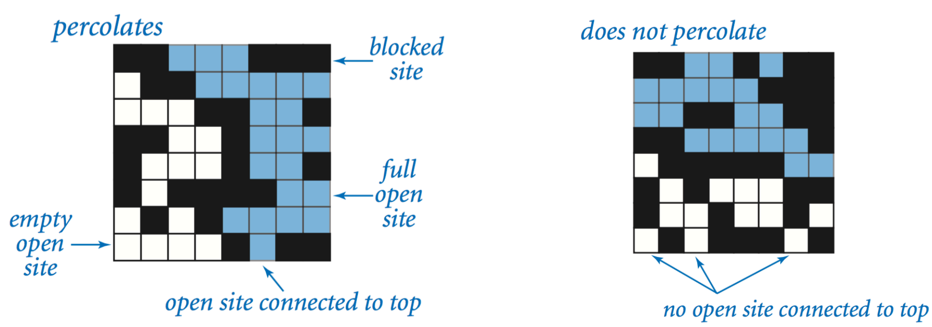
After following these instructions, the commands `javac-algs4` and `java-algs4` will classpath in algs4.jar, which contains Java classes for I/O and all of the algorithms in the textbook. To access a class in `algs4.jar`, you need an `import` statement, such as the ones below:

```
import edu.princeton.cs.algs4.StdRandom;
import edu.princeton.cs.algs4.StdStats;
import edu.princeton.cs.algs4.WeightedQuickUnionUF;
```

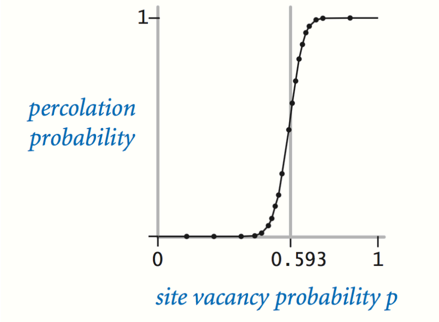
Note that *your* code must be in the *default package*; if you use a `package` statement, the autograder will reject your submission.

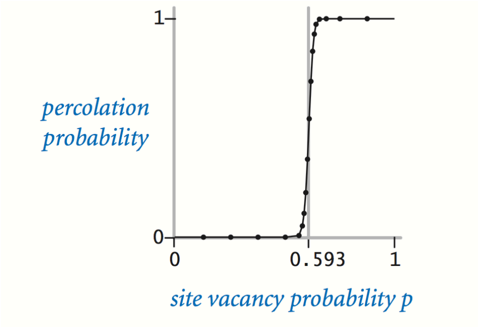
Percolation. Given a composite systems comprised of randomly distributed insulating and metallic materials: what fraction of the materials need to be metallic so that the composite system is an electrical conductor? Given a porous landscape with water on the surface (or oil below), under what conditions will the water be able to drain through to the bottom (or the oil to gush through to the surface)? Scientists have defined an abstract process known as *percolation* to model such situations.

The model. We model a percolation system using an n-by-n grid of *sites*. Each site is either *open* or *blocked*. A *full* site is an open site that can be connected to an open site in the top row via a chain of neighboring (left, right, up, down) open sites. We say the system *percolates* if there is a full site in the bottom row. In other words, a system percolates if we fill all open sites connected to the top row and that process fills some open site on the bottom row. (For the insulating/metallic materials example, the open sites correspond to metallic materials, so that a system that percolates has a metallic path from top to bottom, with full sites conducting. For the porous substance example, the open sites correspond to empty space through which water might flow, so that a system that percolates lets water fill open sites, flowing from top to bottom.)



The problem. In a famous scientific problem, researchers are interested in the following question: if sites are independently set to be open with probability p (and therefore blocked with probability $1 - p$), what is the probability that the system percolates? When p equals 0, the system does not percolate; when p equals 1, the system percolates. The plots below show the site vacancy probability p versus the percolation probability for 20-by-20 random grid (left) and 100-by-100 random grid (right).





When n is sufficiently large, there is a *threshold* value p^* such that when $p < p^*$ a random n-by-n grid almost never percolates, and when $p > p^*$, a random n-by-n grid almost always percolates. No mathematical solution for determining the percolation threshold p^* has yet been derived. Your task is to write a computer program to estimate p^*.

Percolation data type. To model a percolation system, create a data type `Percolation` with the following API:

```
public class Percolation {

    // creates n-by-n grid, with all sites initially blocked
    public Percolation(int n)

    // opens the site (row, col) if it is not open already
    public void open(int row, int col)

    // is the site (row, col) open?
    public boolean isOpen(int row, int col)

    // is the site (row, col) full?
    public boolean isFull(int row, int col)

    // returns the number of open sites
    public int numberOfOpenSites()

    // does the system percolate?
    public boolean percolates()

    // test client (optional)
    public static void main(String[] args)
}
```

Corner cases. By convention, the row and column indices are integers between 1 and n, where $(1, 1)$ is the upper-left site: Throw an `IllegalArgumentException` if any argument to `open()`, `isOpen()`, or `isFull()` is outside its prescribed range. Throw an `IllegalArgumentException` in the constructor if $n \le 0$.

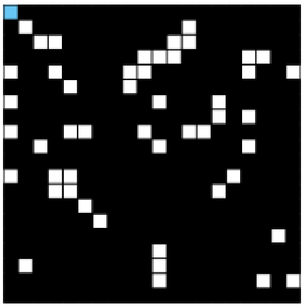
Performance requirements. The constructor must take time proportional to n^2; all instance methods must take constant time plus a constant number of calls to `union()` and `find()`.

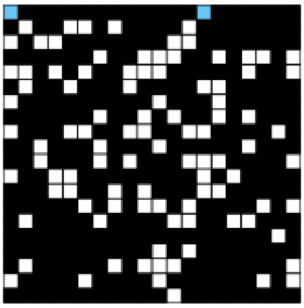
Monte Carlo simulation. To estimate the percolation threshold, consider the following computational experiment:

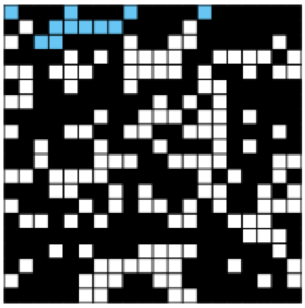
- Initialize all sites to be blocked.
- Repeat the following until the system percolates:
    - Choose a site uniformly at random among all blocked sites.
    - Open the site.

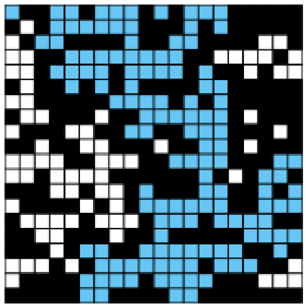
- The fraction of sites that are opened when the system percolates provides an estimate of the percolation threshold.

For example, if sites are opened in a 20-by-20 lattice according to the snapshots below, then our estimate of the percolation threshold is 204/400 = 0.51 because the system percolates when the 204th site is opened.


50 open sites


100 open sites


150 open sites


204 open sites

By repeating this computation experiment T times and averaging the results, we obtain a more accurate estimate of the percolation threshold. Let x_t be the fraction of open sites in computational experiment t. The sample mean $\overline{x}$ provides an estimate of the percolation threshold; the sample standard deviation s; measures the sharpness of the threshold.

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_T}{T}, \qquad s^2 = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_T - \overline{x})^2}{T - 1}$$

Assuming T is sufficiently large (say, at least 30), the following provides a 95% confidence interval for the percolation threshold:

$$\left[\, \overline{x} - \frac{1.96s}{\sqrt{T}}, \;\; \overline{x} + \frac{1.96s}{\sqrt{T}} \,\right]$$

To perform a series of computational experiments, create a data type `PercolationStats` with the following API.

```
public class PercolationStats {

    // perform independent trials on an n-by-n grid
    public PercolationStats(int n, int trials)

    // sample mean of percolation threshold
    public double mean()

    // sample standard deviation of percolation threshold
    public double stddev()

    // low endpoint of 95% confidence interval
    public double confidenceLo()

    // high endpoint of 95% confidence interval
    public double confidenceHi()

   // test client (see below)
   public static void main(String[] args)

}
```

Throw an `IllegalArgumentException` in the constructor if either $n \le 0$ or $trials \le 0$.

Also, include a `main()` method that takes two *command-line arguments* n and T, performs T independent computational experiments (discussed above) on an n-by-n grid, and prints the sample mean, sample standard deviation, and the *95% confidence interval* for the percolation threshold. Use StdRandom to generate random numbers; use StdStats to compute the sample mean and sample standard deviation.

```
~/Desktop/percolation> java-algs4 PercolationStats 200 100
mean                    = 0.5929934999999997
stddev                  = 0.00876990421552567
95% confidence interval = [0.5912745987737567, 0.5947124012262428]

~/Desktop/percolation> java-algs4 PercolationStats 200 100
mean                    = 0.592877
stddev                  = 0.009990523717073799
95% confidence interval = [0.5909188573514536, 0.5948351426485464]

~/Desktop/percolation> java-algs4 PercolationStats 2 10000
mean                    = 0.666925
stddev                  = 0.11776536521033558
95% confidence interval = [0.6646167988418774, 0.6692332011581226]

~/Desktop/percolation> java-algs4 PercolationStats 2 100000
mean                    = 0.6669475
stddev                  = 0.11775205263262094
95% confidence interval = [0.666217665216461, 0.6676773347835391]
```

Analysis of running time and memory usage (optional and not graded). Implement the `Percolation` data type using the *quick find* algorithm in QuickFindUF.

- Use Stopwatch to measure the total running time of `PercolationStats` for various values of n and T. How does doubling n affect the total running time? How does doubling T affect the total running time? Give a formula (using tilde notation) of the total running time on your computer (in seconds) as a single function of both n and T.
- Using the 64-bit memory-cost model from lecture, give the total memory usage in bytes (using tilde notation) that a `Percolation` object uses to model an n-by-n percolation system. Count all memory that is used, including memory for the union–find data structure.

Now, implement the `Percolation` data type using the *weighted quick union* algorithm in WeightedQuickUnionUF. Answer the questions in the previous paragraph.

Web submission. Submit a .zip file containing only Submit only `Percolation.java` (using the weighted quick-union algorithm from WeightedQuickUnionUF) and `PercolationStats.java`. We will supply `algs4.jar`. Your submission may not call library functions except those in StdIn, StdOut, StdRandom, StdStats, WeightedQuickUnionUF, and `java.lang`.

For fun. Create your own percolation input file and share it in the discussion forums. For some inspiration, do an image search for "nonogram puzzles solved."

This assignment was developed by Bob Sedgewick and Kevin Wayne.