# Ex - Sheet 03

08 September 2020 20:21

Exercise 1, Bayes Rule:

a)

$p(z = \text{blue} \mid \text{taxi} = \text{blue}) = 0.75$

$p(z = \text{green} \mid \text{taxi} = \text{green}) = 0.75$

$p(\text{taxi} = \text{green}) = 0.9$

$p(\text{taxi} = \text{blue} \mid z = \text{blue}) = ?$

$$= \frac{p(z = \text{blue} \mid \text{taxi} = \text{blue})\, p(\text{taxi} = \text{blue})}{p(z = \text{blue})}$$

($p(z = \text{blue} \mid \text{taxi} = \text{blue}) \to 0.75$; $p(\text{taxi} = \text{blue}) \to 1 - 0.9 = 0.1$)

$$p(z = \text{blue}) = p(z = \text{blue}, \text{taxi} = \text{blue}) + p(z = \text{blue}, \text{taxi} = \text{green})$$

$$= p(z = \text{blue} \mid \text{taxi} = \text{blue})\, p(\text{taxi} = \text{blue}) + p(z = \text{blue} \mid \text{taxi} = \text{green})\, p(\text{taxi} = \text{green})$$

($0.75$, $0.1$, $0.25$, $0.9$)

$$\Rightarrow p(\text{taxi} = \text{blue} \mid z = \text{blue}) = \frac{0.75 \cdot 0.1}{0.75 \cdot 0.1 + 0.25 \cdot 0.7} = 0.25$$

b) $$= \frac{0.75 \cdot 0.3}{0.75 \cdot 0.3 + 0.25 \cdot 0.7} = 0.5625$$

c) $p(z \mid \text{taxi}) = 0.5 \rightarrow$ doesn't give us any information so the answer wouldn't change from b)

Exercise 2: Bayes Filter

$p(x_{t+1} = \text{clean} \mid x_t = \text{dirty}, u_{t+1} = \text{vaccum clean}) = 0.7$

$p(z = \text{clean} \mid x = \text{dirty}) = 0.3$

$p(z = \text{clean} \mid x = \text{clean}) = 0.9$

No knowledge about the current state of the floor

a) $p(x_{t+1} = \text{dirty} \mid u_{t+1} = \text{vaccum-clean}, x_t = \text{dirty}) = ?$ question is not clear

initial state: $p(x_0 = \text{clean}) = q$ I would assume no knowledge would mean $q = 0.5$

$p(x_1 = c \mid z = c, u = vc)$ we need to use Bayes Filter Formula

$$= Bel(x_1 = c) = \eta\, p(z = c \mid x_1 = c) \sum_{x_0} p(x_1 = c \mid u = vc, x_0)\, Bel(x_0)$$

$Bel(x_0 = c) = q$

$Bel(x_0 = d) = 1 - q$

$$= \eta\, p(z=c \mid x_1=c)\left[ p(x_1=c \mid x_0=c, u=uc)\, q + p(x_1=c \mid x_0=d, u=uc)\,(1-q)\right]$$

($p(z=c \mid x_1=c) = 0.9$, $p(x_1=c \mid x_0=c, u=uc) = 1.0$, $p(x_1=c \mid x_0=d, u=uc) = 0.7$)

$$= \eta\, p(z=c \mid x_1=c)\left[0.7 + 0.3q\right]$$

$$= \eta\, 0.9\left[0.7 + 0.3q\right]$$

$$Bel(x_1=c) + Bel(x_1=d) = 1.0$$

$$Bel(x_1=d) = \eta\, 0.3\left[0.0\, q + 0.3\,(1-q)\right]$$

($0.0\, q \to 0.0$)

$$= 0.09\,\eta\,(1-q)$$

$$\eta\left[0.09(1-q) + 0.9\,(0.7 + 0.3q)\right] = 1 \Rightarrow$$

$$0.09 - 0.09q + 0.63 + 0.27q$$

$$\left[0.72 - 0.18q\right] =$$

$$\Rightarrow \quad \eta = \frac{1}{0.72 - 0.18q}$$

$$Bel(x_1=c) = \frac{0.63 + 0.27q}{0.72 - 0.18q}.$$

$$Bel(x_1=d) = \frac{0.09 - 0.09q}{0.72 - 0.18q}$$