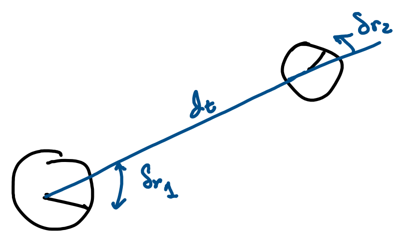
Ex - Sheet 04

08 September 2020   23:08

Exercise 2 :

$$\vec{x}_t = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \quad \vec{u} = \begin{pmatrix} \delta_{r1} \\ \delta_{r2} \\ \delta_t \end{pmatrix} \quad \alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix}$$



$\hat{u} =$ noisy_u(u, α)

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \\ \theta_{t+1} \end{pmatrix} = \begin{pmatrix} x_t + \hat{\delta}_t \cos(\hat{\delta}_{r1} + \theta) \\ y_t + \hat{\delta}_t \sin(\hat{\delta}_{r1} + \theta) \\ \theta_t + \hat{\delta}_{r1} + \hat{\delta}_{r2} \end{pmatrix}$$

new state

Exercise 3: Velocity-Based Motion Model

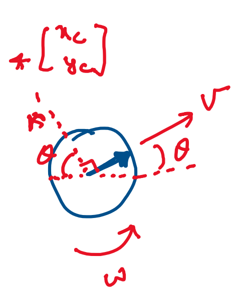
Robot with constant v & ω following a perfect circle

current pose (x, y, θ)

a)

derive :
$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega} \sin\theta \\ \frac{v}{\omega} \cos\theta \end{pmatrix}$$

center of circle



time it takes to make a circle

$$\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{2\pi}{\omega}$$

$$v = \frac{s}{\Delta t} \Rightarrow s = v \Delta t = \frac{v}{\omega} 2\pi$$ $\rightsquigarrow$ perimeter of circle

$$\Rightarrow 2\pi r = \frac{v}{\omega} 2\pi \Rightarrow \boxed{r = \frac{v}{\omega}}$$ radius of circle

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -r \sin\theta \\ r \cos\theta \end{pmatrix}$$

$\left(-\frac{v}{\omega} \sin\theta \right.$ [illegible]

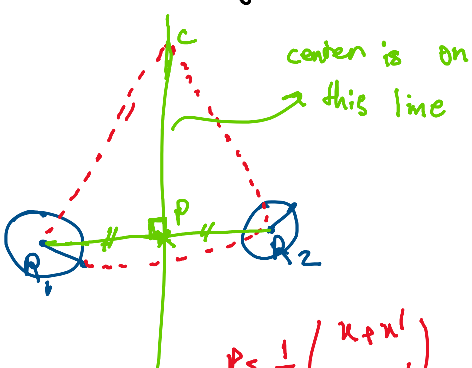
$$= \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{v}{\omega}\sin\theta \\ \frac{v}{\omega}\cos\theta \end{pmatrix}$$

b) start : (x, y, θ) end: (x', y', θ')

circular movement

derive center of circles

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x+x' \\ y+y' \end{pmatrix} + \mu\begin{pmatrix} y-y' \\ x'-x \end{pmatrix}, \mu \in \mathbb{R}$$



$$P = \frac{1}{2}\begin{pmatrix} x+x' \\ y+y' \end{pmatrix}$$

Slope of $\overline{PC}$ line $= \dfrac{-1}{\text{slope of } \overline{R_1R_2}} = -\dfrac{\Delta x}{\Delta y} = \dfrac{-x'+x}{y'-y}$

or $\overrightarrow{R_1R_2} \cdot \overrightarrow{PC} = 0$

$$\Rightarrow C = \frac{1}{2}\begin{pmatrix} x+x' \\ y+y' \end{pmatrix} + \mu\begin{pmatrix} -y'+y \\ -x+x' \end{pmatrix}$$

second way

$$\begin{pmatrix} x'-x \\ y'-y \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$$a(x'-x) + b(y'-y) = 0 \Rightarrow a(x'-x) = (y-y')b$$

$$\Rightarrow \begin{pmatrix} a = y-y' \\ b = x'-x \end{pmatrix}$$

one option

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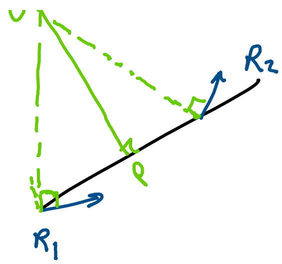
c)



From b)

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x+x' \\ y+y' \end{pmatrix} + \mu\begin{pmatrix} y-y' \\ x'-x \end{pmatrix}$$

From a)

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \lambda\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}, \quad \lambda = \frac{v}{\omega}$$

$$\Rightarrow \begin{cases} x - \lambda\sin\theta = \dfrac{x+x'}{2} + \mu(y-y') \\ y + \lambda\cos\theta = \dfrac{y+y'}{2} + \mu(x'-x) \end{cases}$$

$$\Rightarrow \begin{cases} \lambda = \left[\dfrac{x'-x}{2} + \mu(y-y')\right] \Big/ \sin\theta \\ (2)\quad y + \dfrac{(x'-x)\cos\theta}{2\sin\theta} + \dfrac{\mu(y-y')\cos\theta}{\sin\theta} = \dfrac{y+y'}{2} + \mu(x'-x) \end{cases}$$

$$\mu\left[\frac{(y-y')\cos\theta}{\sin\theta} + (x-x')\right] = \frac{y'-y}{2} + \frac{(x-x')\cos\theta}{2\sin\theta}$$

$$\Rightarrow \mu = \checkmark$$