Homework 1B

Question 6

$$e^{2x} \cdot y'' + 2 \cdot y' + y = 0 \quad ; \quad 0 < x < \infty$$

6.1. Find the normal form of the equation.

6.2. Show that the distance between two successive zeroes of the solution is greater than $\frac{\pi}{\sqrt{2}}$.

Solutions:

6.1. $u'' + \left(\frac{3}{e^{2x}} - \frac{1}{e^{4x}}\right) u = 0 \quad ; \quad y = u(x) \cdot e^{\left(\frac{1}{2 \cdot e^{2x}}\right)}$

Question 7

$$y'' + Q(x) \cdot y = 0 \quad ; \quad Q(x) < 0$$

Show that the solution of the following equation has no more than one zero.

<u>Question 8</u>

Consider an inhomogeneous beam with varying cross sectional moment of inertia $I(x)$ and varying modulus $E(x)$ along the beam, and thus localized flexural stiffness $K(x) = I(x)E(x)$. The beam is subjected to an axial tensile force (N) and to a localized distributed force ($q(x)$). The beam deflection function$w(x)$, is determined by the following fourth order differential equation:

$$\frac{d^2}{dx^2}\left(K \cdot \frac{d^2 w}{dx^2}\right) - N \cdot \frac{d^2 w}{dx^2} = q(x)$$

The differential operator of the beam bending is thus:

$$L = \frac{d^2}{dx^2}\left(K \cdot \frac{d^2}{dx^2}\right) - N \cdot \frac{d^2}{dx^2} \quad ; \quad L[w] = q(x)$$

<u>Find:</u> The adjoint differential operator $\tilde{L}$ and Lagrange identity, such that for $u(x)$ and $w(x)$:

$$u \cdot L[w] - u \cdot \tilde{L}[w] = \frac{d}{dx} F(x, w, u, w', u', w'', u'', w''', u''', w^{IV}, u^{IV})$$

<u>Solutions:</u>
$L = \tilde{L}$; $F = \{u \cdot [(K \cdot w'')' - N \cdot w'] - w \cdot [(K \cdot u'')' - N \cdot u']\} + [(K \cdot u'')w' - u' \cdot (K \cdot w'')]$

<u>Question 9</u>

Find the self-adjoint form of following Laguerre equation of an order n.

$$x \cdot y'' + (1 - x) \cdot y' + n \cdot y = 0 \quad ; \quad n = const$$

<u>Solutions:</u>
$p(x) = xe^{-x}$; $(xe^{-x} \cdot y')' + n \cdot e^{-x} \cdot y = 0$