Homework 2A

Question 1

Find the reoccurrence relation and first four terms of the power series solutions of the following.

1.1. $(2+x^2)y'' - xy' + 4y = 0 \quad ; \quad x_0 = 0$

1.2. $(1+x^2)y'' - 4xy' + 6y = 0 \quad ; \quad x_0 = 0$

1.3. $y'' - 2xy' + \lambda y = 0 \quad , \quad \lambda = const \quad ; \quad x_0 = 0$ (The Hermit Equation)

Solutions:

1.1. $a_{n+2} = -(n^2 - 2n + 4)a_n/[2(n+1)(n+2)]$, $a_2 = -a_0$, $a_3 = -a_1/4$

$\varphi_1 = 1 - x^2 + \frac{1}{6}x^4 - \frac{1}{30}x^4 + \cdots$, $\varphi_2 = x - \frac{1}{4}x^3 + \frac{7}{160}x^5 - \frac{19}{1920}x^4 + \cdots$

1.2. $a_{n+2} = -(n-2)(n-3)a_n/[(n+1)(n+2)]$

$\varphi_1 = 1 - 3x^2$, $\varphi_2 = x - \frac{1}{3}x^3$

1.3 $a_{n+2} = -\frac{\lambda-2n}{(n+2)\cdot(n+1)} \cdot a_n$, $a_2 = -\frac{\lambda}{2}a_0$

$\varphi(x) = a_0\left[1 - \frac{\lambda}{2}x^2 + \frac{(\lambda-4)\cdot\lambda}{4\cdot3\cdot2}x^4 + \frac{(\lambda-8)\cdot(\lambda-4)\cdot\lambda}{6\cdot5\cdot4\cdot3\cdot2}x^6 + \cdots\right] + a_1\left[x - \frac{\lambda-2}{3\cdot2}x^3 + \frac{(\lambda-6)\cdot(\lambda-2)}{5\cdot4\cdot3\cdot2}x^5 + \frac{(\lambda-10)\cdot(\lambda-6)\cdot(\lambda-2)}{7\cdot6\cdot5\cdot4\cdot3\cdot2}x^7 + \cdots\right]$

Question 2

Find the first ***seven*** terms of the power series solution of the following.

$$y'' + 5xy = 2 + 6x^2 \quad ; \quad x_0 = 0$$

Hint: use the same procedure as for homogeneous equations, but equate the power-terms coefficients for both sides of the equation

Solution:

$\varphi(x) = a_0\left[1 - \frac{5}{6}x^3 + \frac{5}{36}x^6 + \cdots\right] + a_1\left[x - \frac{5}{12}x^4 + \frac{5}{432}x^7 + \cdots\right] + 1 \cdot \left[x^2 + \frac{1}{2}x^4 - \frac{1}{4}x^5 - \frac{5}{84}\cdot x^7 + \cdots\right]$