Homework 3B

Question 4

Find the solution of the following inhomogeneous boundary-value problem via the Green's function method.

$$y'' - y = 0$$

$$y(0) = 1 \ \ , \ \ y(1) = 0$$

Solution:

Green's function (homogeneous B.C.):

$$G(x, x_0) = \begin{cases} G_I(x, x_0) & 0 \le x < x_0 \\ G_{II}(x, x_0) & x_0 < x < 1 \end{cases}$$

Where:

$$G_I(x, x_0) = \exp(x_0) \cdot \frac{1 - \exp(1 - x_0)}{\exp(1) - 1} \cdot [1 - \exp(-x)] \qquad , \qquad G_{II}(x, x_0) = \frac{\exp(x_0) - 1}{\exp(1) - 1} \cdot [1 - \exp(1 - x)]$$

The solution:

$$\varphi(x_1) = \int_0^L \tilde{G}(x, x_1) \cdot f(x) dx + \left[F\left(x, \varphi, \frac{d\varphi}{dx}, \tilde{G}, \frac{d\tilde{G}}{dx}\right)\right]_{x=0}^1 = \left[F\left(x, \varphi, \frac{d\varphi}{dx}, \tilde{G}, \frac{d\tilde{G}}{dx}\right)\right]_{x=0}^1 = \frac{1 - \exp(1 - x_1)}{\exp(1) - 1}$$

Question 5

The deflection equation of an elastic beam of flexural stiffness $K = EI$ upon a distributed load $f(x)$ is given by the following 4th order differential equation:

$$K \cdot w^{IV} = f(x)$$

The boundary conditions of a unit-length beam, clamped at $x = 0$ and hinged at $x = 1$ are:

$$w(0) = w'(0) = 0 \ \ ; \ \ w(1) = K \cdot w''(1) = 0$$

5.1. Show that the boundary-value problem above is self-adjoint.

5.2. Find its Green's function, and validate its symmetry.

5.3. Use Green's function to fund the deflation at $x = 1/2$ upon a local unit-force at $x_0 = 1/2$, and compare the solution classical table solutions for beam bending (e.g. Roark's formulae book, table 8.1 case 1c).

<u>*Guidance for 5.2*</u>

Follow the same procedure as for Green's function for 2nd order equations. Solve the homogeneous equation for two regions, for each of which the solution includes 4 undetermined coefficients (total 8 for the two regions). Use the 4 B.C. above, and impose 4 more connection conditions at x_0: Green's function is continues at x_0 up to the 2nd derivative, and experience a unit jump at its 3rd derivative at x_0.

<u>Solution:</u>

Green's function:

$$G(x,x_0)=\begin{cases}G_I(x,x_0) & 0\le x<x_0\\ G_{II}(x,x_0) & x_0<x<1\end{cases}$$

Where:

$$G_I(x,x_0)=\frac{1}{K}\cdot\left[-\left(\frac{1}{12}x_0^3-\frac{1}{4}x_0^2+\frac{1}{6}\right)\cdot x^3+\left(\frac{1}{4}x_0^3-\frac{3}{4}x_0^2+\frac{1}{2}\cdot x_0\right)\cdot x^2\right]$$

$$G_{II}(x,x_0)=\frac{1}{K}\cdot\left[-\left(\frac{1}{12}x^3-\frac{1}{4}x^2+\frac{1}{6}\right)\cdot x_0^3+\left(\frac{1}{4}x^3-\frac{3}{4}x^2+\frac{1}{2}\cdot x\right)\cdot x_0^2\right]$$

Load-point $x_0=1/2$ deflection at $x=1/2$:

$$w\left(x=\frac{1}{2}\right)=G_I\left(x=\frac{1}{2},x_0=1/2\right)\sim\frac{0.0091}{K}$$