

NO.

Date

BFGS

Broyden 拟牛顿法

16) 列举四个具有收敛终止性的算法: Newton, 拟Newton, 拟Newton法, DFP, Broyden

FR, PRP

✓ <7> 对二次函数  $f(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$ , 取初始点  $x_0 = (9, 1)^T$ , 用最速下降法求得下一迭代点

(相梯度搜索)

$$x_1 = \begin{pmatrix} \frac{36}{5} \\ -\frac{4}{5} \end{pmatrix}$$

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$g_k = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$g_0 = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

$$x_1 = x_0 - \frac{g_0^T g_0}{g_0^T G g_0} g_0$$

$$(9, 1)^T - \frac{82}{82} \begin{pmatrix} 9 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 = x_0 - \alpha_0 g_0$$

$$\min f(x) \Rightarrow \alpha_0 = \frac{g_0^T g_0}{g_0^T G g_0} = \frac{82}{82} = 1$$

$$\Rightarrow x_1 = \begin{pmatrix} \frac{36}{5} \\ -\frac{4}{5} \end{pmatrix}^T$$

★ 点  $(a, a)$  为非线性规划  $\min (x_1 - 2)^2 + x_2^2$  的KT点, 则参数  $\alpha = 1$

$$s.t. \ x_1 - x_2 \geq 0 \Rightarrow G$$

$$-x_1 + x_2 \geq 0 \Rightarrow G_2$$

$$-x_1 \nabla f(x) - \lambda_1 \nabla G_1(x) - \lambda_2 \nabla G_2(x) = 0$$

将  $(a, a)$  代入  $G \neq 0$

$$a = 0 \Rightarrow \lambda_1 = 0$$

$$\nabla f(x) - \lambda_2 \nabla G_2(x) = 0 \Rightarrow \begin{pmatrix} 2(a-2) \\ -2a \end{pmatrix} - \lambda_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0$$

$$\begin{aligned} 2a - 4 + \lambda_2 &= 0 \\ -2a - \lambda_2 &= 0 \\ \lambda_2 &= 2a \\ 2a - 4 + 2a &= 0 \end{aligned}$$

$$\alpha(a, a) \Rightarrow \alpha = 1$$

✓ 设  $f(x)$  为凸集  $D \subset \mathbb{R}^n$  上的函数, 其上图为  $epi(f) = \{(x, y) | x \in D, y \in \mathbb{R}, y \geq f(x)\}$

证:  $f(x)$  为凸函数的充要条件是  $epi(f)$  为凸集

证明: 首先证明其充分性: 在  $D$  上任取两点  $x_1, x_2$ , 则对应的在  $epi(f)$  上取点  $(x_1, f(x_1)), (x_2, f(x_2))$

$$\therefore epi(f) \text{ 是凸集} \Rightarrow \alpha(x_1, f(x_1)) + (1-\alpha)(x_2, f(x_2)) = (\alpha x_1 + (1-\alpha)x_2, \alpha f(x_1) + (1-\alpha)f(x_2)) \in epi(f)$$

$$\Rightarrow \alpha f(x_1) + (1-\alpha)f(x_2) \geq f(\alpha x_1 + (1-\alpha)x_2)$$

$\Rightarrow f$  为在凸集  $D$  上的凸函数

再证明必要性:  $\forall (x_1, y_1), (x_2, y_2) \in epi(f), 0 < \alpha < 1$

$$\therefore x_1, x_2 \in D \Rightarrow \alpha x_1 + (1-\alpha)x_2 \in D$$

$$\alpha y_1 + (1-\alpha)y_2 \geq \alpha f(x_1) + (1-\alpha)f(x_2)$$

$$\because f(x) \text{ 在 } D \text{ 上是凸函数, } \alpha f(x_1) + (1-\alpha)f(x_2) \geq f(\alpha x_1 + (1-\alpha)x_2)$$

$$\Rightarrow \alpha y_1 + (1-\alpha)y_2 \geq f(\alpha x_1 + (1-\alpha)x_2)$$

$$\therefore \text{点 } (\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2) \in epi(f)$$

$\Rightarrow epi(f)$  是凸集

Mastino