Date

五、已知矩阵 $A=\begin{pmatrix} 3 & 0 & 8 \\ 3 & -1 & 6 \\ -2 & 0 & -5 \end{pmatrix}$ 求

(1) A的最小多项式，并指出A可否对角化

(2) A的Jordan标准形

(3) e^{At}

解：(1) $|\lambda I-A| = \begin{vmatrix} \lambda-3 & 0 & -8 \\ -3 & \lambda+1 & -6 \\ 2 & 0 & \lambda+5 \end{vmatrix} = (\lambda+1)[(\lambda-3)(\lambda+5)+16] = (\lambda+1)^3$

故最小多项式可能是 $(\lambda+1)$ $(\lambda+1)^2$ $(\lambda+1)^3$

$(A+I) = \begin{pmatrix} 4 & 0 & 8 \\ 3 & 0 & 6 \\ -2 & 0 & -4 \end{pmatrix} \neq 0$

$(A+I)^2 = \begin{pmatrix} 4 & 0 & 8 \\ 3 & 0 & 6 \\ -2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 4 & 0 & 8 \\ 3 & 0 & 6 \\ -2 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$

$\therefore m(\lambda) = (\lambda+1)^2$

$\because$ 最小多项式有重根 $\therefore$ 不能对角化

(2) 初等因子 $(\lambda+1)^2 \cdot (\lambda+1)$

$\therefore J = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(3) 设 $e^{\lambda t} = p(\lambda)m(\lambda) + c_0 + c_1\lambda$.

$\therefore \begin{cases} e^{-t} = c_0 - c_1 \\ te^{-t} = c_1 \end{cases}$ $\begin{cases} c_0 = e^{-t} + te^{-t} \\ c_1 = te^{-t} \end{cases}$

$\therefore e^{\lambda t} = p(\lambda)m(\lambda) + (1+t)e^{-t} + te^{-t}\lambda$

$\therefore e^{At} = (1+t)e^{-t} I + te^{-t} A$

$= \begin{pmatrix} (1+t)e^{-t} & & \\ & (1+t)e^{-t} & \\ & & (1+t)e^{-t} \end{pmatrix} + \begin{pmatrix} 3te^{-t} & 0 & 8te^{-t} \\ 3te^{-t} & -te^{-t} & 6te^{-t} \\ -2te^{-t} & 0 & -5te^{-t} \end{pmatrix}$

$= e^{-t}\begin{pmatrix} 1+4t & 0 & 8t \\ 3t & 1 & 6t \\ -2t & 0 & 1-4t \end{pmatrix}$