(2) 对 $y/3 = x_1 + 5/6x_3 - 1/6x_4$ 得

$\Rightarrow x_1 - x_4 - 1 = \frac{2}{3} - \frac{5}{6}x_3 - \frac{5}{6}x_4 \le 0$ 作为等式割平面方程 $-5x_3 - 5x_4 + x_5 = -4$.

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4. 用PRP方法求解问题: $\min x_1^2 + 2x_2^2 - 2x_1x_2 - 4x_1$, 初始点 $x^{(0)} = (1,1)^T$ P116

解: $g(x) = (2x_1 - 2x_2 - 4, 4x_2 - 2x_1)^T$, $g_0 = (-4, 2) \ne 0$ 取 $p_0 = -g_0 = (4, -2)$. 从 x_0 出发沿 p_0 精确搜索

$\min f(x_0 + \alpha_0 p_0) = 40\alpha^2 - 20\alpha - 3$ 取极值时 $\alpha = \frac{1}{4}$

$\therefore x_1 = x_0 + \alpha_0 p_0 = (2, \frac{1}{2})^T$, $g_1 = (-1, -2)^T$ PRP算法 $\beta_{k-1} = \frac{g_k^T(g_k - g_{k-1})}{g_{k-1}^T g_{k-1}}$

$\therefore \beta_0 = \frac{g_1^T(g_1 - g_0)}{g_0^T g_0} = \frac{(-1,-2)\left[\binom{-1}{-2} - \binom{-4}{+2}\right]}{(-4,2)\binom{-4}{2}} = \frac{1}{4}$

$\therefore p_1 = -g_1 + \beta_0 p_0 = (2, \frac{3}{2})^T$ 从 x_1 出发沿 p_1 精确一维搜索,

$\min f(x_1 + \alpha_1 p_1) = \frac{5}{4}\alpha_1^2 - 5\alpha_1 - 6$ 取极值时 $\alpha_1 = 1$

$\therefore x_2 = x_1 + \alpha_1 p_1 = (4, 2)^T$, $g_2 = (0, 0)^T = 0$

$\therefore x_2 = x^*$

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5. 用内罚函数法(对数罚函数)求解 $\min x_1^2 + x_2^2$

s.t. $x_1 \ge 1$.

解: $B(x_1, x_2, r) = x_1^2 + x_2^2 - r\ln(x_1 - 1)$

令 $\frac{\partial B}{\partial x_1} = 2x_1 - \frac{r}{x_1 - 1} = 0 \Rightarrow x(r) = (\frac{1+\sqrt{1+2r}}{2}, 0)$.

$\frac{\partial B}{\partial x_2} = 2x_2 = 0$ 当 $r \to 0$ 时, $x^* = (1, 0)$, $f^* = 1$.

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6. 验证 $x^* = (1,1,1)^T$ 为下面问题的KT点: $\min f(x) = -3x_1^2 - x_2^2 - 2x_3^2$

s.t. $c_1(x) = x_1^2 + x_2^2 + x_3^2 - 3 = 0$

$c_2(x) = -x_1 + x_2 \ge 0$

$c_3(x) = x_1 \ge 0$

$c_4(x) = x_2 \ge 0$

$c_5(x) = x_3 \ge 0$

解: $I^* = \{1, 2\}$, $\nabla f(x^*) = (-6, -2, -4)^T$, $\nabla c_1(x^*) = (2, 2, 2)^T$, $\nabla c_2(x^*) = (-1, 1, 0)$

令 $\begin{pmatrix}-6\\-2\\-4\end{pmatrix} - \lambda_1\begin{pmatrix}2\\2\\2\end{pmatrix} - \lambda_2\begin{pmatrix}-1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ 解得 $\lambda_1 = -2$, $\lambda_2 = 2$.

即 $\nabla f(x^*) = -2\nabla c_1(x^*) + 2\nabla c_2(x^*)$, $\lambda_2 c_2(x^*) = 0$, $\lambda_2 > 0$

$\therefore x^*$ 为KT点

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7. 若线性规划: $\min x_1 + 2x_2 + x_3$ 的最优解 $(a, b, c)^T$ 其对偶线性规划最优解为 $(\frac{1}{6}, \ldots$

s.t. $3x_1 + 4x_2 = 5$

$x_1 + 2x_3 = u$

$x_1, x_2, x_3 \ge 0$

a, b, c, u 四个常数中,你可以确定哪些?如果有不确定的,确定其范围.

解: 对偶线性规划 $\max 5y_1 + uy_2$

s.t. $3y_1 + y_2 \le 1$

$4y_1 \le 2$

$2y_2 \le 1$

$\therefore$ 对偶线性规划最优值相同

由原线性规划约束可得 $3a + 4b = 5$,

$a + 2c = u$

$a, b, c \ge 0$

由以上条件得 $a = \frac{5}{3}$, $b = 0$, $2c + \frac{5}{3} = u$

其中 $c \ge 0$, $u \ge \frac{5}{3}$.