No. ....

Date

$2x_1 - \lambda + 2\sigma(x_1 - 1) = 0$

$2x_1(1+\sigma) = \lambda + 2\sigma$ $\quad x_1 = \frac{\lambda + 2\sigma}{2+2\sigma}$

等式

(用乘子法) 求: $\min f(x) = x_1^2 + x_2^2$

s.t. $x_1 + x_2 - 2 = 0$

解: 增广拉格朗日函数 $M(x, \lambda, \sigma) = x_1^2 + x_2^2 - \lambda(x_1+x_2-2) + \frac{\sigma}{2}(x_1+x_2-2)^2$.

$\frac{\partial M}{\partial x_1} = 2x_1 - \lambda + \sigma(x_1+x_2-2)$

$\frac{\partial M}{\partial x_2} = 2x_2 - \lambda + \sigma(x_1+x_2-2)$

$\Rightarrow x_1 = x_2 = \frac{2\sigma + \lambda_k}{2\sigma + 2}$

则当 $\lambda \to \lambda^*$ 时, 则 $x \to x^*$. $\quad \lambda_{k+1} = \lambda_k - \sigma c(x_k) = \lambda_k - \sigma(x_1+x_2-2)$

将 $x_1 = x_2 = \frac{2\sigma+\lambda_k}{2\sigma+2}$ 代入 $\quad \lambda_{k+1} = \lambda_k - \left(\frac{4\sigma+2\lambda_k}{2\sigma+2} - 2\right)\sigma$.

$\Rightarrow \lambda_{k+1} = \frac{1}{\sigma+1}\lambda_k + \frac{\sigma \cdot 2}{\sigma+1}\sigma$. 当 $\sigma > 0$ 时 $\{\lambda_k\}$ 收敛, 设 $\lambda_k \Rightarrow \lambda^*$ 取极限

令 $(\lambda_{k+1} - u) = \frac{1}{\sigma+1}(\lambda_k - u)$. $\Rightarrow u = 2$. $\quad \lambda^* = \frac{1}{\sigma+1}\lambda^* + \frac{2}{\sigma+1}\sigma \Rightarrow \lambda^* = 2$

$\Rightarrow \lambda_k \to 2$.

$\therefore$ 当 $\lambda_k = 2$ 时. $x_1 = x_2 = 1$. 则原问题的最优解 $x^* = (1, 1)^T$.

(不等式约束)

内点 $\quad B(x, r) = f(x) + r\tilde{B}(x)$

约束一定是不等式

$\tilde{B}(x) = \sum_{i=1}^{m} \frac{1}{c_i(x)}$ 或 $-\sum_{i=1}^{m} \ln c_i(x)$

令 $\frac{\partial B}{\partial x_i} = 0$. 当 $r \to 0$ 时 $\Rightarrow x^*$

外点 $\quad P(x, \sigma) = f(x) + \sigma\tilde{P}(x)$

(等式时)

$\tilde{P}(x) = \sum_{i=1}^{l} |c_i(x)|^\beta$ $\quad \beta \geq 1$

不等式约束

$\tilde{P}(x) = \sum_{i=1}^{m} |\min\{0, g_i(x)\}|^\alpha$ $\quad \alpha \geq 1$

$\sigma \to +\infty$

$\alpha = 2$

$\beta = 2$

乘子 $\lambda_{k+1} = \lambda_k - \sigma c(x_k)$