Global Epistasis

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Symmetric Matrix Case

Our Fitness is:

$$F = \sum_{i=1}^{L} h_i \sigma_i + \sum_{i \neq j}^{L} \sigma_i J_{ij} \sigma_j$$

Where the size of our system is L spins, with sparsity ρ.

We have $L + L^2\rho - L = L^2\rho$ terms summed in the fitness calculations(non epistatic term minus diagonal).

At any given time we have $N_+(t)$ positive terms and $N_- = L^2\rho - N_+(t)$ negative terms such that:

$$F(t) \approx \sigma_J (N_+ - N_-) = \sigma_J (2N_+ - L^2\rho)$$

Each spin i appears in $2L\rho + 1 \approx 2L\rho$ of these terms. We take the average approximation where all spins have exactly the mean spin energy.

Thus, there are $N_+/2L\rho$ positive terms each spin appears in and $N_-/2L\rho$ negative terms.

The average effect of flipping a spin is:

$$F(t+1) \approx F(t) - \sigma_J \frac{N_+(t)}{2L\rho} + \sigma_J \frac{N_-(t)}{2L\rho}$$

$$< \Delta F(t) >= -\frac{F(t)}{2L\rho}$$

Now, for a case where we do not transform $\tilde{F}(t) = F(t) - F_{off}$ s.t. $\tilde{F}(t = 0) = 1$ there is no constant b:

$$< \Delta F(t) >= b - \frac{F(t)}{2L\rho}$$

Because the constant would be $< \Delta F(t = 0) >= 0$ in the Gaussian J_{ij}, random initial genome case.

If we do transform $\tilde{F}(t) = F(t) - F_{off}$ then $\tilde{F}(t) + F_{off} = F(t)$ so that:

$$< \Delta F(t) >= -\frac{F(t)}{2L\rho} = -\frac{\tilde{F}(t) + F_{off}}{2L\rho} = -\frac{F_{off}}{2L\rho} - \frac{\tilde{F}(t)}{2L\rho}$$

And we get $b = -\frac{F_{off}}{2L\rho}$