We have:

$$
\begin{aligned}
\partial_t P_\sigma(k,t) &= r(-k)P_{-\sigma}(-k,t) - r(k)P_\sigma(k,t) \\
&\quad + D_\sigma(t)\partial_{kk}P_\sigma(k,t) + 2\partial_k \int dk' r(k') \left[R_{\sigma,\sigma}(k,k') + R_{\sigma,-\sigma}(k,k')\right]
\end{aligned}
$$

For values $\sigma = \pm 1$. $R_{\sigma,\sigma'}(k,k')$ is defined as:

$$R_{\sigma,\sigma'}(k,k') = \frac{1}{N}\sum_{i \neq j} \overline{\left\langle \delta_{\sigma,\sigma_i}\delta(k-k_i)\sigma_i J_{ij}\sigma_j \delta_{\sigma',\sigma_j}\delta(k'-k_j)\right\rangle}$$

Now to make an approximation, first we notice the local fields can be separated into the J_{ij} dependent and independent parts:

$$k_i = \sigma_i J_{ij}\sigma_j + \sum_{l \neq j}\sigma_i J_{il}\sigma_l := \sigma_i J_{ij}\sigma_j + k_i^{\neq j}$$

Now we have:

$$R_{\sigma,\sigma'}(k,k') = \frac{1}{N}\sum_{i \neq j} \overline{\left\langle \delta_{\sigma,\sigma_i}\delta(k-k_i^{\neq j}-\sigma_i J_{ij}\sigma_j)\sigma_i J_{ij}\sigma_j \delta_{\sigma',\sigma_j}\delta(k'-k_j^{\neq i}-\sigma_i J_{ij}\sigma_j)\right\rangle}$$

Now using Stein's lemma and the dependence of k_i, k_j on $\sigma_i J_{ij}\sigma_j$:

$$
\begin{aligned}
R_{\sigma,\sigma'}(k,k') &= \frac{1}{N}\sum_{i \neq j} \overline{\left\langle \delta_{\sigma,\sigma_i}\delta(k-k_i)\sigma_i J_{ij}\sigma_j \delta_{\sigma',\sigma_j}\delta(k'-k_j)\right\rangle} \\
&= \frac{1}{N}Var(J_{ij})\frac{\partial}{\partial J_{ij}}\sum_{i \neq j} \overline{\left\langle \delta_{\sigma,\sigma_i}\delta(k-k_i^{\neq j}-\sigma_i J_{ij}\sigma_j)\sigma_i\sigma_j \delta_{\sigma',\sigma_j}\delta(k'-k_j^{\neq i}-\sigma_i J_{ij}\sigma_j)\right\rangle} \\
&= \frac{1}{N^2}\sum_{i \neq j} \overline{\left\langle \sigma_i\sigma_j\delta_{\sigma,\sigma_i}\delta_{\sigma',\sigma_j}\left(\delta(k'-k_j)\frac{\partial}{\partial J_{ij}}\delta(k-k_i^{\neq j}-\sigma_i J_{ij}\sigma_j) + \delta(k-k_i)\frac{\partial}{\partial J_{ij}}\delta(k'-k_j^{\neq i}-\sigma\right.\right.} \\
&= -\frac{1}{N^2}\sum_{i \neq j} \overline{\left\langle \delta_{\sigma,\sigma_i}\delta_{\sigma',\sigma_j}\delta'(k-k_i)\delta(k'-k_j) + \delta_{\sigma,\sigma_i}\delta_{\sigma',\sigma_j}\delta(k-k_i)\delta'(k'-k_j)\right\rangle} \\
&= -\frac{1}{N^2}\left(\partial_k + \partial_{k'}\right)\sum_{i \neq j} \overline{\left\langle \delta_{\sigma,\sigma_i}\delta_{\sigma',\sigma_j}\delta(k-k_i)\delta(k'-k_j)\right\rangle} \\
&= -\left(\partial_k + \partial_{k'}\right)P_\sigma(k,t)P_{\sigma'}(k',t) \\
&= -\left(P_{\sigma'}(k',t)\partial_k P_\sigma(k,t) + P_\sigma(k,t)\partial_{k'}P_{\sigma'}(k',t)\right)
\end{aligned}
$$

Now we have:

$$
\begin{aligned}
\int dk' r(k') \left[R_{\sigma,\sigma}(k,k') + R_{\sigma,-\sigma}(k,k')\right] &= -\int dk' r(k') \left[P_\sigma(k',t)\partial_k P_\sigma(k,t) + P_\sigma(k,t)\partial_{k'} P_\sigma(k',t)\right] \\
&\quad - \int dk' r(k') \left[P_{-\sigma}(k',t)\partial_k P_\sigma(k,t) + P_\sigma(k,t)\partial_{k'} P_{-\sigma}(k',t)\right] \\
&= -\frac{D_\sigma(t)}{2}\partial_k P_\sigma(k,t) - P_\sigma(k,t)\int dk' r(k')\partial_{k'} P_\sigma(k',t) \\
&\quad - \frac{D_{-\sigma}(t)}{2}\partial_k P_\sigma(k,t) - P_\sigma(k,t)\int dk' r(k')\partial_{k'} P_{-\sigma}(k',t)
\end{aligned}
$$

All in all our equation becomes, using $D_\sigma(t) := 2\int dk r(k) P_\sigma(k,t)$:

$$
\begin{aligned}
\partial_t P_\sigma(k,t) &= r(-k)P_{-\sigma}(-k,t) - r(k)P_\sigma(k,t) + D_\sigma(t)\partial_{kk}P_\sigma(k,t) \\
&\quad - 2\partial_k\left(\frac{D_\sigma(t)}{2}\partial_k P_\sigma(k,t) + P_\sigma(k,t)\int dk' r(k')\partial_{k'} P_\sigma(k',t)\right) \\
&\quad - 2\partial_k\left(\frac{D_{-\sigma}(t)}{2}\partial_k P_\sigma(k,t) + P_\sigma(k,t)\int dk' r(k')\partial_{k'} P_{-\sigma}(k',t)\right) \\
&= r(-k)P_{-\sigma}(-k,t) - r(k)P_\sigma(k,t) - 2\partial_k P_\sigma(k,t)\int dk' r(k')\partial_{k'} P_\sigma(k',t) \\
&\quad - D_{-\sigma}(t)\partial_{kk}P_\sigma(k,t) - 2\partial_k P_\sigma(k,t)\int dk' r(k')\partial_{k'} P_{-\sigma}(k',t) \\
&= r(-k)P_{-\sigma}(-k,t) - r(k)P_\sigma(k,t) \\
&\quad - \left(2\sum_{\sigma'}\int dk' r(k')\partial_{k'} P_{\sigma'}(k',t)\right)\partial_k P_\sigma(k,t) - D_{-\sigma}(t)\partial_{kk}P_\sigma(k,t)
\end{aligned}
$$

So we get a drift velocity that depends both on the current sign-distribution and the opposite, and a diffusion term that couples exclusively to opposite sign. Note the following:

$$
\begin{aligned}
\partial_t P_+(k,t) &= r(-k)P_-(-k,t) - r(k)P_+(k,t) \\
&\quad - \left(2\sum_{\sigma'}\int dk' r(k')\partial_{k'} P_{\sigma'}(k',t)\right)\partial_k P_+(k,t) - D_-(t)\partial_{kk}P_+(k,t)
\end{aligned}
$$

$$
\begin{aligned}
\partial_t P_-(k,t) &= r(-k)P_+(-k,t) - r(k)P_-(k,t) \\
&\quad - \left(2\sum_{\sigma'}\int dk' r(k')\partial_{k'} P_{\sigma'}(k',t)\right)\partial_k P_-(k,t) - D_+(t)\partial_{kk}P_-(k,t)
\end{aligned}
$$

Denote $P = P_+ + P_-$ and sum the 2 equations:

$$\partial_t P(k,t) = r(-k)P(-k,t) - r(k)P(k,t) \\ - \left(2\int dk' r(k')\partial_{k'}P(k',t)\right)\partial_k P(k,t) - \partial_{kk}\left(D_-(t)P_+(k,t) + D_+(t)P_-(k,t)\right)$$

So there is no closed form for P with no couplings. Finally, defining $v(t) := 2\int\sum_{\sigma'} dk' r(k')\partial_{k'}P_{\sigma'}(k',t)$ we have:

$$\partial_t P_\sigma(k,t) = r(-k)P_{-\sigma}(-k,t) - r(k)P_\sigma(k,t) - v(t)\partial_k P_\sigma(k,t) - D_{-\sigma}(t)\partial_{kk}P_\sigma(k,t)$$