We begin with the equation:

Look for steady state solutions:

Define new coefficients:

And drop the tildes from here forward.

What happens if we integrate over an region around :

This is the virtual flux of factors through the -plane, and we can turn it into a Robin BC:

And just solve the Laplace equation on the positive half volume.

Assume structure of :

We assume this structure due to the fact that one can deduce the following symmetry from the equation / BC:

And also:

Where is a constant, and there is no way to satisfy the previous condition for besides constant , which cannot be a solution.

The symmetry can be seen as follows - if is a solution, for we have:

And:

Now taking the function , observe that plugging it in to the BC for we get:

It also satisfies the rest of the reflecting BC and is a solution of the Laplace equation as well, thus is also a solution on .

From uniqueness we can deduce and we get our symmetry.

So, we have BC:

And we solve for the region, which dictates the region.

This is the simplest structure that has the potential to adhere to the constraints, where is still seperable in the Poisson equation.

Now solve:

BC 3:

BC 4:

Where we have only odd wavenumbers.

BC 1:

So we now have for :

And our final BC is (on ):

Multiply by and integrate on :

Where we get that for these sines are orthogonal on .

So finally for :

And this solution agrees with the symmetry we found, so this is the solution on as well.

Now, we are most interested at what happens on plane, so:

Taking we get:

Which is exactly the same solution as with absorbing BC at .

We can further simplify this solution:

So that:

We know is the fourier transform of a sin square wave on , so:

With the original notation of .

To take to the continuum limit:

With and being the sin integral and cosine integral, respectively.

Therefore we have:

Observe we have only 1 length scale of ; Let us explore the 2 extreme regimes of :

1. For we have the asymptiotic expansion:

Si( x L D ) &ap; π 2 - cos( x L D ) x L D ;Ci( x ) &ap; sin( x L D ) x L D (47)

Plugging this in to get the behaviour far from the origin (assume x>0 for simplicity):

C( x,z=0 ) &ap; r Ω × ( L D π x ) (48)

And for x<0 :

C( x,z=0 ) &ap; r Ω × ( 1+ L D π x ) (49)

And this is the approximate behaviour for x &gg; L D , which is different then the exponential asymptotic behaviour of the 1D version.

We plot this asymptotic behaviour in [6](#fig__linear_asymptotic).

For x &ll; L D we have the expansions to 1st order:

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Figure 6: Vertical axis is log( -C( x,z=0 ) / r Ω ) +log( π Ω ) . If we plot this in a region of x &gg; L D we expect log( -C( x,z=0 ) / r Ω ) +log( π L D ) &ap; log( π L D ) -log( L D π x ) =log( x ) . Thus, plotting log( -C( x,z=0 ) / r Ω ) +log( π Ω ) for different values of Ω vs log( x ) for x &gg; L D we expect all plots to fall on a 45 degree line, as can be seen.

Si( x L D ) &ap; x L D ;Ci( x ) &ap; ln( |x| L D ) + γ (50)

Where γ &ap; 0.577 is the the Euler-Mascheroni constant. Plugging this in and assuming x>0 for brevity, we get:

C( x,0 ) &ap; r Ω π × ( sign( x ) [ π 2 - |x| L D ] + x L D ( ln( |x| L D ) + γ ) ) (51)

Therefor, for 0<x &ll; L D :

C( x,0 ) &ap; r Ω π × ( π 2 + x L D ( ln( x L D ) + γ -1 ) ) (52)