**The concentration distribution of -factor without Bar1**

According to our experimental setup, -factors are secreted at the plane for at a rate . They can diffuse into the positive half-plane and are taken up by *a*-cells at both and on the plane at a rate . Assuming translational symmetry along the y direction (parallel to the interface), we can write the concentration of the -factor in the plane as follows

With a boundary condition

where is the Heaviside step function and is the Dirac delta function.

In the steady state, where , we have

It is easy to show that has the following symmetry

If one substitutes into assuming is a solution, one finds is also a solution, and thus from uniqueness we can conclude . Specifically, we notice

And this will be of use later. We now take and rewrite the production and degradation terms as a boundary condition at

And use this together with to solve for the steady state. At , we have

The above Laplace equation can be solved by separation of variables:

But this simple form of cannot satisfy our boundary conditions, as we saw that which would lead to constant . We use to state the ansatz

With the idea that , and we will soon see this is the solution. The most general form of this series solution is

We use and reflecting boundary conditions on to determine , and , and finally obtain:

With

In Fig. , we show the heat-map of and the corresponding flow field . Clearly, the uptake of the -factor is not only due to the flow along the plane; the space also plays an essential role, leading to a quantitatively different result than a model of diffusion along . The importance of the dimensionality on diffusion in such problems is systemically studied in [2,3].

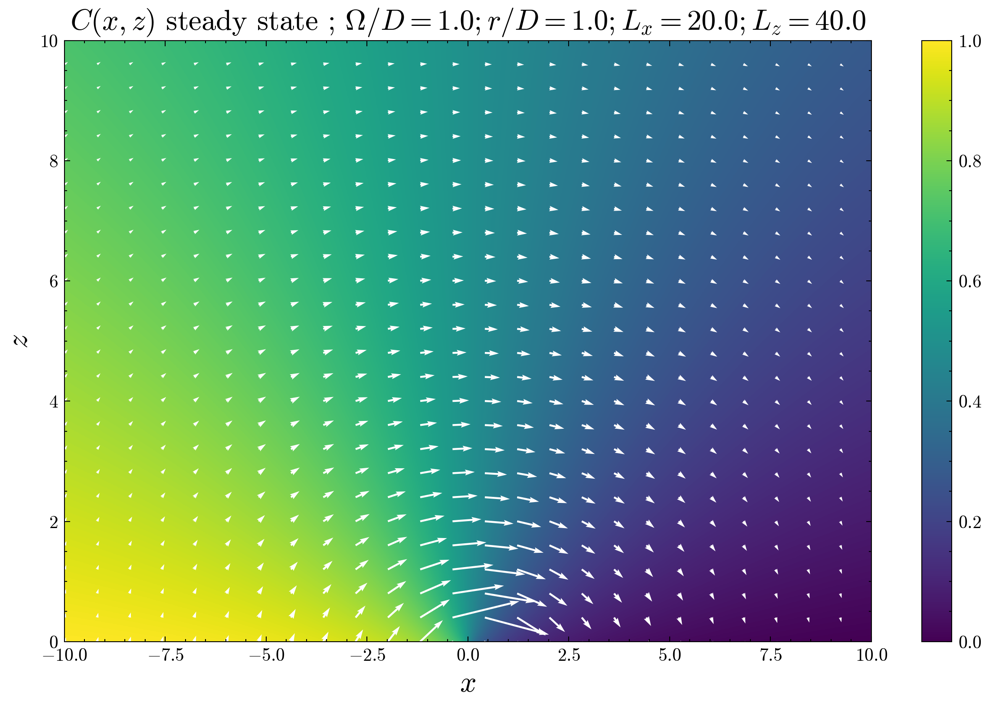


Fig. : Heat-map of and the corresponding field flow

For , in the limit and using the known Fourier series expansion for square waves, we have the solution

We can then consider the continuous limit of as , we obtain

where and are the sine and cosine integrals, respectively. Clearly there is a length scale , which is different from the case diffusion occurring along the axis. When , we obtain

decays as in the far-field instead of exponential decay for the case in 1D diffusion.

When , we have

where is the Euler-Mascheroni constant.

# Bibliography

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[3] P. B. Dieterle and A. Amir, Diffusive wave dynamics beyond the continuum limit, Phys. Rev. E 104, 014406 (2021)