A DENOTATIONAL APPROACH TO RELEASE/ACQUIRE CONCURRENCY

GALOP, POPL 2024

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GOAL

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Design a standard, monad-based denotational semantics (Moggi [1991])

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Using Brookes-style [1996], totally-ordered traces

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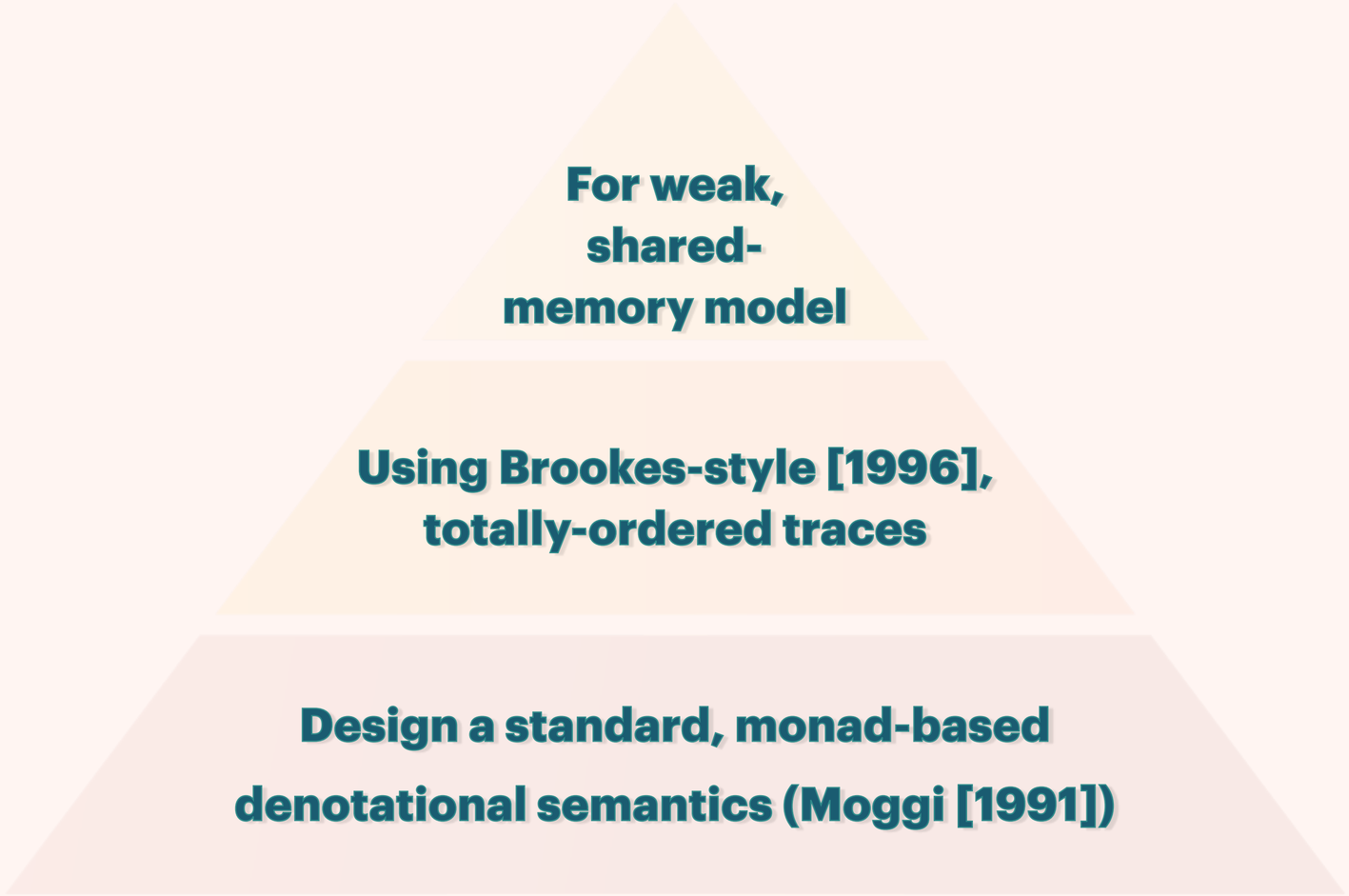
For weak,
shared-
memory model
Using Brookes-style [1996],
totally-ordered traces
Design a standard, monad-based
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GOAL

Release/Acquire

For weak, shared-memory model

Using Brookes-style [1996], totally-ordered traces

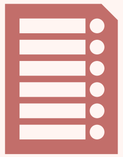
Design a standard, monad-based denotational semantics (Moggi [1991])

WHY RELEASE/ACQUIRE?

RA is an important fragment of C/C++, enables decentralized architectures (POWER)

First adaptation of Brookes's traces to a software model (compositional parallelism)

Intricate causal semantics, not overwhelmingly detailed



Threads can disagree about the order of writes (non-multi-copy-atomic)

Supports flag-based synchronization (e.g. for signaling a data structure is ready)

Supports important transformations (e.g. thread sequencing, write-read-reorder)

Supports read-modify-write atomicity

TRACE-BASED SEMANTICS

Brookes [1996]

Main ingredient: linearly-ordered traces of state-transitions that sequence and interleave

$$\langle \mu_1, \varrho_1 \rangle \langle \mu_2, \varrho_2 \rangle \cdots \langle \mu_{n-1}, \varrho_{n-1} \rangle \langle \mu_n, \varrho_n \rangle$$

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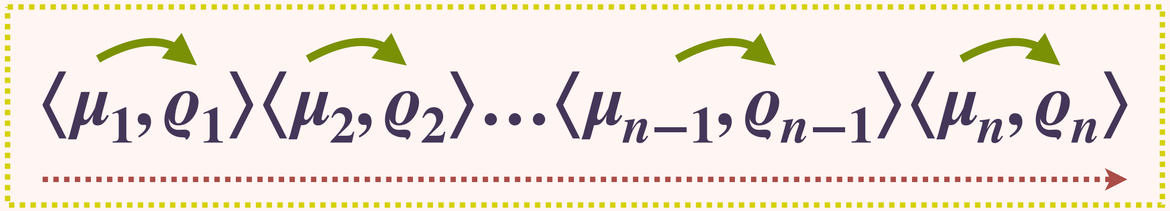

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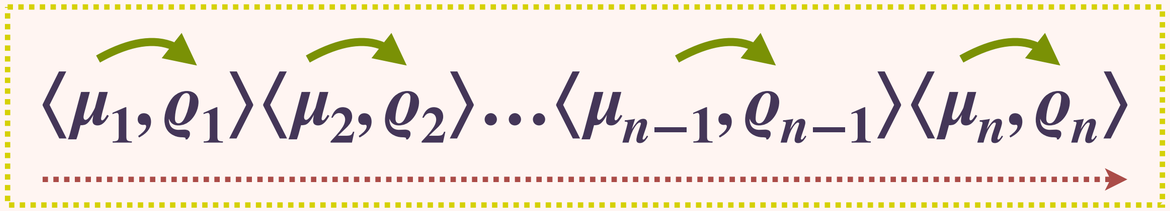

$$\langle \mu_1, \mu_1' \rangle \ \langle \mu_2, \mu_2' \rangle \ \ldots \ \langle \mu_n, \mu_n' \rangle \ \langle \varrho_1, \varrho_1' \rangle \ \langle \varrho_2, \varrho_2' \rangle \ \ldots \ \langle \varrho_n, \varrho_n' \rangle$$

SEQUENCE

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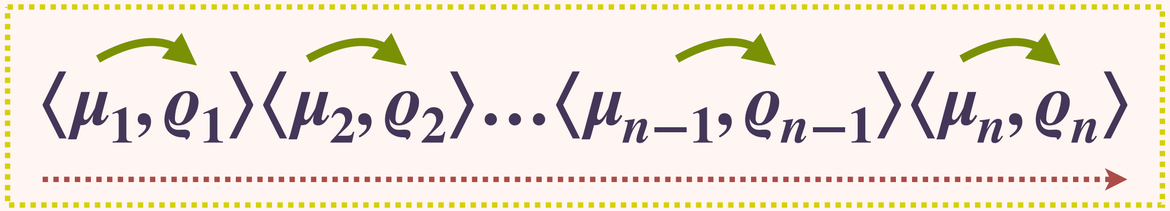
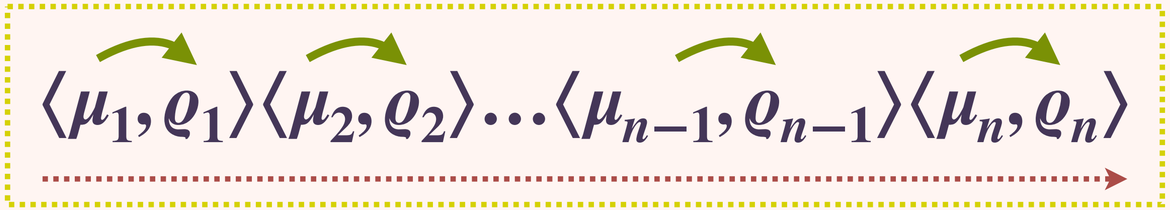


$$\frac{\langle \varrho_1, \varrho_1' \rangle \ \langle \mu_1, \mu_1' \rangle \ \langle \mu_2, \mu_2' \rangle \ \langle \varrho_2, \varrho_2' \rangle \ \ldots \ \langle \mu_n, \mu_n' \rangle \ \langle \varrho_n, \varrho_n' \rangle}{}$$

INTERLEAVE

TRACE-BASED SEMANTICS

Main ingredient: linearly-ordered traces of state-transitions that sequence and interleave



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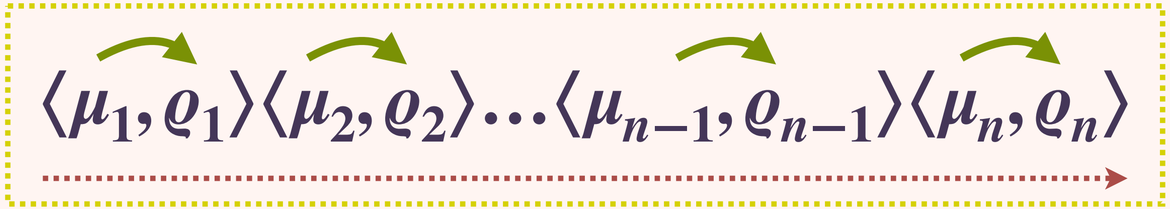

TRACE-BASED SEMANTICS

Brookes [1996]

- Denotational semantics $[\![-]\!]$ for concurrency
- Idealized model - Sequential Consistency (SC)
- Follows operational semantics

Main ingredient: linearly-ordered traces of state-transitions that sequence and interleave

$$\langle \mu_1, \varrho_1 \rangle \langle \mu_2, \varrho_2 \rangle \dots \langle \mu_{n-1}, \varrho_{n-1} \rangle \langle \mu_n, \varrho_n \rangle$$

Jagadeesan, Petri, Riely [2012]

- Adapts traces to TSO (hardware model)
- Follows operational semantics too
  - Relatively close to SC

This work

- Adapts traces to RA (software model)
- Kang et al. [2017] operational presentation
  - Much more complex notion of state

CONTRIBUTION

Directionally Adequate

Refinement: $[\![M]\!] \supseteq [\![K]\!]$ $\Longrightarrow$ Transformation: $M \twoheadrightarrow K$

denotational semantics for RA based on linearly-ordered traces

Standard (CbV) semantics [Moggi 1991]

- enables structural transformations (e.g. $[\![K; (M; N)]\!] = [\![(K; M); N]\!]$)
- has higher-order functions for free
- etc.

Abstract enough to justify every transformation discussed in the literature that we know of (but no full-abstraction)

New challenge — non-operational interpretation:

each trace represents a possible behavior as a Rely/Guarantee sequence

RELEASE/ACQUIRE

TYPICAL EXAMPLES

Store Buffering

$$x := 0; y := 0;$$
$$x := 1; \quad \| \quad y := 1;$$
$$y? \quad\quad\quad\quad x?$$

Message Passing

$$x := 0; y := 0;$$
$$x := 1; \quad \| \quad y?;$$
$$y := 1 \quad\quad\quad\quad x?$$

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TYPICAL EXAMPLES

Propagation is not instant

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$$x := 0; y := 0;$$
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$$y? \quad //0 \quad \| \quad x? \quad //0$$

Message Passing

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$$x := 0; y := 0;$$
$$x := 1; \;\|\; y := 1;$$
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Message Passing

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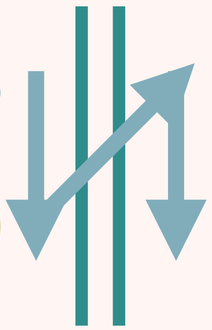

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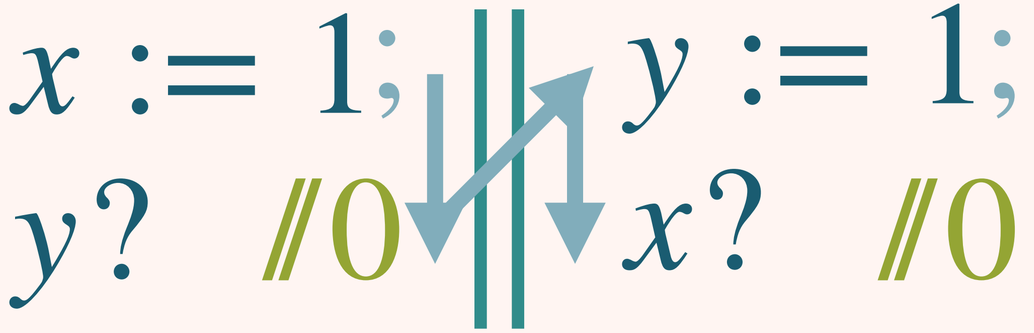
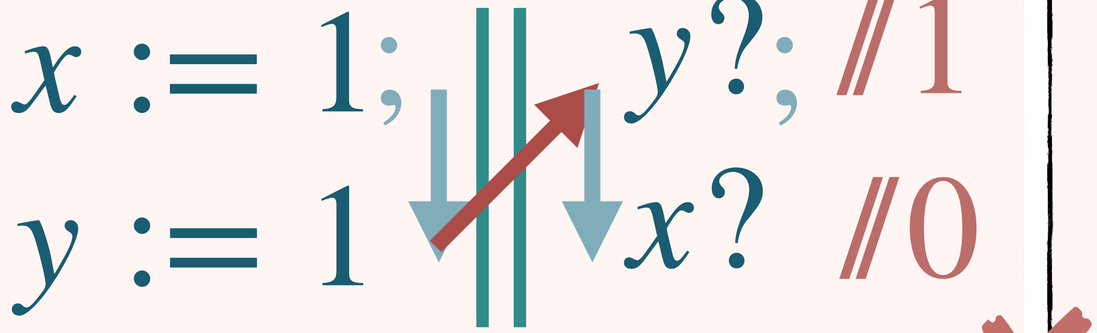


Message Pas

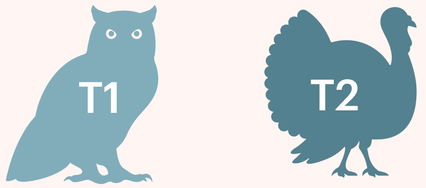
Propagation respects causality

$$x := 0; y := 0;$$



RELEASE/ACQUIRE VIEW-BASED OPERATIONAL SEMANTICS

Kang et al. [2017]



- **Memory**: Timeline per location (e.g. x, y, z)
- Populated with immutable messages (e.g. x0, y0, z0)
- Each thread's view points to a msgs on each timeline (e.g. T1)
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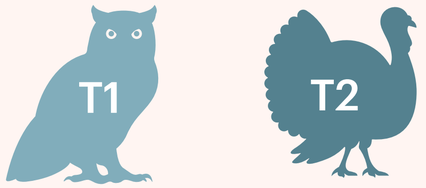

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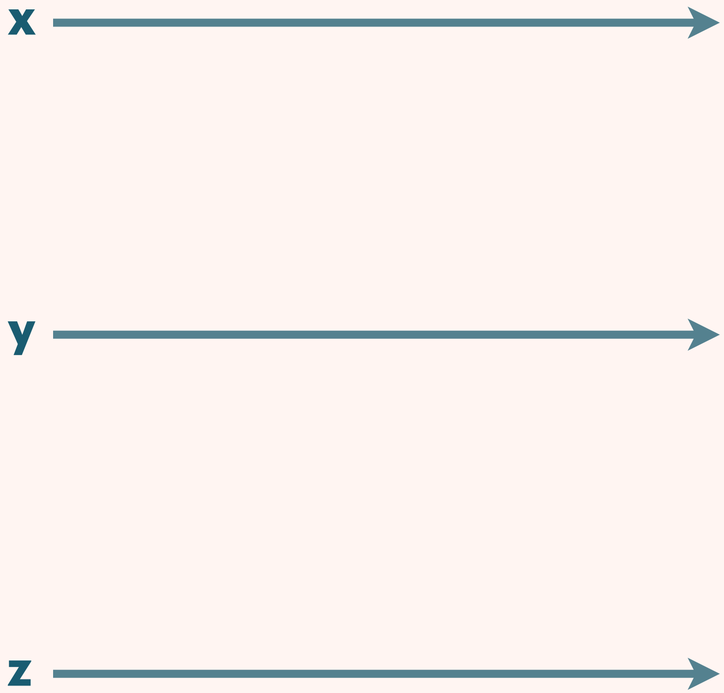

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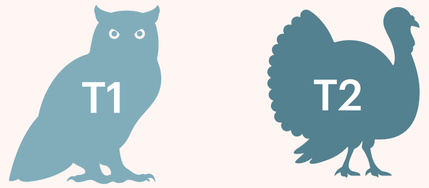

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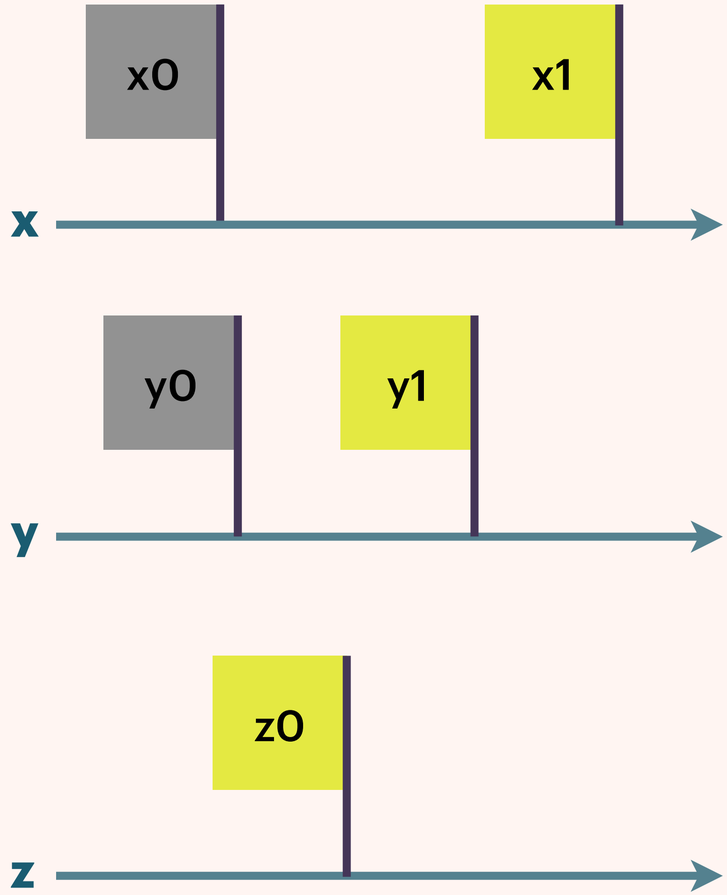

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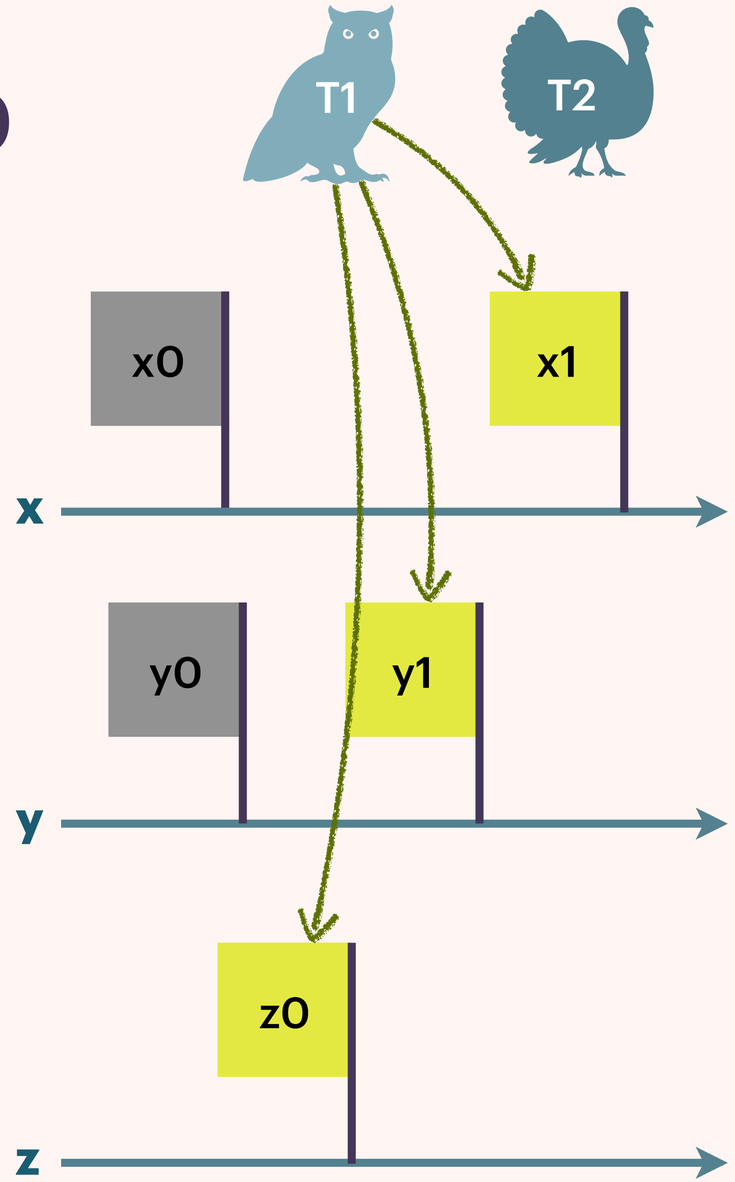

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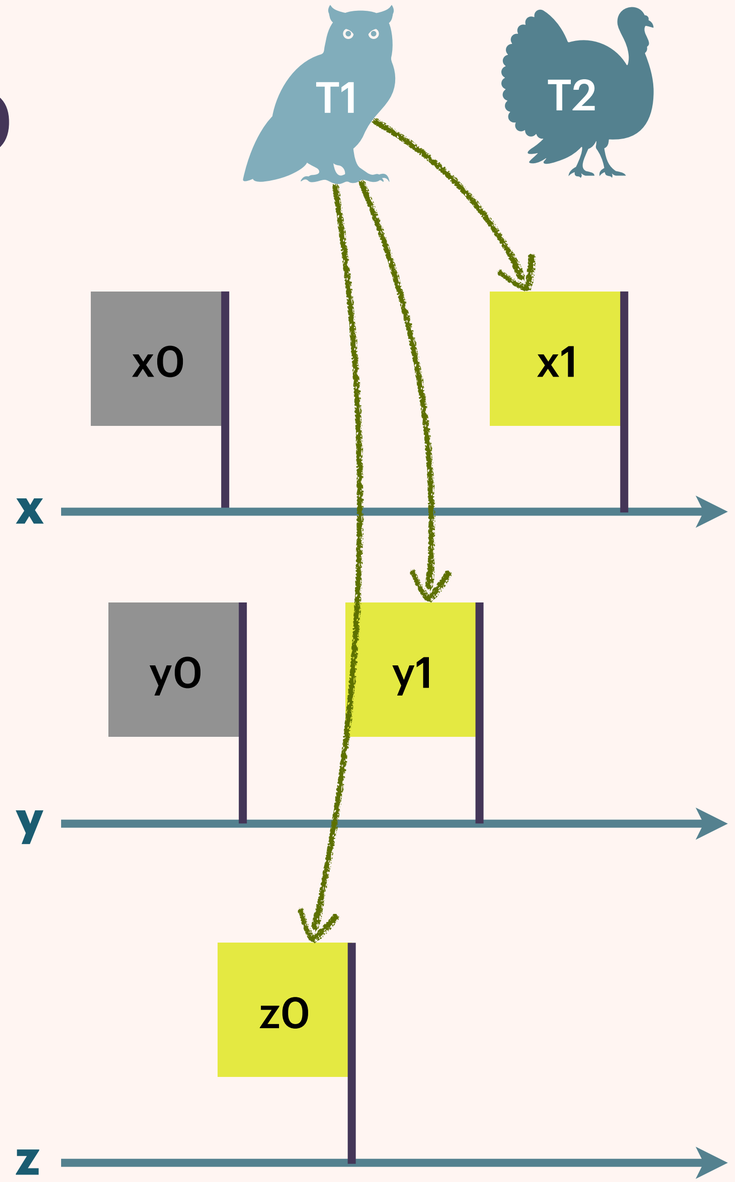

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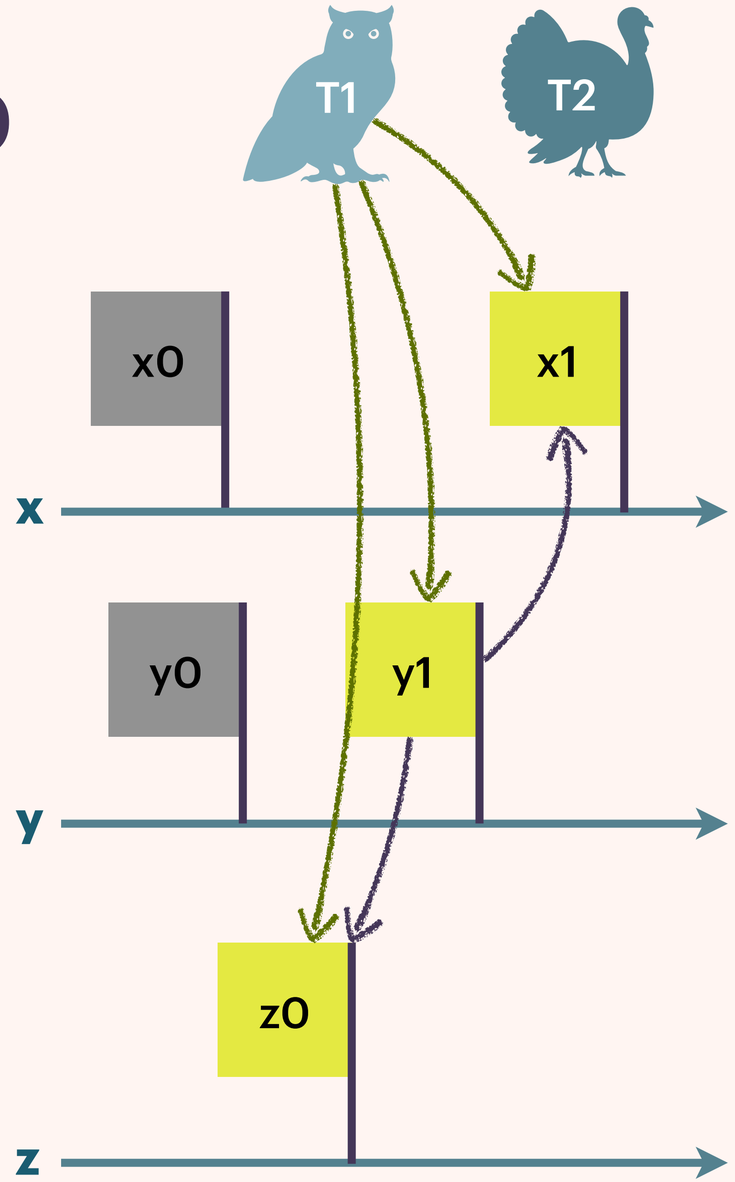
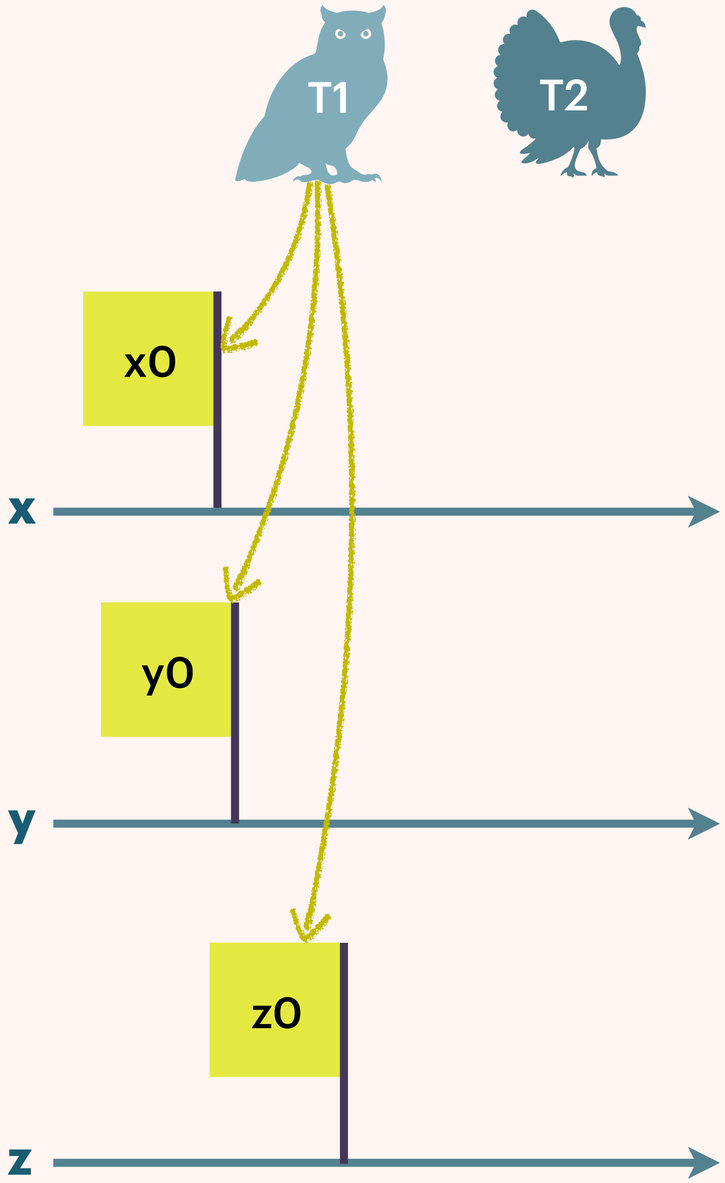
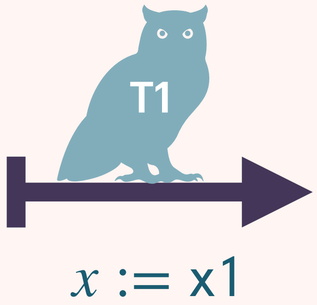
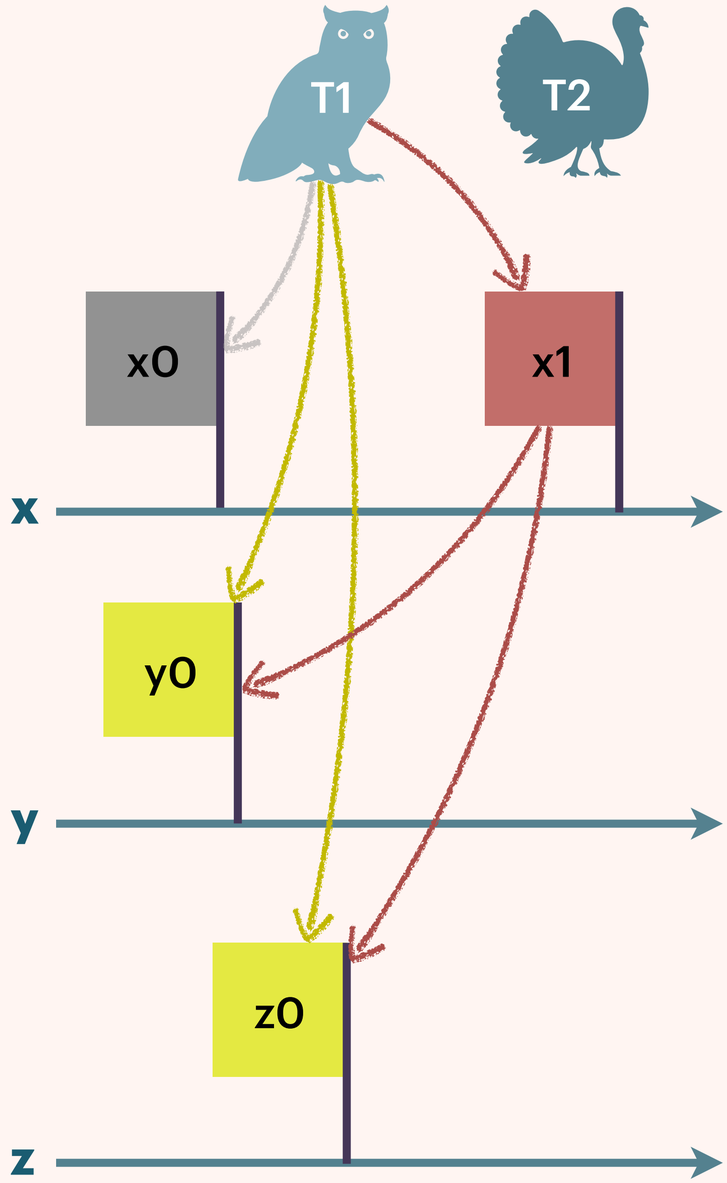


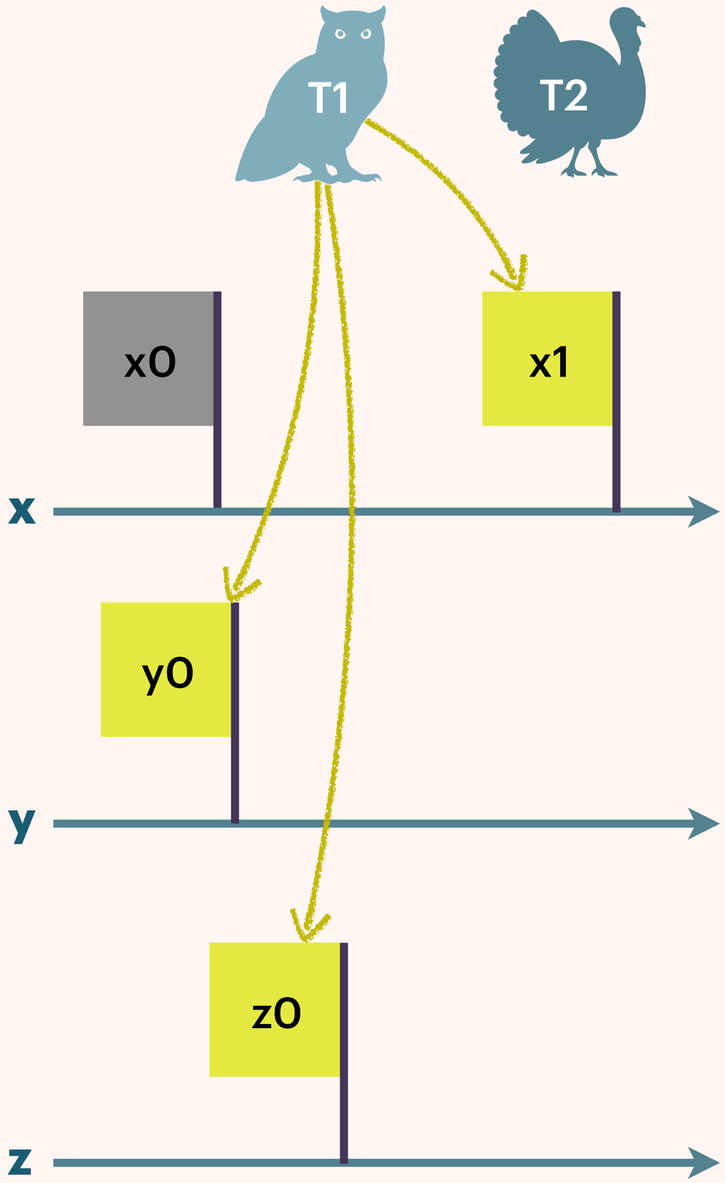


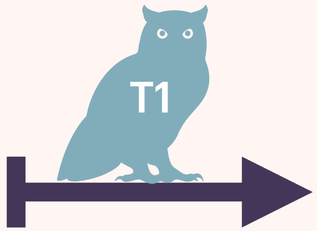


When writing, the message:

- must be placed after thread's view
- may be placed before others
- copies thread's view







$y := y1$

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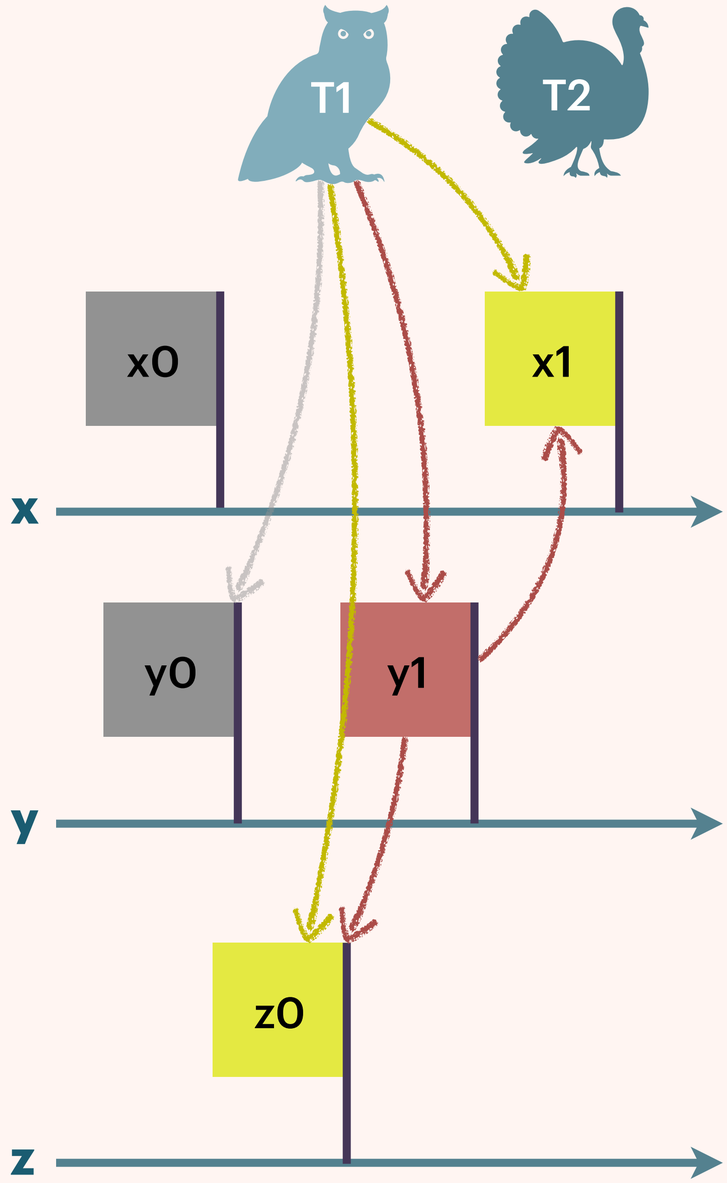

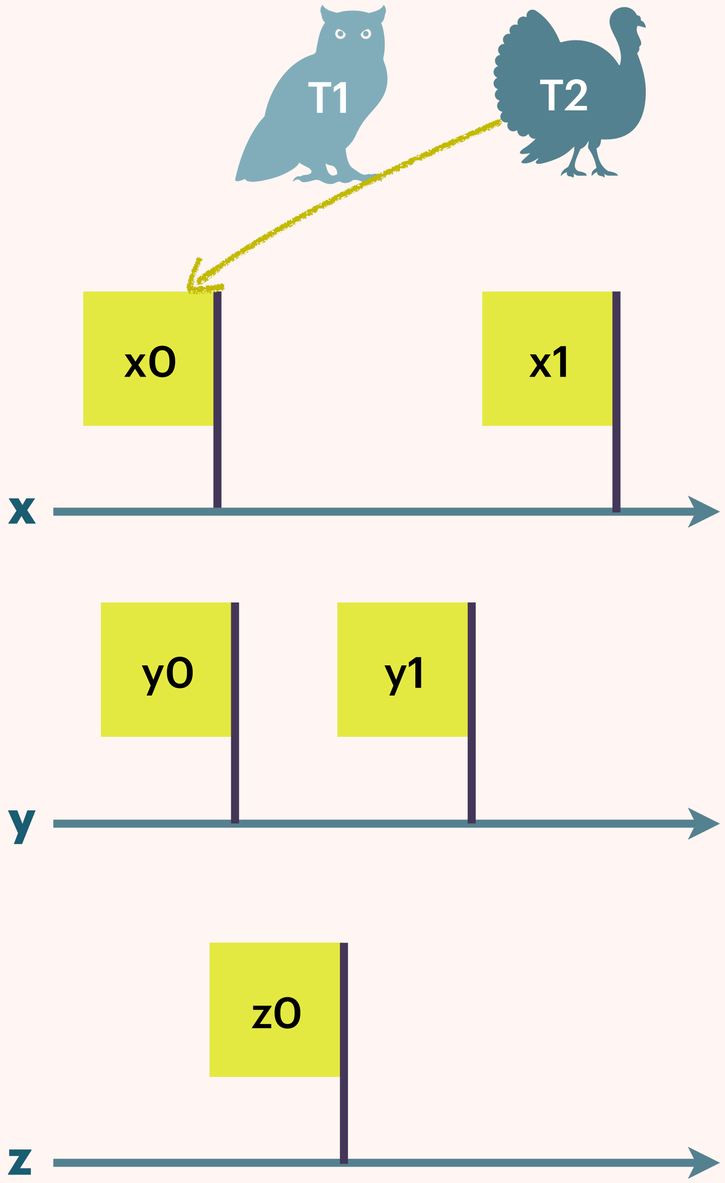

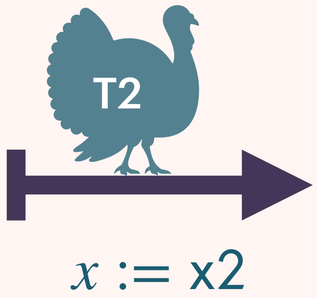

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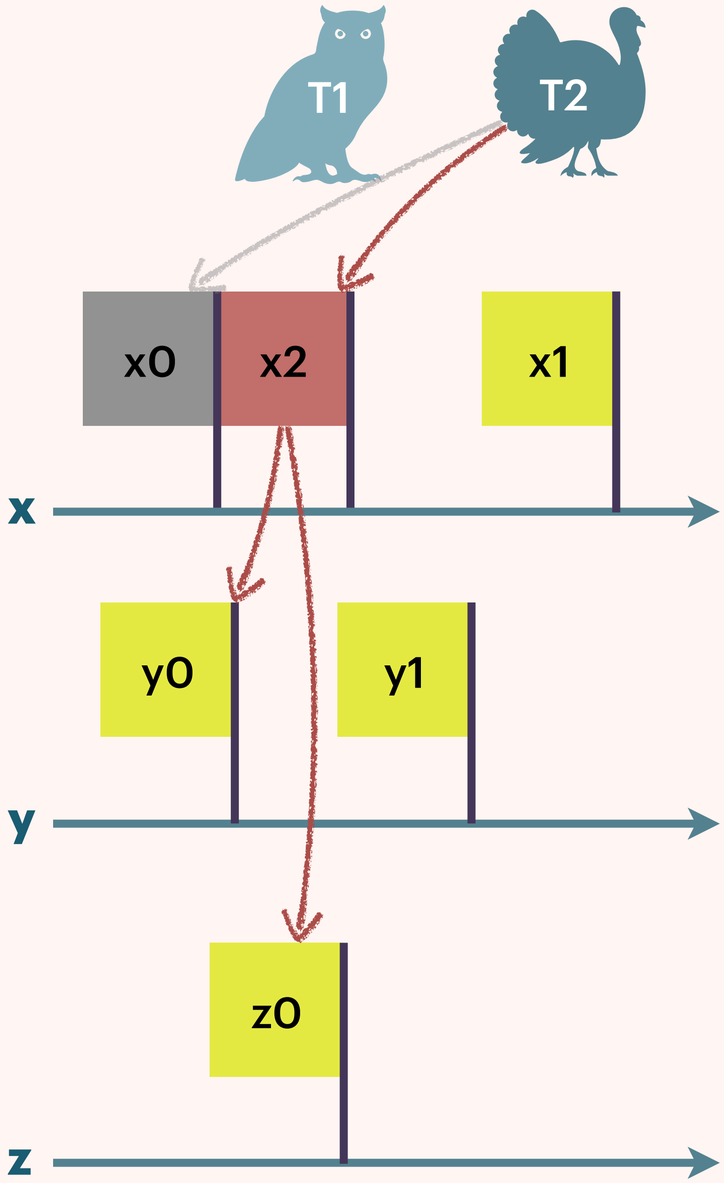
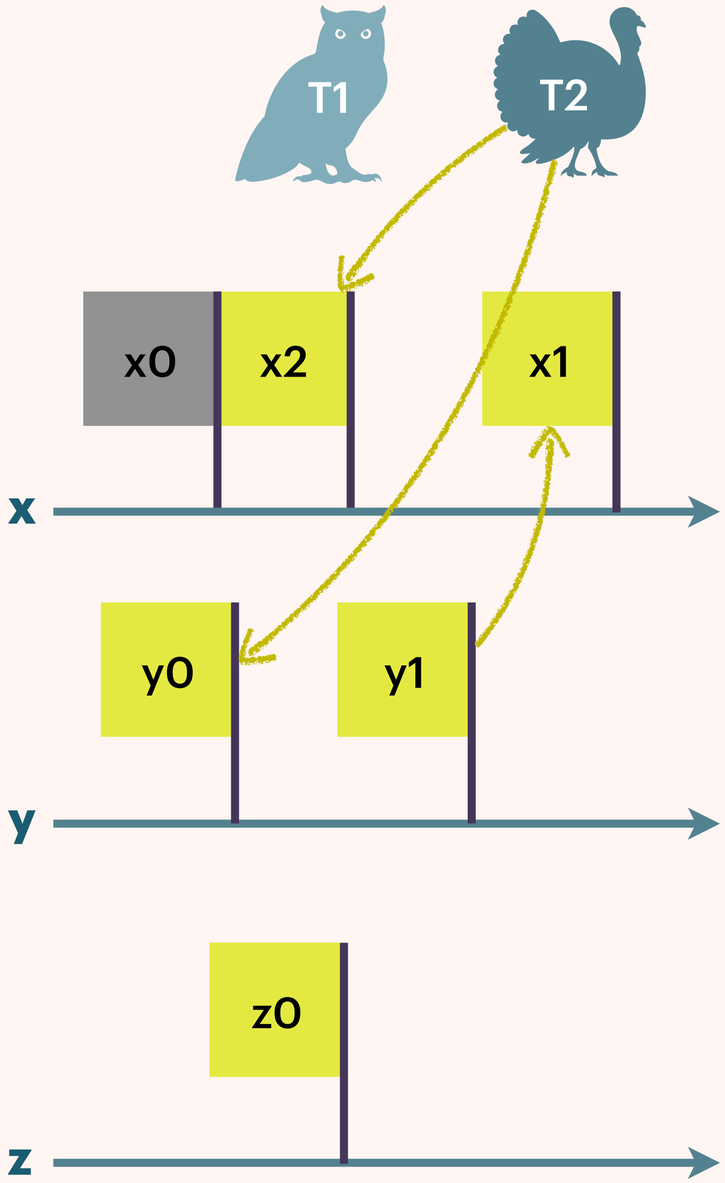
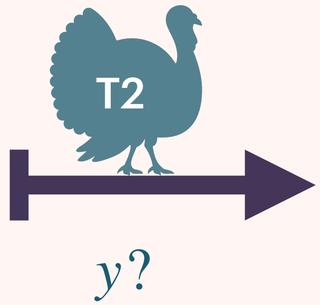




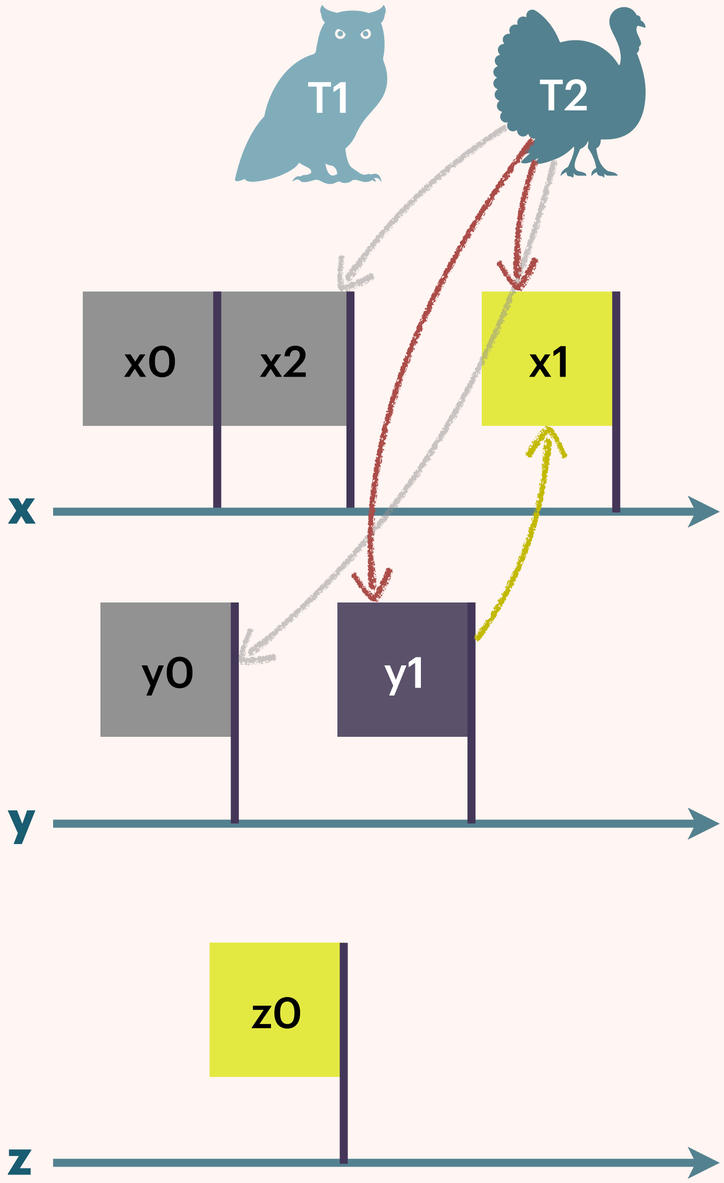


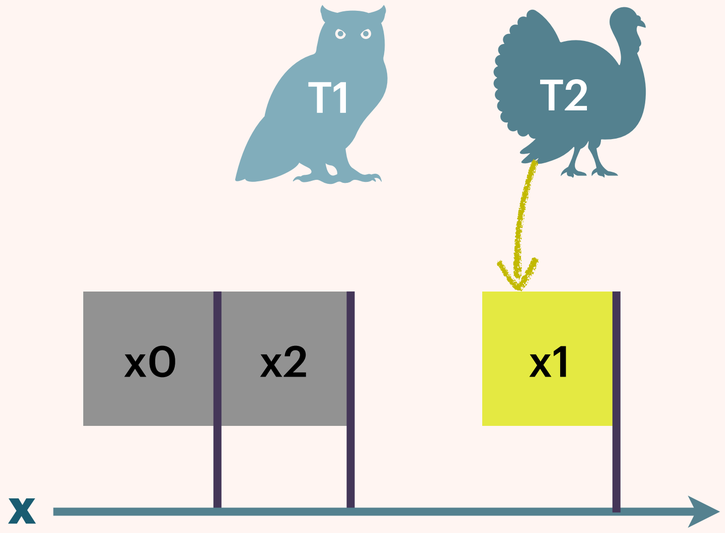
When reading, the message:

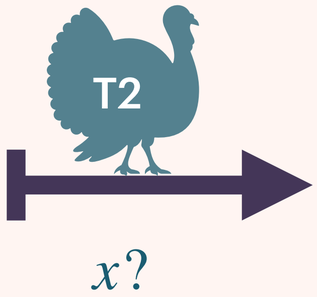
- **cannot be before thread's view**
- **may be before others**

and the thread:

- **inherits the copy of the view**





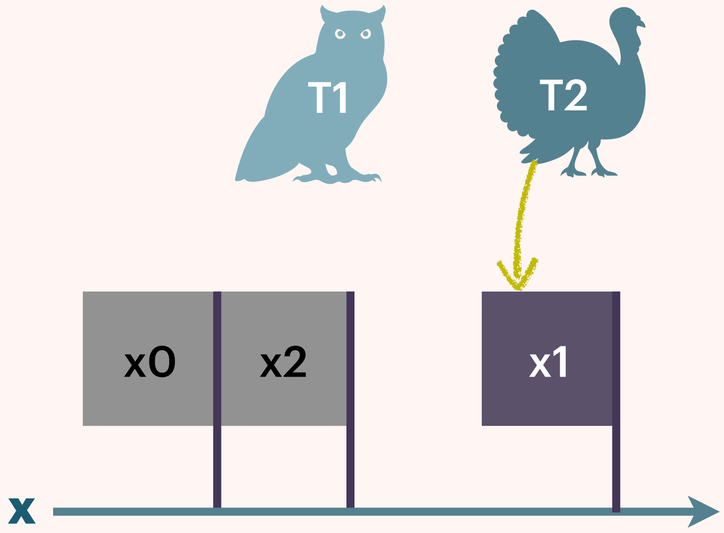


When reading, the message:

- **cannot be before thread's view**
- **may be before others**

and the thread:

- **inherits the copy of the view**



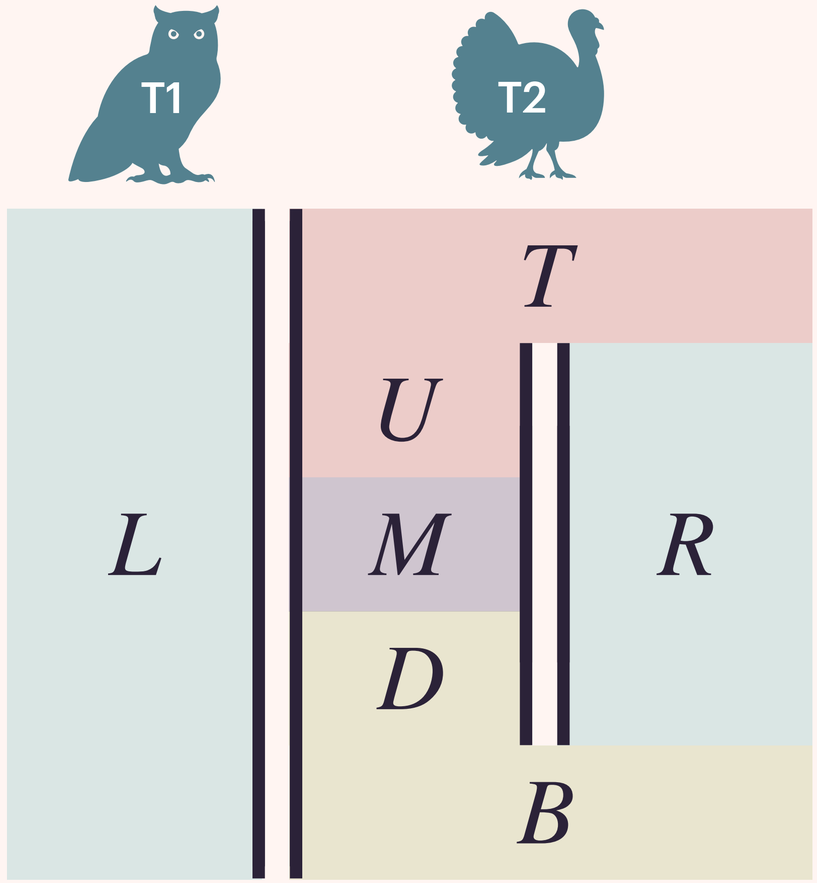
CAUSALITY AND COMPOSITION

With first class parallelism

$$L \parallel \Big(T; \big((U; M; D) \parallel R \big); B \Big)$$

Generalized Sequencing

$$(M_1; M_2) \parallel (K_1; K_2) \twoheadrightarrow (M_1 \parallel K_1); (M_2 \parallel K_2)$$



TRACE-BASED SEMANTICS

TRACE-BASED SEMANTICS IN RA

Terms denote sets of traces

$$[\![M]\!] \ni \tau$$

Each trace represents a possible behavior as a Rely/Guarantee sequence

$$\alpha \langle \mu_1, \varrho_1 \rangle \langle \mu_2, \varrho_2 \rangle \ldots \langle \mu_{n-1}, \varrho_{n-1} \rangle \langle \mu_n, \varrho_n \rangle \omega \therefore r$$

Initial View — **Sequence of Transitions** — **Final View** — **Returns**

RA RA

TRACE-BASED SEMANTICS IN RA

Rely On μ_1

To Guarantee ϱ_1

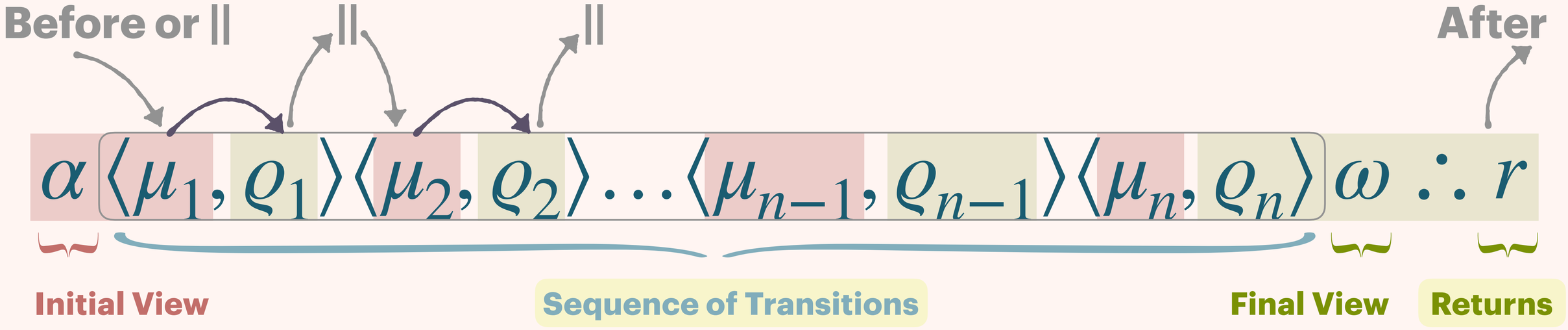
Then

Rely On μ_2

To Guarantee ϱ_2

Then...

Guarantee to the **sequential** environment to return r



TRACE-BASED SEMANTICS IN RA

Rely on the sequential environment to reveal messages before α

Guarantee to the sequential environment to reveal messages before ω

Before

After

$$\alpha \langle \mu_1, \varrho_1 \rangle \langle \mu_2, \varrho_2 \rangle \dots \langle \mu_{n-1}, \varrho_{n-1} \rangle \langle \mu_n, \varrho_n \rangle \omega \therefore r$$

Initial View

Sequence of Transitions

Final View

Returns

Analogous to Brookes's

TRANSITION CLOSURES

Stutter

$$\frac{\alpha\,\xi\eta\,\omega \therefore r \in [\![M]\!]}{\alpha\,\xi\langle\mu,\mu\rangle\eta\,\omega \therefore r \in [\![M]\!]}$$

Propagate Reliance as a Guarantee

Mumble

$$\frac{\alpha\,\xi\langle\mu,\rho\rangle\langle\rho,\theta\rangle\eta\,\omega \therefore r \in [\![M]\!]}{\alpha\,\xi\langle\mu,\theta\rangle\eta\,\omega \therefore r \in [\![M]\!]}$$

Rely on an omitted Guarantee

Specific to RA

VIEW CLOSURES

Rewind

$$\alpha' \leq \alpha \qquad \alpha \; \xi \; \omega \therefore r \in [\![M]\!]$$

$$\alpha' \; \xi \; \omega \therefore r \in [\![M]\!]$$

Relying on more being revealed

Forward

$$\alpha \; \xi \; \omega \therefore r \in [\![M]\!] \qquad \omega \leq \omega'$$

$$\alpha \; \xi \; \omega' \therefore r \in [\![M]\!]$$

Guaranteeing less being revealed

COMPOSITION

Sequential

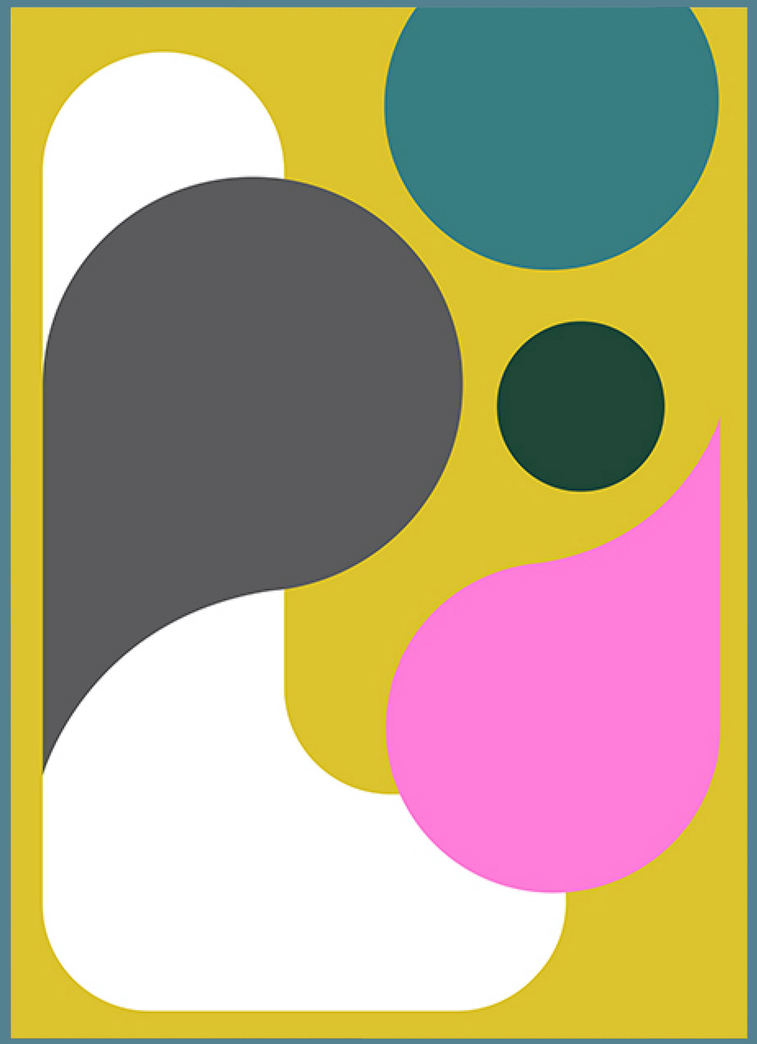
$$\alpha\,\xi_1\,\kappa \therefore r_1 \in [\![M_1]\!] \qquad \kappa\,\xi_2\,\omega \therefore r_2 \in [\![M_2]\!][x \mapsto r_1]$$

$$\alpha\,\xi_1\xi_2\,\omega \therefore r_2 \in [\![\mathbf{let}\, x = M_1 \,\mathbf{in}\, M_2]\!]$$

Parallel

$$\forall i \in \{1,2\}.\ \ \alpha\,\xi_i\,\omega \therefore r_i \in [\![M_i]\!] \qquad \xi \in \xi_1 \parallel \xi_2$$

$$\alpha\,\xi\,\omega \therefore \langle r_1, r_2 \rangle \in [\![M_1 \parallel M_2]\!]$$



ABSTRACTION

WHAT WE CAN JUSTIFY

with Stutter, Mumble, Rewind, and Forward

Standard Semantics

- **Structural equivalences, e.g. if K is effect-free then**

$$[\![\,\mathbf{if}\, K \,\mathbf{then}\, M; P_1 \,\mathbf{else}\, M; P_2 \,]\!] = [\![\, M; \mathbf{if}\, K \,\mathbf{then}\, P_1 \,\mathbf{else}\, P_2 \,]\!]$$

First-class parallelism

- **Laws of Parallel Programming, e.g. Generalized Sequencing**

$$[\![\,(M_1; M_2) \parallel (K_1; K_2) \,]\!] \supseteq [\![\,(M_1 \parallel K_1); (M_2 \parallel K_2) \,]\!]$$

Some memory access related transformations, e.g. Read-Read Elimination

$$[\![\,\mathbf{let}\, a = x? \,\mathbf{in}\, \mathbf{let}\, b = x? \,\mathbf{in}\, \langle a, b \rangle \,]\!] \supseteq [\![\,\mathbf{let}\, c = x? \,\mathbf{in}\, \langle c, c \rangle \,]\!]$$

SEMANTIC INVARIANTS ON TRACES

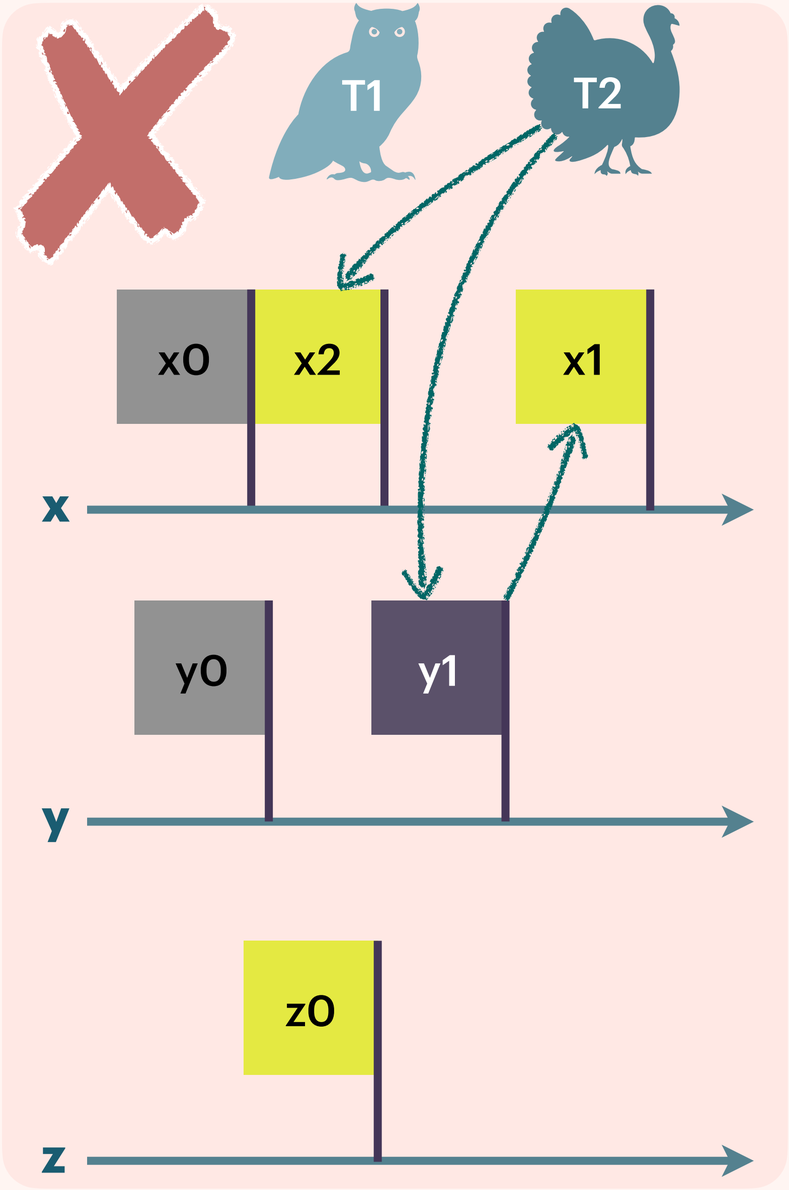
Read Elimination

$$x?; M \twoheadrightarrow M$$

operational invariant becomes denotational requirement

views point to messages that carry a smaller view

$$\kappa \boxed{\langle \mu, \mu \rangle} \kappa \therefore \langle\rangle \in [\![\langle\rangle]\!] \implies \exists v \,.\, \kappa \boxed{\langle \mu, \mu \rangle} \kappa \therefore v \in [\![x?]\!]$$



MORE CLOSURES

RA Specific
Compiler Optimization

- **Some transformations are valid even without preserving state**
- **Traces cannot strictly correspond to operational semantics (e.g. Transition ≡ exec. steps)**

Write-Read Reorder

$$x := 1; \quad \mathbf{let}\ a = y? \;\twoheadrightarrow\; \mathbf{let}\ a = y?\ \mathbf{in}\ x := 1;$$

$$\mathbf{in}\ M \qquad\qquad M$$

$$\alpha\ \langle \mu_1, \varrho_1 \rangle \langle \mu_2, \varrho_2 \rangle \ldots \langle \mu_{n-1}, \varrho_{n-1} \rangle \langle \mu_n, \varrho_n \rangle\ \omega \therefore r$$

$$\ldots\ \langle \mu_2, - \rangle, M_1 \rightarrow^* \langle \rho_2, - \rangle, M_2\ \ldots$$

$\leq$

View in message at x

ABSTRACT CLOSURES

Specific to RA

Rewrite

$$\frac{\pi \in [\![M]\!] \quad \pi \longmapsto \tau}{\tau \in [\![M]\!]}$$

- ***Absorb*** **a redundant local message into a following one**
  (e.g. $[\![x := 0; x := 1]\!] \supseteq [\![x := 1]\!]$**)**
- ***Dilute*** **a message by a redundant local message**
  (e.g. $[\![x?]\!] \supseteq [\![\mathrm{FAA}[x](0)]\!]$**)**
- ***Tighten*** **the encumbering view that a local message carries**
  (e.g. $[\![x := 1; y?]\!] \supseteq [\![(x := 1 \parallel y?).\mathrm{snd}]\!]$**)**

ABSTRACT CLOSURES

Specific to RA

Rewrite

$$\frac{\pi \in [\![M]\!] \quad \pi \longmapsto \tau}{\tau \in [\![M]\!]}$$

- ***Absorb*** **a redundant local message into a following one**
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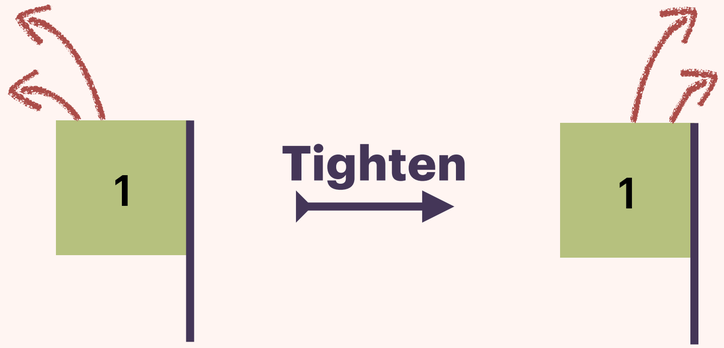
- ***Tighten*** **the encumbering view that a local message carries**
  (e.g. $[\![x := 1; y?]\!] \supseteq [\![(x := 1 \parallel y?).\mathrm{snd}]\!]$**)**

ABSTRACT REWRITE RULES

Write-Read Deorder + LoPP + Struct ⇒ Write-Read Reorder



← ***GUARANTEE IS WEAKER BECAUSE LOADING THIS MESSAGE OBSCURES MORE***

$$[\![\, x := 1; y? \,]\!] \supseteq [\![\, (x := 1 \parallel y?).\mathrm{snd} \,]\!]$$

NEW ADEQUACY PROOF IDEA

- Because traces are not operational, the adequacy proof is more nuanced:
  - We define a similar denotational semantics $[\![M]\!]$ but without the abstract rules
  - We show it is adequate (easier because it has an operational interpretation)
  - We show $[\![M]\!] = [\![M]\!]^{\dagger}$ — it is enough to apply the closure on top
  - We show that the abstract closures preserve observations

Laws of Parallel Programming

Symmetry	$M \parallel N$	$\twoheadrightarrow$	$\mathbf{match}\, N \parallel M\, \mathbf{with}\, \langle y, x\rangle.\, \langle x, y\rangle$	
Generalized Sequencing				
	$(\mathbf{let}\, x = M_1\, \mathbf{in}\, M_2) \parallel (\mathbf{let}\, y = N_1\, \mathbf{in}\, N_2)$	$\twoheadrightarrow$	$\mathbf{match}\, M_1 \parallel N_1\, \mathbf{with}\, \langle x, y\rangle.\, M_2 \parallel N_2$	

Eliminations

Irrelevant Read	$\ell\mathbf{?}\,;\langle\rangle$	$\twoheadrightarrow$	$\langle\rangle$	
Write-Write	$\ell := v\,;\ell := w$	$\overset{\mathsf{Ab}}{\twoheadrightarrow}$	$\ell := w$	
Write-Read	$\ell := v\,;\ell\mathbf{?}$	$\twoheadrightarrow$	$\ell := v\,; v$	
Write-FAA	$\ell := v\,;\mathrm{FAA}\,(\ell, w)$	$\overset{\mathsf{Ab}}{\twoheadrightarrow}$	$\ell := (v + w)\,; v$	
Read-Write	$\mathbf{let}\, x = \ell\mathbf{?}\, \mathbf{in}\, \ell := (x + v)\,; x$	$\twoheadrightarrow$	$\mathrm{FAA}\,(\ell, v)$	
Read-Read	$\langle \ell\mathbf{?}, \ell\mathbf{?}\rangle$	$\twoheadrightarrow$	$\mathbf{let}\, x = \ell\mathbf{?}\, \mathbf{in}\, \langle x, x\rangle$	
Read-FAA	$\langle \ell\mathbf{?}, \mathrm{FAA}\,(\ell, v)\rangle$	$\twoheadrightarrow$	$\mathbf{let}\, x = \mathrm{FAA}\,(\ell, v)\, \mathbf{in}\, \langle x, x\rangle$	
FAA-Read	$\langle \mathrm{FAA}\,(\ell, v)\,, \ell\mathbf{?}\rangle$	$\twoheadrightarrow$	$\mathbf{let}\, x = \mathrm{FAA}\,(\ell, v)\, \mathbf{in}\, \langle x, x + v\rangle$	
FAA-FAA	$\langle \mathrm{FAA}\,(\ell, v)\,, \mathrm{FAA}\,(\ell, w)\rangle$	$\overset{\mathsf{Ab}}{\twoheadrightarrow}$	$\mathbf{let}\, x = \mathrm{FAA}\,(\ell, v + w)\, \mathbf{in}\, \langle x, x + v\rangle$	

Others

Irrelevant Read Introduction	$\langle\rangle$	$\twoheadrightarrow$	$\ell\mathbf{?}\,;\langle\rangle$	
Read to FAA	$\ell\mathbf{?}$	$\overset{\mathsf{Di}}{\twoheadrightarrow}$	$\mathrm{FAA}\,(\ell, 0)$	
Write-Read Deorder	$\langle (\ell := v)\,, \ell'\mathbf{?}\rangle$	$\overset{\mathsf{Ti}}{\twoheadrightarrow}$	$(\ell := v) \parallel \ell'\mathbf{?}$	$(\ell \neq \ell')$
Write-Read Reorder	$\langle (\ell := v)\,, \ell'\mathbf{?}\rangle$	$\overset{\mathsf{Ti}}{\twoheadrightarrow}$	$\mathbf{let}\, x = \ell'\mathbf{?}\, \mathbf{in}\, (\ell := v)\,; x$	$(\ell \neq \ell')$

CONCLUSION

CONCLUSION

- **Standard, adequate and fully-compositional denotational semantic for RA**
- **More nuanced traces**
- **Sufficiently abstract: validates all RA transformations that we know of**
  (memory access, laws of parallel programming, structural transformations)
- **Extended RA view-based machine with compositional (i.e. first-class) parallelism**
  (weak-memory models are usually studied with top-level parallelism)

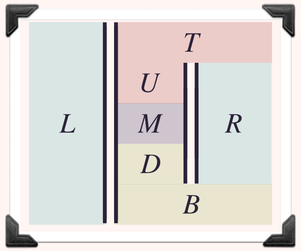
LIMITATIONS

- Parsimonious in features (e.g. no recursion)
- No type-and-effect system
- No algebraic presentation
- No non-atomics, not the full C/C++ model
- No full abstraction theorem even for first-order

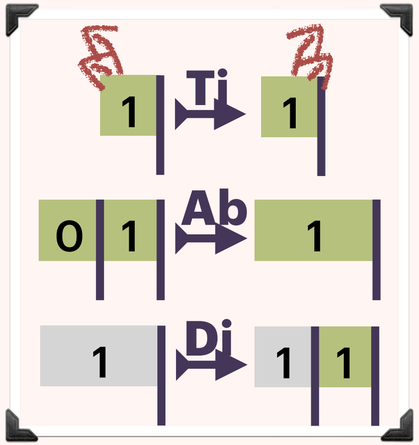
FUTURE DIRECTIONS

- **Address the mentioned limitations, e.g. promising semantics to cover more of C/C++**
- **Algebraic effects as Rely/Guarantee traces**

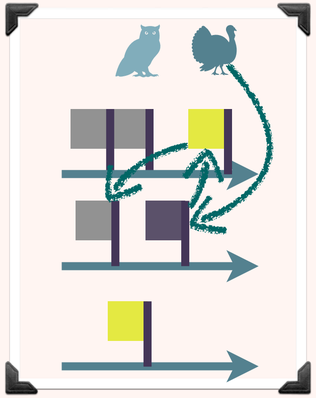
$$
\begin{aligned}
(\!|-|\!) &: \mathbf{Term}_{\{\mathrm{L},\mathrm{U}\}} X \to \mathcal{P}_{\mathrm{fin}}\left(\mathbb{T} X\right) \\
(\!|x|\!) &:= \{\langle\rangle \mathrel{\therefore} x\} \\
(\!|\mathrm{L}_\ell \left\langle t_v \right\rangle_{v \in \mathbf{Val}}|\!) &:= \bigcup_{v \in \mathbf{Val}} \{(\mathrm{R}_{\ell,v} :: \mathtt{t}) \mathrel{\therefore} x \mid \mathtt{t} \mathrel{\therefore} x \in (\!|t_v|\!)\} \\
(\!|\mathrm{U}_{\ell,v} t|\!) &:= \{(\mathrm{G}_{\ell,v} :: \mathtt{t}) \mathrel{\therefore} x \mid \mathtt{t} \mathrel{\therefore} x \in (\!|t|\!)\}
\end{aligned}
$$



1ST-CLASS PARALLELISM



ABSTRACT CLOSURES

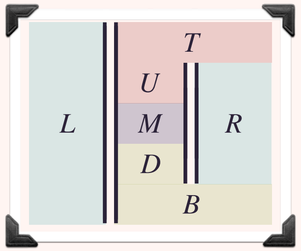


OPERATIONAL SEMANTICS

ADEQUACY PROOF

$$\underbrace{\alpha}_{\text{Initial View}}\ \underbrace{\langle \mu_1, \varrho_1\rangle\langle \mu_2, \varrho_2\rangle \ldots \langle \mu_{n-1}, \varrho_{n-1}\rangle\langle \mu_n, \varrho_n\rangle}_{\text{Sequence of Transitions}}\ \underbrace{\omega}_{\text{Final View}}\ \therefore\ \underbrace{r}_{\text{Returns}}$$

RELY/GUARANTEE TRACES



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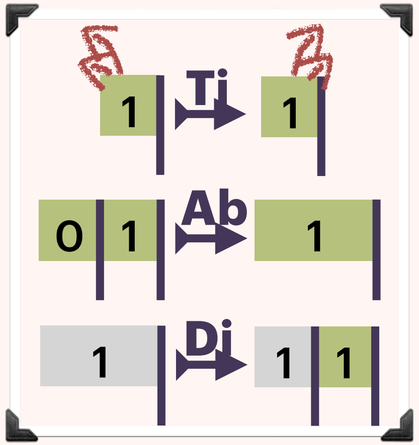
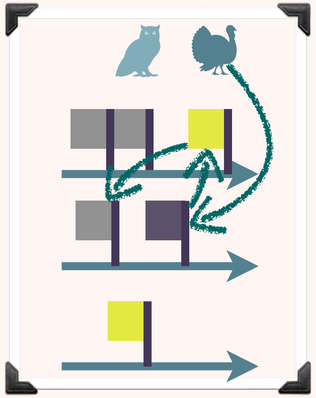
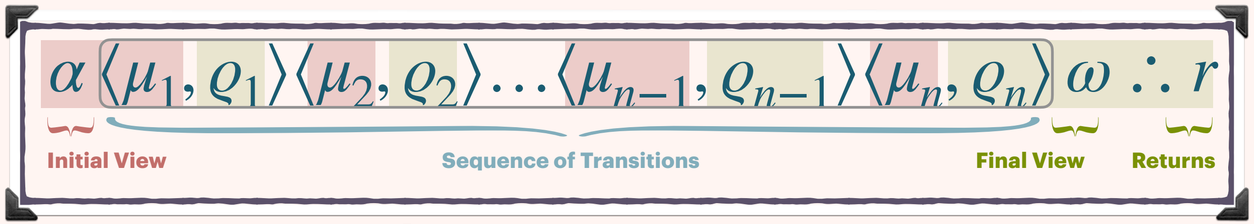


ABSTRACT CLOSURES



OPERATIONAL SEMANTICS

ADEQUACY PROOF



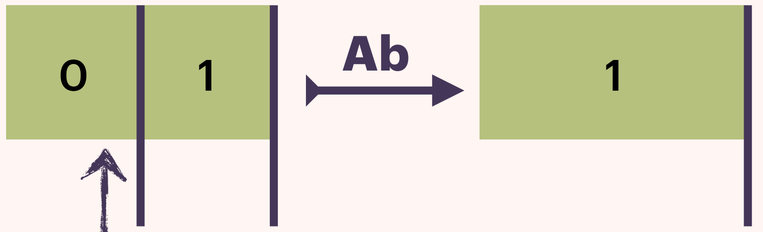
RELY/GUARANTEE TRACES

REWRITE RULE: ABSORB

Write Eliminations

$$x := 0; x := 1 \twoheadrightarrow x := 1$$

$$x := 0; CAS[x](0,1) \twoheadrightarrow x := 1$$



Eliminate redundant message

REWRITE RULE: DILUTE

Write Eliminations

$$x? \twoheadrightarrow CAS[x](1,1)$$

$$CAS[x](1,1) \twoheadrightarrow FAA[x](0)$$